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Chaos synchronization of Yin and Yang T-S fuzzy models of Hénon map system

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Abstract

Based on the Chinese philosophy, Yin is the decreasing, negative, historical or feminine principle in nature, while Yang is the increasing, positive, contemporary or masculine principle in nature. Yin and Yang are two fundamental opposites in Chinese philosophy. Since the discrete Hénon map system is an invertible map, Yin–Yang T-S fuzzy model of chaotic Hénon map systems with increasing and decreasing argument can be studied. The Yang T-S fuzzy model of chaotic Hénon map system is presented. The Yin T-S fuzzy model of chaotic Hénon map system and Yin T-S fuzzy rules are obtained by invertible matrix in linear system theory. Chaos synchronization of Yang T-S fuzzy model of Hénon map systems is achieved by parallel distributed compensation technique, and the fuzzy controller for chaos synchronization of Yin T-S fuzzy model of Hénon map systems is also obtained by invertible matrix in linear system theory. The design of the Yin fuzzy controller is fleetly obtained by the inverse of Yang fuzzy controller, the concern is the Yin chaotic system must be an inverse of the Yang chaotic system. T-S fuzzy model scheme is used in the nonlinear discrete chaotic map system, so the nonlinear discrete chaotic map system can be analyzed by linear system theory.

Keywords

Chinese philosophy, Yin chaos, Yang chaos, Yin–Yang chaotic Hénon map system, inversed Hénon map system, Yin–Yang T-S fuzzy model, chaos synchronization, inverse matrix theory

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Introduction

Chaos is an interesting nonlinear phenomenon, has been intensively investigated in the last three decades;[1–7] it has many characteristics, such as sensitive dependence on initial conditions and parameters, mixing randomness in the time domain, broadband power spectrum, ergodicity, etc.[8] Chaos is a common phenomenon in discrete time nonlinear systems[9,10] such as Hénon map,[11–13] logistic map[14–16] and generalized Hénon map, where the generalized Hénon map system has hyperchaotic behavior when it has one positive Lyapunov exponent more than Hénon map system.[17–20] In 1990, Pecora and Carroll[21] showed the possibility of chaotic synchronization and started a new research interest. The applications of chaos synchronization have found in the fields of engineering and science such as in secure communications, chemical reactions, power converters, biological systems, and information processing, etc.[22] In secure communication field, the main purpose for chaos and hyperchaos phenomenon in discrete time dynamical systems is to send a secret message from a transmitter to the receiver through a public channel and the secret message is sent safely when they are Synchronized.[23,24] In many papers, chaos synchronization[25–29] of two-dimensional Hénon map system are studied, of which the Yang chaos is also studied.[30] In 1985, Takagi–Sugeno (T-S) fuzzy model was firstly proposed.[31] Recently, it has become quite popular to adopt T-S fuzzy models to represent a nonlinear system. T-S fuzzy model can be used for nonlinear system, the state spaces are represented by linear models. So the conventional linear system theory can be applied to the analysis and synthesis of chaos synchronization.[32–35]

In this article, Yin chaos and the Yin T-S fuzzy model of the inverse Hénon map system are firstly proposed. Since the Hénon map system is an invertible map, the Yin T-S fuzzy model and Yin chaos synchronization of Hénon map system are acquired by invertible matrix in linear system theory.

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The design of the Yin fuzzy controller can be fleetly obtained by the inverse of Yang fuzzy controller, the concern is the Yin chaotic system must be an inverse of the Yang chaotic system. The controller design by parallel distributed compensation (PDC) technology to achieve chaos synchronization of T-S fuzzy model of Yin Hénon map system that can be acquired by the inverse of the controller for chaos synchronization of Yang Hénon map system. It is more efficient than other approaches.

This article is organized as follows. In the following section, the two-dimensional invertible maps are introduced. In the next section, the Yin chaos of inversed Hénon map system is presented and the Yang chaos of Hénon map system is also shown for comparison. In the later section, T-S fuzzy model scheme is given. Further, Yin–Yang T-S fuzzy models of the chaotic Hénon map systems and controller design by PDC technique for chaos synchronization of Yang T-S fuzzy model of Hénon map system are presented, and a simple method for Yin fuzzy controller is given. In the final sections, numerical analysis is given and conclusions are drawn.

Two-dimensional invertible maps

Consider the general two-dimensional map form as follows

$$\begin{aligned} x_1[n_1+1] &= f_1\{x_1[n_1], x_2[n_1]\} \\ x_2[n_1+1] &= f_2\{x_1[n_1], x_{21}[n_1]\} \end{aligned} \tag{1}$$

where $x_1[n_1+1]$, $x_2[n_1+1]$ are the functions of $x_1[n_1]$, $x_2[n_1]$, n_1 is the positive argument, i.e. $n_1 = 0, 1, 2, 3, \ldots, n$. The map is invertible[36] if equation (1) can be solved uniquely for $x_1[n_1]$, and $x_2[n_1]$ as functions of $x_1[n_1+1]$ and $x_2[n_1+1]$. The inversed map system can be written from equation (1)

$$\begin{aligned} \hat{x}_1[n_2-1] &= g_1\{\hat{x}_1[n_2], \hat{x}_2[n_2]\} \\ \hat{x}_2[n_2-1] &= g_2\{\hat{x}_1[n_2], \hat{x}_2[n_2]\} \end{aligned} \tag{2}$$

where n_2 is the negative argument, i.e. $n_2 = 0, -1, -2, -3, \ldots, -n$. The chaos obtained for negative argument is called Yin chaos, while that obtained for positive argument is called Yang chaos. Yin chaos for map system is studied in this article firstly. In Ge and Li,[37] the chaos obtained for negative time for differential equations is also called Yin chaos, while that obtained for positive time for differential equations is called Yang chaos.

Consider the Hénon map system[30]

$$\begin{aligned} x_1[n_1+1] &= -a_1 x_1^2[n_1] + x_2[n_1] + 1 \\ x_2[n_1+1] &= b_1 x_1[n_1] \end{aligned} \tag{3}$$

The Hénon map system is invertible if $b_1 \neq 0$. The inversed Hénon map system can be written by equation (3)

$$\begin{aligned} \hat{x}_1[n_2-1] &= \frac{1}{b_1}\hat{x}_2[n_2] \\ \hat{x}_2[n_2-1] &= \hat{x}_1[n_2] + \frac{a_1}{b_1^2}\hat{x}_2^2[n_2] - 1 \end{aligned} \tag{4}$$

The Yang chaos of Hénon map system and Yin chaos of inversed Hénon map system

The system parameters for equation (3) are: $a_1 = 1.4$, $b_1 = 0.3$. The Yang chaotic behaviors of Hénon map system with $a_1 = 1.4$, $b_1 = 0.3$ are quoted[38] by phase portrait in Figure 1.

The system parameters for equation (4) are replaced as $b_2 = \frac{1}{b_1}$, $a_2 = \frac{a_1}{b_1^2}$. The Yin chaotic behaviors of inversed Hénon map system with $b_2 = 0.3$, $a_2 = 1.4$ are shown by the phase portrait in Figure 2.

Comparing Figures 1 and 2, it is surprisingly noted that Figure 2 gives many new informations for famous Hénon system. Since 1976,[30] traditional studies of Hénon system have only devoted to its behaviors with positive argument (Yang chaos). Now it is discovered that with negative argument or negative time (Yin chaos), a new continent is waiting for us in the future study of either nonlinear map systems or nonlinear continuous systems.

T-S fuzzy model

T-S fuzzy model was given by Takagi and Sugeno.[31] The T-S fuzzy model can represent a general nonlinear system and the T-S fuzzy rules are obtained by IF-THEN fuzzy rules.

Consider the following discrete time T-S fuzzy rules

$$\begin{aligned} &R^i: \text{IF } p_1(n) \text{ is } M_{i1}, \ldots, \text{ and } p_q(n) \text{ is } M_i q, \\ &\text{THEN } x(n+1) = A_i x(n) + c, \end{aligned} \tag{5}$$

where $i = 1, 2, \ldots, r$ (r is the number of fuzzy rules), $x(n) \in R^j$ represents the state vector, $A_i \in R^{j \times j}$ are known matrix, $p_1(n), p_2(n), \ldots, p_q(n)$ are premise variables, M_{ih} is a fuzzy set ($h = 1, 2, \ldots, q$), $c \in R^j$ is a constant vector. Equation (5) represents local linear models by T-S fuzzy rules. The final discrete time T-S fuzzy system is inferred by the fuzzy strategies of singleton fuzzier, product fuzzy inference and weighted average defuzzier as follows

$$x(n+1) = \frac{\sum_{i=1}^{r} \omega_i(p(n))(A_i x(n) + c)}{\sum_{i=1}^{r} \omega_i(p(n))} \tag{6}$$

where

$$\omega_i(p(n)) = \prod_{h=1}^{q} M_{ih}(p_h(n)) \tag{7}$$

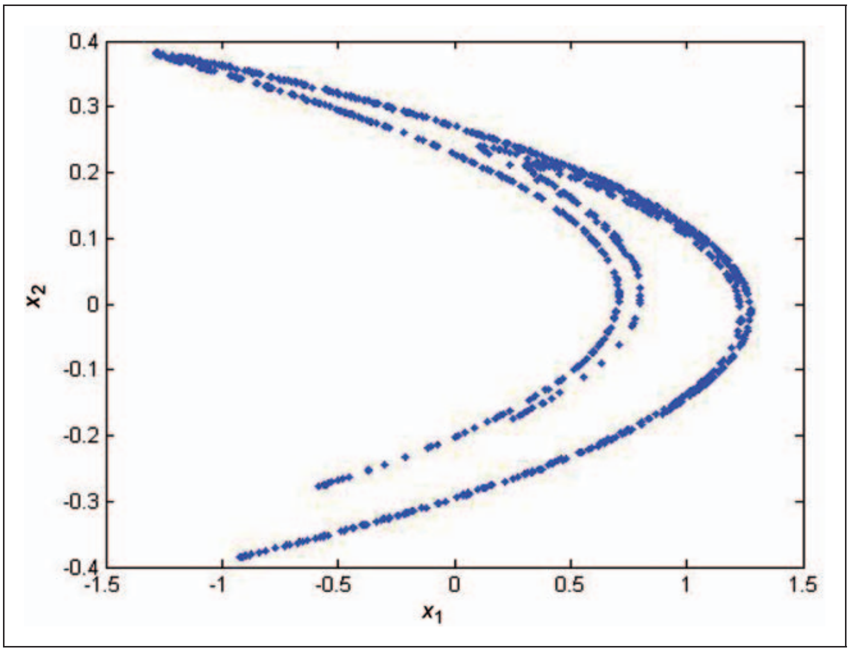

Figure 1. Phase portrait of the chaotic Yang Hénon map system with $a_1 = 1.4$, $b_1 = 0.3$.

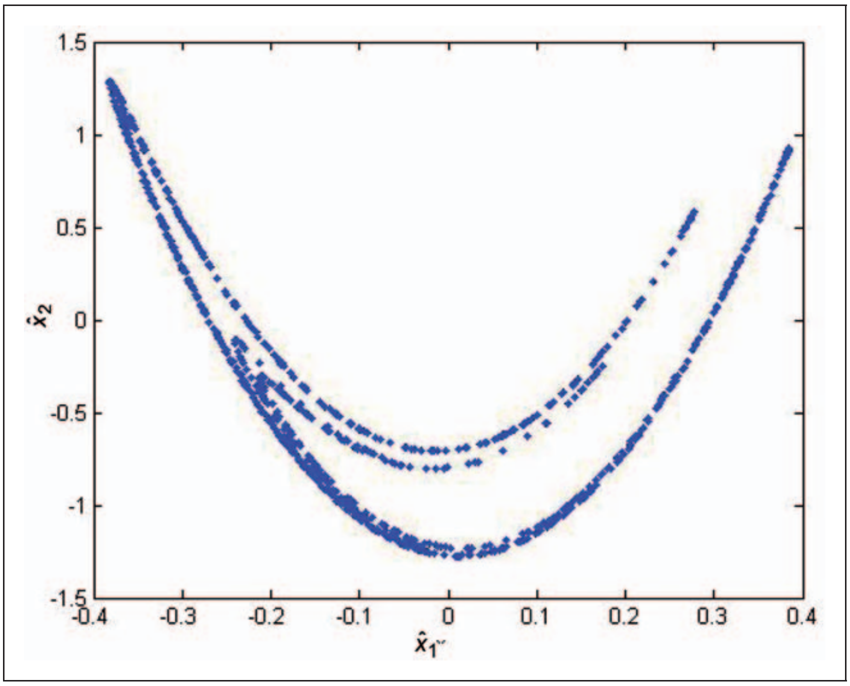

Figure 2. Phase portrait of the chaotic Yin Hénon map system with $a_2 = 1.4$, $b_2 = 0.3$.

in which $M_{ih}(p_h(n))$ is the degree of membership of $p_h(n)$ in M_{ih}. The following conditions must be satisfied

$$\begin{cases} \sum_{i=1}^{r} \omega_i(p(n)) > 0, \\ \omega_i(p(n)) \geqslant 0, \end{cases} \quad i = 1, 2, \ldots, r. \tag{8}$$

Let $\mu_i(p(n)) = \omega_i(p(n)) / \sum_{i=1}^{r} \omega_i(p(n))$, equation (6) can be rewritten as

$$x(n+1) = \sum_{i=1}^{r} \mu_i(p(n))(A_i x(n) + c) \tag{9}$$

Note that

$$\begin{cases} \sum_{i=1}^{r} \mu_i(p(n)) = 1, \\ \mu_i(p(n)) \geqslant 0, \end{cases} \quad i = 1, 2, \ldots, r. \tag{10}$$

Yin and Yang T-S fuzzy models of the chaotic Hénon map system

From the phase portraits of Yang Hénon map system in Figure 1, we can get the range of the state $x_1(n)$ is

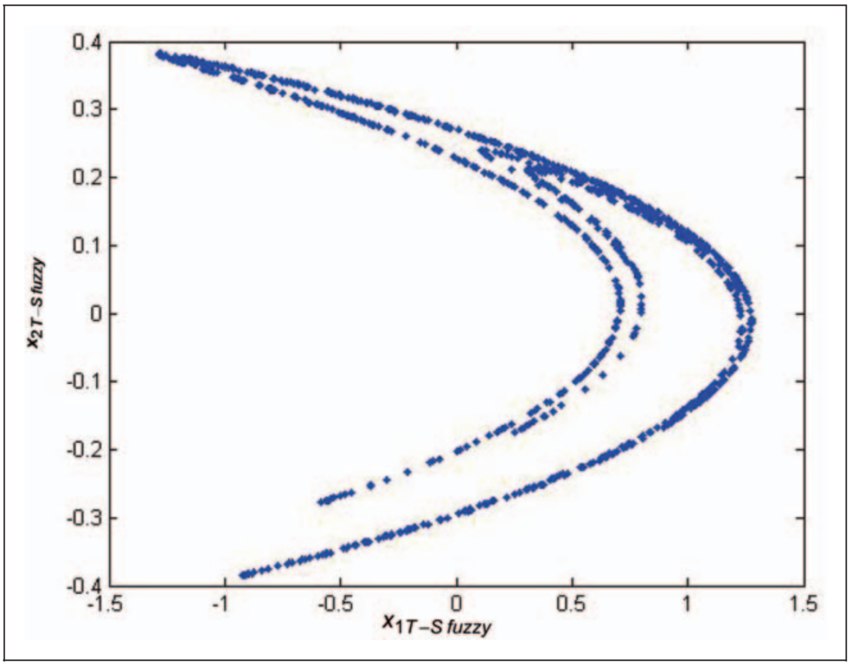

Figure 3. Phase portrait of Yang T-S fuzzy model of Hénon map system with $a_1 = 1.4$, $b_1 = 0.3$.

from -1.4 to 1.4 ($x_1(n) \in [-1.4, 1.4]$). The Yang T-S fuzzy model of chaotic Hénon map system can be obtained as follows

$$\begin{aligned} &R^1: \text{ IF } x_1(n_1) \text{ is } M_{11}, \\ &\text{THEN } x(n_1+1) = A_1 x(n_1) + c \\ &R^2: \text{ IF } x_1(n_1) \text{ is } M_{21}, \\ &\text{THEN } x(n_1+1) = A_2 x(n_1) + c \end{aligned} \tag{11}$$

where $x(n_1) = (x_1(n_1), x_2(n_1))^T$, $n_1 = 0, 1, 2, 3, \ldots, n$, $A_1 = \begin{bmatrix} -a_1(1.4) & 1 \\ b_1 & 0 \end{bmatrix}$, $A_2 = t\begin{bmatrix} -a_1(-1.4) & 1 \\ b_1 & 0 \end{bmatrix}$, $c = (1, 0)^T$.

The membership functions are chosen as

$$M_{11}(x_1(n_1)) = \frac{1}{2}\left(1 + \frac{x_1(n_1)}{1.4}\right) \tag{12}$$

$$M_{21}(x_1(n_1)) = \frac{1}{2}\left(1 - \frac{x_1(n_1)}{1.4}\right) \tag{13}$$

Then, applying the product-inference rule, singleton fuzzifier, and the center of gravity defuzzifier to the above fuzzy rule base, the overall Yang fuzzy chaotic map can be inferred as

$$x(n_1+1) = \sum_{i=1}^{2} \mu_i(x_1(n_1))(A_i x(n_1) + c) \tag{14}$$

where

$$\mu_i(x_1(n_1)) = \frac{M_{i1}(x_1(n_1))}{\sum_{i=1}^{2}(M_{i1}(x_1(n_1)))} \tag{15}$$

The Yang T-S fuzzy model of chaotic Hénon map system is given in Figure 3. From Figure 3, it is clearly to see that the derived Yang T-S fuzzy model is equivalent to the original chaotic map in Figure 1.

Since the Hénon map system is a nonlinear invertible map and the Yang T-S fuzzy model of the chaotic map system in different state space regions are represented by linear model, the Yin T-S fuzzy model of chaotic Hénon map system can be inferred by inverse matrix in linear system theory as follows

$$\begin{aligned} \hat{A}_1 = A_1^{-1} &= \frac{1}{\det(A_1)}\begin{bmatrix} 0 & -1 \\ -b_1 & -a_1(1.4) \end{bmatrix} \\ &= \begin{bmatrix} 0 & \frac{1}{b_1} \\ 1 & \frac{a_1}{b_1}(1.4) \end{bmatrix} = \begin{bmatrix} 0 & b_2 \\ 1 & a_2(1.4) \end{bmatrix} \end{aligned} \tag{16}$$

$$\begin{aligned} \hat{A}_2 = A_2^{-1} &= \frac{1}{\det(A_2)}\begin{bmatrix} 0 & -1 \\ -b_1 & a_1(1.4) \end{bmatrix} \\ &= \begin{bmatrix} 0 & \frac{1}{b_1} \\ 1 & -\frac{a_1}{b_1}(1.4) \end{bmatrix} = \begin{bmatrix} 0 & b_2 \\ 1 & a_2(-1.4) \end{bmatrix} \end{aligned} \tag{17}$$

where $b_2 = \frac{1}{b_1}$, $a_2 = \frac{a_1}{b_1}$.

From the matrix $\hat{A}_1$, $\hat{A}_2$, and phase portraits of Yin Hénon map system ($x_2(n) \in [-1.4, 1.4]$) in Figure 2, the Yin T-S fuzzy rules are as follows

$$\begin{aligned} &R^1: \text{IF } \hat{x}_2(n_2) \text{ is } \hat{M}_{11}, \\ &\text{THEN } \hat{x}(n_2-1) = \hat{A}_1\hat{x}(n_2) + \hat{c}, \\ &R^2: \text{IF } \hat{x}_2(n_2) \text{ is } \hat{M}_{21}, \\ &\text{THEN } \hat{x}(n_2-1) = \hat{A}_2\hat{x}(n_2) + \hat{c}, \end{aligned} \tag{18}$$

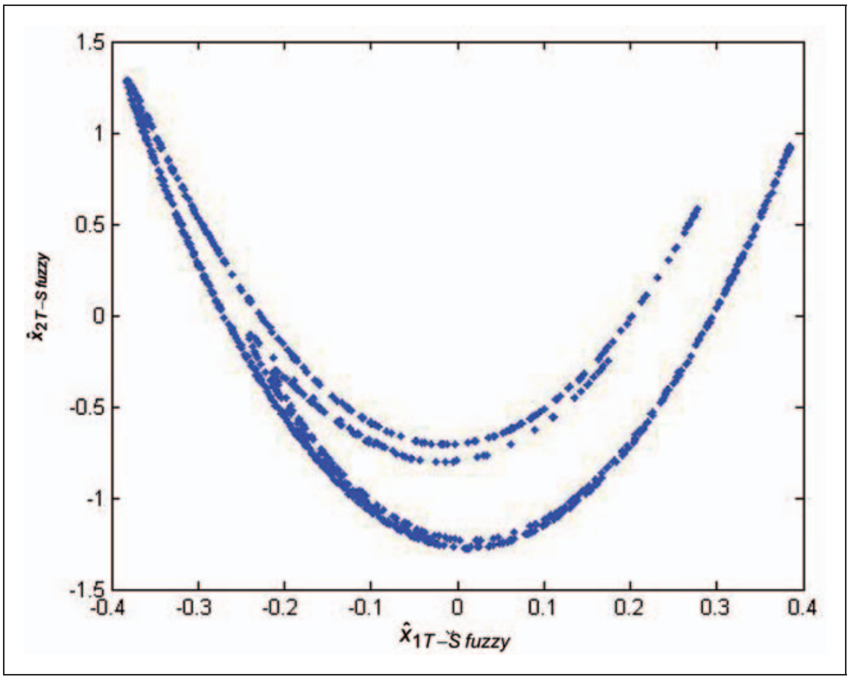

Figure 4. Phase portrait of Yin T-S fuzzy model of Hénon map system with $a_2 = 1.4$, $b_2 = 0.3$.

where

$$\hat{x}(n_2) = (\hat{x}_1(n_2), \hat{x}_2(n_2))^T,\quad n_2 = 0, -1, -2, -3, \ldots, -n.$$

The membership functions are chosen as

$$\hat{M}_{11}(\hat{x}_2(n_2)) = \frac{1}{2}\left(1 + \frac{\hat{x}_2(n_2)}{1.4}\right) \tag{19}$$

$$\hat{M}_{11}(\hat{x}_2(n_2)) = \frac{1}{2}\left(1 - \frac{\hat{x}_2(n_2)}{1.4}\right) \tag{20}$$

The Yin T-S fuzzy model of chaotic Hénon map system is given in Figure 4 and is equivalent to the original chaotic map in Figure 2.

Controller design for chaos synchronization of Yin and Yang T-S fuzzy model of Hénon map system

Let equation (14) as the drive system and the response system as follows

$$y(n+1) = \sum_{i=1}^{2} \mu_i(y_1(n))(A_i y(n) + c) + u(n) \tag{21}$$

The target for the design of a fuzzy controller $u(n)$ is synchronized the two discrete time chaotic map systems by the PDC technique as follows

$$u(n) = \sum_{i=1}^{2} \mu_i(x_1(n))K_i x(n) - \sum_{i=1}^{2} \mu_i(y_1(n))K_i y(n) \tag{22}$$

Define the error dynamics

$$e(n) = x(n) - y(n) \tag{23}$$

From equations (21) and (22), the closed-loop synchronization error dynamics is arranged as

$$e(n+1) = \sum_{i=1}^{2} \mu_i(x_1(n))(A_i + K_i)x(n) - \sum_{i=1}^{2} \mu_i(y_1(n))(A_i + K_i)y(n) \tag{24}$$

According to stability theory for discrete linear system, it is known that a matrix H if and only if the eigenvalues of matrix H are less than 1 in absolute value,[39] the error dynamics (equation (23)) is asymptotically stable, in other words, the chaos synchronization is achieved. Let a Schur stable matrix H satisfied the following equation

$$A_1 + K_1 = A_2 - K_2 = H \tag{25}$$

From equation (25), equation (24) can be rewritten as follows

$$e(n+1) = \sum_{i=1}^{2} \mu_i(x_1(n))Hx(n) - \sum_{i=1}^{2} \mu_i(y_1(n))Hy(n) \tag{26}$$

Since the system is an invertible map, the Yin fuzzy controller can be obtained by invertible matrix theorem as follows

$$\hat{K}_1 = K_1^{-1}, \hat{K}_2 = K_2^{-1} \tag{27}$$

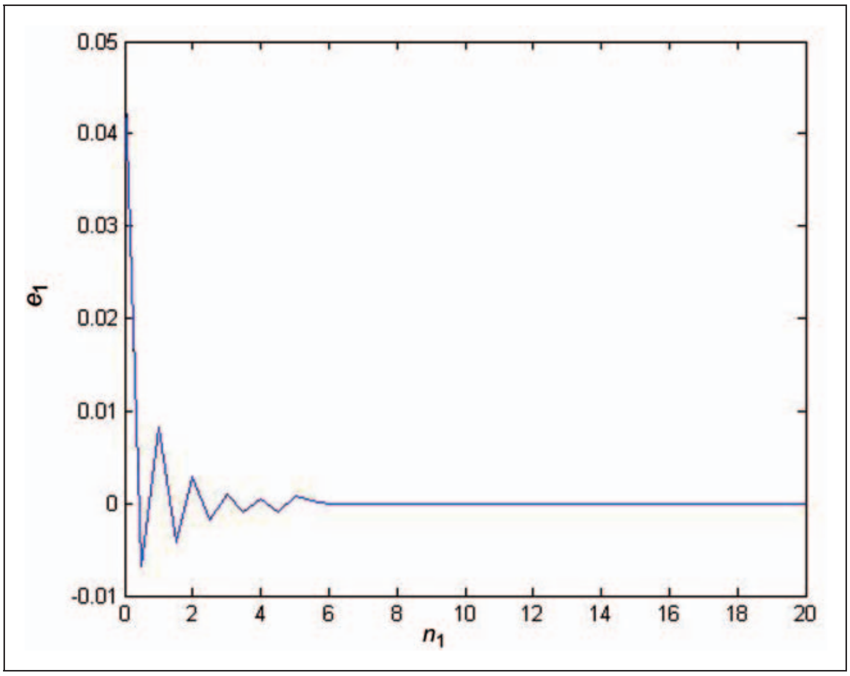

Figure 5. Error dynamics e_1 for chaos synchronization of Yang T-S fuzzy model of Hénon map system.

Simulation results

In order to apply the proposed Yin–Yang chaos synchronization scheme by PDC technique, the T-S fuzzy model is used. Yang T-S fuzzy model of Hénon map system as drive system is described in equations (11) to (15). Yang T-S fuzzy model of Hénon map system as response system is described by equations (21) and (22)

$$y(n_1+1)=\sum_{i=1}^{2}\mu_i(y_1(n_1))(A_i y(n_1)+c)+u(n_1)$$

where

$$A_1=\begin{bmatrix}-a_1(1.4) & 1\\ b_1 & 0\end{bmatrix},\ A_2=\begin{bmatrix}-a_1(-1.4) & 1\\ b_1 & 0\end{bmatrix},$$
$$c=[1,0]^T,$$

and

$$u(n)=\sum_{i=1}^{2}\mu_i(x_1(n_1))K_i x(n_1)-\sum_{i=1}^{2}\mu_i(y_1(n_1))K_i y(n_1)$$

According to the proposed Yin–Yang chaos synchronization by PDC technique, the stable matrix H are chosen as

$$H=\begin{bmatrix}0.27 & 0\\ 0 & 0.1\end{bmatrix}$$

The Yang fuzzy controller can be obtained from equation (25) as follows

$$K_1=\begin{bmatrix}2.23 & -1\\ -0.3 & 0.1\end{bmatrix},$$
$$K_2=\begin{bmatrix}1.69 & 1\\ 0.3 & -0.1\end{bmatrix},$$

Yang T-S fuzzy model of chaotic Hénon map system as drive system and Yang T-S fuzzy model of chaotic Hénon map system as response system is simulated with the parameters $a_1=1.4$, $b_1=0.3$, and the initial conditions $[x_1(0)\quad x_2(0)]^T=[0.63\quad 0.19]^T$, $[y_1(0)\quad y_2(0)]^T=[0.4\quad 0.1]^T$.

Simulation results show that the error dynamics approach to asymptotically stable in Figures 5 and 6. The chaos synchronization of Yang T-S fuzzy model of Hénon map system is achieved by PDC technique. The discrete time chaotic Hénon map system is a nonlinear invertible map, the Yin–Yang T-S fuzzy model of chaotic Hénon map system can be represented as a fuzzy aggregation of some local linear systems. So linear theory adopt to the T-S fuzzy model of chaotic Hénon map system, the Yin–Yang chaotic Hénon map systems are invertible systems for each other, the Yin fuzzy controller can be obtained by invertible matrix theorem by equation (27).

$$\hat{K}_1=K_1^{-1}=\begin{bmatrix}-1.29 & -12.98\\ -3.89 & -28.96\end{bmatrix},$$
$$\hat{K}_2=K_2^{-1}=\begin{bmatrix}-0.76 & -7.63\\ -2.29 & -11.14\end{bmatrix}$$

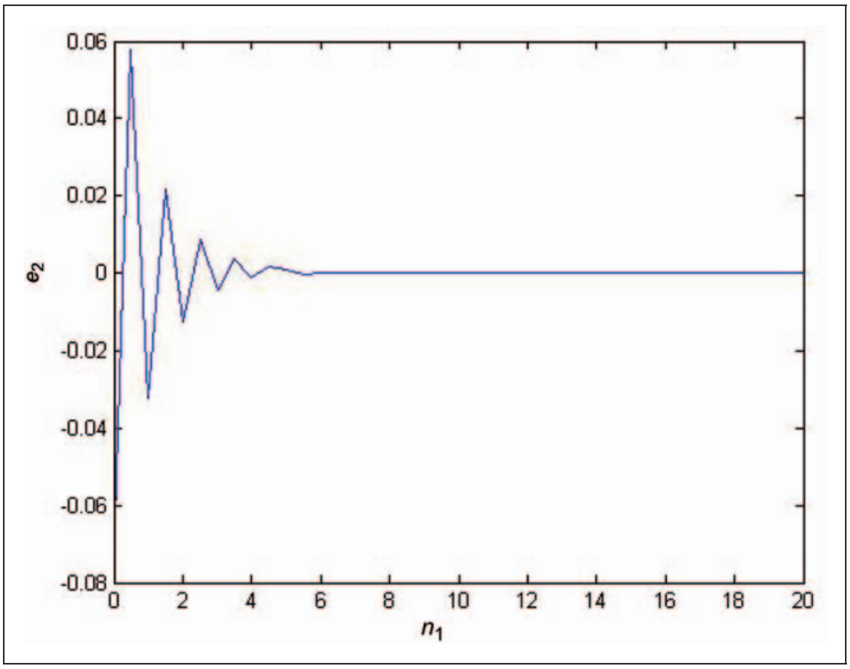

Figure 6. Error dynamics e_2 for chaos synchronization of Yang T-S fuzzy model of Hénon map system.

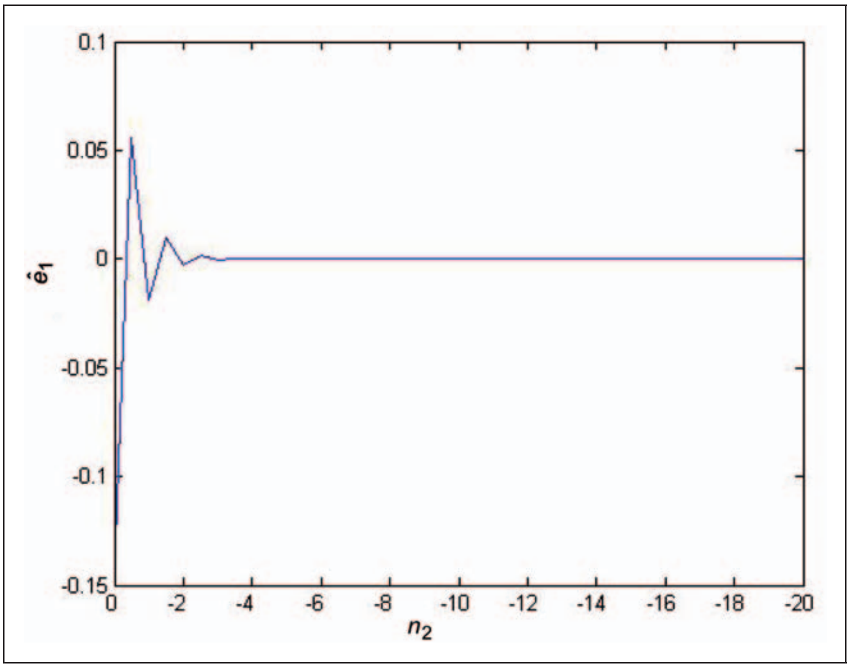

Figure 7. Error dynamics $\hat{e}_1$ for chaos synchronization of Yin T-S fuzzy model of Hénon map system.

Simulation results show that the error dynamics approach to asymptotically stable in Figures 7 and 8. The chaos synchronization of Yin T-S fuzzy model of Hénon map system is achieved by PDC technique.

Conclusions

Yin chaos of inversed Hénon map system and the Yin T-S fuzzy model of chaotic Hénon map system are firstly proposed. The T-S fuzzy model of a chaotic system can exactly be represented as a fuzzy aggregation of some local linear systems. As a result, the conventional linear system theory can be applied. Since nonlinear discrete time Hénon map is an invertible map, we develop the Yang T-S fuzzy model of Hénon map system, then the invertible matrix theorem in linear system is used to get the Yin T-S fuzzy model of Hénon map system. Correspondingly, the Yin fuzzy controller is designed by invertible matrix

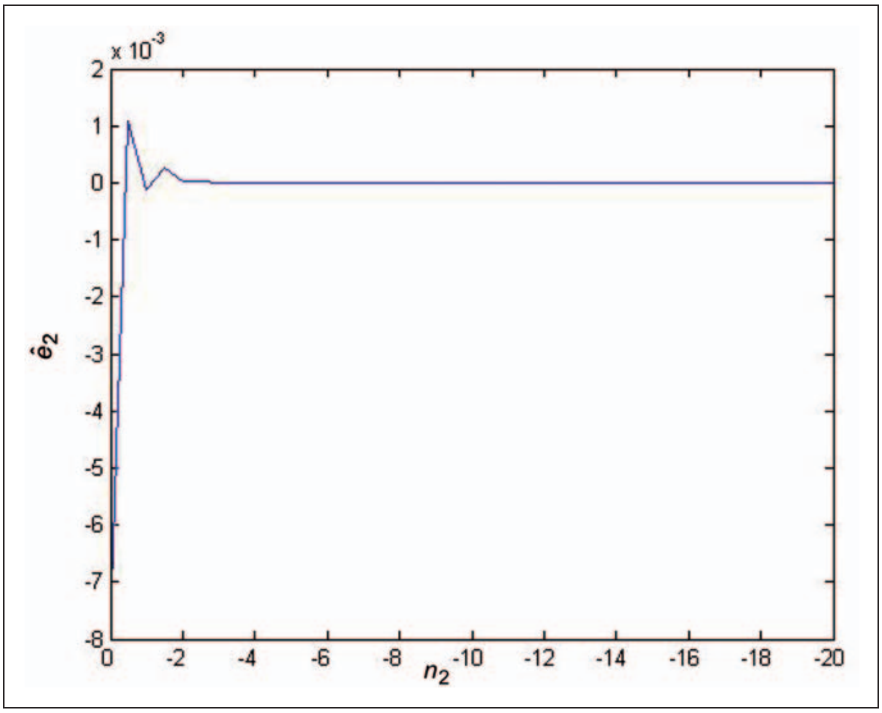

Figure 8. Error dynamics $\hat{e}_2$ for chaos synchronization of Yin T-S fuzzy model of Hénon map system.

theorem. The design of the Yin fuzzy controller is fleetly obtained by the inverse of Yang fuzzy controller, the concern is that the Yin chaotic system must be an inverse of the Yang chaotic system. According PDC technology, if the control parameters make Schur matrix stable, and the other set of system parameters have chaotic behavior, the chaos synchronization is achieved even if the Yin and Yang T-S fuzzy models of Hénon map system include the small parameters. In secure communication, the chaotic system as transmitter and the inverse chaotic system as receiver are hard to achieve due to the nonlinearity, the Yin and Yang T-S fuzzy models of Hénon map system can overcome these difficulties.

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