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## Revised DEMATEL: Resolving the Infeasibility of DEMATEL



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## ABSTRACT

Decision Making Trial and Evaluation Laboratory (DEMATEL) has been applied in many situations, such as marketing strategies, control systems, safety problems, developing the competencies of global managers and group decision making. It has been incorporated into other methods such as Analytical Network Process (ANP), Multiple Criteria Decision Making (MCDM), fuzzy set theory, etc., to vitalize these traditional methods and explore new applications for the hybrid methods. DEMATEL models the influences of components of a system with an initial direct relation matrix. Influences of components can ripple transitively to other components, which is modeled by raising the initial direct relation matrix to powers. The total influence is computed by summing up matrices of all powers based on the assumption that the matrix raising to the power of infinity would converge to zero. The current paper shows that raising the initial relation matrix to the power of infinity may not converge to zero and hence total influence may not converge. The current paper also shows that our revised DEMATEL guarantees that the initial direct-relation matrix to infinite power will converge to zero and the total influence can be obtained accordingly. The newly developed approach is illustrated with numerical examples.

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## 1. Introduction

In practice, the Decision Making Trial and Evaluation Laboratory (DEMATEL) method [1–3] has been applied to illustrate the interrelations among criteria and to find the central criteria to represent the effectiveness of factors/aspects. It has also been applied in many situations, such as marketing strategies, control systems, safety problems [4,5], development of the competencies of global managers, and group decision making [4–9]. Furthermore, hybrid models combining the DEMATEL and other methods have been widely used in various fields, for example, e-learning evaluation [10], airline safety measurement [5], and innovation policy portfolios for Taiwan's SIP Mall [11]. Wu and Lee proposed an effective method combining fuzzy logic and the DEMATEL to segment required competencies for better promoting the competency development of global managers which involves the vagueness of human judgments [12]. Yang et al. used DEMATEL not only to detect complex relationships and build an impact-relation map (IRM) of the criteria, but also to obtain the influence levels of each element over others; they then adopted these influence level values as the basis of the normalization supermatrix for determining ANP weights to obtain the relative importance [13]. The ANP [14–17], which is the general form of analytic hierarchy (AHP) [18], has been applied successfully in many practical decision-making problems, such as project selection, product planning, green supply chain management, and optimal scheduling problems [19–21].

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0307-904X/\$ - see front matter © 2013 Elsevier Inc. All rights reserved. http://dx.doi.org/10.1016/j.apm.2013.01.016 Wei et al. proposed SEM modified by DEMATEL technique as the causal model of Web-advertising effects [22]. Wu et al. applied the DEMATEL method to not only evaluate the importance of the criteria but also construct the causal relationships among the criteria to evaluate the outreach personnel program [9]. Yang and Tzeng proposed an integrated multiple criteria decision making (MCDM) technique which combines the DEMATEL and a novel cluster-weighted ANP method in their work [23], in which the DEMATEL method is used to visualize the structure of complicated causal relationships between criteria of a system and obtain the influence level of these criteria. Buyukozkan and Ozturkcan developed a novel approach based on a combined ANP and DEMATEL technique to help companies determine critical Six Sigma projects and identify the priorities of these projects especially in logistics companies [24].

The complex interaction among components of a system can be modeled by DEMATEL. The initial influence of a component on another component is represented by a value between 0 and 1. Zero means that the component exercises no influence, and one means that it exercises an absolute influence. A matrix with such entries is used to represent the initial influence between components of a system. The complication of the interaction among components results from the assumption that the influence may ripple transitively. The transitive influence is modeled by matrix multiplication. The initial matrix represents the direct influence, and the multiplication of the matrix n times represents the *n*-indirect influence exercised by the components of a system. For example, 2-indirect influence, which is obtained by raising the initial matrix to the power of two, represents the influence exercised by a component after a ripple of length two; moreover, the 1-indirect influence is the same as the initial direct influence. The total influence exercised by a component is obtained by summing up the direct initial influence and the indirect influence of all lengths; therefore, the total influence is the sum of an infinite series.

The current paper finds that the infinite series of the total influence using the conventional DEMATEL might not converge under some circumstances. A sufficient condition for the infinite series to be convergent is identified in this paper. Based on such sufficient condition, we proposed a new version of DEMATEL, which guarantees the convergence of the infinite series.

A simple guideline for readers to choose DEMATEL or our revised DEMATEL is to check the normalized initial direct-relation matrix. If each column sum of the matrix is less than one, then DEMATEL is applicable. Otherwise, DEMATEL is not applicable and our revised DEMATEL should be used.

The rest of the paper is organized as follows. Section 2 presents the original version of DEMATEL. Section 3 identifies the infeasibility of the original DEMATEL by giving a counter example to show the divergence resulted from the original DEM-ATEL. Section 4 presents a sufficient condition under which the total influence, the sum of an infinite series, will converge and a new version of DEMATEL. Section 5 applies the newly developed approach to cases for which the original DEMATEL is infeasible and feasible. We demonstrate how our new proposed approach works and how close our solution is to that of the original DEMATEL. Section 6 discusses the potential risk might be met in the original DEMATEL computation. Finally, conclusions are presented in Section 7.

## 2. The original DEMATEL

For clarity, the original DEMATEL method is reiterated and summarized in the following:

## Step 1: Find the average matrix A

Suppose we have *H* experts to provide their opinions and *n* factors to be considered. Each stakeholder is asked to indicate the degree to which he or she believes the factor *i* affects the factor *j*. These pairwise comparison between the *i*th factor and the *j*th factor given by *k*th expert is denoted as  $b_{ij}^{(k)}$ , which takes an integer score ranging from 0, 1, 2, 3, and 4, representing 'No influence (0),' 'Low influence (1),' 'Medium influence (2),' 'High influence (3),' and 'Very high influence (4),' respectively. The scores given by each expert will form a  $n \times n$  non-negative answer matrix  $B^{(k)} = [b_{ij}^{(k)}]_{n \times n}$ , with  $1 \le k \le H$ . Thus  $B^{(1)}, B^{(2)}, \ldots, B^{(H)}$  are the answer matrices of *H* experts. The diagonal elements of each answer matrix  $B^{(k)}$  are all set to zero, which means no influence is given by itself. We can then compute the  $n \times n$  average matrix *A* for all experts by averaging the *H* experts' scores as follows:

$$a_{ij} = \frac{1}{H} \sum_{k=1}^{H} b_{ij}^{(k)}.$$
(1)

The average matrix  $A = [a_{ij}]_{n \times n}$  is also called the initial direct relation matrix. The matrix A shows the initial direct effects that a factor exerts on and receives from other factors. Furthermore, we can map out the causal effect between each pair of factors in a system by drawing an influence map. Fig. 1 is an example of such an influence map. Here, each letter represents a factor in the system. An arrow from c to d shows the effect that c exercises on d, and the strength of its effect is four. DEM-ATEL can convert the structural relations among the factors of a system into an intelligible map of the system.

## Step 2: Calculate the normalized initial direct-relation matrix D

The normalized initial direct-relation matrix  $D = [d_{ij}]_{n \times n}$  is obtained by normalizing the average matrix A in the following way:

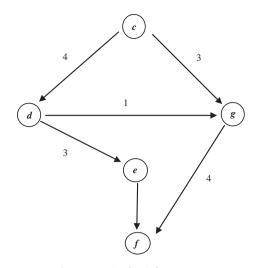


Fig. 1. Example of an influence map.

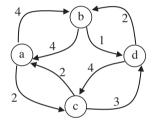


Fig. 2. The intelligible map of the first expert.

Let 
$$s = \max\left(\max_{1 \le i \le n} \sum_{j=1}^{n} a_{ij}, \max_{1 \le j \le n} \sum_{i=1}^{n} a_{ij}\right),$$
 (2)  
Then  $D = \frac{A}{s}.$  (3)

Since the sum of each row *i* of the matrix *A*,  $\sum_{j=1}^{n} a_{ij}$ , represents the total direct effect that the factor *i* gives to other factors,  $\max_{1 \le i \le n} \sum_{j=1}^{n} a_{ij}$  represents the largest total direct effect of all factors. Likewise, since the sum of each column *j* of the matrix *A*,  $\sum_{i=1}^{n} a_{ij}$ , represents the total direct effect received by the factor *j*,  $\max_{1 \le i \le n} \sum_{i=1}^{n} a_{ij}$  represents the largest total direct effect received for all factors. The positive scalar *s* takes the larger of the two as the scaling factor, and the matrix *D* is obtained by dividing each element of *A* by the scalar *s*. Note that each element  $d_{ij}$  of matrix *D* is between zero and one.

#### Step 3: Compute the total relation matrix

The power of the normalized initial direct-relation matrix D,  $D^m$ , which is called *m*-indirect influence, can be used to represent the effect of length *m* or the effect propagated after m - 1 intermediates. The total influence or total relation can be obtained by summing up D,  $D^2$ ,  $D^3$ , ...,  $D^\infty$ . The original DEMATEL assumes that  $D^m$  would converge to zero matrix and the total relation matrix  $T = D + D^1 + D^2 + ... + D^\infty$  can be obtained by

$$T = \lim_{n \to \infty} (D + D^2 + \ldots + D^m) = D(I - D)^{-1}.$$
(4)

However, the assumption that  $\lim_{m\to\infty} D^m = [0]_{n\times n}$  is incorrect, which is to be shown in the next section. Therefore,  $T = D + D^1 + D^2 + \cdots + D^\infty$  might not exist.

Once  $T = [t_{ij}]_{n \times n}$  is obtained, we can define *r* and *c* as  $n \times 1$  vectors representing the sum of rows and the sum of columns of the total relation matrix *T* as follows:

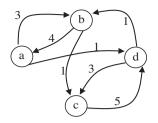


Fig. 3. The intelligible map of the second expert.

$$r = [r_i]_{n \times 1} = \left(\sum_{j=1}^n t_{ij}\right)_{n \times 1},$$
(5)  

$$c = [c_j]'_{1 \times n} = \left(\sum_{i=1}^n t_{ij}\right)'_{1 \times n},$$
(6)

where the superscript ' denotes transpose.

Let  $r_i$  be the sum of the *i*-th row of the matrix *T*. Then  $r_i$  shows the total effect, both direct and indirect, given by the factor *i* to other factors. Let  $c_j$  denotes the sum of the *j*-th column of the matrix *T*. Then  $c_j$  shows the total effect, both direct and indirect, received by the factor *j* from other factors. Thus when j = i, the sum  $(r_i + c_i)$  gives us an index representing the total effect both given and received by the factor *i*. In other words,  $(r_i + c_i)$  shows the degree of the importance (total sum of the effects given and received) that the factor *i* plays in the system. In addition, the difference  $(r_i - c_i)$  shows the net effect that the factor *i* is a net causer, and when  $(r_i - c_i)$  is negative, the factor *i* is a net receiver [10,25].

#### Step 4: Set a threshold value and obtain the impact-relations-map

In order to explain the structural relation among the factors while keeping the complexity of a system to a manageable level, it is necessary to set a threshold value *p* to filter out some negligible effect in the matrix *T*. While each factor of the matrix *T* provides information on how one factor affects another, the decision-maker must set a threshold value in order to reduce the complexity of the structural relation model implied by the matrix *T*. Only some factors, whose effect in the matrix *T* is greater than the threshold value, should be chosen and shown in an impact-relations-map (IRM) [10].

#### 3. Infeasibility of DEMATEL

To demonstrate the infeasibility of the original DEMATEL, let us consider the following example.

**Example 1.** Assume two intelligible maps of as system are given by two experts as shown in Figs. 2 and 3. The answer matrices corresponding to the intelligible maps are as follows:

	٢0	4	2	[0	]	0	3	0	ן 1
<b>D</b> (1)	4	0	0	1	and $B^{(2)} =$	4	0	1	0
$B^{(1)} = \begin{bmatrix} \\ \\ \end{bmatrix}$	2	0	0	3		0	0	0	5
	L0	2	4	0		0	1	3	0

The initial direct relation matrix, which is obtained by averaging the answer matrices, is as following:

	٥٦	3.5	1	0.5	
A =	4	3.5 0 0 1.5	0.5	0.5	
A =	1	0	0	4	•
	0	1.5	3.5	0	

By finding the maximum of the row sums, which is five, and the maximum of the column sums, which is also five, the normalized initial direct-relation matrix *D* is given by dividing the initial direct relation matrix by five, which is

$$D = \begin{bmatrix} 0 & 0.7 & 0.2 & 0.1 \\ 0.8 & 0 & 0.1 & 0.1 \\ 0.2 & 0 & 0 & 0.8 \\ 0 & 0.3 & 0.7 & 0 \end{bmatrix}.$$

According to DEMATEL, to compute the total relation matrix T,  $D^{\infty}$  must be computed and be a null matrix such that the

DEMATEL can successfully work. However,  $D^{\infty} = \begin{bmatrix} 0.169447 & 0.169447 & 0.169447 & 0.169447 \\ 0.075505 & 0.075505 & 0.075505 & 0.075505 \\ 0.410887 & 0.410887 & 0.410887 & 0.410887 \\ 0.344162 & 0.344162 & 0.344162 & 0.344162 \end{bmatrix}$ , which is not as expected

as a null matrix  $[0]_{n \times n}$ . Therefore, for this example, the total relation matrix *T* cannot be obtained because  $T = D + D^1 + D^2 + \cdots + D^\infty$  does not converge. The infeasibility of DEMATEL is summarized in the Theorem 2.

One may argue that the above example is too artificial because each column of the matrix sums to unity. However the following theorem shows that even if some columns of *D* sum to unity and some sum to number less than 1,  $D^{\infty}$  still might not converge to zero matrix.

**Theorem 1.** Let *D* be a normalized initial-direct relation matrix, some of whose columns sum to unity and some of whose columns sum less than one; then,  $\lim D^k = [0]_{n \times n}$  might or might not hold.

Proof. Consider the following matrix

$$D = \begin{bmatrix} 0 & 0 & 0.4 & 0.2 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0.4 & 0 & 0.2 \\ 0.3 & 0 & 0.3 & 0 & 0 & 0.4 \\ 0 & 0.7 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.4 & 0 & 0 \end{bmatrix}.$$
  
Its power of infinity is  $D^{\infty} = \begin{bmatrix} 0.25 & 0 & 0.25 & 0.25 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0 & 0.25 \\ 0.25 & 0 & 0.25 & 0.25 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0 & 0.25 \end{bmatrix}.$ 

That is, it does not converge to  $[0]_{n \times n}$ .

```
If
```

$D = \begin{bmatrix} 0 & 0.7 & 0 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0.5 & 0 & 0.3 & 0 \\ 0.4 & 0 & 0 & 0.5 & 0 & 0.1 \\ 0 & 0 & 0.2 & 0 & 0.6 & 0.1 \\ 0.3 & 0.2 & 0 & 0 & 0 & 0.4 \\ 0.3 & 0.1 & 0.3 & 03 & 0 & 0 \end{bmatrix}$
$D = \begin{vmatrix} 0.4 & 0 & 0 & 0.5 & 0 & 0.1 \\ 0 & 0 & 0.2 & 0 & 0.6 & 0.1 \\ 0.3 & 0.2 & 0 & 0 & 0 & 0.4 \end{vmatrix}$
$D = \begin{vmatrix} 0 & 0 & 0.2 & 0 & 0.6 & 0.1 \\ 0.3 & 0.2 & 0 & 0 & 0.4 \end{vmatrix}$
0.3 0.2 0 0 0 0.4
0.3 0.1 0.3 03 0 0
0 0 0 0 0 0
$D = \left[ \begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 \end{array} \right]^{-1}$
0 0 0 0 0 0
$D^{\infty} = egin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \ 0 & 0 & 0 & 0 &$

Hence if each column of *D* has a sum less than or equal to one,  $D^{\infty}$  may or may not converge to  $[0]_{n \times n}$ .  $\Box$ 

**Theorem 2.** For some cases,  $\lim_{m\to\infty} D^m$  may not converge to null matrix  $[0]_{n\times n}$ ; therefore,  $T = D + D^1 + D^2 + \dots + D^\infty$  might not converge. That is, DEMATEL is infeasible when  $\lim_{m\to\infty} D^m$  does not converge to the null matrix.

## 4. The revised DEMATEL

In this paper, we present a sufficient condition under which the infinite-direct influence will become a null matrix such that the sum of the infinite series, the total influence, will converge.

**Theorem 3.** Let *D* be a normalized initial-direct relation matrix having the column sum of each column less than one; then  $\lim_{k \to 0} D^k = [0]_{n \times n}$ .

**Proof.** Let  $D = [d_{ij}]_{n \times n}$  and  $D^k = [d_{ij}^{(k)}]_{n \times n}$ . Let  $d_i^{(k)}$  denote the *i*-th row of  $D^k$ . Let  $m_i^{(k)} = \max_{j=1}^n \{d_{ij}^{(k)}\}$  denote the maximum value in  $d_i^{(k)}$ . We want to show that  $m_i^{(k)} > m_i^{(k+1)}$  for all  $k \ge 1$  if  $m_i^{(k)} > 0$ .

Assume  $m_i^{(k)} > 0$ . Since  $\forall j, \sum_{p=1}^n d_{pj} < 1$ ,  $d_{ij}^{(k+1)} = \sum_{p=1}^n d_{ip}^{(k)} d_{pj} \leq \sum_{p=1}^n m_i^{(k)} d_{pj} = m_i^{(k)} \sum_{p=1}^n d_{pj} < m_i^{(k)}$  for all j. That is  $m_i^{(k+1)} < m_i^{(k)}$ . Hence the maximum value of the *i*th row of the matrix to a power will strictly decrease if it is not zero as the power increases  $\Box$ 

Based on Theorem 3, we propose a new version of DEMATEL as follows.

1. Calculate the initial average matrix

Let  $A = [a_{ii}]_{n \times n}$  be an average matrix of the respondents' direct matrices in which the entry (i, j) indicates the direct influence the factor *i* exerts on the factor *j*. The initial average matrix  $A = [a_{ij}]_{n \times n}$  is given by

$$A = \frac{1}{H} \sum_{k=1}^{H} B^{(k)},$$
(7)

where  $B^{(k)}$  is the answering matrix of the *k*-th respondent.

2. Calculate the normalized initial-direct relation matrix X.

It is calculated by

 $X = \frac{A}{c}$ 

where

$$s = \max\left(\max_{1 \le i \le n} \sum_{j=1}^{n} a_{ij}, \varepsilon + \max_{1 \le j \le n} \sum_{i=1}^{n} a_{ij}\right),\tag{9}$$

and  $\varepsilon$  is a very small positive number, for example,  $10^{-5}$ .

3. Derive the total influence matrix S.

All indirect influence matrices are  $X^2, X^3, \dots, X^k, \dots, X^\infty$ .

The total influence matrix  $S = X + X^2 + \ldots + X^{\infty} = \sum_{k=1}^{\infty} X^k$ , which is equal to

$$S = X(I - X)^{-1}.$$
 (10)

Similar to the original DEMATEL, the revised DEMATAL also requires the infinite power of the normalized initial-direct relation matrix become a null matrix, which is not guaranteed by the original DEMATEL but guaranteed by our revised DEM-ATEL. The fact of convergence is shown in the following theorem.

**Theorem 4.** The term of the total influence infinite series,  $X^N$ , will approach null matrix  $[0]_{n \times n}$  as N approaches infinity. That is,  $\lim_{N \to \infty} X^N = [0]_{n \times n}.$ 

**Proof.** Since  $x_{ij} = a_{ij} / \max(\max_{1 \le i \le n} \sum_{j=1}^n a_{ij}, \varepsilon + \max_{1 \le j \le n} \sum_{i=1}^n a_{ij})$ , we have  $\forall j, \sum_{p=1}^n x_{pj} < 1$ . According to Theorem 3, we have  $\lim_{N} X^{N} = [0]_{n \times n}. \square$ 

Since the infinite-indirect influence  $X^{\infty}$  is zero, the infinite series of the total influence can be obtained according to the following theorem.

**Theorem 5.**  $S = X(I - X)^{-1}$ .

**Proof.** Let  $S_N = \sum_{k=1}^N X^k$ . Then  $S_N - XS_N = X - X^{N+1}$ .

It follows:  $S_N(I - X) = X(I - X^N)$ .

Since the sum of each column of X is less than one, it is easy to show that I - X is diagonalizable. That is, I - X is invertible. Therefore, Multiplying both sides of the above equation by  $(I - X)^{-1}$ , we have

$$S_N = X(I - X^N)(I - X)^{-1}.$$

Since  $\lim_{N \to \infty} X^N = [0]_{n \times n}$ ,  $S = \lim_{N \to \infty} S_N = X(I - X)^{-1}$ .

## 5. Illustration

Four examples are illustrated in this section. The first example is to demonstrate that the case which is infeasible for the original DEMATEL becomes feasible for our revised DEMATEL. The second example shows that, for those cases that are

(8)

feasible for the original method, the solution of the revised DEMATEL is quite close to the solution of original DEMATEL if  $\varepsilon$  is small enough. Two empirical examples are used to illustrate our method. The third example applies our revised method to the data used by Shieh et al. [19]. And in the last example, the data used by Chen and Chen [2] to develop to an innovation support system for Taiwan higher education is adopted to verify our revised DEMATEL.

#### 5.1. Example 2

Let us revisit Example 1 by the revised DEMATAL as follows. Step 1. The initial average influence matrix is

$$A = \begin{bmatrix} 0 & 3.5 & 1 & 0.5 \\ 4 & 0 & 0.5 & 0.5 \\ 1 & 0 & 0 & 4 \\ 0 & 1.5 & 3.5 & 0 \end{bmatrix}.$$

Step 2. Let  $\varepsilon = 10^{-5}$ . The initial influence matrix is

```
0.6999986 0.1999996 0.0999998
                  0 0.0999998
0 0
0.2999994 0.6999986
       0.7999984
0.1999996
                              0.0999998 0.0999998
  X =
                                         0.7999984
                                             0
, we have S = X + X^1 + X^2 + \dots + X^{\infty} =
                   0 0 0
    125000.0321 125000.397 124999.8209 124999.7497
    125000.5204 125000.026 124999.7497 124999.7043
                                           125000.5204
    124999.7303 124999.717 125000.0321
    124999.7173
                  124999.86
                              125000.3973
                                          125000.0256
```

## 5.2. Example 3

Let Figs. 4 and 5 be intelligible maps of a system are given by two experts. The answer matrices corresponding to the intelligible maps are as follows:

$$B^{(1)} = \begin{bmatrix} 0 & 3 & 2 & 0 \\ 4 & 0 & 0 & 1 \\ 2 & 0 & 0 & 3 \\ 0 & 2 & 4 & 0 \end{bmatrix} \text{ and } B^{(2)} = \begin{bmatrix} 0 & 3 & 0 & 1 \\ 4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 1 & 3 & 0 \end{bmatrix}$$

The initial direct relation matrix, which is obtained by averaging the answer matrices, is as following:

$$A = \begin{bmatrix} 0 & 3 & 1 & 0.5 \\ 4 & 0 & 0.5 & 0.5 \\ 1 & 0 & 0 & 3.5 \\ 0 & 1.5 & 3.5 & 0 \end{bmatrix}.$$

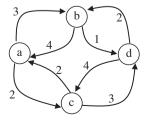


Fig. 4. The intelligible map of the first expert in example 3.

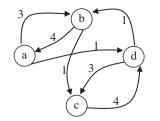


Fig. 5. The intelligible map of the second expert in example 3.

By finding the maximum of the row sums, which is five, and the maximum of the column sums, which is also five, the normalized initial direct-relation matrix *D* is given by dividing the initial direct relation matrix by five, which is

$$D = \begin{bmatrix} 0 & 0.6 & 0.2 & 0.1 \\ 0.8 & 0 & 0.1 & 0.1 \\ 0.2 & 0 & 0 & 0.7 \\ 0 & 0.3 & 0.7 & 0 \end{bmatrix}.$$

$$T = D + D^{1} + D^{2} + \ldots + D^{\infty} = D(I - D)^{-1} = \begin{bmatrix} 4.543478 & 4.565217 & 4.456522 & 4.130435 \\ 5.338164 & 4.507246 & 4.661836 & 4.347826 \\ 4.371981 & 4.057971 & 4.6278091 & 4.782609 \\ 4.661836 & 4.492754 & 5.338164 & 4.652174 \end{bmatrix},$$

which is the solution of the original DEMATEL.

Let us take a look at the solution of our revised method. According to the step 2 of our method,  $s = \max(\max_{1 \le i \le n} \sum_{i=1}^{n} a_{ij}, \varepsilon + \max_{1 \le j \le n} \sum_{i=1}^{n} a_{ij}) = 5.00001$  if  $\varepsilon = 0.00001$ . Hence

<i>X</i> =	0	0.5999988	0.1999996	0.0999998	ľ
	0.7999984	0	0.0999998	0.0999998	
	0.1999996	0	0	0.6999986	
	0	0.2999994	0.699999	0	

and

$$S = X + X^{1} + X^{2} + \ldots + X^{\infty} = X(I - X)^{-1} = \begin{bmatrix} 4.543301676 & 4.56505235 & 4.456344425 & 4.130268239 \\ 5.337975653 & 4.5070711 & 4.66164726 & 4.347649062 \\ 4.371803828 & 4.05780587 & 4.627839374 & 4.782438967 \\ 4.661646052 & 4.49257645 & 5.337971064 & 4.651992691 \end{bmatrix}$$

which is very close to *T*. Therefore, for the cases where the original DEMATEL is feasible, our method provides solutions very close to those of the original DEMATEL.

## 5.3. Example 4

Shieh et al. [26] try to identify the key success factors of hospital service quality. In their study, they first use SERVQUAL model to identify seven major criteria from patients' or their families' viewpoints at Show Chwan Memorial Hospital in Changhua City, Taiwan. When the key criteria were found, the second survey developed for applying decision-making trial and evaluation laboratory (DEMATEL) method was issued to the hospital management to evaluate the importance of the seven criteria. The seven major criteria are well-equipped medical equipment, service personnel with good communication skills, trusted medical staff with professional competence of health care, service personnel with immediate problem-solving abilities, detailed description of the patient's condition by the medical doctor, medical staff with professional abilities, and pharmacist's advices on taking medicine. The average matrix *A* obtained by averaging the questionnaires from the 21 managerial personnel is as follows:

	F 0	1.5789	2.0526	1.7895	2.2632	2	1.3158 ]
	1.5263	0	2.0526	2.3684	2.3684	2.0526	1.6316
	1.9474	1.9474	0	2.0526	2.4211	2.5263	1.9474
A =	1.3684	2.2632	2.1053	0	2.2105	2.2632	1.5789
	1.8421	2	2.2105	1.7895	0	2.2105	1.5789 1.4737
	2.0526	1.8421	2.1579	1.8421	2.2105	0	1.6842
	1.0526	1.7368	1.8421	1.6316	1.5263	1.7368	0 ]

The normalized initial direct-relation matrix D is

	F 0	0.121454	0.157892	0.137654	0.174092	0.153846	0.101215	
	0.117408	0	0.157892	0.182185	0.182185	0.157892	0.125508	
	0.1498	0.1498	0	0.157892	0.186238	0.157892	0.1498	
D =	0.105262	0.174092	0.161946	0	0.170038	0.174092	0.121454	
	0.1417	0.153846	0.170038	0.137654	0	0.170038	0.113362	
	0.157892	0.1417	0.165992	0.1417	0.170038	0	0.129554	
	0.080969	0.1336	0.1417	0.125508	0.117408	0.1336	0	

The total influence matrix T obtained by original DEMATEL is

	0.829827	1.046086	1.147538	1.063649	1.203058	1.138035	0.901401]
	0.998533	1.01196	1.226558	1.171749	1.29103	1.219942	0.984036
	1.042554	1.163119	1.113665	1.174872	1.318062	1.24287	1.021259
T =		1.146203					
	0.988254	1.109906	1.198775	1.103991	1.098902	1.191951	0.945345
		1.116469					
	0.803486	0.941642	1.01149	0.94018	1.030594	0.999579	0.711547

We apply our revised DEMATEL to matrix A with  $\varepsilon$  = 0.00001. Then we obtain

	Γ 0	0.121454	0.157892	0.137654	0.174092	0.153846	0.101215	
	0.117408	0	0.157892	0.182184	0.182184	0.157892	0.125508	
	0.1498	0.1498	0	0.157892	0.186238	0.157892	0.1498	
X =	0.105261	0.174092	0.161946	0	0.170038	0.174092	0.121454	,
	0.1417	0.153846	0.170038	0.137654	0	0.170038	0.113361	
	0.157892	0.1417	0.165992	0.1417	0.170038	0	0.129554	
		0.1336					0	

and

	$S = X(I - X)^{-1}$										
	0.829821	1.046079	1.147531	1.063642	1.20305	1.138027	0.901395				
	0.998527	1.011952	1.22655	1.171742	1.291022	1.219934	0.98403				
		1.163111									
=	0.977454	1.146196	1.214446	1.003527	1.266533	1.216755	0.969261				
		1.109898									
	1.014099	1.116462	1.212803	1.122417	1.261553	1.063444	0.971105				
	0.80348	0.941636	1.011483	0.940174	1.030587	0.999573	0.711542				

It is obvious that the total influence matrix obtained by our revised DEMATEL, *S*, is very close to the total influence matrix obtained by the original DEMATEL, *T*.

5.4. Example 5

Chen and Chen [27] use a novel conjunctive MCDM approach based on DEMATEL as an innovation support system for Taiwanese higher education. The innovation support system consists of seven evaluating dimensions, which are academic research, administrative process, faculty and staff, market development, organizational structure, organizational culture, and leadership style. The DEMATEL is employed to determine the weights of the evaluating dimensions. Sixty-six

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educational experts were asked to specify the relationships between the seven measurement dimensions. The initial direct-relation matrix is obtained by averaging the matrices from the sixty-six experts, which is as follows:

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	[ 0	0.12	1.35	1.62	0.27	0.33	0.03
	1.24	0	2.33	0.57	1.13	0.06	0.71
	3.91	3.76	0	2.97	1.19	0.23	0.04
A =	3.29	0.24	0.26	0	0.3	1.75	1.22
	1.07	2.93	3.35	1.1	0	3.63	1.32
	3.01	1.25	2.63	2.77	1.29	0	1.1
	2.98	3.03	3.42	2.2	3.78	3.89	0.03 0.71 0.04 1.22 1.32 1.1 0

The normalized initial direct-relation matrix D is

	Γ 0	0.006218	0.069948	0.083938	0.01399	0.017098	0.001554	
	0.064249	0	0.120725	0.029534	0.058549	0.003109	0.036788	
				0.153886				
D =							0.063212	
				0.056995				
				0.143523				
	0.154404	0.156995	0.177202	0.11399	0.195855	0.201554	0	

The total influence matrix *T* obtained by original DEMATEL is

	0.049719	0.035013	0.091969	0.112033	0.029483	0.037752	0.01436 0.053546	
	0.137041	0.059668	0.169497	0.086241	0.089115	0.04303	0.053546	
	0.293518	0.240862	0.084998	0.217738	0.099816	0.064436	0.035829	
T =	0.23654	0.059028	0.078883	0.0643	0.052555	0.127855	0.080861	
	0.221279	0.25678	0.289185	0.180271	0.076878	0.249012	0.109629	
	0.282	0.149741	0.223415	0.233091	0.11929	0.069716	0.090271	
	0.362748	0.301651	0.343735	0.273028	0.277176	0.302953	0.065856	

Applying our revised DEMATEL to matrix A with  $\varepsilon$  = 0.00001, we obtain

	Γ 0	0.006218	0.069948	0.083938	0.01399	0.017098	0.001554	
	0.064249	0	0.120725	0.029534	0.058549	0.003109	0.036788	
		0.194819						
X =							0.063212	
	0.05544	0.151813	0.173575	0.056995	0	0.188083	0.068394	
		0.064767						
	0.154404	0.156995	0.177202	0.11399	0.195855	0.201554	0	

The total influence matrix *S* obtained by our revised DEMATEL is

	0.049719	0.035013	0.091969	0.112033	0.029483	0.037752	0.01436	
	0.137041	0.059668	0.169497	0.086241	0.089115	0.04303	0.053546	
	0.293518	0.240862	0.084998	0.217738	0.099816	0.064436	0.035829	
S =	0.23654	0.059028	0.078883	0.0643	0.052555	0.127855	0.080861	
		0.25678						
		0.149741					I	
	0.362748	0.301651	0.343735	0.273028	0.277176	0.302953	0.065856	

Table 1
Summary of the examples.

	DEMATEL	Revised DEMATEL	Result comparison
Example 2	Infeasible	Feasible	Not available
Example 3	Feasible	Feasible	Very close
Example 4	Feasible	Feasible	Very close
Example 5	Feasible	Feasible	The same

Note that the total influence matrix obtained by our revised DEMATEL, S, is the same as the total influence matrix obtained by the original DEMATEL, T.

The results of these four examples are summarized in Table 1. The second column of the Table 1 reports that example 2 is infeasible under DEMATEL and examples 3 to 5 are feasible under DEMATEL. The third column of the Table 1 reports that all examples are feasible under the revised DEMATEL. The fourth column of the Table 1 reports that the results of the revised DEMATEL are either very close or equal to those of DEMATEL except example 2 where DEMATEL is infeasible.

## 6. Discussion

As we show in Theorem 1, that the relation matrix to infinite power might or might not converge to zero matrix if some columns of the matrix have column sums equal to 1 and other column sums are less than one. In what follows, we are going to show the potential risk in the original DEMATEL when the total relation matrix is computed directly by (4).

Consider the following example. Assume the normalized initial-direct relation matrix is given by

	Γ0	0	0.7	0	0.3	ך 0					
D	0	0	0	0.3	0	0.4					
	0.3	0	0	0	0.7	0					
D =	0	0.2	0	0	0	0.5	•				
	0.7	0	0.3	0	0	0					
	0 0.1 0 0.6		0	0 ]							
				0.33	3333	0	0.333333	0	0.333333	0]	
				0		0	0	0	0	0	
Its power of infinity is $D^{\infty} =$			0.333333		0	0.333333	0	0.333333	0		
			0		0	0	0	0	0   .		
			0.333333		0	0.333333	0	0.333333	0		
					0		0	0	0	0	0]

That is, it does not converge to  $[0]_{n \times n}$ . The total relation matrix  $T = \lim_{m \to \infty} (D + D^2 + \ldots + D^m)$  will not converge and therefore does not exist.

However,  $(I - D)^{-1}$  exists and

$$(I-D)^{-1} = \begin{bmatrix} 4.5E+15 & 0 & 4.5E+15 & 0 & 4.5E+15 & 0 \\ 0 & 1.303538 & 0 & 1.005587 & 0 & 1.024209 \\ 4.5E+15 & 0 & 4.5E+15 & 0 & 4.5E+15 & 0 \\ 0 & 0.465549 & 0 & 1.787709 & 0 & 1.080074 \\ 4.5E+15 & 0 & 4.5E+15 & 0 & 4.5E+15 & 0 \\ 0 & 0.409683 & 0 & 1.173184 & 0 & 1.750466 \end{bmatrix}.$$

Hence  $D(I - D)^{-1}$  exists and

	ך4.5 <i>E</i> + 15	0	4.5E + 15	0	4.5E + 15	ך 0	
$D(I-D)^{-1} =$	0	0.303538	0	1.005587	0	1.024209	
	4.5 <i>E</i> + 15	0	4.5E + 15	0	4.5E + 15	0	
	0	0.465549	0	1.787709	0	1.080074	•
	4.5 <i>E</i> + 15	0	4.5E + 15	0	4.5E + 15	0	
	LO	0.409683	0	1.173184	0	0.750466	

In this case,  $\lim(D + D^2 + \ldots + D^m)$  does not exist but  $D(I - D)^{-1}$  does exist. That is  $\lim(D + D^2 + \ldots + D^m) \neq D(I - D)^{-1}$ . In other words, if we compute  $D(I - D)^{-1}$  directly as the total relation matrix, the result might be wrong depending on whether  $D^{\infty}$  converges to zero matrix or not.

## 7. Conclusions

The current paper revises DEMATEL by providing an approach for addressing the infeasibility issue in the original DEM-ATEL method which has been widely used in many applications. We show that the infeasibility might occur in some cases and revise the method so that infeasibility can be avoided. We provide a proof to show that the revised DEMATEL is sound and applicable to all situations. For the cases that are infeasible, our approach can provide a feasible solution; however, for those cases that are feasible for the original method, our method yields a solution which is very close to that of the original

DEMATEL. How closely our method would approach to the original DEMATEL depends on the choice of the small positive constant epsilon. The smaller the epsilon is, the closer the solution is. Our method provides a more rigid approach for those applications to which original DEMATEL is applied.

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