Precoder/two-stage equaliser for block-based single-carrier transmission with insufficient guard interval

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Proposed is a zero-inserting precoder and a two-stage linear equaliser, to shorten the guard interval in block-based single-carrier modulation. The first-stage equaliser consists of a linear single-tapper-subcarrier frequency-domain equaliser. The second-stage equaliser maximises the SINR, in the time-domain, based on the interference-plus-noise estimated from the zero-padded sub-intervals of the single-carrier modulation. This proposed scheme is applicable even without cyclic prefixing.

Review of block-based cyclically-prefixed singe-carrier transmission through time-dispersed channel: The information-bearing symbols $\{u(j), \forall j\}$ are segmented at the transmitter into blocks of N symbols. Represent the kth block as an N-element vector, $\boldsymbol{u}(k) = [u_{-N/2+1}(k), \ldots, u_0(k), \ldots, u_{N/2}(k)]^T$, where $u_n(k) = u(kN + N/2 - 1 + n)$, for $n = -N/2 + 1, \ldots, N/2$. Prefix $\boldsymbol{u}(k)$ with a length-G guard interval, which could be a cyclic prefix (CP), i.e. a replication of the last v entries of $\boldsymbol{u}(k)$. Mathematically, this cyclic-prefixing operation equals the multiplication of $\boldsymbol{u}(k)$ into an $(N+G)\times N$ cyclic-prefix-insertion matrix $\boldsymbol{T}_{cp} = \begin{bmatrix} \mathbf{0}_{G\times(N-G)} & \mathbf{I}_G \\ \mathbf{I}_N & \end{bmatrix}$, to produce the $(N+G)$-element vector, $\tilde{\boldsymbol{u}}(k) = \boldsymbol{T}_{cp}\boldsymbol{u}(k)$. This CP serves to reduce or to eliminate up to G taps of inter-block interference (IBI), caused by a frequency-selective fading-channel. The guard interval need not be a cyclic prefix as above, but could be entirely zero-energy symbols, or some mix of the two.

Consider a frequency-selective but time-invariant channel of order Q, with the discrete-time impulse-response taps of $h(0)$, $h(1)$, …, $h(Q)$. This channel's output is modelled as corrupted by additive noise, symbolised by the $(N+G)$-element noise-vector $\eta(k)$, which is zero-mean, characterised by a prior known temporal correlation matrix of $\boldsymbol{R}_{\boldsymbol{\eta}(k)\boldsymbol{\eta}(k)}$, and is statistically independent from $\boldsymbol{u}(k)$. Hence, the received data have the kth symbol-block equal to $\tilde{\boldsymbol{r}}(k) = \boldsymbol{H}_0 \underbrace{\boldsymbol{T}_{cp}\boldsymbol{u}(k)}_{\tilde{\boldsymbol{u}}(k)} + \boldsymbol{H}_1 \underbrace{\boldsymbol{T}_{cp}\boldsymbol{u}(k-1)}_{\tilde{\boldsymbol{u}}(k-1)} + \boldsymbol{\eta}(k)$, where $\boldsymbol{H}_0 \in \mathbb{C}^{N+G)\times(N+G)}$ represents a lower triangular Toeplitz matrix, with its first column being $[h(0), h(1), \cdots, h(Q), 0, \cdots, 0]^T$; and $\boldsymbol{H}_1 \in \mathbb{C}^{(N+G)\times(N+G)}$ denotes an upper triangular Toeplitz matrix, with its first row as $[0, \cdots, 0, h(Q), \cdots, h(1)]$.

The receiver removes the cyclic prefix, via $\boldsymbol{R}_{cp} = [\mathbf{0}_{N\times G}\ \mathbf{I}_{N\times N}]$, from the received signal to yield the N-element vector

$$\begin{aligned}
\boldsymbol{x}(k) &= \boldsymbol{R}_{cp}\underbrace{[\boldsymbol{H}_0\tilde{\boldsymbol{u}}(k) + \boldsymbol{H}_1\tilde{\boldsymbol{u}}(k-1) + \boldsymbol{\eta}(k)]}_{=\tilde{\boldsymbol{r}}(k)} \\
&= \boldsymbol{R}_{cp}\boldsymbol{H}_0\boldsymbol{T}_{cp}\boldsymbol{u}(k) + \underbrace{\boldsymbol{R}_{cp}\boldsymbol{H}_1\boldsymbol{T}_{cp}}_{=\boldsymbol{H}_{IBI}}\boldsymbol{u}(k-1) + \boldsymbol{R}_{cp}\boldsymbol{\eta}(k) \\
&= \underbrace{(\boldsymbol{R}_{CP}\boldsymbol{H}_0\boldsymbol{T}_{cp} + \boldsymbol{H}_{ISI})}_{=\boldsymbol{C}}\boldsymbol{u}(k) \\
&\quad - \boldsymbol{H}_{ISI}\boldsymbol{u}(k) + \boldsymbol{H}_{IBI}\boldsymbol{u}(k-1) + \boldsymbol{R}_{cp}\boldsymbol{\eta}(k)
\end{aligned} \tag{1}$$

with the $N\times N$ inter-block interference (IBI) matrix $\boldsymbol{H}_{IBI} \triangleq \boldsymbol{R}_{cp}\boldsymbol{H}_1\boldsymbol{T}_{cp}$, the $N\times N$ inter-symbol interference (ISI) matrix $\boldsymbol{H}_{ISI} = \boldsymbol{H}_{IBI}\boldsymbol{P}$, and the permutation matrix $\boldsymbol{P} = \begin{bmatrix} \mathbf{0}_{G\times(N-G)} & \mathbf{I}_G \\ \mathbf{I}_{N-G} & \mathbf{0}_{(N-G)\times G} \end{bmatrix}$. The $N\times N$ matrix $\boldsymbol{C} = \boldsymbol{W}_N^H\boldsymbol{D}\boldsymbol{W}_N$ in (1) is circulant, regardless of the relative magnitudes of G and Q. Moreover, the $N\times N$ matrix $\boldsymbol{D}$ signifies the channel transfer-function matrix, which is diagonal for $G \geq Q$, with its (k, k)th entry equal to the kth DFT coefficient of the channel impulse response $\{h(0), h(1), \cdots, h(Q)\}$ appended by $(N-Q-1)$ zeros, i.e. $[\boldsymbol{D}]_{k,k} = \sum_{q=0}^{Q} h(q)e^{-j(2\pi/N)kq}$, $k = 0, \cdots, N-1$.

Proposed zero-inserting precoder: To suppress the ISI and IBI, but with a length-G insufficient cyclic prefix: [1] proposes inserting $2(Q-G)$ zero-energy symbols to correspond to the $Q-G$ non-zero columns in $\boldsymbol{H}_{IBI}$ plus the $Q-G$ non-zero columns in $\boldsymbol{H}_{ISI}$. The present scheme will not incur this $2(Q-G)$-symbol overhead, but deploys a guard interval (comprising zero-energy symbols, plus an optional cyclic prefix) that may be shorter than the channel impulse response. From the data received during the zero-energy symbol intervals, the proposed scheme estimates the combined effects of the signal-of-interest's self-interference, of any multiple-access-user interference, of any overlaid interference, and of the additive noises. These denigrating effects are then 'subtracted' from the information-bearing parts of the symbol block, via a SINR-maximiser in the receiver. This interference-suppression approach philosophically resembles the null-subcarriers-based methods in [2, 3] for OFDM, though the system architectures and the algorithmic details are very different. The present scheme can operate with any non-zero number of zero-energy symbols, with or without a cyclic prefix.

This zero-inserting precoding can be realised by an $N\times(N-P)$ precoding matrix $\boldsymbol{T}_{zero}$, formed by inserting P number of all-zero rows into an $(N-P)\times(N-P)$ identity matrix. For example, appending all these zeros would require a precoding matrix of $\boldsymbol{T}_{zero} = \begin{bmatrix} \mathbf{I}_{(N-P)\times(N-P)} \\ \mathbf{0}_{P\times(N-P)} \end{bmatrix}$.

Proposed two-stage equaliser: At the receiver, (1) remains valid despite the zero-inserting precoder, but now has $\boldsymbol{u}(k) = \boldsymbol{T}_{zero}\boldsymbol{s}(k)$. The proposed linear equaliser involves a post-FFT linear single-tap-per-subcarrier frequency-domain equaliser (FDE) $\boldsymbol{W}$, followed by a post-IFFT signal-to-interference-and-noise (SINR) maximiser in the time-domain. These are shown in Fig. 1.

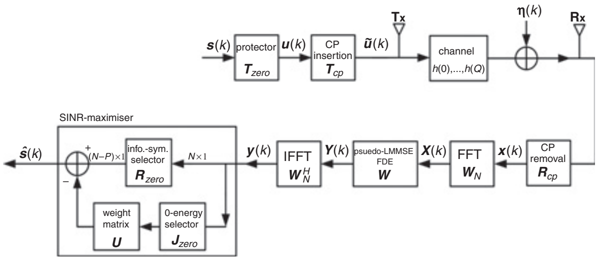

Fig. 1 *Proposed zero-inserting precoder and proposed two-stage equaliser*

The first stage is a single-tap-per-subcarrier frequency-domain linear equaliser (FDE):

$$\boldsymbol{W} = \boldsymbol{D}^H\left(\boldsymbol{D}\boldsymbol{D}^H + \frac{1}{\mathrm{SNR}}\mathbf{I}_N\right)^{-1} \tag{2}$$

where superscript ^{H} denotes complex-conjugate transposition, $\mathrm{SNR} \stackrel{\text{def}}{=} \sigma_s^2/\sigma_n^2$, σ_s^2 refers to the signal power, and σ_n^2 symbolises the noise power. The $N\times N$ diagonal $\boldsymbol{W}$ of (4) reduces the signal-of-interest's energy in the zero-energy symbol-intervals. (This $\boldsymbol{W}$ would constitute a linear minimum-mean-square-error (LMMSE) equaliser, if no interference existed and if $G \geq Q$.) The output of $\boldsymbol{W}$ equals

$$\boldsymbol{y}(k) = \boldsymbol{W}_N^H\boldsymbol{W}\boldsymbol{W}_N\underbrace{\{\boldsymbol{C}\boldsymbol{u}(k) - \boldsymbol{H}_{ISI}\boldsymbol{u}(k) + \boldsymbol{H}_{IBI}\boldsymbol{u}(k-1) + \boldsymbol{n}(k)\}}_{=\boldsymbol{x}(k)} \tag{3}$$

For the second stage:

(a) Form a $P\times N$ 'zero-selection' matrix, to block all information-bearing symbol-intervals (which have non-zero energy at transmission); e.g. $\boldsymbol{J}_{zero} = [\mathbf{0}_{P\times(N-P)}|\mathbf{I}_{P\times P}]$ would be compatible with the earlier defined $\boldsymbol{T}_{zero}$.
(b) Also form a (N − P) × N 'zero-removal' matrix, to remove the precoder-inserted zeros; e.g. $\boldsymbol{R}_{zero} = [\mathbf{I}_{(N-P)\times(N-P)}|\mathbf{0}_{(N-P)\times P}]$ would be compatible with the earlier defined $\boldsymbol{T}_{zero}$ and $\boldsymbol{J}_{zero}$.

Next, form the $(N-P)\times P$ matrix $\boldsymbol{U}$, to minimise the mean-squared error ξ between the signal-output from $\boldsymbol{R}_{zero}$ and $\boldsymbol{J}_{zero}$, i.e.

$$\xi_{\min} = \min_{\boldsymbol{U}} \underbrace{E[\|\boldsymbol{i}(k) - \boldsymbol{U}\boldsymbol{J}_{zero}\boldsymbol{y}(k)\|_2^2]}_{\triangleq\xi} \tag{4}$$

where $\boldsymbol{i}(k) \triangleq \boldsymbol{R}_{zero}\boldsymbol{W}_N^H\boldsymbol{W}\boldsymbol{W}_N[-\boldsymbol{H}_{ISI}\boldsymbol{u}(k) + \boldsymbol{H}_{IBI}(\boldsymbol{u})(k-1) + \boldsymbol{n}(k)]$ represents the interference and noise in the information-bearing symbol durations. The optimisation in (6) can be solved via the principle of orthogonality, i.e. $E[\boldsymbol{U}\boldsymbol{J}_{zero}\boldsymbol{y}(k)(\boldsymbol{i}(k) - \boldsymbol{U}\boldsymbol{J}_{zero}\boldsymbol{y}(k))^H] = \mathbf{0}$, to yield $\boldsymbol{U} = \boldsymbol{R}_{zero}\boldsymbol{R}_{\boldsymbol{i}(k)\boldsymbol{i}(k)}\boldsymbol{J}_{zero}^H[\boldsymbol{J}_{zero}\boldsymbol{R}_{\boldsymbol{y}(k)\boldsymbol{y}(k)}\boldsymbol{J}_{zero}^H]^{-1}$ which may be pre-calculated offline, using the prior knowledge that

$$\begin{aligned}\boldsymbol{R}_{\boldsymbol{i}(k)\boldsymbol{i}(k)} \triangleq \boldsymbol{W}_N^H \boldsymbol{W}\boldsymbol{W}_N\{&\boldsymbol{H}_{ISI}\boldsymbol{R}_{\boldsymbol{u}(k)\boldsymbol{u}(k)}(\boldsymbol{W}_N^H\boldsymbol{W}\boldsymbol{W}_N\boldsymbol{H}_{ISI})^H \\ &+ \boldsymbol{H}_{IBI}\boldsymbol{R}_{\boldsymbol{u}(k-1)\boldsymbol{u}(k-1)}(\boldsymbol{W}_N^H\boldsymbol{W}\boldsymbol{W}_N\boldsymbol{H}_{IBI})^H \\ &+ \boldsymbol{R}_{\boldsymbol{n}(k)\boldsymbol{n}(k)}(\boldsymbol{W}_N^H\boldsymbol{W}\boldsymbol{W}_N)^H\}\end{aligned} \quad (5)$$

Lastly, the $(N-P)\times 1$ transmitted symbol vector $s(k)$ is estimated by the receiver as $\hat{\boldsymbol{s}}(k) = (\boldsymbol{R}_{zero} - \boldsymbol{U}\boldsymbol{J}_{zero})\boldsymbol{y}(k)$.

The real-time computational complexity of this proposed precoder/equaliser scheme is compared in Table 1 against the customary LMMSE-FDE (i.e. (4) alone, without the precoder and without the SINR-maximiser) in terms of N and P. As $\boldsymbol{W}$ and $\boldsymbol{U}$ may be pre-computed offline, while $\boldsymbol{T}_{zero}$, $\boldsymbol{R}_{zero}$ and $\boldsymbol{J}_{zero}$ involve no multiplication nor addition, these do not contribute to the real-time computational load.

Table 1: Proposed scheme's computational complexity against that of the customary LMMSE-FDE

Number	LMMSE-based FDE	Proposed two-stage equaliser
Number of complex-value multiplications	$N\log_2 N + N$	$N\log_2 N + N + (N-P)P$
Number of complex-value additions	$2N\log_2 N$	$2N\log_2 N + (N-P)P$

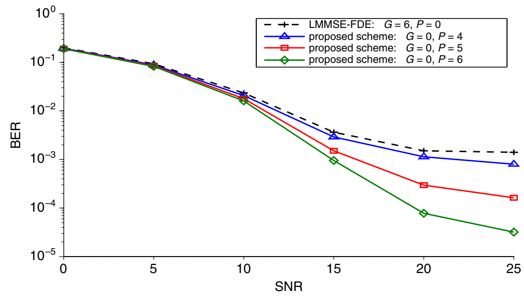

Fig. 2 *BER performance of proposed algorithm with P zero-energy symbols inserted at end of symbol-block against MMSE-FDE with length of G = 6 CP inserted where P ≤ G*

Channel has exponential decay with $T_s/T_{rms} = 1/4$

Monte Carlo simulations: The information-bearing symbols $\{s(k)\}$ are modulated with equi-probable QPSK-symbols. The transmitted signal has $N = 64$. The channel impulse response has $Q+1 = 11$ complex-valued taps, each randomly generated and not cross-correlated among themselves. Each tap's real-value part and imaginary-value part are not cross-correlated. Each tap is Gaussian, zero-mean. The qth tap has an exponentially decaying variance of $\sigma_q^2 = (1 - e^{-T_S/T_{RMS}})e^{-qT_s/T_{RMS}}$, $\forall q = 0, \ldots, Q$, where T_s denotes the sampling period, and T_{RMS} symbolises the root-mean-square delay-spread of the channel. The additive noise is complex-value, temporally uncorrelated, zero-mean, Gaussian, with a noise power of $\sigma^2_{\boldsymbol{\eta}(k)\boldsymbol{\eta}(k)}$.

Consider these two curves in Fig. 2:

(i) the curve at $G = 6$ and $P = 0$ (i.e. an insufficient CP but no zero-inserting precoding), against
(ii) the curve at $G = 0$ and $P = 6$ (i.e. no CP but six zeros inserted by the precoder, as proposed in this Letter).

These two curves both incur the overhead of $P + G = 6$ symbols, but the proposed scheme lowers the BER by $1 - 9\times 10^{-4}/3.8\times 10^{-3} = 76\%$ at SNR = 15 dB, and by $1 - 3.2\times 10^{-5}/1.4\times 10^{-3} = 98\%$ at SNR = 25 dB. Alternatively, if the transmission overhead is lightened to just four inserted zeros (i.e. $(6+64)-(4+64)/6+64 = 2.9\%$ reduction overhead on the data-rate) but no cyclic prefix, then the proposed scheme can still lower the BER by $1 - 2.9\times 10^{-3}/3.8\times 10^{-3} = 24\%$ at SNR = 15 dB, and by $1 - 7.8\times 10^{-4}/1.4\times 10^{-3} = 44\%$ at SNR = 25 dB.

In terms of computational complexity, the proposed scheme would increase the popular LMMSE-FDE method's number of complex-value multiplications by 60% and the number of complex-value additions by 45%, at $N = 64$ and $P = 6$ as for the bottom curve in Fig. 2.

Conclusion: For cyclic-prefixed block-based single-carrier-based communication systems, this proposes a zero-inserting time-domain precoder and an accompanying two-stage equaliser, to allow an insufficient guard interval, in order to reduce the transmission overhead. This proposed precoder is predefined offline and requires no iteration, no feed-forward, and no decision feedback.

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One or more of the Figures in this Letter are available in colour online.

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