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Single- and multi- user uplink energy-efficient resource allocation algorithms with users' power and minimum rate constraint in OFDMA cellular networks

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Abstract In this paper, single- and multi- user Resource Allocation (RA) optimization problems considering transmit power and minimum rate constraint of Mobile Station (MS) for maximizing MS' energy efficiency, measured as bits-per-joule (bpj), are addressed. Assume channel state information of all MSs is known by base station. We propose uplink RA algorithms, performing subcarrier assignment and power allocation, for optimizing bpj of MS in a single-cell OFDMA-based cellular network for both single- and multi- user scenarios. In the single-user case, we propose RA algorithms, which utilize the closed-form solution derived by applying Lambert-W function and an iterative approach based on Karush–Kuhn–Tucker conditions respectively to achieve optimal bpj of MS. In the multi-user case, centralized iterative multi-user RA algorithms for maximizing sum of MS' bpj, performing joint subcarrier assignment and power allocation iteratively, are proposed by utilizing the proposed single-user RA schemes. In particular, tradeoffs between energy efficiency and spectral efficiency are fully investigated, and the influence of MS' power and minimum rate constraints on bpj performance is also studied. The effectiveness of proposed algorithms is presented by numerical experiments. Numerical results demonstrate the proposed algorithms can enhance bpj significantly with limited loss of total throughput compared to the sum-rate maximization algorithm (in Moretti et al., IEEE Trans Veh Technol 60(4):1788–1798, 2011).

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1 Introduction

In order to fulfill the fast-growing demand of high data-rate wireless applications as well as the increasing needs and desire for being able to access those applications ubiquitously for a substantial period of time, the wireless cellular technologies are required to continuously evolve as fast and good as possible. For many reasons, orthogonal frequency division multiple access (OFDMA) has been selected as the multiple access technologies for state-of-the-art wireless systems such as LTE and WiMAX. OFDMA is considered as a promising technology for wireless broadband systems due to many of its advantages, e.g. robustness against inter-symbol interference and multipath fading and relatively simple equalizations. With the use of OFDMA technology, there are still technical challenges needed to be solved to meet requirements of subscribers and wireless applications in different kind of scenarios, e.g. high-rate requirements, low power consumption, and etc. With the fact that many wireless applications are highly energy-consuming and require large bandwidth, one of the major challenges is to fulfill the needs of every user, e.g. QoS requirements, in the system while keeping them with good energy utilization which directly leads to longer battery lifetime. In order to resolve it, the resource allocation (RA) technique can be applied to achieve substantial improvement on the related performance of the OFDMA cellular system. In this paper, we focus on the uplink RA with the objective of optimizing the overall uplink energy efficiency of the network. The multi-user RA problem in the uplink could be more complex than that in the downlink due to the distributed power sources. Briefly speaking, in contrast to downlink RA with a single power source from the base station (BS), each Mobile Station (MS) has its own power budget in uplink RA. Figure 1 illustrates the underlying framework of the uplink RA in an OFDMA network, where the central BS, aiming to optimize the uplink energy efficiency, is in charge of assigning radio resources and allocating transmit power of users who provides BS the knowledge of necessary information, such as channel status and other particular required parameters, and their own QoS requirements, e.g. rate and power constraint. Under the control of BS, each user transmits on the assigned resources with the specified transmit power for each assigned resource.

The OFDMA RA problems can be classified into downlink RA and uplink RA in a single- or multi- cell network. Most studies of RA problems in OFDMA systems aim at optimizing the following aspects: (weighted) sum-rate maximization with users' power constraint, transmit power minimization, sum-rate maximization with fairness constraints, and min–max problems. In [1–8], downlink RA algorithms are proposed in OFDMA networks to maximize various objectives, e.g. sum-capacity, weighted sum of minimal rates, and etc., under different constraints. Uplink RA problems with similar objectives as in the downlink are investigated in [9–15]. Besides the various objectives studied in the downlink and uplink RA problems [1–15], another crucial factor dramatically affecting performances of wireless devices, energy efficiency, becomes an attractive topic for researchers. Bits-per-energy efficiency has been studied in a notable literature, [16], where an information-theoretic characterization for single-user, multiple-access, and interference, is presented. In [17–25], various RA problems are formulated and solved for enhancing energy utilization of wireless transmission with different definition of energy efficiency, assumptions, and system architecture. A more detailed review is given in Section II. Compared with the above existing works, the focus and advantages of this paper have its uniqueness, and the distinct contributions of this paper lie in the following aspects:

- In the uplink of an OFDMA-based network, we address uplink RA optimization problems imposed with multiple MS' maximum transmit power constraint and minimum rate requirement for maximizing MS' energy efficiency, measured as total bits transmitted per joule of energy consumed (namely instantaneous *bpj*), and sum of MS' *bpj* in the single- and multi- user scenario respectively. Several centralized single- and multi- user RA algorithms are proposed to resolve the formulated problem. In the proposed algorithms, BS with the knowledge of MSs' channel information is in charge of deciding which MS can transmit on each subcarrier as well as allocating the radio transmit power of the MS on each subcarrier for the purpose of optimizing the formulated objective, i.e. instant energy efficiency of single- or multi- user in bpj.
- In the case of single-user network, the closed-form solution of MS' optimal total transmit power for maximizing MS' bpj is firstly derived by applying Lambert-W function [27]. Based on that, a power allocation (PA) scheme is developed with the water-filling algorithm. Another PA scheme with a low-complexity iterative approach is proposed by applying Karush–Kuhn–Tucker (KKT) conditions. These two schemes resolves the bpj maximization problem with MS' maximum power constraint. Finally, with the derived minimum required power for achieving a certain rate, an extended PA scheme is proposed to

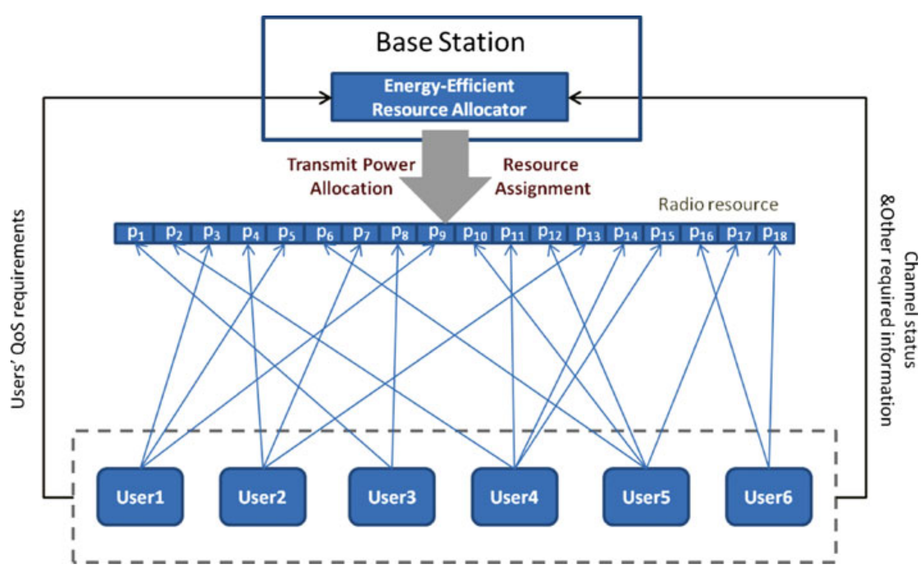

Fig. 1 The uplink resource allocation framework in an OFDMA-based network. The assigned resources for each user are pointed by the *arrow lines*, and p_1–p_{18} means the specified transmit power of users on each resource

solve the same problem imposed with both MS' power and rate constraints.

- In the case of multi-user network, the *bpj* optimization problem becomes complex to solve due to the involvement of subcarrier assignment along with transmit power allocation (PA) for all MSs which results in the non-convex objective function. Therefore, two effective iterative RA algorithms are proposed to achieve the sub-optimal solution, where the proposed SA and PA optimization are executed iteratively. In each iteration, a better RA is guaranteed to be found to improve the system energy efficiency computed in last iteration, and the RA result will finally converge and approach the sub-optimal solution.
- In particular, tradeoffs between energy efficiency and spectral efficiency are illustrated for both the single- and multi-user scenario. The influence of MS' maximal transmit power, minimal rate constraint, and circuit power consumption on performance of *bpj* is fully investigated. Additionally, the advantage of the proposed schemes is shown via performance comparison with other related multi-user RA scheme and RA scheme for maximizing sum capacity in terms of *bpj* and throughput. It shows the enhancement of *bpj* is significant with respect to many parameters while the decrease of rate is relatively marginal.

The rest of this paper is organized as follows. In Sect. 2, a review of related works is given. In Sects. 3 and 4, single-user *bpj* optimization problems with MS' transmit power constraint and minimal rate requirement are addressed, and three PA schemes are developed to obtain optimal *bpj*. In Sects. 5 and 6 multi-user *bpj* optimization problems with MS' power and minimal rate constraint are addressed with proposed iterative joint SA and PA algorithms. Finally, numerical experiments illustrate tradeoffs between energy efficiency and spectral efficiency in both single- and multi-user scenarios, and show the enhancement of the bpj performance by comparing with the other multi-user RA algorithm maximizing sum of users' rate in Sect. 7. The paper is concluded in Sect. 8.

2 Related works

In [1], authors propose a subcarrier, bit, power allocation algorithm to minimize the total transmit power subject to users' rate requirement in the downlink. Similar problems in the downlink of a multi-cell OFDMA system are also addressed in [2, 3]. In [4], a downlink sub-optimal RA algorithm is proposed to maximize sum capacity while maintaining proportional fairness. In [5], an iterative RA algorithm is proposed to maximize the weighted sum of the minimal user rates of coordinated cells by applying duality-based approaches. In [6], a max–min RA problem is addressed to achieve the maximum fairness among users. In [7], capacity maximization RA scheme applying water-filling PA is proposed in the downlink of an OFDMA system. In [8], authors proposed a resource allocation structure which performs iterative RA in a distributed manner to minimize interference and packet scheduling to guarantee fairness of resource sharing between users in the multi-cell downlink OFDMA system. The studies of [1–8] address RA problems in the downlink OFDMA system where the methodologies and ideas cannot be applied to the uplink transmission directly due to the multiple access nature of uplink OFDMA where each MS has its individual requirements rather than the BS-centralized control in the downlink system. In [9, 10], authors propose a sub-optimal uplink RA scheme for maximizing sum capacity which greedily allocates subcarriers to users achieving maximal Signal-to-Noise Ratio (SNR) by performing water-filling for power allocation. In [11], utility maximization uplink RA schemes are proposed for optimizing sum rate, proportional fairness, and max–min fairness. In [12], the additional fairness constraint is added into the approach of [9] to maximize sum of users' rate. A dual decomposition approach is proposed to solve the uplink instantaneous rate maximization problem in [13]. In [14], the uplink PA problem is modeled as a non-cooperative game while subcarrier assignment (SA) is done similarly as that in [9]. An uplink ergodic weighted sum-rate maximization RA problem is solved in [15]. The literatures [9–15] studying uplink RA problems focus on typical objectives without considering any type of energy efficiency optimization. However, performance of these kinds of schemes in terms of spectral and energy efficiency will be compared to show the advantages of the proposed scheme. In [17–19], game-theoretic approaches are applied for maximizing energy efficiency. In [17] and [18], authors proposed a game-theoretic model of joint power and rate control with packet-delay constraints for maximizing bits-per-joule (*bpj*) in the uplink of CDMA systems. In [19], an energy-efficient power control scheme modeled as a non-cooperative game in multicarrier CDMA systems is presented. The proposed methods in [17–19] cannot be applied to the OFDMA network since there has no issue of subcarrier-based allocation involved in CDMA systems where users transmit in the same band with different codes. Additionally, the transmit power and rate in [17–19] has no inter-relationship and can be separately chosen. That is quite different from the model used in the paper, where the achievable rate from Shannon-Capacity formula is applied. Tradeoffs between energy efficiency and delay in the single-user case are studied in [20], where only a single transmitter sending fixed amount of data is considered in the one-dimensional time-varying channel. The assumptions and system model are totally different from the multi-user OFDMA network assumed in this paper, and no QoS requirement is considered. In [21], authors proposed a two-user game-theoretic PA approach for maximizing bits-per-energy in user-cooperation networks, where it assumes two-user uplink transmission, and each user can decide whether to relay the packet for the other or not. The transmission architecture assumed is simple, and no subcarrier allocation and QoS requirement are involved. In [22], a link adaptation scheme for single-user uplink transmission is proposed to optimize bpj in an OFDM system where the only one user uses the whole bandwidth. The subcarrier allocation is not available for a single-user OFDM system, and no QoS requirement is imposed. In [23], energy-efficient schemes for improving average energy utilization of MS in the uplink of OFDMA networks are investigated, where the energy efficiency in *bpj* is defined as the ratio of time-averaged rate to time-averaged power including transmit power and circuit power consumption. In [24], the RA algorithm satisfying additional MSs' fairness constraints with specially-designed subcarrier ordering and assignment methods is proposed for optimizing time-averaged bpj in OFDMA networks. In [25], another downlink RA approach using standard optimization methods is proposed to maximize instantaneous *bpj* with the assumption of flat fading across subcarriers in an OFDMA network. In addition to the fact that the scheme in [25] is addressed in the centralized downlink OFDMA system which is very different from the uplink case and no constraint is given, the assumption that each user experiences flat-fading across all subcarriers is also too simple. In [26], a joint RA and relay selection scheme is proposed to maximized time-averaged bpj in cooperative-relay OFDMA-based networks where the cooperative relaying with maximum ratio combining is utilized to further improve energy efficiency. Although in [23–26], the time-averaged bpj optimization RA problems are investigated ([26] is for cooperative-relay network) in multi-user uplink OFDMA networks, they do not consider MS' power constraint and QoS requirement in the problem while both MSs' maximal transmit power and minimum rate constraint are considered in this paper. Furthermore, there are some disadvantages of the consideration of time-averaged bpj instead of instantaneous bpj, which is shown and discussed in Section of numerical results and discussion.

3 Single-user energy-efficient resource allocation with MS' transmit power constraint

3.1 System model

Consider the uplink transmission of a single-cell OFDMA network with K subcarriers, indexed by $k \in \mathcal{K} = \{1, 2, 3, \ldots, K\}$. Assume Channel State Information (CSI) is perfectly known by both MS and BS. Let $s_k(t)$ denote the received signal transmitted by MS on subcarrier k at time t, $P_k(t)$ and $x_k(t)$ denote the transmit power and the transmitted signal with unit energy on subcarrier k at time t respectively, and $h_k(t)$ and $n_0(t)$ denote the channel fading coefficient (gain) of the link between MS and BS on subcarrier k and the Addictive White Gaussian Noise (AWGN) at time t respectively. Then, the received signal can be written as:

$$s_k(t) = \sqrt{P_k(t)}h_k(t)x_k(t) + n_0(t) \tag{1}$$

With Shannon Capacity formula, we can express the achievable rate on subcarrier k at time t as:

$$r_k(t) = \log_2\left(1 + \frac{P_k(t)|h_k(t)|^2}{N_0 B}\right) \tag{2}$$

where N_0 and B denotes the noise power spectral density and subcarrier bandwidth respectively.

3.2 Problem formulation

In this paper, the instantaneous energy efficiency is defined as number of bits transmitted by MS when consuming one joule of its energy, namely bits-per-joule, expressed as:

$$\begin{aligned} EE &= \frac{\textit{number of bits transmitted}}{\textit{consumed energy}} = \frac{\Delta B}{\Delta E} = \frac{R\Delta t}{P\Delta t} = \frac{R}{P} \\ &= \frac{R}{P_t + P_c} \end{aligned} \tag{3}$$

where Δt, R, and P denote transmit time duration, transmit rate, and consumed power of MS respectively. The consumed power includes transmit power, denoted by P_t, and circuit power, P_c, which is assumed as a constant. From Eq. (3), it is clear to see that bits-per-joule is equivalent to the ratio of MS' transmit rate to MS' consumed power.

Thus, the energy efficiency of MS at time t can be expressed as the ratio of sum of MS' achievable rate on each subcarrier to MS' consumed power:

$$\begin{aligned} EE(t) &= \frac{R(t)}{P_t(t) + P_c} = \frac{\sum_{k=1}^{K} r_k(t)}{\left(\sum_{k=1}^{K} P_k(t) + P_c\right)} \\ &= \frac{\sum_{k=1}^{K} \log_2\left(1 + \frac{P_k(t)|h_k(t)|^2}{N_0 B}\right)}{\left(\sum_{k=1}^{K} P_k(t) + P_c\right)} \end{aligned} \tag{4}$$

The single-user RA optimization problem is formulated as follows:

$$\max_{\{P_k(t),\forall k\}} EE(t)\; s.t. \sum_{k=1}^{K} P_k(t) \le P_n; P_k(t) \ge 0, \quad \forall k \tag{5}$$

Our goal is to optimize MS' bits-per-joule in the current OFDMA frame through allocating the transmit power on each subcarrier. The total transmit power can't exceed the upper bound, P_n, and power on each subcarrier must be positive. Note that since the proposed schemes are performed on a per-frame basis, the time domain expression is neglected in the following sections, which means, for instance, $EE(t)$ and $P_k(t)$ are expressed as EE and P_K respectively.

3.3 Optimal RA with Lambert-W function

Firstly, we simplify Problem (5) into Problem (6) by considering fixed transmit power constraint:

$$\max_{\{P_k,\forall k\}} EE\, s.t \sum_{k=1}^{K} P_k = P \tag{6}$$

According to Eq. (4), the solution of Problem (6) is equivalent to that of Problem (7), aiming to maximizing throughput with a total power constraint:

$$\max R = \sum_{k=1}^{K} \log_2\left(1 + \frac{P_k|h_k|^2}{N_0B}\right) s.t. \sum_{k=1}^{K} P_k = P \tag{7}$$

Problem (7) can be resolved by applying the conventional water-filling scheme:

$$P_k = \left(\frac{1}{\lambda \ln 2} - \frac{N_0B}{|h_k|^2}\right)^+, \text{where } (x)^+ = \begin{cases} x & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where λ is the Lagrange Multiplier, and the term, $\frac{1}{\lambda \ln 2}$, in Eq. (8) denotes the water-level in the water-filling scheme, which must be chosen to satisfy $\sum_{k=1}^{K} P_k = P$. Let $\mathcal{K}^+ \subset \mathcal{K}$ denotes the resultant subcarrier index set after performing the standard water-filling scheme, in which all the subcarriers are allocated positive power in (8), namely $P_{k'} > 0$, for $\forall k' \in \mathcal{K}^+$. With $\mathcal{K}^+$, we can obtain λ by solving $\sum_{k=1}^{K} P_k = P$ as:

$$\lambda = \frac{K'}{\left(P + \sum_{k' \in \mathcal{K}^+} \frac{N_0B}{|h_{k'}|^2}\right) \ln 2} \tag{9}$$

where K' denotes number of subcarriers in $\mathcal{K}^+$. By substituting Eq. (8) into Eq. (7), we can have the maximum achievable rate as follows:

$$\begin{aligned} R &= \sum_{k=1}^{K} r_k = \sum_{k=1}^{K} \log_2\left(1 + \frac{P_k|h_k|^2}{N_0B}\right) \\ &= \sum_{k' \in \mathcal{K}^+} \log_2\left[\frac{|h_{k'}|^2}{N_0BK'}\left(P + \sum_{k' \in \mathcal{K}^+} \frac{N_0B}{|h_{k'}|^2}\right)\right] \end{aligned} \tag{10}$$

By substituting Eq. (10) into Eq. (4), the solution of Problem (6) can be expressed as:

$$EE = \frac{\sum_{k' \in \mathcal{K}^+} \log_2\left[\frac{|h_{k'}|^2}{N_0BK'}\left(P + \sum_{k' \in \mathcal{K}^+} \frac{N_0B}{|h_{k'}|^2}\right)\right]}{(P + P_c)} \tag{11}$$

In order to solve Problem (5) where the total power constraint is not fixed and an upper bound of that is given, we can utilize Eq. (11) and treat the total transmit power, P, in Eq. (11) as a variable. It can be shown that MS' energy efficiency, Eq. (11), is a strictly quasi-concave function with respect to P, plotted in Fig. 2. Thus, a globally optimal P exists for maximizing EE. The proof of the strict quasi-concavity of Eq. (11) is given in "Appendix 2".

Therefore, we first derive the optimal total transmit power by solving:

$$\frac{\partial EE}{\partial P} = 0 \tag{12}$$

Assume $H' = \sum_{k' \in \mathcal{K}^+} \frac{N_0B}{|h_{k'}|^2}$, and we can have:

$$\frac{\partial EE}{\partial P} = \frac{\frac{K'}{(P+H')\ln 2}(P+P_c) - \left[\sum_{k' \in \mathcal{K}^+} \log_2\left(\frac{|h_{k'}|^2}{\sigma^2 K'}\right) + K' \log_2(P+H')\right]}{(P+P_c)^2} = 0,$$

and then have $\frac{1}{(P+H')}(P + P_c) - \ln(P + H') = \frac{(\ln 2)}{K'} \sum_{k' \in \mathcal{K}^+} \log_2\left(\frac{|h_{k'}|^2}{\sigma^2 K'}\right)$, where $\sigma^2 = N_0B$. Let $X = P + H'$, and with some mathematical minipulations, we can have:

$$\frac{P_c - H'}{X} - \ln X = \frac{(\ln 2)}{K'} \sum_{k' \in \mathcal{K}^+} \log_2\left(\frac{|h_{k'}|^2}{\sigma^2 K'}\right) - 1 \tag{13}$$

Let $a = P_c - H'$ and $b = \frac{(\ln 2)}{K'} \sum_{k' \in \mathcal{K}^+} \log_2\left(\frac{|h_{k'}|^2}{\sigma^2 K'}\right) - 1$, and substitute a and b into Eq. (13).

Then, we have $\ln X = \frac{a}{X} - b$, which can also be written as:

$$e^b = \frac{1}{X} e^{\frac{a}{X}} \tag{14}$$

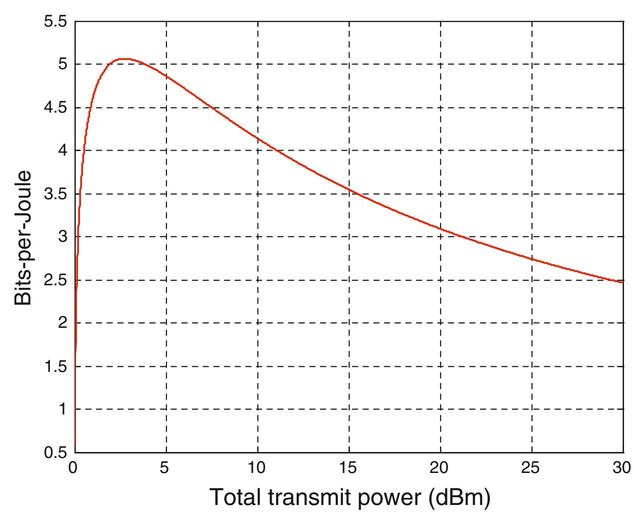

Fig. 2 *EE*, bits-per-joule, of MS by performing water-filling with different total transmit power, *P*

Multiply the left and right side of Eq. (14) by parameter a, and then we have:

$$ae^b = \frac{a}{X} e^{\frac{a}{X}} \tag{15}$$

With Eq. (15), the closed-form solution can be obtained by applying *Lambert-W function* [27]:

$$\begin{cases} X = \frac{a}{W(ae^b)}, & \textit{if } ae^b \in \left\{-\frac{1}{e}\right\} \bigcup [0,\infty) \\ X = \begin{cases} \frac{a}{W(ae^b)} \\ \frac{a}{W_{-1}(ae^b)} \end{cases}, & \textit{if } ae^b \in \left(-\frac{1}{e}, 0\right) \\ N/A, \textit{if } ae^b \in \left(-\infty, -\frac{1}{e}\right) \end{cases}, \tag{16}$$

where $W(x) = \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} x^n$ represents *Lambert-W function*.

From Eq. (16) the optimal total transmit power, denoted as P^*, for maximizing energy efficiency, *bpj*, can be written as:

$$\begin{cases} P^* = \frac{a}{W(ae^b)} - \sum\limits_{k' \in \mathcal{K}^+} \frac{N_0 B}{\left|h_{k'}\right|^2}, & \textit{if } ae^b \in \left\{-\frac{1}{e}\right\} \bigcup [0,\infty) \\ P^* = \max\left((P^*)^+\right), P^* = \begin{cases} \frac{a}{W(ae^b)} - \sum\limits_{k' \in \mathcal{K}^+} \frac{N_0 B}{\left|h_{k'}\right|^2} \\ \frac{a}{W_{-1}(ae^b)} - \sum\limits_{k' \in \mathcal{K}^+} \frac{N_0 B}{\left|h_{k'}\right|^2}, \end{cases} & \textit{if } ae^b \in \left(-\frac{1}{e}, 0\right) \\ N/A, & \textit{if } P_c < \sum\limits_{k' \in \mathcal{K}^+} \frac{N_0 B}{\left|h_{k'}\right|^2} \end{cases} \tag{17}$$

where $(\cdot)^+$ means to choose the positive value. Therefore, if $P^* \le P_n$, the maximum bits-per-joule of Problem (5) can be found by substituting P^* into Eq. (11), expressed as:

$$EE = \frac{\sum_{k' \in \mathcal{K}^+} \log_2 \left[\frac{\left|h_{k'}\right|^2}{N_0 B K'} \left(P^* + \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{\left|h_{k'}\right|^2} \right) \right]}{(P^* + P_c)} \tag{18}$$

The corresponding transmit power on each subcarrier can be obtained from (8), where

$$\lambda = \frac{K'}{\left(P^* + \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{\left|h_{k'}\right|^2} \right) \ln 2} \tag{19}$$

It can be shown that Eq. (11) is monotonically increasing with respect to the total transmit power (shown in Fig. 2) while the power is smaller than the optimal transmit power. Hence, in the case of $P^* > P_n$, the optimal total transmit power achieving maximum bits-per-joule is P_n, so by applying water-filling, PA for subcarriers can be expressed as:

$$P_k = \left(\frac{1}{\lambda \ln 2} - \frac{N_0 B}{\left|h_k\right|^2} \right)^+ \tag{20}$$

where $\frac{1}{\lambda \ln 2}$ is chosen such that $\sum_{k=1}^{K} P_k = P_n$. We can obtain: $\lambda = \frac{K'}{\left(P_n + \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{\left|h_{k'}\right|^2} \right) \ln 2}$.

The proposed PA algorithm applying the closed-form solution to solve Problem (5) is described as follows:

MaxEE Algorithm 1

Step 1: With given channel information, $\{h_k | k \in [1, K]\}$, examine the availability of Eq. (17). (The initial $\mathcal{K}^+ = \mathcal{K}$)

Step 2: If the solution is available, compute the optimal total power, P^*, by Eq. (17).

Step 3: Compare the resultant P^* with the power constraint, P_n. If $P^* \le P_n$, compute the final power allocation by Eqs. (8 and 19). Otherwise, PA can be obtained from Eq. (20). (If the resultant PA with $\mathcal{K}^+$ contains zero power on any subcarrier, according to standard water-filling procedure, those should be removed from $\mathcal{K}^+$ and restart from Step 1.)

3.4 Optimal RA with an iterative approach

The defect of the Lambert-W method is the optimal power, P^*, isn't always available if the required condition, $P_c < \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{\left|h_{k'}\right|^2}$, is not met. Hence, we use KKT conditions to further develop an iterative algorithm iteratively computing the transmit power on each subcarrier until it converges. The formulated RA problem is the same as Problem (5). The Lagrangian function can be written as:

$$L(P_k) = \frac{\sum_{k=1}^{K} \log_2 \left(1 + \frac{P_k |h_k|^2}{N_0 B} \right)}{\left(\sum_{k=1}^{K} P_k + P_c \right)} + \lambda \left(\sum_{k=1}^{K} P_k - P_n \right) - \mu_k P_k \tag{21}$$

Thus, the KKT conditions can be written as:

$$\frac{\partial L}{\partial P_k} = \frac{1}{(\ln 2)\left(\sum_{k=1}^{K} P_k + P_c \right)} \frac{1}{\sigma^2 / |h_k|^2 + P_k} - \frac{R}{\left(\sum_{k=1}^{K} P_k + P_c \right)^2} + (\lambda - \mu_k) = 0 \tag{22}$$

$$\lambda\left(\sum_{k=1}^{K} P_k - P_n\right) = 0, \lambda \geq 0 \quad (23)$$

$$\sum_{k=1}^{K} \mu_k P_k = 0, \mu_k \geq 0, \quad \forall k \quad (24)$$

where λ and μk are non-negative Lagrangrian multipliers.

From Eq. (22), we can derive:

$$\frac{R}{\left(\sum_{k=1}^{K} P_k + P_c\right)} - (\lambda - \mu_k)\left(\sum_{k=1}^{K} P_k + P_c\right) = \frac{1}{\ln 2}\frac{1}{\sigma^2/|h_k|^2 + P_k}$$

$$EE = \frac{R}{\left(\sum_{k=1}^{K} P_k + P_c\right)} = \frac{1}{\ln 2}\frac{1}{\sigma^2/|h_k|^2 + P_k} + (\lambda - \mu_k)\left(\sum_{k=1}^{K} P_k + P_c\right) \quad (25)$$

From Eq. (24) and $P_k \geq 0, \forall k$, we can have: $\mu_k = 0, \forall k$.

Furthermore, because the optimal transmit power may not be the maximal power according to Eq. (11), λ must be zero. Thus, from Eq. (25) the maximum *EE* and the corresponding optimal transmit power on subcarrier k can be derived as:

$$EE = \frac{1}{\ln 2}\frac{1}{\sigma^2/|h_k|^2 + P_k}$$
$$P_k = \frac{1}{EE \times \ln 2} - \frac{\sigma^2}{|h_k|^2} \quad (26)$$

Equations (25) and (26) show that the optimal energy efficiency and transmit power on subcarrier k intervene with each other, so the maximal *EE*, *bpj*, and corresponding optimal PA, $\{P_k, \forall k\}$, cannot be computed directly. Therefore, an iterative approach, *MaxEE Algorithm 2*, to jointly solve Eqs. (25 and 26) is proposed and depicted in the following, and the effectiveness of the proposed algorithm is shown in Lemma 1.

MaxEE Algorithm 2

Initialization: Allocate equal power on subcarriers while satisfying the power constraint. Compute the initial energy efficiency, EE_0, by Eq. (4). Set the iteration index, i, to be 1.■

While

Step 1: Compute the power of subcarrier k of the ith iteration, denoted as P_k^i, with EE_{i-1} for $\forall k$ by Eq. (26)

Step 2: Compute EE_i with the resultant power allocation of **Step 1**, $\{P_k^i, \forall k\}$, by Eq. (4)

Step 3: Compute the difference between the *bpj* in the current and previous iteration, denoted as ΔEE_i, which can be expressed as:

if $\Delta EE_i < \varepsilon(\varepsilon > 0)$

MaxEE Algorithm 2 continued

Then, the optimal *bpj* is EE_i, and $P^{opt} = \sum_{k=1}^{K} P_k^i$, and $\boldsymbol{P}^* = \{P_k^* | P_k^* = P_k^i, \forall k\}$.

(P^{opt} and $\boldsymbol{P}^*$ denote the optimal total transmit power and optimal power allocation respectively.)

break while

end of if ($i = i+1$ for the next iteration count)

End of while

Step 4: If $P^{opt} \leq P_n$, the optimal solution is the final result. Otherwise, the final PA can be obtained by Eq. (20).

Lemma 1 (MaxEE Algorithm 2) *With sufficient iterations, the energy efficiency, EE_i, will converge to the optimum solution of Problem* (5), *which is the solution of Eq.* (25).

Proof See "Appendix 1".

4 Single-user energy-efficient resource allocation with MS' power and minimum rate constraint

4.1 Problem formulation and proposed algorithm

In fact, achieving higher *bpj* might result in reduction of the data rate possibly causing violation of user's QoS. Therefore, in addition to the power constraint, MS' minimum transmit rate is also a key factor to be considered. The problem of *bpj* optimization is formulated as follow:

$$\max_{\{P_k, \forall k\}} EE. s.t. \sum_{k=1}^{K} P_k \leq P_n; P_k \geq 0, \quad \forall k; \sum_{k=1}^{K} r_k(t) \geq R_n \quad (28)$$

The rate constraint implies that sum of the achievable rate on each subcarrier, $r_k(t)$ for $\forall k$, must be equal or larger than the minimum rate requirement, denoted as R_n. Firstly, the min-rate constraint can be converted into the minimum transmit power requirement by exploiting the water-filling solution. Equation (10) can be illustrated as the maximum rate which can be achieved with the total transmit power P. Thus, we can obtain the minimum required transmit power, denoted as P_r, achieving R_n by solving Eq. (10) with the input of R_n, which is written as:

$$P_r = \left(2^{R_n} \Big/ \prod_{k' \in \mathcal{K}^+} |h_{k'}|^2\right)^{\frac{1}{K'}} K' N_0 B - \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{|h_{k'}|^2} \quad (29)$$

P_r denotes the minimum required transmit power for achieving R_n. In other words, R_n is just achieved by water-filling PA with $\sum_{k=1}^{K} P_k = P_r$. Note that during the computation of P_r if the resultant PA, computed from (8) with $\mathcal{K}^+$ used in (29) and $P = P_r$, contains zero power on any

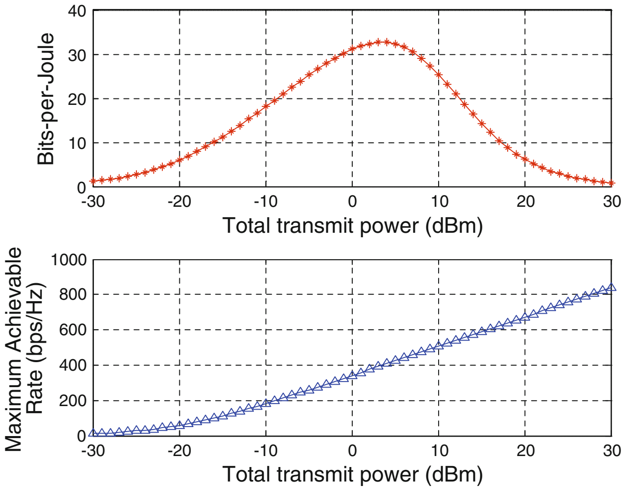

Fig. 3 R, Eq. (10), and EE, Eq. (11), with respect to P. Assume $P_c = 10$ dBm, $K = 50$, and $N_0B = -120$ dB

subcarrier in $\mathcal{K}^+$, they are removed from $\mathcal{K}^+$, and then P_r is re-computed with updated $\mathcal{K}^+$. The final resultant set $\mathcal{K}^+$ can only include the subcarriers with positive transmit power in Eq. (8). Figure 3 illustrates the relationship between the optimal achievable rate computed with the water-filling scheme and its corresponding bpj with respect to MS' total transmit power, described in Eqs. (10) and (11), respectively. It shows that the bpj curve has a peak (maximal) value with a specific transmit power, namely P^*, while the maximum achievable rate is monotonically increasing with respect to MS' total transmit power. The derived P_r and the maximal transmit power, P_n, can be either smaller or larger than P^*. Thus, to solve Problem (28), *MaxEE Algorithm 3* is developed with the joint consideration of P_r, P^*, and P_n.

MaxEE Algorithm 3

Step 1: Use *MaxEE Algorithm 1* or *2* to calculate the optimal transmit power, $P^* = \sum_{k=1}^{K} P_k^*$.

Step 2: Calculate P_r by Eq. (29)

Step 3: *if* $P^* < P_r < P_n$, *then* $P = P_r$, *else if* $P_r < P^* < P_n$, *then* $P = P^*$, *else if* $P_r < P_n < P^*$, *then* $P = P_n$, *else*, Problem (28) has no solution. [P denotes the resultant optimal total transmit power for Problem (28)]

Step 4: Compute the final PA with P by applying the standard water-filling scheme.

5 Multi-user energy-efficient resource allocation with MS' power constraint

5.1 Problem formulation

In this section, the problem of single-user RA for maximizing energy efficiency is extended to the multi-user case in a single-cell OFDMA network. Firstly, we measure the energy efficiency of the uplink multi-user network as sum of MS' bits-per-joule, which can be written as:

$$EE = \sum_{n=1}^{N} EE_n = \sum_{n=1}^{N} \frac{R_n}{P_{t,n} + P_c} = \sum_{n=1}^{N} \frac{\sum_{k=1}^{K} C_{k,n} \log_2\left(1 + \frac{P_{k,n}|h_{k,n}|^2}{N_0B}\right)}{\left(\sum_{k=1}^{K} P_{k,n} + P_c\right)} \tag{30}$$

where EE_n denotes the energy efficiency of MS n, and $C_{k,n}$ denotes the assignment indicator which represent 1 if subcarrier k is assigned to MS n, otherwise it is set to 0. The optimization problem can be formulated as follows:

$$\max_{\{P_{k,n}, C_{k,n}\}} \ EE \quad s.t. \sum_{k=1}^{K} P_{k,n} \le P_n, \quad \forall n; P_{k,n} \ge 0, \quad \forall k, \forall n; \sum_{n=1}^{N} C_{k,n} \le 1, \quad \forall k; C_{k,n} \in \{0,1\}, \quad \forall k \text{ and } \forall n \tag{31}$$

Our goal is to maximize the so-called *System Energy Efficiency* (*SEE*), sum of MS' *bpj*, subject to MS' transmit power budget. The last constraint implies that each subcarrier must be assigned to one MS at most to prevent from intra-cell interference. We can see that Problem (31) is a problem of constrained non-linear programming which includes both integer and continuous variables. In addition, the objective function is a non-concave function, which means that the global optimum is quite difficult to find because it might have more than one local optimum.

5.2 Iterative multi-user energy-efficient RA

Therefore, by applying the single-user *bpj* optimization addressed previously, we proposed an iterative multi-user RA algorithm performing subcarrier assignment and power allocation optimization alternately to approach the optimum. The overall algorithm is described as follows:

MU-MaxEE Algorithm 1

Initialize 1: (Initial subcarrier allocation) Randomly assign subcarriers to MSs. One subcarrier can only be assigned to one MS. Set the initial iteration time, j, to 1.

Initialize 2: (Initial PA optimization) With the given initial SA, use either *MaxEE Algorithm 1* or *2* to obtain the initial optimal PA and *bpj*, $\{EE_n^0, \forall n\}$, for each MS, and then compute the initial *SEE*, denoted as SEE^0. ■

While

for i = 1:K

Step 1: Choose a subcarrier k randomly from the subcarrier set, $\mathcal{K}$. $\mathcal{K} = \mathcal{K} - \{k\}$.

MU-MaxEE Algorithm 1 continued

Step 2: With subcarrier k, execute Joint SA and PA Algorithm 1, depicted in the following Sub-Section C, to obtain SEE_i^j, denoting SEE of the ith iteration in the inner loop and the jth iteration in the outer loop, and the new subcarrier and power allocation for the (i + *1*)th iteration.

end of for

Step 3: Compute SEE^j by Eq. (38), and

if $\left\| SEE^j - SEE^{j-1} \right\| < \delta$ *or* $j = J$

(*NOTE:* $\delta > 0$. SEE^j *denotes the SEE of the* j*th iteration in the outer loop, and J is the pre-defined maximum times of iteration to prevent the infinite loop situation.)*

break while

end of if ($j = j$+*1* for the next iteration count)

end of while

5.3 Joint SA and PA optimization

The main idea of this approach is to re-assign subcarriers based on the current subcarrier and power allocation in order to improve the *SEE* gradually. Each subcarrier is re-assigned to the MS which provide the maximum margin for *SEE*. Firstly, let $I_n = \left\{ \mathrm{i} \,|\, C_{i,n} = 1, \forall i \in \mathcal{K} \right\}$ denote the index set of assigned subcarriers for MS n. We define the incremental set, $I_n^{'}$, which adds subcarrier k chosen in Step 1 into I_n for $\forall n$, expressed as:

$$I_n^{'} = I_n \cup \{k\}, \quad \forall n \neq m \tag{32}$$

For MS m, which subcarrier k is originally assigned to, namely $k \in I_m$, $I_m^{'}$ is obtained by removing subcarrier k from I_m. That can be written as:

$$I_m^{'} = I_m - \{k\} \tag{33}$$

The difference between the optimal *bpj* with I_n, denoted as EE_n, and that with $I_n^{'}$, denoted as $EE_n^{'}$, can be written as:

$$\Delta EE_n = EE_n^{'} - EE_n \tag{34}$$

where ΔEE_n represents the changed amount of *bpj* of MS n after adding the subcarrier to I_n. $EE_n^{'}$ and EE_n can be obtained by using *MaxEE Algorithm 1* or *2* with the input of I_n and $I_n^{'}$ respectively. Therefore, after re-assigning subcarrier k, originally assigned to MS m, to MS n, the overall difference of *SEE*, denoted as η_n^k, can be written as:

$$\eta_n^k = \Delta EE_n + \Delta EE_m, \quad \forall n \neq m \tag{35}$$

η_m^k denotes subcarrier k is re-assigned to the original MS, so $\eta_m^k = 0$. Consequently, subcarrier k should be re-assigned to MS n achieving maximum η_n among all in order to obtain the maximum improvement on *SEE*. The decision rule can be expressed as:

$$\begin{cases} I_n = I_n \cup \{k\},\ I_m = I_m - \{k\} \\ C_{k,n} = 1,\ C_{k,m} = 1 \end{cases} \text{if } n = \arg \max_{\mathrm{s}} \eta_s^k \tag{36}$$

After re-assigning subcarrier k in the ith iteration of the inner loop, SEE_i^j, denoting *SEE* of the ith iteration in the inner loop and the jth iteration in the outer loop, can be expressed as:

$$SEE_i^j = SEE_{i-1}^j + \eta_n^k, i \in [1, K] \tag{37}$$

Note that when $i = 0$, $SEE_0^j = SEE^{j-1}$. Thus, after all subcarriers are re-assigned, the *SEE* of the jth iteration, SEE^j, can be written as:

$$SEE^j = SEE^{j-1} + \sum_{k=1}^{K} \eta_n^k \tag{38}$$

Joint SA and PA Algorithm 1

Step 1: Obtain $I_n^{'}$ for $\forall n \neq m$ and $I_m^{'}$ by Eqs. (32 and 33)

Step 2: Obtain $EE_n^{'}$ and EE_n by *MaxEE Algorithm 1* or *2*, and compute ΔEE_n and η_n^k for $\forall n$ by Eqs. (34 and 35)

Step 3: Re-assign the subcarrier by Eq. (36)

Step 4: Update *bpj* of MS n and MS m as $EE_n^{'}$ and $EE_m^{'}$ respectively, and compute SEE_i^j by Eq. (37).

From Eq. (38), we know that $SEE^j \geq SEE^{j-1}$ due to $\eta_n^k \geq 0$, $\forall k$. Thus, it is clear to see the proposed *MU-MaxEE Algorithm 1* can at least converge to the sub-optimal solution, which will be shown in the numerical results.

6 Multi-user energy-efficient resource allocation scheme with MS' power and minimum rate constraint

6.1 Problem formulation

In this section, the user's minimum rate constraint is included in the multi-user *bpj* optimization problem. The achievable rate of MS n can be written as:

$$r_n = \sum_{\mathrm{k}=1}^{\mathrm{K}} C_{k,n} r_{n,k} = \sum_{\mathrm{k}=1}^{\mathrm{K}} C_{k,n} \log_2 \left(1 + \frac{P_{k,n} \left| h_{k,n} \right|^2}{N_0 B} \right) \tag{39}$$

The problem is formulated as follows:

$$\max_{\{P_{k,n}, C_{k,n}\}} EE.s.t. \sum_{k=1}^{K} P_{k,n} \leq P_n, \quad \forall n; P_{k,n} \geq 0, \forall k,n; \sum_{n=1}^{N} C_{k,n} \leq 1, \quad \forall k,\ C_{k,n} \in \{0,1\}, \quad \forall k,n; r_n \geq R_n, \quad \forall n \tag{40}$$

With the rate constraints included, Problem (40) becomes more complicated than Problem (31). The global optimum solution is difficult to find with common optimization approaches. Thus, an iterative RA algorithm with a similar idea as MU-*MaxEE Algorithm 1* is proposed to resolve it. The proposed algorithm can be divided into the initialization phase and the RA optimization phase. In the initialization phase, a RA approach is proposed to perform subcarrier assignment and allocate transmit power on each subcarrier in order to satisfy all MS' rate constraint. In the phase of RA optimization, an iterative approach performing joint subcarrier assignment and power allocation iteratively is proposed to obtain the optimum RA solution of Problem (40).

6.2 Initial resource allocation

The initial RA algorithm, which gives initial subcarrier and power allocation satisfying all MSs' rate constraint, performs subcarrier exchanging iteratively between MSs to compensate the rate of MSs who have not yet met the rate constraint until the rate constraint of each MS is met. It is described as follows:

Initial Resource Allocation Algorithm

Step 1: Randomly assign subcarriers to MS.

Step 2: For each MS, with channel gain information of the assigned subcarriers, compute the minimum required power, $P_{r,n}$, by Eq. (29) and compare with the maximum transmit power, P_n. Find the set of MSs which cannot meet the rate constraint, written as: $\mathcal{U} = \{n|P_n < P_{r,n}, \forall n\}$, where $\mathcal{U}$ denotes the index set of MSs violating their rate constraint.

Step 3: For each MS $n \in \mathcal{U}$, additional subcarriers from other MSs are given to it to compensate its achievable rate for just meeting the rate constraint. Note that we have to make sure that the chosen candidates of MSs, which are going to give away their assigned subcarriers to MS $n \in \mathcal{U}$, still meet their rate constraint after giving their subcarriers away. Besides, the given subcarrier is selected randomly from candidates of MSs. This process continues until all MSs meet their minimum rate constraint. The pseudo code of this process is listed as follows:

While $\mathcal{U} \neq \emptyset$

Step 1: Select a MS $n \in \mathcal{U}$

Step 2: Randomly select a subcarrier k ($k \in \mathcal{K}$, $k \notin \mathrm{I_n} \forall \mathrm{n} \in \mathcal{U}$). Remove it from $\mathcal{K}$, namely $\mathcal{K} = \mathcal{K} - \{k\}$.

Step 3: Examine whether the original owner of subcarrier k, denoted as MS q, can still meet the rate constraint with the rest of subcarriers, denoted as $\hat{I}_q = I_q - \{k\}$.

If $\hat{P}_{r,q} \leq P_q$, (Re-assign subcarrier k)

$I_n = I_n \cup \{k\}$, $C_{k,n}$=1; $I_q = I_q - \{k\}$, $C_{k,q} = 0$

Else if $\mathcal{K} = \emptyset$

Break while and go back to **Step 1** of the initial RA algorithm to re-assign subcarriers again.

Else, (It means the chosen subcarrier isn't able to be re-assigned) go back to **Step 2**.

End of if

Step 4: Compute the new minimum required power, $P_{r,n}$ and compare with P_n.

If $P_n \geq P_{r,n}$, (The rate constraint of MS n is now met.)

Remove MS n from $\mathcal{U}$, $\mathcal{U} = \mathcal{U} - \{n\}$, and go back to **Step 1**.

Initial Resource Allocation Algorithm continued

Else, (It means the rate of MS n is still insufficient) go back to **Step 2**

End of if

End of While

End of Algorithm

6.3 Iterative joint subcarrier and power allocation with MS' minimal rate constraints

We propose an iterative RA algorithm by applying similar philosophy of *MU-MaxEE Algorithm 1* which optimizes *SEE* without MS' rate constraint. The proposed algorithm taking advantage of *MaxEE Algorithm 3* can gradually enhance *SEE* by performing joint SA and PA iteratively while maintaining the satisfaction of all MS' rate constraint which is initially met by *Initial RA Algorithm*. The whole algorithm is described as follows:

MU-MaxEE Algorithm 2

Initialization: Perform *Initial Resource Allocation Algorithm* to obtain initial subcarrier and power allocation. Set the initial iteration time, *j*, to 1. ■

While

for i = 1:K

Step 1: Choose a subcarrier k randomly from $\mathcal{K}$. Remove subcarrier k from $\mathcal{K}$, namely $\mathcal{K} = \mathcal{K} - \{k\}$.

Step 2: With the chosen subcarrier k, execute *Joint SA and PA Algorithm 2* described below to obtain SEE_i^j and the new subcarrier and power allocation for the $(i+1)$th iteration.

end of for

Step 3: Compute SEE^j by Eq. (38).

If $SEE^j - SEE^{j-1} < \delta$ *or* $j = J$, ***break while***.

end of if ($j = j+1$ for the next iteration count)

end of while

Joint SA and PA Algorithm 2

Step 1: Obtain $I_n^{'}$ $\forall n \neq m$ and $I_m^{'}$ by Eqs. (32 and 33). Compute the minimum required power for $I_n^{'}$ and $I_m^{'}$, $\forall n \neq m$, denoted as $P_{r,n}^{'}$ and $P_{r,m}^{'}$ respectively.

If $P_{r,m}^{'} > P_m$, (That means subcarrier k can't be re-assigned because the rate constraint cannot be met if subcarrier k, originally assigned to MS m, is re-assigned to other MS.)

Don't re-assign the subcarrier and go back to **Step 1** of *MU-MaxEE Algorithm 2* for the next iteration.

End of if

Step 2: Obtain $EE_n^{'}$, $EE_m^{'}$, EE_n, and EE_m by *MaxEE Algorithm 3*. Compute ΔEE_n and η_n^k for $\forall n$ by Eqs. (34 and 35)

Step 3: Re-assign the subcarrier by Eq. (36)

Step 4: Update *bpj* of MS n and MS m as $EE_n^{'}$ and $EE_m^{'}$ respectively, and compute SEE_i^j by Eq. (37).

7 Numerical result and discussion

We consider a single cell OFDMA network with 1,000-m radius. MSs are randomly distributed within the cell. Assume the noise power spectral density, N_0, and subcarrier bandwidth, B, are -174 dBm/Hz and 10 kHz respectively. Each subcarrier suffers from i.i.d. Rayleigh fading. Lognormal shadowing with zero mean and the standard deviation of 8 dB is considered. The transmitted signal experiences path loss modeled as: $PL(dB) = 30.18 + 26\log_{10} d$, where PL and d denote the path-loss coefficient and the distance (meter) between MS and BS respectively. Monte-Carlo simulations are performed in all figures. Figures 4 and 5 compare the proposed single-user RA algorithm with/without MS' rate constraint (*MaxEE 1*, *2*, and *3*) and the water-filling scheme, which optimizes the total achievable rate with given transmit power, in terms of performance of bits-per-joule and bps/Hz. Additionally, it also illustrates the tradeoff between energy efficiency and spectral efficiency. In Figs. 4 and 7, when the maximum transmit power, P_n, is less than the optimal power, P^*, performance of *bpj* and achievable rate of the water-filling and proposed scheme w/o MS' rate constraint are the same. That is because of the feature of quasi-concavity of bits-per-joule and the monotonic increase of optimum rate with respect to the transmit power, P, as shown in Fig. 3. Since the proposed schemes keep using P^* as its total transmit power even if the given power budget is larger than the optimal one, the performance of *bpj* and *bps/Hz* remain unchanged. For *MaxEE 3* considering R_n, while $P_n < P_r$, the rate constraint cannot be met even when MS transmits at P_n. In Figs. 4 and 7, because P_r for the two cases, $R_n = 150, 170$ bps/Hz, is larger than P*, their best transmit power for optimizing bpj is P_r. Thus, they have worse *bpj* but better achievable rate than *MaxEE 1* and *2*. Figure 6 compares the optimal bpj of four different schemes, the two proposed schemes and water-filling with different P_n, with respect to MS' circuit power, P_c. The two proposed schemes overwhelmingly outperform the water-filling scheme while P_c is small. However, when the circuit power increases, it dominates the total power consumption, which results in the decrease of the performance gap between the proposed schemes and water-filling. Figure 7 provides an overall view on how the *bpj* performance varies with MS' maximum transmit power, P_n, and minimum rate constraint, R_n. While the optimal transmit power, P^*, falls between P_n and P_r, the optimal bpj can be achieved. Performance of bpj decreases while $P_n < P^*$, or R_n becomes too large, resulting in P_r exceeds P^*. Note that the area where

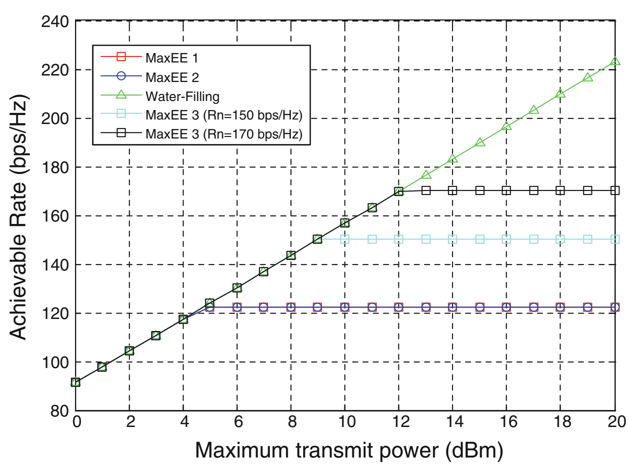

Fig. 4 Achievable rate in bps/Hz with respect to P_n in dBm. Assume $P_c = 10$ dBm, $K = 20$. The corresponding P_r for the line in *cyan* and *black* are 9 and 12 dBm respectively

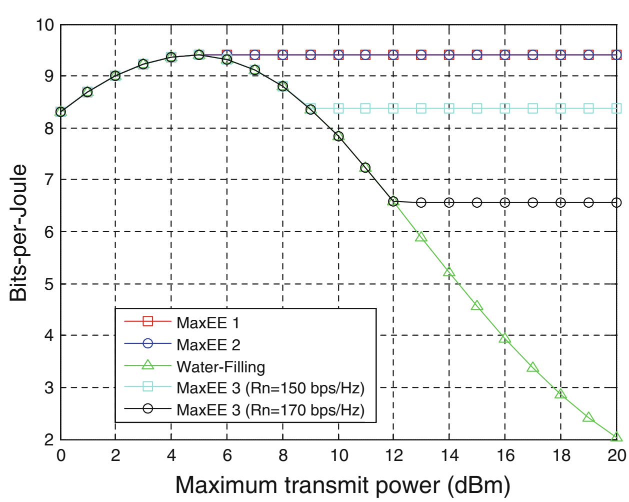

Fig. 5 Performance of bits-per-joule with respect to P_n in dBm. Assume $P_c = 10$ dBm, $K = 20$

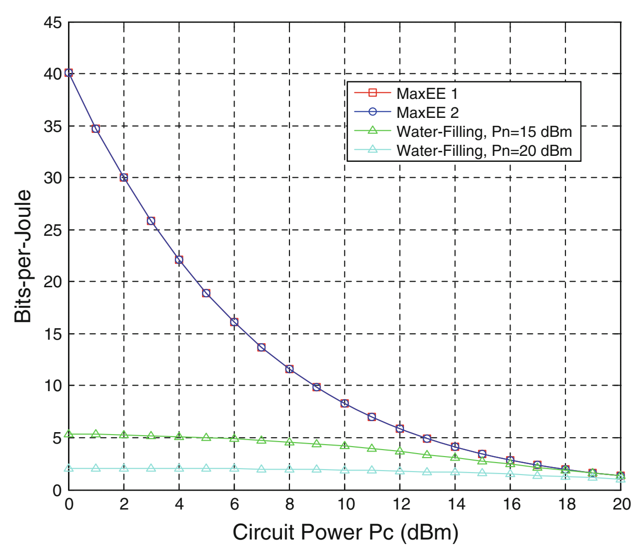

Fig. 6 Performance of *bpj* with respect to P_c in dBm. Assume $K = 20$ and $P_n = 15$ dBm for *MaxEE 1* and *2*

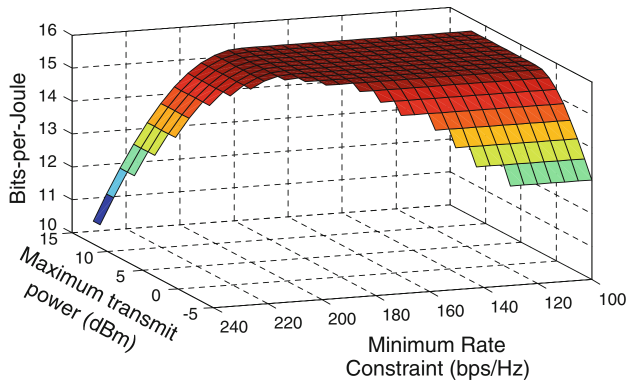

Fig. 7 *bpj* of *MaxEE 2* with respect to P_n and R_n. Assume $P_c = 10$ dBm

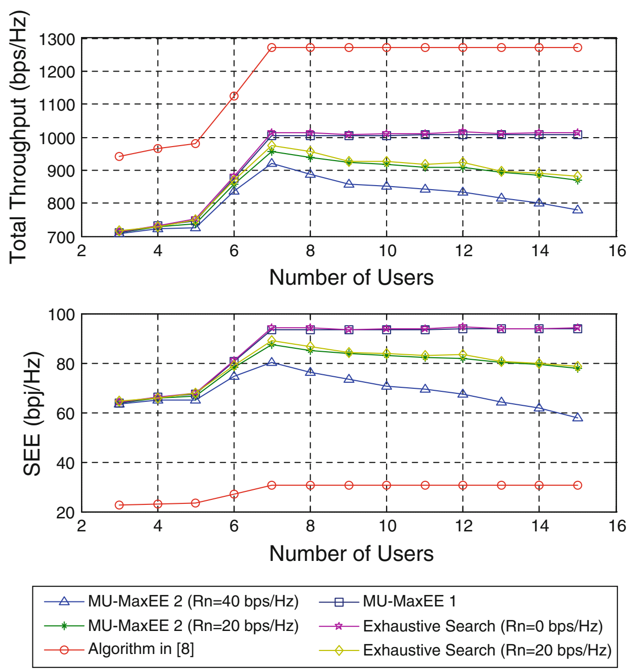

Fig. 8 Performance of total throughput and *SEE* with respect to *N*. Assume $K = 50$, $P_c = 10$ dBm, and $P_n = 15$ dBm

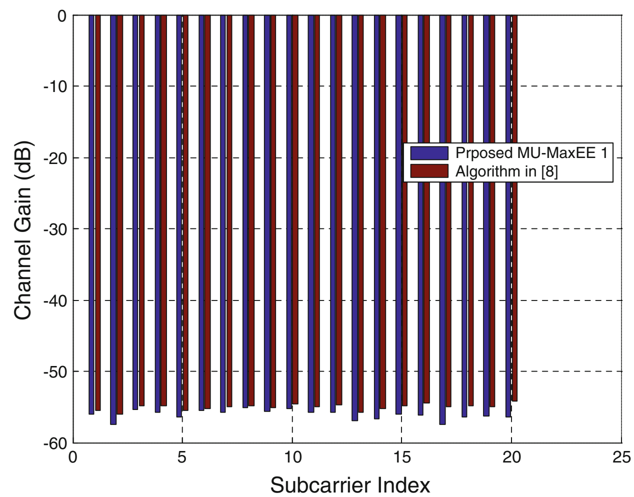

Fig. 9 Channel gain of the assigned MS for each subcarrier averaged through Monte-Carlo simulation

there's no value of *bpj* (Z-axi) indicates MS cannot meet the minimum rate constraint with the given P_n, $P_n < P_r$.

For the multi-user case, we compare performances of *bpj* and total throughput of *MU-MaxEE 1* and *2* with/without minimum rate constraints and the algorithm proposed in [9] which can achieve the sub-optimal solution for maximizing sum of users' rate without considering users' minimum rate constraint in the uplink of an OFDMA network. In Fig. 8, it shows that Algorithm in [9] performs better in total throughput but worse in bits-per-joule than the proposed algorithms. Figure 9 reveals that the channel coefficient (gain) of the assigned MS on each subcarrier for the proposed algorithm is lower than that for Algorithm in [9]. That's because the proposed algorithm allocates resources based on the capacity of MS' energy efficiency determined by its channel characteristics and the ratio of rate to power while the sum-rate optimization method (Algorithm in [9]) assigns resources by taking users' SNR as the main consideration. This phenomenon clearly explains the tradeoff between energy efficiency and spectral efficiency. By comparing performance of the proposed algorithms and Algorithm in [9], it is observed that the improvement on *bpj* is much larger than the degradation of total throughput in percentage (%). In addition, *MU-MaxEE 1* achieves almost same results as the optimum (Exhaustive Search) while *MU-MaxEE 2* can still achieve about 96–99 % of the optimum. In Fig. 8, both the performance of total throughput and bits-per-joule of the proposed algorithm decreases with the increase of the minimum rate constraint, R_n. Briefly speaking, that is because the proposed algorithm has to sacrifice the opportunity of assigning a subcarrier to a better MS to enhance *bpj* for meeting every MS' rate constraint. *MU-MaxEE 2* must satisfy all MS' rate constraint even if some of them have poor channel quality, which results in the decrease of both sum of *bpj* and total throughput. Moreover, *bpj* performance of *MU-MaxEE 2* rises at first, and starts to decrease while the number of MSs is greater than 7. Similarly, that is because to satisfy every MS' min-rate requirement results in reduction of the total throughput especially when *N* increases. In Fig. 10, the bpj curves of *MU-MaxEE 2* intersects with that of Algorithm in [9] since many more subcarriers are needed in order to meet R_n of each MS, which might lead to huge reduction of *bpj* when P_n becomes too low. Figure 11 demonstrates similar concepts as the single user case, which indicates that the gap of *bpj* performance between the proposed algorithms and Algorithm in [9] reduces when the circuit power becomes

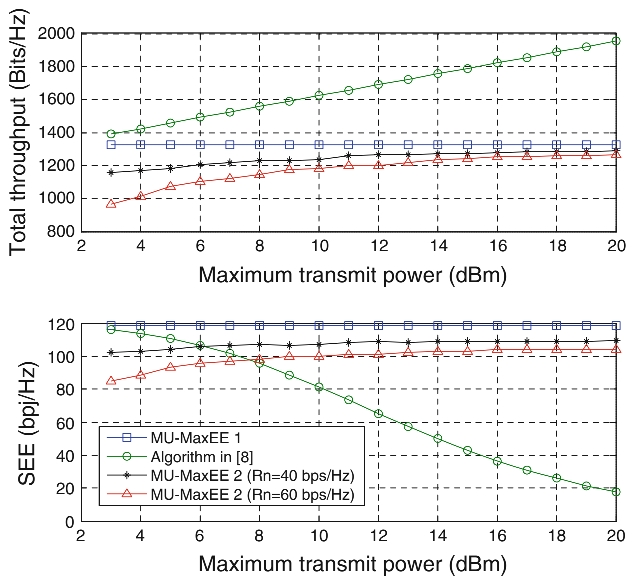

Fig. 10 Total throughput and *SEE* with respect to P_n. Assume $K = 100$, $N = 7$, and $P_c = 10$ dBm

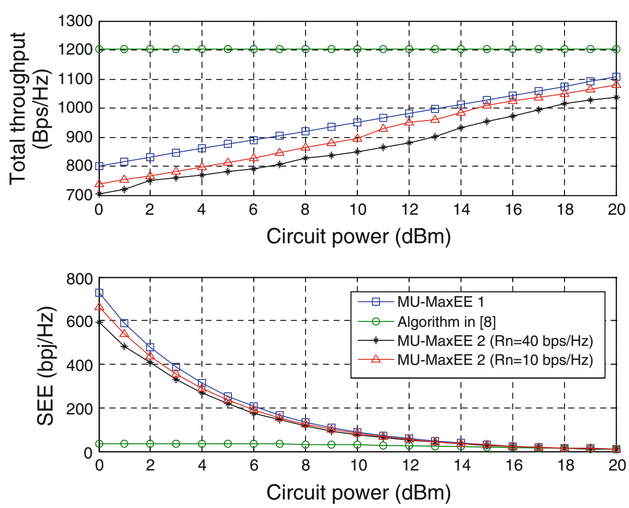

Fig. 11 Performance of total throughput and *SEE* with respect to P_c. Assume $K = 50$, $N = 10$, and $P_n = 15$ dBm

more and more dominant in total power consumption of MS. Note that bpj of the proposed algorithms is still about 12 % better than that of Algorithm in [9] even when $P_c = 20$ dBm. Figure 12 shows how much different initial SA influences the final bpj result and demonstrates the effectiveness of the proposed iterative algorithms with and without R_n (*MU-MaxEE 1* and *2* respectively). Each point in Fig. 12 represents the variance of *bpj* computed from the numerous results generated by Monte-Carlo simulation with different initial SA. It illustrates that *MU-MaxEE 1* is almost irrelevant to initial SA with respect to both K/N and P_c while *MU-MaxEE 2* have reasonably small variance, which rapidly decreases when P_c increases. In Figs. 13, 14,

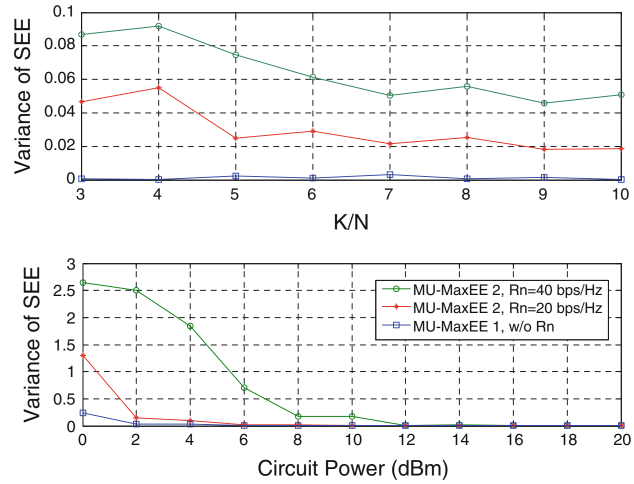

Fig. 12 Variance of *SEE* with respect to P_c and the ratio of number of subcarrier to number of MSs

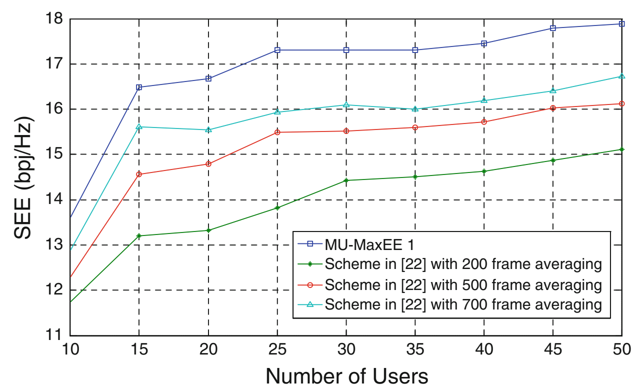

Fig. 13 SEE comparison with the scheme in [23], $P_c = 10$ dBm, $P_{max} = 15$ dBm

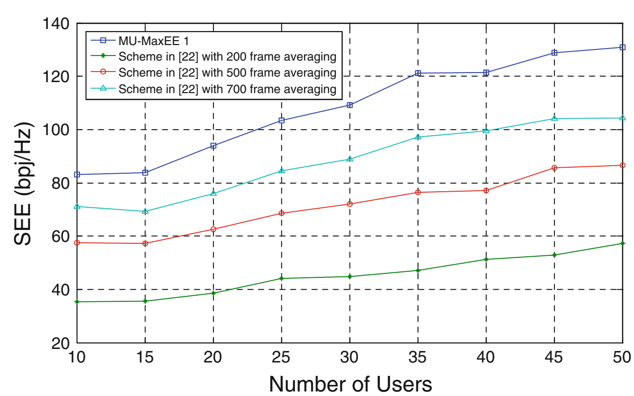

Fig. 14 SEE comparison with the scheme in [23], $P_c = 0$ dBm, $P_{max} = 15$ dBm

15, performance comparison with the scheme proposed in [23], where the similar methodology and definition of energy efficiency have also been applied in [24–26], is

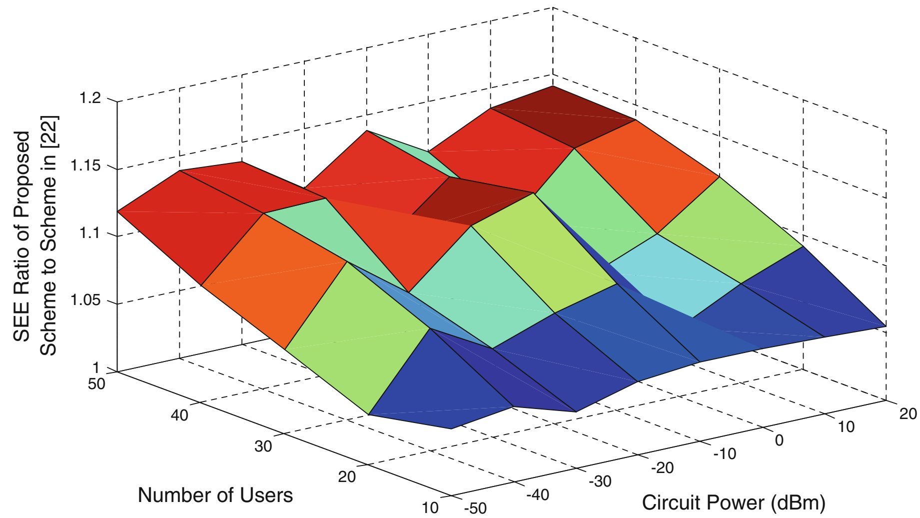

Fig. 15 The ratio of SEE of "MU-MaxEE 1" to SEE of the scheme in [23], $P_{max} = 15$ dBm. For each simulation point, the SEE of [23] is its stable value which is converged from its initial

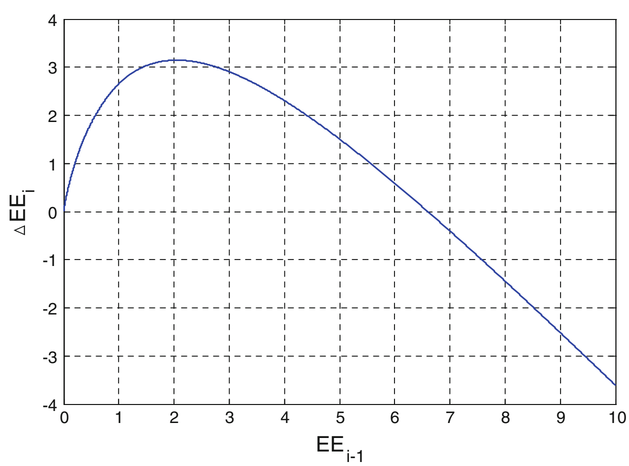

Fig. 16 ΔEE_i vs. EE_{i-1}

presented. While our proposed scheme can optimize instantaneous energy efficiency by tracking users' channel condition on a per-frame basis, the major defect of the scheme in [23] which optimizes time-averaged energy efficiency, defined as the ratio of time-averaged rate to time-averaged power, instead of instantaneous one needs a long period, ranging from roughly 100 to 500 frames depending on the averaging window size, to converge from the initial bpj to a stable (sub-optimal) value. Therefore, we can see that in Figs. 13 and 14, where we compare bpj of proposed MU-MaxEE 1 with bpj of Scheme in [23], averaging over the first 200, 500 and 700 OFDMA frames respectively, the proposed scheme outperforms Scheme in [23] about 6–24 %. In Fig. 15, MU-MaxEE 1 is compared directly with the converged stable value achieved by Scheme in [23] w.r.t. P_c and N. Even we compare with the converged result of [23], the proposed scheme can still outperform [23] by 5–14 %. In addition, the proposed scheme performs even better when number of users is very high due to the advantage of instant bpj optimization. Note that since [23] doesn't have any power or QoS constraint, we don't enforce MSs' min-rate constraint for fair comparison.

8 Conclusion

In OFDMA-based cellular networks, the uplink RA problems for maximizing MS' bits-per-joule subject to MS' transmit power constraint and minimum rate requirement are addressed in both single- and multi-user scenario. Two single-user RA algorithms based on the derived closed-form solution and an iterative approach applying KKT conditions are proposed to achieve optimal transmission in terms of *bpj*. The RA algorithm considering additional minimum rate requirement is proposed. For the multi-user case, we propose two iterative RA algorithms, which perform joint RA and SA optimization iteratively and achieve sub-optimal solution. Numerical results present the tradeoff between energy efficiency and spectral efficiency, and show great improvement on *SEE* with limited loss of total throughput compared to the sum-rate maximization scheme [9]. It also highlights how the power and minimal rate constraint affect performance of *bpj* and demonstrates the effectiveness of proposed algorithms.

Appendix 1

Proof By Eq. (26), the transmit power on subcarrier k of the ith iteration can be written as:

$$P_k^i = \frac{1}{EE_{i-1}\ln 2} - \frac{\sigma^2}{|h_k|^2} \tag{41}$$

By Eq. (4), the energy efficiency of the ith iteration can be written as:

$$EE_i = \frac{\sum_{k=1}^{K} \log_2\left(1 + \frac{P_k^i |h_k|^2}{N_0 B}\right)}{\left(\sum_{k=1}^{K} P_k^i + P_c\right)}$$

By substituting Eq. (28) into EE_i, we can express EE_i as a function of EE_{i-1}:

$$EE_i = \frac{K \log_2\left(\frac{1}{EE_{i-1} \ln 2}\right) + \sum_{k=1}^{K} \log_2\left(\frac{|h_k|^2}{N_0 B}\right)}{\frac{K}{EE_{i-1} \ln 2} - \sum_{k=1}^{K} \frac{N_0 B}{|h_k|^2} + P_c} \tag{42}$$

Then, by substituting Eq. (42) into Eq. (27), we can have ΔEE_i as:

$$\Delta EE_i = \frac{K \log_2\left(\frac{1}{EE_{i-1} \ln 2}\right) + \sum_{k=1}^{K} \log_2\left(\frac{|h_k|^2}{N_0 B}\right)}{\frac{K}{EE_{i-1} \ln 2} - \sum_{k=1}^{K} \frac{N_0 B}{|h_k|^2} + P_c} - EE_{i-1} \tag{43}$$

Figure 3 shows the curve of Eq. (43) as a function of EE_{i-1}. It indicates that the intersection point of Eq. (43) and the x-axi, meaning $EE_i = EE_{i-1}$, represents the optimal solution. Therefore, from Fig. 16, it is clear to see that no matter where the initial value located, the value of energy efficiency, EE_i, will be getting closer to the intersection point at each iteration. As a result, we can conclude that EE_i will finally converge to the optimal solution, which achieves $\Delta EE_i = 0$, with sufficient iterations. ∎

Appendix 2

Proof The function of MS' energy efficiency function (11) is written as:

$$EE(P) = \frac{\sum_{k' \in \mathcal{K}^+} \log_2\left[\frac{|h_{k'}|^2}{N_0 B K'}\left(P + \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{|h_{k'}|^2}\right)\right]}{(P + P_c)},$$

where P denotes MS' total transmit power. We know from Proposition C.9 in [28] that $EE(P)$ is a strictly quasi-concave function with respect to P if and only if the upper contour set, defined as $\Phi_\delta = \{P \geq 0 | EE(P) \geq \delta\}$, is strictly convex for any real number δ. For $\delta < 0$, it is obvious that no solution of $EE(P) < 0$ exists. For $\delta = 0$, the only solution is $P = 0$ which makes $\mathcal{K}^+ = \emptyset$ and $EE(P) = 0$. Thus, we can see that Φ_δ is strictly convex for $\delta \leq 0$. For $\delta > 0$, we can first express Φ_δ as:

$$\Phi_\delta = \{P \geq 0 | \delta(P + P_c) - \sum_{k' \in \mathcal{K}^+} \log_2\left[\frac{|h_{k'}|^2}{N_0 B K'}\left(P + \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{|h_{k'}|^2}\right)\right] \leq 0\} \tag{43}$$

Let $f(P) = \delta(P + P_c) - \sum_{k' \in \mathcal{K}^+} \log_2\left[\frac{|h_{k'}|^2}{N_0 B K'}\left(P + \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{|h_{k'}|^2}\right)\right]$, and then we can obtain the second order derivative of $f(P)$ w.r.t. P as:

$$\frac{\partial^2 f(P)}{\partial P^2} = f(P)'' = \frac{K'}{\ln 2}\left(P + \sum_{k' \in \mathcal{K}^+} \frac{N_0 B}{|h_{k'}|^2}\right)^{-2} \tag{44}$$

From (44), we can see that $f(P)'' > 0$. Therefore, Φ_δ is also strictly convex for $\delta > 0$, and then the strict quasi-concavity of (11) is proved.

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