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A closed-form phase-comparison ML DOA estimator for automotive radar with one single snapshot

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Abstract: In automotive radar systems, only a small number of snapshots are available for direction-of-arrival (DOA) estimation in high mobility scenarios. We here propose a closed-form single-snapshot maximum likelihood (ML) DOA estimator based on the phase-comparison technique. The estimator can be effective in a wide field-of-view (FOV) scenario and is robust to gain-mismatch effects among antenna elements. Computer simulations are conducted to confirm the effectiveness of the proposed method.

Keywords: direction-of-arrival (DOA), maximum likelihood (ML), FMCW, antenna array, automotive radar
Classification: Microwave and millimeter wave devices, circuits, and systems

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1 Introduction

Millimeter-wave radars are employed today in many active automotive safety applications for crash mitigation and collision avoidance. One of the key performance limitations of today's radar techniques is the azimuth resolution [1]. Various direction-of-arrival (DOA) estimation methods are available in the literatures [2, 3, 4]. The super-resolution approach has received considerable attention as a possible solution, which utilizes eigen-space decomposition and sub-space projection techniques to deliver high resolution estimation. Its major drawbacks include high computational cost and the requirement for more than one snapshot for constructing the correlation matrix. These pose the disadvantage to practical real-time embedded systems [3]. As an alternative, amplitude-comparison monopulse technique requires lower complexity but its performance is sensitive to inhomogeneous antenna patterns caused by manufacture processes [4].

The maximum likelihood (ML) method is well known to be a good candidate for DOA estimation which can asymptotically achieve the Cramér-Rao bound [5]. However, the ML estimator usually does not have a closed-form solution and thus requires a computationally intensive searching procedure. In [6], authors show that adopting more antenna elements can potentially obtain a solution with lower mean square error (MSE). Besides, many key components and system-level solutions for automotive radar are easily accessible nowadays. In mature modern applications [7, 8], three to four antenna elements are commonly adopted to achieve the accuracy requirement. A phase-comparison ML DOA estimator using a single snapshot was proposed in [9]. Unfortunately, the proposed estimator is only applicable for $N = 2$ receive array elements configuration.

However, a more general solution is not available in the literature. The development of more generalized solutions is crucial in meeting the present and future specifications. In this letter, a low complexity ML DOA estimator based on the phase-comparison technique is proposed to provide a promising solution for practical automotive radar applications. The proposed estimator can effectively overcome the inherent physical limitations and is robust to antenna pattern mismatch.

2 Method

In a frequency modulated continuous wave (FMCW) radar system with multiple targets, the received signal exhibits a spectrum peaks in the frequency domain, known as the beat frequency, which is a function of the relative velocity and distance of the target [5]. By a proper waveform

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design, these targets can be distinguished with high probability according to their different beat frequencies. In this case, DOA information of each detected target can be extracted at the corresponding beat frequency using array processing techniques. Therefore, the requirement where the number of antenna elements must be larger than the number of targets can be relaxed. The DOA estimator considered in this letter involves one transmit antenna and one receive antenna array consisting of N identical elements with a uniform spacing of d and the transmitted waveform is narrowband with a wavelength of λ. All targets are in the far field of the receive array with the azimuth angle denoted by ϕ. In the scenario where the radar system demands high resolution specifications for safety reasons, it is difficult to collect more than one snapshot for angle estimation due to that the velocity resolution is inversely proportional to the observation interval [3, 5]. Therefore, the main challenge to the proposed estimator is to obtain precise angle information for each detected target with a single snapshot. We here develop a generalized closed-form solution for $N \geqq 3$. The overall procedure of the proposed DOA estimator and complexity analysis are described as follows.

First, an $N\times 1$ snapshot vector is collected from all receive antenna outputs at the beat frequency associated with a detected target. For the kth detected target, the received snapshot vector $\mathbf{r}_k \in \mathbb{C}^{N\times 1}$ can be represented as [2]

$$\mathbf{r}_k = \alpha_k \mathbf{a}(\phi_k) g(\phi_k) s_k + \mathbf{n}, \tag{1}$$

where α_k is the complex amplitude of the kth detected target, $\mathbf{a}(\phi_k) \in \mathbb{C}^{N\times 1}$ is the signal steering vector, $g(\phi_k)$ is the gain response of the antenna toward ϕ_k, s_k is FMCW echoed signal from the kth detected target, and $\mathbf{n} \in \mathbb{C}^{N\times 1}$ is spatially white and complex Gaussian noise with zero mean and covariance matrix $\sigma_n^2 \mathbf{I}_N$ at the same beat frequency of the target.

For brevity of notation, the index k is omitted. The optimal angle estimate can be obtained by the ML DOA estimator, which performs an exhaustive search over all possible combinations of α and ϕ within the field-of-view (FOV) via the following criterion:

$$\hat{\phi} = \underset{\{\alpha,\phi\in FOV\}}{\arg\max} \left\{ \frac{1}{\left(2\pi\sigma_n^2\right)^N} \exp\left[-\frac{\left\|\mathbf{r} - \alpha\mathbf{a}(\phi)g(\phi)s\right\|^2}{2\sigma_n^2} \right] \right\}. \tag{2}$$

The computational complexity in Eq. (2) would be prohibitively high for high angle resolution applications. As a remedy, the following equivalent ML problem and closed-form solution are obtained by substituting the optimal α given ϕ in Eq. (2) and then with a lengthy mathematical derivation:

$$\begin{aligned} \hat{\phi} &= \underset{\phi\in FOV}{\arg\max} \left[\left| \sum_{i=1}^{N} r_i^* e^{j\frac{2\pi}{\lambda}(i-1)d\sin\phi} \right|^2 \right] \\ &= -\sin^{-1}\left[\frac{\lambda}{2d\pi} \cdot \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(j-i)\varphi_{i,j}}{\sum_{i=1}^{N-1}\sum_{j=1}^{N-i} j^2} \right], \end{aligned} \tag{3}$$

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where r_i is ith entry of $\mathbf{r}$ and $\varphi_{i,j} = ARG\left(r_i^* \cdot r_j\right)$. The angle estimation provided by the closed-form estimator in Eq. (3) is not always accurate because only the principal values of $\varphi_{i,j}$ can be obtained. To avoid obtaining wrap-around values in evaluating $\varphi_{i,j}$, the following inequality must be satisfied:

$$-\frac{\lambda}{2(N-1)d} < \sin(\theta_{FOV}) < \frac{\lambda}{2(N-1)d}. \tag{4}$$

In practice, in order to meet the safety requirement for automotive radars, the FOV needs to be wide enough to observe all possible traffic activities on the road. On the other hand, in order to have sufficient antenna gains and lower sidelobes, each antenna element is constructed by using an antenna array. Therefore, the size of the antenna spacing cannot be very small due to the area size of the antenna array adopted. When the antenna spacing is large or the FOV is wide, it is difficult to maintain the condition in Eq. (4). Consequently, the angle estimation will likely to be in error. To tackle the above problem, we re-formulate Eq. (3) and propose an alternative solution such that the aforementioned problem can be resolved without sacrificing the estimation performance.

As a remedy, the term $\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(j-i)\varphi_{i,j}$ should be replaced with $\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(j-i)\varphi_{i,j} + 2p\pi$ to solve the ambiguity, where $\{p\}$ is the set containing all possible integer vales. Furthermore, in dealing with the wrap-around phenomenon for each $\varphi_{i,j}$ term, the circumstance only depends on the corresponding antenna distance between the selected signal pair (r_i, r_j). It is evident that the all possible values of $|i-j|$ are from 1 to $(N-1)$. When evaluating $\varphi_{i,j}$ for $|i-j| = q$, the maximal possible wrap-around time is $\left\lfloor \frac{\theta_{FOV}}{\sin^{-1}(\lambda/2qd)} \right\rfloor + 1$, where $\lfloor \bullet \rfloor$ denotes the Gaussian integer operator. Therefore, the number of all possible outcomes of p can be formulated as $\prod_{i=1}^{N-1}\left(\left\lfloor \frac{\theta_{FOV}}{\sin^{-1}(\lambda/2id)} \right\rfloor + 1\right)$. For p being a positive integer number, the $\{p\}$ set can be formulated as $p = \left\{0, \pm 1, \ldots, \pm \prod_{i=1}^{N-1}\left(\left\lfloor \frac{\theta_{FOV}}{\sin^{-1}(\lambda/2id)} \right\rfloor + 1\right)\right\}$ to contain all possible combinations.

By substituting the aforementioned observation into Eq. (3), the alternative ML closed-form can be obtained as follows

$$\begin{aligned} \theta_p &= -\sin^{-1}\left[\frac{\lambda}{2d\pi} \cdot \frac{\sum\limits_{i=1}^{N-1}\sum\limits_{j=i+1}^{N}(j-i)\varphi_{i,j}}{\sum\limits_{i=1}^{N-1}\sum\limits_{j=1}^{N-i} j^2} + 2p\pi\left(\frac{\lambda}{2d\pi\sum\limits_{i=1}^{N-1}\sum\limits_{j=1}^{N-i} j^2}\right)\right] \\ &= \sin^{-1}\left[\sin\left(\hat{\phi}\right) - 2\pi p\left(\frac{\lambda}{2\pi d\sum\limits_{i=1}^{N-1}\sum\limits_{j=1}^{N-i} j^2}\right)\right], \end{aligned} \tag{5}$$

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where $p = \left\{0, \pm 1, \ldots, \pm \prod_{i=1}^{N-1} \left(\left\lfloor \frac{\theta_{FOV}}{\sin^{-1}(\lambda/2id)} \right\rfloor + 1 \right) \right\}$. Finally, all possible angles in $\{\theta_p\}$ are examined, and the angle yielding the maximum value of the objective function in Eq. (6) is the actual ML solution:

$$\hat{\phi} = \arg\max_{\phi \in \{\theta_p\}} \left[\left| \sum_{i=1}^{N} r_i^* e^{j\frac{2\pi}{\lambda}(i-1)d\sin\phi} \right|^2 \right]. \tag{6}$$

By doing so, the correct target angle estimate can be determined without ambiguity.

The computational complexity is a critical factor for real-time applications. For complexity analysis, the computational complexity is measured in terms of the number of floating point operations (flops). All real additions, multiplications, division, triangular functions, and comparison are equally treated as flops. The total flop requirement of the proposed algorithm and exhaustive search can be expressed as $5N \cdot (N-1) + 3|p| - 1$ and $(\lfloor \mathrm{FOV}/\delta\theta \rfloor + 1) \cdot (12N + 3) - 1$, respectively, where $\delta\theta$ is the requested angle resolution and $|p|$ is the size of set p. The above results show that the proposed algorithm can provide significant complexity reduction particularly in high resolution scenario.

3 Result and discussion

In the following, computer simulations are conducted to evaluate the effectiveness of the proposed estimator. For all simulations, the number of receive antennas N is 3, antenna spacing d is 0.6λ, FOV regime is $[-\pi/4\ \pi/4]$, the $\delta\theta$ is 1°, and the antenna pattern as depicted in Fig. 1 is adopted for each array element. A target is measured at different distances corresponding to different SNRs according to the radar equation [5]. All simulation results are obtained by averaging over 10^3 independent trials.

In the above configuration, the closed-form estimator in Eq. (3) cannot preserve a one-to-one mapping, and there are five possible angles, i.e. $|p| = 5$, associated with $p = 0, \pm 1, \pm 2$ to be evaluated. Fig. 2(a) and Fig. 2(b) show

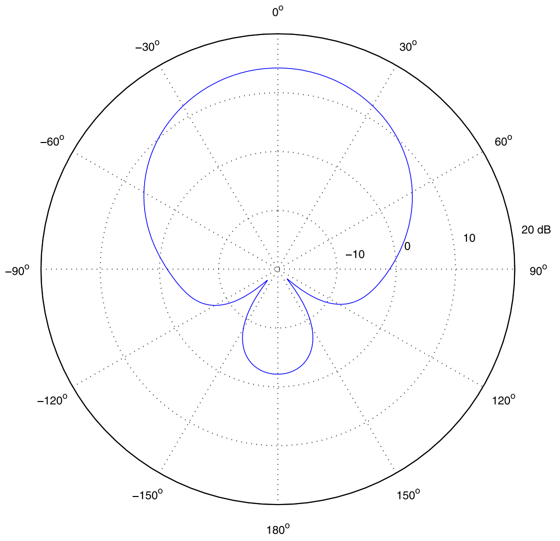

Fig. 1. Radiation pattern of each antenna array element for performance evaluation.

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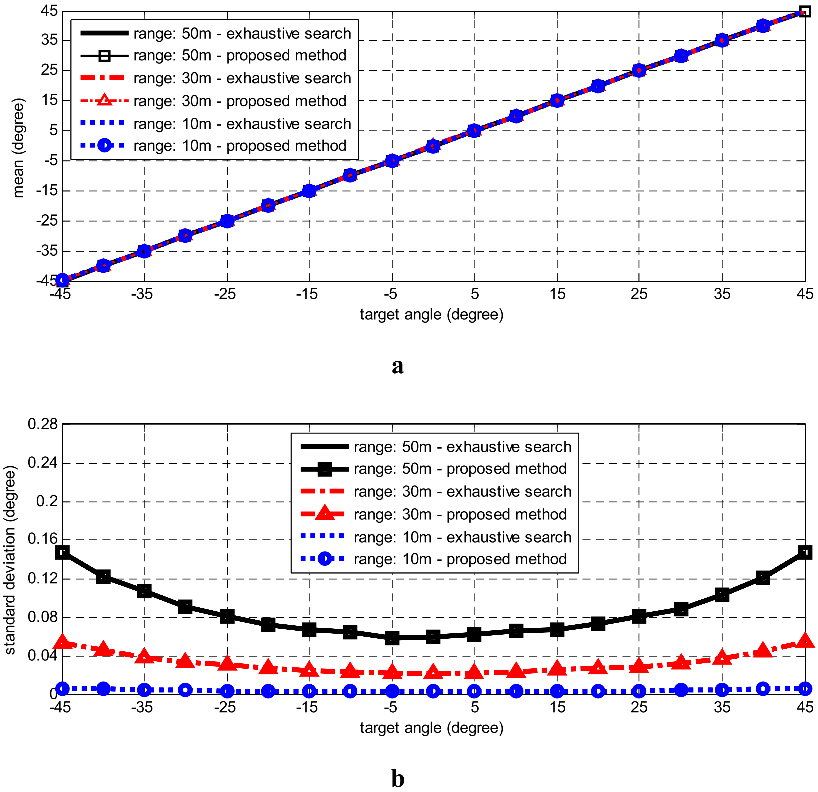

a

b

Fig. 2. Sample mean (Fig. 2a) and standard deviation (Fig. 2b) of exhaustive search based and proposed DOA estimators; the target is measured at a distance of 10 m, 30 m and 50 m, respectively.

the sample mean and standard deviations, respectively, of the DOA estimates obtained by the proposed estimator and equivalent ML estimator in Eq. (3). For the latter, an exhaustive search is performed over the FOV. It is observed that the proposed estimator is unbiased and achieves the ML performance with exhaustive search. In Fig. 2(b), the sample standard deviations increase at both ends because the antenna pattern gain exhibits a slight attenuation near the edge of FOV, which reduces the effective SNR. Furthermore, comparing the computational complexities, the proposed algorithm can reduce 98.75% complexity requirements for each detected target. The results confirm that the proposed algorithm can provide significantly low complexity at identical performance comparing with

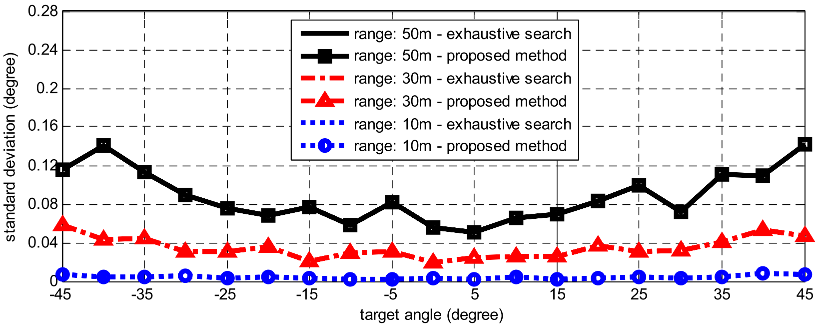

Fig. 3. Sample standard deviation of exhaustive search based and proposed DOA estimators with pattern gain mismatch; the target is measured at a distance of 10 m, 30 m and 50 m, respectively.

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exhaustive search and is applicable to practical implementation.

Next, we evaluate the effect of gain mismatch between antenna elements by adding artificial disturbances to the gain patterns. Each disturbance term is i.i.d. normal distribution with zero mean and a variance of 3 dB. The sample standard deviations shown in Fig. 3 reveal that the estimation performance is not degraded by the gain mismatch, except that local fluctuations occur due to the variation of effective SNR.

Conclusions

In this letter, a closed-form ML angle estimator with N receive antenna elements is derived. To resolve the ambiguity due to a large FOV and antenna spacing, a simple criterion is introduced to determine the correct angle estimator. The corresponding complexity requirement is also evaluated with a closed-form expression. Simulation results show that the incident angle of a detected target can be accurately estimated with a wide FOV and/or large antenna spacing. The results also confirm that the proposed angle estimator can provide a low-complexity ML solution and is robust to gain mismatch among antenna elements as often encountered in practical automotive radar systems.

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