Majorana states in a p-wave superconducting ring

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Abstract – The spectrum of excitations of the chiral superconducting ring with internal and external radii R_i, R_e (comparable with coherence length ξ) trapping a unit flux Φ_0 is calculated. We find within the Bogoliubov-de Gennes approach that there exists a pair of *precisely* zero-energy states when $2k_\perp(R_e - R_i)/\pi$ is integer (here $k_\perp$ is the momentum component in the disk plane while $k_\perp\xi > 1$). They are not protected by topology, but are stable under certain deformations of the system. We discuss the ways to tune the system so that it grows into such a "Majorana disk". This condition has a character of a resonance phenomenon.

Introduction. – Spin-triplet *p*-wave superfluids, both neutral, such as liquid He^3, Li^6 or K^{40} and charged, such as the superconducting material Sr_2RuO_4 and possibly the heavy fermion UPt_3 have resulted in very rich physics [1]. The condensate is described by a generally tensorial complex order parameter Δ exhibiting a great variety of broken symmetries ground states. The broken symmetry and boundary conditions give rise to the continuous configuration of the order parameter as nontrivial topological excitations [2]. Especially interesting is the case of the so-called topological superconductors, characterized by the presence of electron-hole symmetry and the absence of both time-reversal and spin-rotation symmetry. Realizations of topological *p*-wave superfluids are the chiral superconductors like Sr_2RuO_4, with order parameter of the $p_x \pm ip_y$ symmetry type [3] and the ABM-phase [1] of superfluid He^3 and other fermionic cold atoms, as well as the topological superconductor $Cu_xBi_2Se_3$ that produces an equivalent pseudospin system on its surface [4].

Generally a magnetic field in type-II superconducting films easily creates a stable set of topological defects —Abrikosov vortices [5]. In the simplest vortex the phase of the order parameter rotates by 2π around the vortex and each vortex carries a unit of magnetic flux Φ_0 with superfluid density depleted in the core of the size of the coherence length ξ. Quasiparticles near the vortex core "feel" the phase wind by creating a set of discrete low-energy Andreev bound states. An unpinned vortex in an *infinite p*-wave superconductors exhibits a remarkable topological feature, *i.e.* appearance of the zero-energy

mode in the vortex core [6] (for each value of momentum $k_\perp$ perpendicular to the field direction). The zero mode represents a condensed-matter analog of the Majorana fermion [7]. Its topological nature ensures robustness against perturbations from deformations of order parameters and nonmagnetic impurities. Due to possible applications of the Majorana states in quantum computing it is important to ensure a relatively large minigap E_{mg} separating the Majorana states from charged excitations. It was proposed to enlarge the minigap from [8] Δ^2/E_F to order Δ by pinning vortices on inclusions of small radius [9,10] $R_i \sim \xi$.

In this letter we study the influence of size effects on the appearance of the Majorana states in a ring made of the chiral superconductor. It was noted that finite samples of size $L \gg \xi$ presented a *pair* of "nearly" Majorana states which constitute one charged fermion degree of freedom like in the 1D Kitaev model [11,12]. However these states are no longer topologically protected and naively one would expect that the degeneracy is removed with small splitting energy $\delta E \propto e^{-L/\xi}$ [13,14] due to tunneling from the core to the surface of the sample. For systems of small size $L \sim \xi$ the situation might be different. In particular it is not clear whether the splitting always occurs at all and how it depends on the sample geometry.

To investigate the survival of Majorana states we calculate exactly the spectrum of excitations in the chiral superconducting ring with internal and external radii R_i, R_e of order ξ trapping a unit flux Φ_0 (see fig. 1) within the Bogoliubov-de Gennes approach.
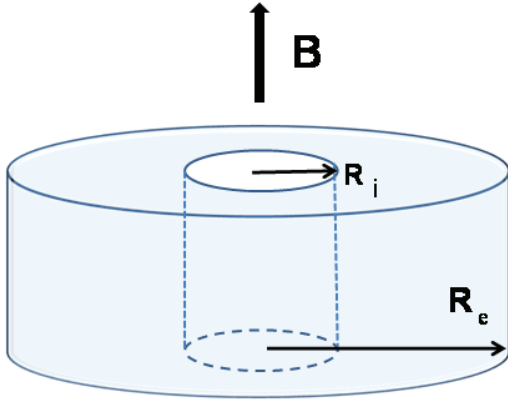
Fig. 1: (Colour on-line) A *p*-wave superconducting mesoscopic disk with internal and external radii R_i and R_e subject to a magnetic field perpendicular to the disk.

A surprising finding is that although the splitting is generally of order of the bulk energy gap Δ, there exists a pair of *precisely* zero-energy states for the special relation $R_e - R_i \approx \pi n/2k_\perp$ for any integer n. Then, using the perturbation theory, we generalize the consideration to other geometries.

Basic equations. – We start with the Bogoliubov-de Gennes (BdG) equations for the $p_x + ip_y$ superconductor in the presence of a single pinned vortex. The vector potential $\mathbf{A}$ in polar coordinates r, φ has only an azimuthal component $A_\varphi(r)$ and in the London gauge consists of the singular part $A_\varphi^s = hc/2er$ and the regular part of the vector potential that can be neglected for a type-II superconductor [15]. In the operator matrix form for a two-component amplitude the BdG equations read

$$\begin{pmatrix} \hat{H}_0 & L \\ L^+ & -\hat{H}_0^* \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}, \tag{1}$$

where for the anisotropic dispersion

$$H_0 = -\frac{\hbar^2}{2m_\perp}\nabla_\perp^2 - \frac{\hbar^2}{2m_\shortparallel}\nabla_\shortparallel^2 - E_F;$$

$$L = -\frac{\Delta}{k_F}\left\{ s\left(\mathbf{r}\right)e^{i\varphi}\left(i\boldsymbol{\nabla}_x - \boldsymbol{\nabla}_y\right) \right.$$

$$\left. +\frac{1}{2}\left[\left(i\boldsymbol{\nabla}_x - \boldsymbol{\nabla}_y\right)\left(s\left(\mathbf{r}\right)e^{i\varphi}\right)\right]\right\}, \tag{2}$$

with Δ being the "bulk gap" of order T_c (neglecting the small inhomogeneity of the superfluid density within the ring). The dimensionless profile of the order parameter $s(\mathbf{r})$ is defined to represent the gap function $\Delta(\mathbf{r}) = \Delta s(\mathbf{r})$. In principle it should be determined self-consistently, however, for a sufficiently thin homogeneous disk we initially take $s(\mathbf{r}) = 1$. This is justified in the bulk since the sample size is of order ξ. Although Andreev's bound states are typically inhomogeneous, the effect of their inhomogeneity on the order parameter is still smaller than

that of the continuum states even for small sizes [9]. We will later investigate the stability of the solutions with respect to variations to $s(\mathbf{r})$.

The equations possess the rotational and the electron-hole symmetries and eigenstates can be found in the form

$$u = \frac{1+i}{\sqrt{2}}f(r)e^{il\varphi}e^{ik_\shortparallel z}$$

$$v = \frac{1-i}{\sqrt{2}}g(r)e^{i(l-2)\varphi}e^{ik_\shortparallel z}. \tag{3}$$

For any angular momentum l and momentum along the field $k_\shortparallel$, there are radial excitation levels. In a dimensionless form eq. (1) is written as

$$-\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{l^2}{r^2} + \frac{1}{4\gamma^2}\right)f - \left(2\frac{\partial}{\partial r} - \frac{2l-3}{r}\right)g = \varepsilon_{lk_\shortparallel}f;$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{(l-2)^2}{r^2} + \frac{1}{4\gamma^2}\right)g + \left(2\frac{\partial}{\partial r} + \frac{2l-1}{r}\right)f = \varepsilon_{lk_\shortparallel}g, \tag{4}$$

with the dimensionless energy $\varepsilon_{lk_\shortparallel} = E_{lk_\shortparallel}/\gamma\Delta$. Here distances are in units of ξ. In the clean limit BCS (applicable to SrRu$_2$O$_4$) $\xi = \hbar k_\perp/m_\perp\Delta$, where

$$k_\perp^2/2m_\perp = E_F/\hbar^2 - k_\shortparallel^2/2m_\shortparallel, \tag{5}$$

and for given $k_\shortparallel$ there is just one dimensionless parameter

$$\gamma = 1/2k_\perp\xi = m_\perp\Delta/2\hbar^2k_\perp^2 \ll 1. \tag{6}$$

The Ansatz, eq. (3), was chosen in such a way that the equations become real. In the microscopic theory of the superconductor-insulator interface, (see [16]), the order parameter rises abruptly from zero in a dielectric, where amplitudes of normal excitations $f(R_e) = g(R_i) = 0$, to a finite value inside the superconductor within an atomic distance a from the interface, namely with a slope $\propto 1/a$. This means that the boundary condition on the amplitudes is consistent with a zero-order parameter at the boundary point in the self-consistency equation (see details in ref. [9] and references therein).

Majorana vs. non-Majorana rings. – Let us first determine under what conditions zero-energy (Majorana) states appear. It was shown [14,15] that they appear only for $l = 1$ and might contain two possible states $g_+ = f_+$ or/and $g_- = -f_-$. The corresponding equations simplify and differ just by the transformation $r \to -r$:

$$\left(\frac{\partial^2}{\partial r^2} + \left(\frac{1}{r} + 2\right)\frac{\partial}{\partial r} + \frac{1}{r} - \frac{1}{r^2} + \frac{1}{4\gamma^2}\right)f_+ = 0,$$

$$\left(\frac{\partial^2}{\partial r^2} + \left(\frac{1}{r} - 2\right)\frac{\partial}{\partial r} - \frac{1}{r} - \frac{1}{r^2} + \frac{1}{4\gamma^2}\right)f_- = 0. \tag{7}$$

The solutions are

$$f_\pm = e^{\mp r}\left[c_\pm J_1\left(-r\sqrt{\frac{1}{4\gamma^2} - 1}\right) + d_\pm Y_1\left(r\sqrt{\frac{1}{4\gamma^2} - 1}\right)\right], \tag{8}$$
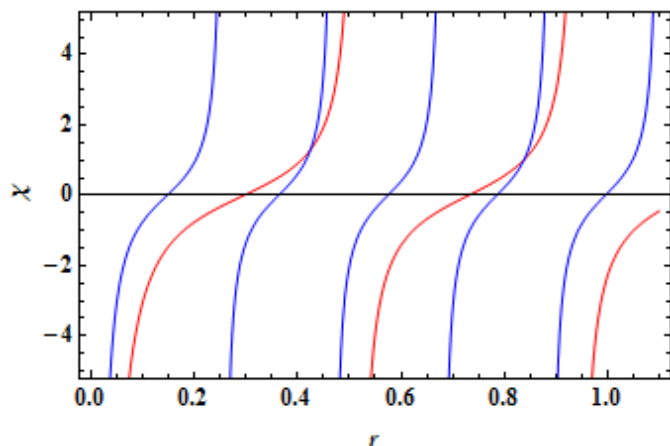
Fig. 2: (Colour on-line) The resonance condition for the Majorana disk. The function $\chi(r)$ for two values of the parameter $\gamma = 1/2\xi k_\perp$, $\gamma = 1/30$ (blue line), and $\gamma = 1/15$ (red line). To obey the resonance condition the internal and external radii R_i and R_e should satisfy $\chi(R_i) = \chi(R_e)$.

where J_1 and Y_1 are the Bessel functions.

Defining the ratio $\chi(r) = \dfrac{J_1(r\sqrt{1/4\gamma^2 - 1})}{Y_1(-r\sqrt{1/4\gamma^2 - 1})}$ plotted in fig. 2, the boundary conditions for both f_+ and f_- read

$$\chi(R_i) = \chi(R_e) = -d_\pm/c_\pm = \chi. \tag{9}$$

Therefore eq. (9) gives a relation between R_i, R_e and γ of the ring when two Majorana states exist *simultaneously*, see the blue line in fig. 2 and table 1 for $\gamma = 1/30$ and $R_i = 0.1$.

The superfluid density

$$\rho(r) = r|f_\pm(r)|^2 \tag{10}$$

of the two Majorana states for $\gamma = 1/30$ and $R_i = 0.1$, $R_e = 0.953$ is presented in fig. 3.

One observes that due to the exponential dependence in eq. (8) $f_\pm$ is located mostly near the internal (external) surface —the red and green lines, respectively.

Using the approximate periodicity of the Bessel functions in eq. (8) with period π one obtains for small $\gamma \ll 1$:

$$R_e - R_i = 2\pi n\gamma\xi = \pi n/2k_\perp, \tag{11}$$

where n is an integer. A finite system that conforms to these conditions will be denoted as "Majorana ring" in what follows.

If the condition eq. (9) is violated (hence the rings will be termed "non-Majorana") the would-be Majorana fermion ($l = 1$) states acquire a nonzero energy that oscillates around zero as a function of R_e for fixed R_i and γ. This is exemplified in fig. 4 where energies of the $l = 1$ states calculated numerically are given for $\gamma = 1/30$ and $R_i = 0.1$ and a range of $0.76\xi < R_e < 1.2\xi$. The calculation utilizes the NAG Fortran Library Routine Document F02EBF. It computes all the eigenvalues, and

Table 1: External radius R_e of the Majorana Ring with parameters $R_i = 0.1\xi$ and two values of γ. The radii are identified by the integer n.

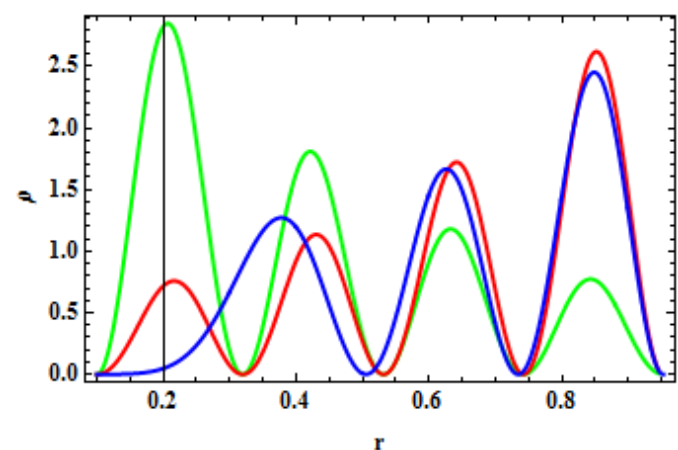| n	1	2	3	4
$\gamma = 1/30$	0.256	0.531	0.742	0.9526
$\gamma = 1/15$	0.559	0.987	1.411	1.835

Fig. 3: (Colour on-line) Superfluid density (defined in eq. (10)) of low-energy states in the Majorana ring with internal and external radii $R_i = 0.1\xi$ and $R_e = 0.953\xi$, respectively and $\gamma = 1/30$. The two Majorana $l = 1$ states are the internal or core state (red line) and the external surface state (green line) while the lowest-energy excited charged state is with $l = 4$ (blue line). The excited state clearly resembles the surface states of p-wave superconductors.

optionally all the eigenvectors, of a real general matrix. One observes that at the "Majorana geometry" the energy of the electron branch (red line) changes sign. Similarly the hole branch has the opposite sign and the same absolute value of energy. This exhibits a phenomenon of "level crossing" in the BdG equation. Note that for a small sample and $R_e - R_i = \pi(n + 1/2)/2k_\perp$ the energy of the $l = 1$ states might even exceed the gap magnitude Δ, see the green lines in fig. 4.

Excited states in Majorana ring. – For a vortex in an infinite s-wave superconductor, when the vortices are unpinned (freely moving) the low-lying spectrum of quasiparticle and hole excitations is equidistant, $E_l = l\omega$, where the angular momentum l takes half-integer values. The "minigap" E_{mg} in the s-wave superfluids is of order of $\omega = \Delta^2/E_F \ll \Delta$, where Δ is the energy gap and E_F is the Fermi energy. In the bulk chiral p-wave the spectrum of the low-energy excitations remains equidistant, $E_l = (l - 1)\omega$, but now l is integer [15].

For a Majorana ring there are excited states well below the continuum at $l \neq 1$ typically close to the surface. The origin of their small energy is the vanishing of the superconducting gap on the nodes of the p-wave
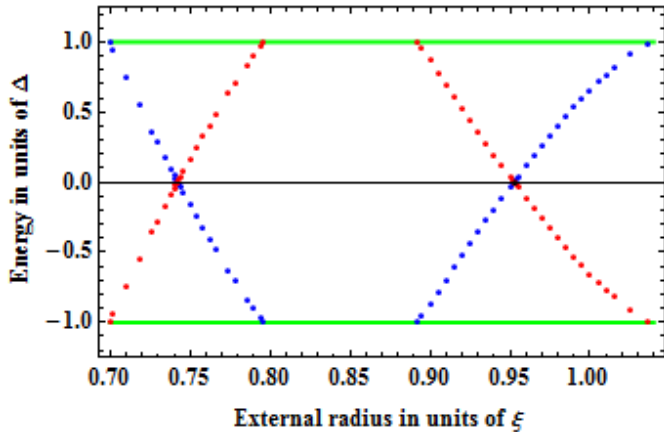
Fig. 4: (Colour on-lilne) Violation of the resonance condition. The Majorana states are split into positive and negative charged branches. The splitting energy in mesoscopic samples (like the one presented with $R_i = 0.1\xi$ and R_e in the range 0.7ξ–1.3ξ) can be as large as the superconducting gap (green line) at $R_e = 0.8$. One observes that for geometries far from the Majorana ring condition, $R_e = 0.742, 0.953$ (for $\gamma = 1/30$). Note the oscillation of the splitting energy of the quasiparticle and the hole.

superconductor. Such a state with minimal energy defines the "minigap" that protects the Majorana states from interferences due to the excitations. The excitation energies below the threshold (namely Andreev states) were calculated numerically for $\gamma = 1/30, R_i = 0.1$ and $R_e = 0.98$, see table 2.

Only the lowest (highest) energy E_+ (E_-) for quasiparticles (holes) for each angular momentum is given (typically other excitations are beyond the threshold). At certain l there are no Andreev states.

In the case of the small ring considered here one observes that the minigap $E_{mg} = 0.0135$ appears at $l = 4$.

The energy is just a fraction of Δ, still an order of magnitude larger than Δ^2/E_F for Sr_2RuO_4. The superfluid density ρ defined in eq. (10) corresponding to its wave function is presented in fig. 3 (blue line). The wave function even for such a small ring is peaked near the outer surface. To understand the relatively small value of the minigap compared to the one that protects the core Majorana states in the infinite system considered [17], $E_{mg} = 0.25\Delta$, we have simulated finite large rings with $R_e = 10, 20$. Of course the order parameter is no longer constant over such sizes and we have used $\Delta(r) = \Delta \tanh(r/\xi)$ for the order parameter distribution. One observes that there are surface modes with energies $E(20\xi) = 0.01\Delta$, $E(10\xi) = 0.01\Delta$. Therefore the energy of these modes is almost independent of R_e.

Stability with respect to change of the order parameter rotation symmetry. – We conjecture that by varying the parameters of the system, like the order parameter that depends on the local temperature etc., one can still tune the system into a Majorana ring. To support this conjecture we calculate in perturbation

Table 2: Energy of excited states in the Majorana ring with parameters $R_i = 0.1\xi$, $R_e = 0.953\xi$, $\gamma = 1/30$. Energies of both the quasiparticles (E_+) and holes (E_-) are given in units of Δ.

l	1	2	3	4	5	6
E_+	$2.6297 \cdot 10^{-4}$	0.1552	–	0.0135	0.5903	–
E_-	$-2.6297 \cdot 10^{-4}$	–	-0.0437	–	–	-0.1967

theory the correction to the Majorana solution of the simple model eq. (8) in which the order parameter was approximated by a constant. Now we assume that the order parameter, in addition to a homogeneous rotation of the phase, depends on the location, $s(\mathbf{r}) = s(r, \varphi) = 1 + \psi(r, \varphi)$, with the inhomogeneous part assumed to be small and $\int \psi(r, \varphi) \, d\varphi = 0$. The only component of the BdG operator in eq. (1) that is corrected is the off-diagonal one, see eq. (2):

$$L[s(r,\varphi)] = -\frac{\Delta}{2k_F} \left\{ 2se^{i\varphi}(i\boldsymbol{\nabla}_x - \boldsymbol{\nabla}_y) + \left[(i\boldsymbol{\nabla}_x - \boldsymbol{\nabla}_y)se^{i\varphi}\right] \right\} \tag{12}$$

$$= -\frac{i\Delta}{2k_F} e^{2i\varphi} \left\{ 2s\left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \varphi}\right) + \frac{1}{r}\left(i\frac{\partial s}{\partial \varphi} - s\right) + \frac{\partial s}{\partial r} \right\}. \tag{13}$$

The energy correction to the $l = 1$ Majorana states given in eq. (8) to leading order in ψ is

$$E_M^{ab} = \langle f_a | H[s] - H[1] | f_b \rangle = \int \begin{pmatrix} u_a^* & v_a^* \end{pmatrix}$$
$$\times \begin{pmatrix} 0 & L[s] - L[1] \\ L[s]^+ - L[1]^+ & 0 \end{pmatrix} \begin{pmatrix} u_b \\ v_b \end{pmatrix}, \tag{14}$$

where $a, b = \pm$ (for the internal or external surface Majorana states of the unperturbed system). Using

$$L[s] - L[1] = -\frac{i\Delta}{k_F} e^{2i\varphi} \left\{ \psi\left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \varphi}\right) + \frac{1}{2r}\left[i\frac{\partial\psi}{\partial\varphi} - \psi\right] + \frac{1}{2}\frac{\partial\psi}{\partial r} \right\}, \tag{15}$$

the diagonal elements are

$$E_M^{++} = E_M^{--} = i\int_{\varphi,r} r f_+(r) e^{-i\varphi} (L[s] - L[1]) \left[f_+(r) e^{i\varphi}\right]$$
$$+ cc = 0. \tag{16}$$

The off-diagonal terms read

$$E_M^{-+} = E_M^{+-} = -\frac{\Delta}{\pi k_F} \int_r r f_-(r) f_+(r) \int_\varphi \psi(r,\varphi) = 0. \tag{17}$$

So there is no splitting of the Majorana states.

Discussion and conclusions. – To summarize, the spectrum of excitations of a single vortex in a chiral superconducting ring with internal and external radii R_i,

R_e comparable with the coherence length ξ was calculated. There is a pair of *precisely* zero-energy states for $R_e - R_i = \pi n/2k_\perp$ for any integer n for $k_\perp \xi > 1$ where $k_\perp = \sqrt{2m_\perp E_F/\hbar^2 - m_\perp/m_\parallel k_\parallel^2}$ is the momentum in direction perpendicular to the magnetic field. Therefore a certain combination of geometrical factors, order parameter (dependent on temperature and material parameters) and magnetic-field distribution make a "Majorana ring". They are not protected by topology, but are stable under certain deformations of the system. This condition has the character of a resonance phenomenon.

The quantized geometry of the Majorana ring is not evidently expected to be robust against perturbations like disorder, shape change, magnetic-field distribution. However, we guess that the pair of exact Majorana states is separated by a minigap of order $\Delta/70$ from charged Andreev surface states, still an order of magnitude larger than the Caroli, Matricon, deGennes minigap [8] Δ^2/E_F for the superconductor Sr_2RuO_4 with $\Delta = 2\,\text{K}$, $E_F = 10^3\,\text{K}$. An analogous situation arises in the multivortex system comprehensively studied by Mizushima and Machida [18] under the assumption that the distance between the vortices L is much larger than ξ, so that tunneling between them can be treated as a perturbation. In this case the pair of Majorana states belong to two different vortices and they found, using a variational state made of the core Majorana states of each of the vortices, that the splitting energy of the two nearly Majorana states oscillates as $E_\pm \sim \Delta\sqrt{k_F L}\cos(k_F L + \pi/4)e^{-L/\xi}$. Therefore at $L = \pi(n + 1/4)/k_F$ one would get a degeneracy. This is reminiscent of our formula with two important differences. First, our splitting is generally of order Δ, not $\Delta\sqrt{k_F L}e^{-L/\xi}$ and second, more importantly, the Majorana states highly overlap, so that n is small, while for two vortices $n > Lk_F \gg 1$.

Practical proposals will depend on the thickness of the film. In the 2D limit $k_\perp = k_F$ (the condition is determined below) and the Majorana condition is $R_e - R_i = \pi n/2k_F$. For Sr_2RuO_4 with [3], $\xi = 65\,\text{nm}$ and the ring width should be quantized in $\pi/2k_F = 4\,\text{nm} \approx \xi/30$ with minigap of order $1\,\text{K}$. Moreover deviations from the Majorana condition lead to a very sharp splitting of order Δ, see fig. 3. When the thickness of the film is D new channels open for the Andreev bound states: $k_\parallel = 2\pi n_\parallel/D$, where $n_\parallel$ is integer. The new condition on the width becomes $R_e - R_i = \frac{\pi}{2k^*}[1 - \alpha(2\pi n_\parallel/Dk^*)^2]^{-1/2}$, where $\alpha = m_\perp/m_\parallel = 0.03$ is the anisotropy and $k^* = \sqrt{2m_\perp E_F}/\hbar$. The second channel $n_\parallel = 1$ will enter for $R_e - R_i \sim \xi$ at width $D = 2\pi\alpha^{1/2}/k^* = 5\,\text{nm}$. Therefore the film is 2D only when it is thinner than

that, while for thicker films, when several new channels are open, one of them can harbor Majorana states.

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