# A 81-dB Dynamic Range 16-MHz Bandwidth $\Delta\Sigma$ Modulator Using Background Calibration

Su-Hao Wu, Student Member, IEEE, and Jieh-Tsorng Wu, Senior Member, IEEE

Abstract—A fourth-order discrete-time delta-sigma modulator (DSM) was fabricated using a 65-nm CMOS technology. It combines low-complexity circuits and digital calibrations to achieve high speed and high performance. The DSM is a cascade of two second-order loops. It has a sampling rate of 1.1 GHz and an input bandwidth of 16.67 MHz with an oversampling ratio of 33. It uses high-speed opamps with a dc gain of only 10. Two different types of digital calibrations are used. We first employ the integrator leakage calibration to correct the poles of the integrators. We then apply the noise leakage calibration to minimize the leaking quantization noise from the first loop. The noise leakage calibration also relaxes the component-matching requirements. Both calibrations can operate in the background without interrupting the normal DSM operation. The chip's measured signal-to-noise-and-distortion ratio and dynamic range are 74.32 and 81 dB, respectively. The chip consumes 94 mW from a 1 –V supply. The active area is  $0.33 \times 0.58 \, \text{mm}^2$ .

Index Terms—Analog-digital conversion, analog-to-digital converter (ADC), calibration, delta-sigma modulation, oversampling, switched-capacitor circuits.

#### I. INTRODUCTION

THE delta-sigma modulator (DSM) is an analog-to-digital conversion technique that uses oversampling and noise shaping to enhance the conversion resolution. Compared with Nyquist-rate ADCs that offer similar input signal bandwidth, the DSMs operate at a much higher circuit speed. The performance of a wide-band discrete-time (DT) DSM is usually limited by its internal opamps that realize the integrator function. For an input bandwidth of 20 MHz and an over-sampling ratio (OSR) of 32, the corresponding sampling rate is  $f_s=1.28~\mathrm{GHz}$ , and the required opamp unity-gain frequency is about  $5f_s=6.4~\mathrm{GHz}$ . It is difficult for an opamp with such a speed to have a decent dc gain. Circuit-level gain enhancement techniques, such as multiple-stage configuration [1], correlated double sampling [2], and correlated level shifting [3], all sacrifice the speed.

In a DT DSM, an integrator realized with a low-gain opamp loses some of its ability to suppress in-band quantization noises. The integrator transfer function is also more sensitive to process-voltage-temperature (PVT) variations. The dc gain requirement for the opamps can be relaxed by increasing

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The authors are with the Department of Electronics Engineering and Institute of Electronics, National Chiao Tung University, Hsin-Chu 300, Taiwan (e-mail: jt.wu@g2.nctu.edu.tw).

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the order of the DSM loop, or by employing the multi-stage noise-shaping (MASH) structure [4]–[7]. A higher-order DSM may require more quantization levels from its internal analog-to-digital converter (ADC) and digital-to-analog (DAC) to stabilize the loop [8], yielding complex circuits. On the other hand, the MASH modulators may require calibration to correct the effects of component mismatches and integrator variations [9], [10]. There is an alternative MASH structure that can mitigate the matching requirements [11].

This paper describes a DT DSM that combines low-complexity circuits and digital calibration to achieve wide bandwidth and large dynamic range. It is a MASH modulator consisting of two cascaded second-order loops. The number of the quantization levels of its internal ADCs and DACs is only 4. The internal integrators are realized with high-speed opamps with a dc gain of only 10. Two different types of digital calibrations are applied. We first employ the integrator leakage calibration to correct the poles of the integrators. We then use the noise leakage calibration to minimize the quantization noise from the first loop leaking to the DSM combined output. The noise leakage calibration also relaxes the component matching requirements for the MASH structure. Since each calibration adjusts only one parameter, it is robust. All calibration can proceed in the background without interrupting the normal DSM operation. The calibration processors are simple digital circuits. They do not include any complex filter. The modulator was fabricated using a 65 nm CMOS technology. It has a sampling rate of 1.1 GHz and an input bandwidth of 16.67 MHz with an oversampling ratio (OSR) of 33. The measured signal-to-noise-and-distortion ratio (SNDR) and dynamic range (DR) are 74.32 and 81 dB, respectively. The chip consumes 94 mW from a 1-V supply. The active area is  $0.33 \times 0.58 \text{ mm}^2$ .

The remainder of this paper is organized as follows. Section II describes the DSM architecture and its design parameters. Section III describes the integrator leakage calibration and its design consideration. Section IV describes the noise leakage calibration. Section V describes the design of the crucial circuits. Section VI shows the experimental results. Section VII draws conclusions. In addition, Appendices A and B analyze the transient behavior and the fluctuation of the integrator leakage calibration, respectively.

#### II. DSM ARCHITECTURE

Fig. 1 shows the reported fourth-order DT DSM architecture. In its core is a MASH modulator consisting of two stages of second-order modulation loops. There are four integrators,  $H_1$  to  $H_4$ . Each integrator is modeled with a gain factor  $\alpha$  and a pole  $\beta$ . Table I shows the design values for  $\alpha$  and  $\beta$ . For an ideal integrator with an opamp dc gain  $A_0 = \infty$ ,  $\beta = 1$ .

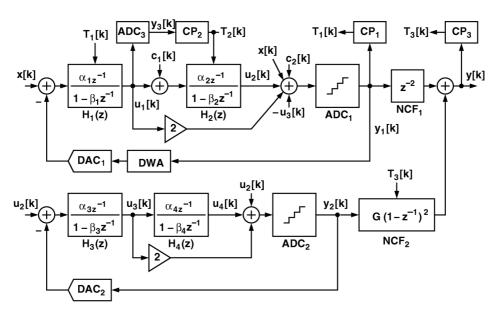


Fig. 1. Fourth-order MASH DSM with digital calibrations.

TABLE I  $\label{eq:table_eq} \text{Integrator Variations due to opamp DC Gain } A_0$ 

	$A_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$
ĺ	$\infty$	0.410	0.455	1	1	1	1	1	1
Ì	10	0.359	0.396	0.833	0.833	0.964	0.957	0.917	0.917

There are two ADCs (digitizers), ADC<sub>1</sub> and ADC<sub>2</sub>. Each one is a 2-bit flash ADC comprising three comparators. There are two corresponding DACs, DAC<sub>1</sub> and DAC<sub>2</sub>. DAC<sub>1</sub> covers an output range of  $\pm V_R$ , while DAC<sub>2</sub> covers an output range of  $\pm (1/2)V_R$ . The adders preceding the two ADCs are passive switched-capacitor circuits. In the first loop, a data-weighted averaging (DWA) dynamic element matching logic [12] is added before DAC<sub>1</sub> to mitigate its conversion errors. A full-scale sine-wave input is defined as  $V_R \sin \omega_i t$ , which has a signal power of  $P_S = (1/2)V_R^2$ . In our design,  $V_R = 1$  V. The use of multibit ADCs leads to smaller quantization errors and stability improvement. The combination of filter feedforward and multibit ADCs relaxes the linearity and voltage swing requirements for the analog circuitry [13].

As shown in Fig. 1, the raw digital outputs from the two modulation loops are  $y_1$  and  $y_2$ , respectively. Due to the use of feedforward, both loops have a signal transfer function (STF) of 1, i.e.,  $\mathrm{STF}_1 = Y_1/X = 1$  and  $\mathrm{STF}_2 = Y_2/U_2 = 1$ . The two digitizers  $\mathrm{ADC}_1$  and  $\mathrm{ADC}_2$  introduce quantization noises, denoted as  $e_1$  and  $e_2$ , respectively. The two noise transfer functions are defined as  $\mathrm{NTF}_1 = Y_1/E_1$  and  $\mathrm{NTF}_2 = Y_2/E_2$ .  $\mathrm{NTF}_1$  is a function of  $H_1$  and  $H_2$ , while  $\mathrm{NTF}_2$  is a function of  $H_3$  and  $H_4$ . Applying the  $\alpha$  and  $\beta$  parameters listed in Table I with an opamp dc gain  $A_0 = \infty$ , the resulting noise transfer functions are  $\mathrm{NTF}_1 = (1-z^{-1})^2/(1-1.18z^{-1}+0.37z^{-2})$  and  $\mathrm{NTF}_2 = (1-z^{-1})^2$ . Two digital noise-cancellation filters  $\mathrm{NCF}_1$  and  $\mathrm{NCF}_2$  combine the two outputs  $y_1$  and  $y_2$  to generate the final DSM output y, which can be expressed as

$$Y = X \times \text{NCF}_1 + E_1 \times \text{NLF} + E_2 \times \text{NTF}_2 \cdot \text{NCF}_2 \quad (1)$$

where the noise leakage transfer function NLF is defined as

$$NLF = \frac{Y}{E_1} = NTF_1 \times (NCF_1 - H_1 \cdot H_2 \cdot NCF_2).$$
 (2)

If we choose the digital filters  $NCF_1 = z^{-2}$  and  $NCF_2 = G(1-z^{-1})^2$  with  $G = 1/(\alpha_1\alpha_2) \approx 5.36$ , then, in the DSM combined output y,  $e_1$  is completely eliminated, and  $e_2$  is shaped by a fourth-order function  $(1-z^{-1})^4$ .

In the wide-bandwidth applications, the DSM uses high-speed opamps to implement the integrators. In our design, the opamps in the integrator configuration have a unity frequency of 6 GHz, but have a dc gain  $A_0$  of only 10. Table I shows the effect of  $A_0$  on the integrators. Their gain factors  $\alpha$  change and their poles  $\beta$  become less than 1. When placing these integrators in the DSM, the corresponding NTF exhibits a degraded capability of suppressing quantization noise in the signal band. Besides the  $A_0$  effect on  $\alpha$  and  $\beta$ , the capacitor mismatch in an integrator also causes a change in  $\alpha$ . Both the  $\alpha$  and the  $\beta$  variations yield NLF  $\neq$  0. This phenomenon is called noise leakage, when a portion of  $e_1$  leaks out to the DSM output y.

Fig. 2 shows the effect of  $\beta$  on the noises  $e_1$  and  $e_2$  appearing in the output y. It plots the ratio of noise power to the signal power of a full-scale sine-wave input,  $P_S = 0.5 V_R^2$ . The noise power includes only the frequency components within the signal band. An OSR of 33 is assumed.  $P_{e1}$  is the noise power of  $e_1$  in y and  $P_{e2}$  is the noise power of  $e_2$  in y. It is assumed that all integrators have the same  $\beta$ . In our design,  $P_{e2}$  is a weak function of  $\beta$ . It can be neglected if the expected signal-to-noise ratio (SNR) of the entire DSM is 80 dB. On the other hand,  $P_{e1}$  is a strong function of  $\beta$ . It requires  $|1 - \beta| < 1.4 \times 10^{-3}$ , so that  $P_{e1}$  is 85 dB below  $P_S$ .

From (1) and (2), and since  $NCF_1 = z^{-2}$  is a simple delay, the digital filter  $NCF_2$  must match the analog integrators  $H_1$  and  $H_2$  to reduce  $P_{e1}$ . The filter  $NCF_2$  can become adaptive to accommodate the variations in  $H_1$  and  $H_2$  [14]. However, if the  $\beta$  parameters in  $H_1$  and  $H_2$  are away from 1, the calibration

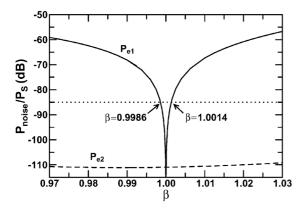


Fig. 2. Effect of integrator pole  $\beta$  on the noises  $e_1$  and  $e_2$  in the DSM output.

for  $\operatorname{NCF}_2$  are complex. In our design, we first calibrate integrators  $H_1$  and  $H_2$  to simplify the requirement for  $\operatorname{NCF}_2$ . We then calibrate  $\operatorname{NCF}_2$  to minimize the NLF of (2). We apply the integrator leakage calibration described in Section III to integrators  $H_1$  and  $H_2$  to recover their capability of noise suppression and make their  $\beta$  approximate 1. We simplify the second noise-cancellation filter as  $\operatorname{NCF}_2 = G(1-z^{-1})^2$ , which has only one adaptive parameter G. We then apply the noise leakage calibration described in Section IV to find G. All calibrations are operated in the background without interrupting the normal DSM function.

The  $\mathrm{ADC}_1$  quantization noise  $e_1$  may contain harmonic tones or idle tones. These tones may show up in the signal band of the DSM output y. Furthermore, these tones may correlate with the calibration signals introduced by the aforementioned calibrations, corrupting the calibration process. Therefore, as shown in Fig. 1, a dithering signal  $-u_3$  is added to the input of  $\mathrm{ADC}_1$  to randomize  $e_1$ . This dithering signal is taken from the output of the integrator  $H_3$ , which is the quantization noise  $e_2$  with first-order noise shaping. The dithering signal is not included in the following analyses since its effect is minuscule.

## III. INTEGRATOR LEAKAGE CALIBRATION

The reported DSM includes four switched-capacitor (SC) integrators. Each SC integrator contains an opamp. Neglecting its settling behavior, the integrator transfer function is  $H(z)=\alpha z^{-1}/(1-\beta z^{-1})$ . If the dc gain of the opamp is finite, then  $\beta<1$ , and the integrator becomes lossy. If the input of this integrator is 0, then its output can then be expressed as  $V_o[k+1]=\beta V_o[k]$ . Its output loses an amount of  $(1-\beta)V_o$  for every clock cycle. The issue is known as integrator leakage.

Fig. 3 shows the integrator with leakage compensation. It is driven by two nonoverlapping clocks  $\phi_1$  and  $\phi_2$ . The switches labeled with 1 are turned on when  $\phi_1=1$ . The switches labeled with 2 are turned on when  $\phi_2=1$ . The circuit is a conventional noninverting integrator with an additional  $C_f$  positive feedback for leakage compensation [15]. The capacitor  $C_f$  puts back  $C_f V_o$  amount of charge into  $C_i$  for every clock cycle. The  $\beta$  of the integrator becomes

$$\beta = \frac{1 + \frac{C_f}{C_i} + \frac{1}{A_0} \left( 1 + \frac{C_p}{C_i} \right)}{1 + \frac{1}{A_0} \left( 1 + \frac{C_s + C_f + C_p}{C_i} \right)} \tag{3}$$

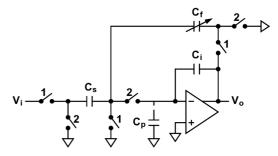


Fig. 3. Integrator with leakage compensation.

where  $A_0$  is the opamp dc gain, and  $C_p$  is the parasitic capacitance at the opamp input. To obtain a lossless integrator, we want  $C_f = C_s/(A_0-1)$  so that  $\beta = 1$ . Comparing with  $C_s$ ,  $C_f$  is relatively small. The capacitor  $C_f$  itself and its associated switches add minuscule loading and noise to the integrator. Since the optimal value for  $C_f$  depends on voltage gain  $A_0$ , which is sensitive to PVT variations. The capacitor  $C_f$  is automatically adjusted by the calibration described below.

As shown in Fig. 1, the integrator leakage calibration is applied to the integrators  $H_1$  and  $H_2$  of the first modulation loop. Consider the calibration of the first integrator  $H_1$ . The  $C_f$  capacitor in  $H_1$  is controlled by a digital signal  $T_1$  such that

$$C_{f1} = C_{f1,0} + \Delta C_{f1} \times T_1 \tag{4}$$

where  $T_1$  is an integer,  $\Delta C_{f1}$  is the  $C_f$  digital control step size and  $C_{f1,0}$  is the  $C_f$  capacitance when  $T_1=0$ . From (3), the corresponding  $\beta_1$  of  $H_1$  is approximated by

$$\beta_1 = \beta_{1,0} + \Delta \beta_1 \times T_1 \tag{5}$$

where  $\Delta\beta_1 \approx \Delta C_{f1}/C_{i1}$ . The control signal  $T_1$  is generated from a calibration processor,  $\operatorname{CP}_1$ . To calibrate  $H_1$ , a calibration signal  $c_1$  is added to the input of the second integrator  $H_2$ .  $\operatorname{CP}_1$  receives the  $\operatorname{ADC}_1$  output  $y_1$ , and detects the  $\beta_1$  of  $H_1$  from the  $c_1$ -related signal embedded in  $y_1$ . It then adjusts  $T_1$  to make  $\beta_1$  approximate 1.

Fig. 4 shows the  $\operatorname{CP}_1$  block diagram and its input components. At the  $\operatorname{CP}_1$  input, the digital stream  $y_1$  is a summation of (1) the input x, (2) the  $\operatorname{ADC}_1$  quantization noise  $e_1$  shaped by the noise transfer function  $\operatorname{NTF}_1$ , and (3) the calibration signal  $c_1$  shaped by the calibration-signal transfer function  $\operatorname{CTF}_1$ , which is defined as  $\operatorname{CTF}_1 = Y_1/C_1$ . The calibration signal  $c_1$  is a square wave with  $f_{c1}$  frequency,  $V_{c1}$  amplitude, and 50% duty cycle. The square wave  $c_1$  excites  $\operatorname{CTF}_1$ , yielding  $d_1$ . Thus, embedded in  $y_1, d_1$  is the step response of  $\operatorname{CTF}_1$  triggered by  $c_1$ . This step response settles toward a final value of

$$V_{cf1} \approx V_{c1} \times \frac{1 - \beta_1}{\alpha_1}.$$
(6)

 $V_{cf1}$  shows the same polarity as  $1 - \beta_1$ . Thus,  $CP_1$  can determine if  $\beta_1$  is above or below 1 by detecting the polarity of  $V_{cf1}$ .

As shown in Fig. 4,  $CP_1$  extracts the  $V_{cf1}$  information from  $y_1$  by correlating  $y_1$  with a triple-valued sequence  $g_1 \in \{-1,0,+1\}$ . This  $g_1$  waveform has the same polarity as  $c_1$ , but its value is set to 0 during the initial transition phase of  $d_1$ . The resulting signal sequence r is accumulated on an

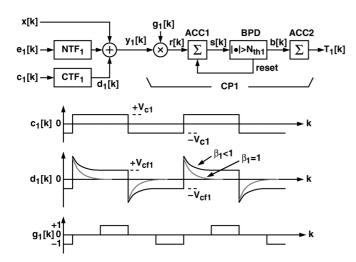


Fig. 4. Calibration of integrator  $H_1$ 

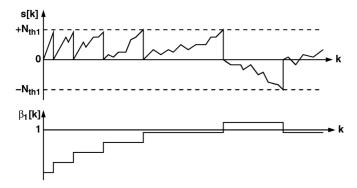


Fig. 5. AAR operation.

accumulator ACC1 followed by a binary peak detector (BPD). Together they perform the accumulation-and-reset (AAR) operation [16]–[18] to guess the polarity of  $V_{cf1}$  while removing the perturbations caused by x and  $e_1$ . The AAR operates as follows. The accumulator ACC1 accumulates the r sequence. Its output s is monitored by the BPD with a threshold  $N_{\text{th}1} > 0$ . Whenever s reaches either  $+N_{\rm th1}$  or  $-N_{\rm th1}$ , the BPD issues an output b = +1 or b = -1 for one clock cycle respectively and reset accumulator output s to 0. The BPD output b remains at 0 when  $-N_{\rm th1}$  < s <  $+N_{\rm th1}$ . The BPD output b is an estimate of the polarities of  $V_{cf1}$  and  $1 - \beta_1$ . CP<sub>1</sub> uses it to increase or decrease the control signal  $T_1$ . Thus, following b is another accumulator, ACC2, that accumulates the b sequence. Its output  $T_1$  controls the capacitor  $C_{f1}$  of the integrator  $H_1$ , thus adjusts its  $\beta_1$ . Fig. 5 illustrates the time-domain waveform of the ACC1 output s, and the waveform of the resulting  $\beta_1$ . When  $\beta_1$  approaches 1, both  $|1 - \beta_1|$  and  $V_{cf1}$  become smaller, and it takes a longer time to activate the BPD.

The above calibration scheme involves signal correlation and AAR operation. For this calibration to be effective, it requires that the calibration signal  $c_1$  has no correlation with other signals in  $y_1$ , including the input x and the quantization noise  $e_1$ . Since  $e_1$  is out of the signal band of x, there is no correlation between  $e_1$  and  $e_1$ . Assume  $e_1$  is a white noise. There are frequency components in  $e_1$  that can have correlation with  $e_1$ . However, this correlation is weak since those frequency components have

randomly varying phases. The effect of this correlation can be overcome by choosing a large BPD threshold  $N_{\rm th1}$ .

This calibration scheme has five design parameters, including the  $c_1$  amplitude,  $V_{c1}$ , the  $c_1$  frequency,  $f_{c1}$ , the  $g_1$  duty ratio,  $D_g$ , the BPD threshold,  $N_{\rm th1}$ , and the  $T_1$  control step size,  $\Delta C_{f1}$ . Referring to Fig. 4, the duty ratio  $D_g$  is defined as the ratio of the time for  $g_1=+1$  to the time for  $c_1=+V_{c1}$ . The duty ratio for  $g_1=-1$  and  $c_1=-V_{c1}$  is assumed to be the same as  $D_g$ .

As shown in Fig. 4, the calibration square wave  $c_1$  triggers a step response  $d_1$ . Let  $c_1$  have a frequency of  $f_{c1}$ , a corresponding period of  $T_{c1}=1/f_{c1}$ , and a duty cycle of 50%. We want  $f_{c1}$  to be larger than the signal bandwidth so that it can be removed by the decimation filter following the DSM. We also want  $T_{c1}/2$  to be longer than the time required for  $d_1$  to settle so that its final value  $V_{cf1}$  can be extracted by correlating  $d_1$  with  $g_1$ . In our design,  $T_{c1}=64T_s$  and  $D_g=1/2$ , so that, in each  $d_1$  transient,  $d_1$  has a period of 16 clock cycles to settle before  $g_1$  is activated for 16 clock cycles. The frequency of  $c_1$  is  $f_{c1}=f_s/64$ . As long as OSR >32, the frequency components of  $d_1$  is outside the signal band.

The injection of  $c_1$  increases the signal ranges of the integrators' outputs,  $u_1$  and  $u_2$ . A larger  $u_1$  and larger  $u_2$  raise the nonlinearity effect of the integrators. A large  $u_2$  may even overload the second modulation loop, yielding large  $e_2$ . Thus, an increase in the  $c_1$  amplitude,  $V_{c1}$ , degrades the SNDR of the DSM. On the other hand, from (6), if  $V_{c1}$  is too small, the corresponding  $V_{cf1}$  is too small to ensure a robust calibration. We choose  $V_{c1}$  by using simulations. In the simulations, all integrators are assumed to have a pole at  $\beta = 1$ . Without the injection of  $c_1$ , the peak SNDR of the entire DSM is 88.5 dB occurring at an input level of -0.25 dBFS. The peak SNDR is degraded by more than 6 dB if  $V_{c1} > 0.2V_R$ . For our design, we choose  $V_{c1} = 0.08V_R$ . The resulting peak SNDR is maintained at 88 dB. Without  $c_1$ , the signal standard deviation for  $u_1$  and  $u_2$  are  $\sigma(u_1) = 0.085 V_R$  and  $\sigma(u_2) = 0.075 V_R$ , respectively. When  $c_1$  with  $V_{c1} = 0.08V_R$  is injected,  $\sigma(u_1)$  and  $\sigma(u_2)$  become  $0.115V_R$  and  $0.160V_R$ , respectively.

Fig. 5 illustrates the transient response of  $\beta_1$  during the calibration. As analyzed in Appendix A, this averaged transient behavior can be modeled as a first-order linear system. The transient response of  $\beta_1$  can be expressed as

$$\beta_1[k] = 1 - (1 - \beta_1[0]) \times e^{-k/\tau_1} \tag{7}$$

where the time constant  $\tau_1$  is

$$\tau_1 = \frac{N_{\text{th1}}}{D_{g1}} \cdot \frac{V_R}{V_{c1}} \cdot \frac{\alpha_1}{\Delta \beta_1}.$$
 (8)

It is assumed the digitizer  $ADC_1$  has an analog-to-digital conversion gain of  $1/V_R$ . From (8), a smaller  $N_{\rm th1}$  and a larger  $\Delta\beta_1$  lead to smaller  $\tau_1$ , yielding a faster calibration speed.

Fig. 5 shows that, as the calibration process converges, the behavior of  $\beta_1$  becomes a discrete random fluctuation around 1. Referring to Fig. 4, both the input x and the quantization noise  $e_1$  induce this fluctuation. Their effects are diminished by the AAR operation. A larger  $N_{\rm th1}$  and a smaller  $\Delta\beta_1$  lead to a smaller fluctuation in  $\beta_1$ , yielding the better SNDR performance for the DSM. As  $N_{\rm th1}$  increases, the standard deviation of  $\beta_1$ 

fluctuation,  $\sigma(\beta_1)$ , converges to an averaged value that can be expressed as [17]

$$\sigma(\beta_1) = \frac{\Delta \beta_1}{\sqrt{6}}.\tag{9}$$

Appendix B contains a simplified derivation of the above equation. As shown in Fig. 2, the deviation of  $\beta$  from 1 increase the noise power  $P_{e1}$ . we want  $3\sigma(\beta) < 1.4 \times 10^{-3}$  so that  $P_{e1}$  is 85 dB below  $P_S$ . In this design, we choose  $\Delta\beta_1 = 1.126 \times 10^{-3}$  and  $N_{th1} = 32$ , yielding a time constant  $\tau_1 = 2.481 \times 10^5$ . If the clock rate  $f_s = 1/T_s = 1$  GHz, the physical time constant is  $\tau \times T_s = 248.1~\mu s$ .

Referring to Fig. 1, to calibrate the second integrator  $H_2$ , a calibration square wave  $c_2$  is added to the input of  $ADC_1$ . An additional digitizer  $ADC_3$  is added to convert  $u_1$ , the output of the first integrator  $H_1$ , into a digital stream  $y_3$ . ADC3 is also a flash ADC, comprising three comparators with thresholds at  $\{0, \pm (1/9)V_R\}$ . The calibration processor  $CP_2$  receives  $y_3$  and generates  $T_2$  to adjust the  $\beta_2$  of  $H_2$ . The control signal  $T_2$  adjusts  $\beta_2$  by controlling the  $C_f$  capacitor in  $H_2$ . The control mechanism is similar to (4) and (5).

The ADC<sub>3</sub> output  $y_3$  comprises: 1) the ADC<sub>3</sub> quantization noise  $e_3$ ; 2) the ADC<sub>1</sub> quantization noise  $e_1$  shaped by CTF<sub>2</sub>; and 3) the calibration signal  $c_2$  shaped by CTF<sub>2</sub>, where CTF<sub>2</sub> =  $U_1/C_2$  is the transfer function from  $c_2$  to  $u_1$ . The calibration signal  $c_2$  is a square wave with  $f_{c2}$  frequency,  $V_{c2}$  amplitude, and 50% duty cycle. Similar to the  $c_1$ -to- $d_1$  response shown in Fig. 4,  $c_2$  triggers the step response of CTF<sub>2</sub>. This step response settles toward a final value of

$$V_{cf2} \approx V_{c2} \times -\frac{1-\beta_2}{\alpha_2}.\tag{10}$$

This  $V_{cf2}$  is extracted by  $\mathrm{CP}_2$  to detect the polarity  $1-\beta_2$ . The operation of  $\mathrm{CP}_2$  is identical to that of  $\mathrm{CP}_1$ . It correlates  $y_3$  with a triple-valued sequence  $g_2 \in \{-1,0,+1\}$ , which has a duty ratio of  $D_{g2}$ . The AAR eliminates the perturbation caused by  $e_1$  and  $e_3$ . The AAR has a BPD threshold of  $N_{th2}$ . Comparing to the  $V_{cf1}$  of (6),  $V_{cf2}$  has an opposite polarity. Thus, comparing with the  $g_1$  of Fig. 4, the polarity of  $g_2$  is inverted. For our design,  $c_2$  has a period of  $T_{c2}=64T_s$  and an amplitude of  $V_{c2}=0.11V_R$ . The duty ratio of  $g_2$  is  $D_{g2}=1/2$ . The BPD threshold for the AAR is  $N_{th2}=32$ . The  $\beta_2$  control step size is  $\Delta\beta_2=2.24\times10^{-3}$ . The above design parameters result in a calibration time constant of  $\tau_2=1.013\times10^5$  or  $\tau_2\times T_s=101.3~\mu\mathrm{s}$  with a 1-GHz clock. When the calibration signal  $c_2$  is injected into the DSM, the signal standard deviations for  $u_1$  and  $u_2$  are  $\sigma(u_1)=0.095V_R$  and  $\sigma(u_2)=0.120V_R$ , respectively.

The  $H_1$  calibration and the  $H_2$  calibration are executed sequentially. They do not interfere with each other. The operations are robust. The calibration signals  $c_1$  and  $c_2$  are easy to generate. The calibration processors  $\mathrm{CP}_1$  and  $\mathrm{CP}_2$  can be realized with simple digital circuits. They use the masking signals  $g_1$  and  $g_2$  to extract the calibration data. Complex filter is not needed.

## IV. NOISE LEAKAGE CALIBRATION

From (1) and (2), the ADC<sub>1</sub> quantization noise  $e_1$  can leak into the DSM output y if the noise leakage transfer function NLF  $\neq 0$ . After the integrator leakage calibration described in Section III, the  $\beta_1$  of  $H_2$  and the  $\beta_2$  of  $H_2$  are close to 1.

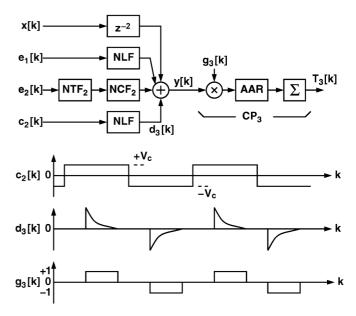


Fig. 6. Noise leakage calibration and calibration processor CP<sub>3</sub>.

However, the gain factors  $\alpha_1$  and  $\alpha_2$  are still subjected to PVT variations. With  $\beta_1 \approx 1$  and  $\beta_2 \approx 1$ , the second noise-cancellation filter can be simplified as  $\mathrm{NCF}_2 = G(1-z^{-1})^2$ , where G is a gain factor. Since  $\mathrm{NCF}_1 = z^{-2}$ , the noise leakage function for  $e_1$  can be approximated by

$$NLF \approx NTF_1 \times z^{-2} (1 - \alpha_1 \alpha_2 \cdot G). \tag{11}$$

We want  $G \approx 1/(\alpha_1 \alpha_2)$  to minimize NLF. As shown in Fig. 1, the gain factor G is adjusted by a digital control  $T_3$ , such that

$$G = G_0 + \Delta G \times T_3 \tag{12}$$

where  $T_3$  is an integer,  $\Delta G$  is the control step size, and  $G_0$  is the value of G when  $T_3 = 0$ . The control  $T_3$  is generated from the calibration processor  $\mathrm{CP}_3$ .

During the integrator leakage calibration of  $H_2$ , a calibration square wave  $c_2$  is added to the input of  $ADC_1$ . Similar to the  $ADC_1$  quantization noise  $e_1$ ,  $c_2$  passes through NLF and appears in the DSM combined output y. Thus,  $CP_3$  can observe y, extract the  $c_2$ -related signal in y, and then adjust G through  $T_3$  to make  $c_2$  disappear from y.

Fig. 6 shows the CP<sub>3</sub> block diagram and its input components. The DSM output y contains: 1) the input x delayed by the filter  $\mathrm{NCF}_1=z^{-2};$  2) the quantization noise  $e_1$  shaped by NLF; 3) the quantization noise  $e_2$  shaped by NTF<sub>2</sub> × NCF<sub>2</sub>; 4) the calibration signal  $c_2$  shaped by NLF. The calibration square wave  $c_2$  excites the NLF filter, yielding  $d_3$ . Thus, embedded in  $y, d_3$  is the step response of NLF triggered by  $c_2$ . As expressed in (11), the NLF filter is a NTF<sub>1</sub> filter with a time delay of two clock cycles and a gain factor of  $1 - \alpha_1 \alpha_2 G$ . The NTF<sub>1</sub> filter is a high-pass filter with two zeros close to dc. Thus, the time-domain step response of NLF is a spike and settles toward zero.  ${\rm CP}_3$  detects the energy of those spikes and then adjusts G to eliminate the spikes. The operation of  $CP_3$  is similar to those of  $CP_1$  and  $CP_2$ . It correlates y with a triple-valued sequence  $g_3 \in \{-1, 0, +1\}$  with a duty cycle of  $D_{q3}$ . The sequence  $g_3$  is aligned with the spikes in  $d_3$ . Its active region overlaps the major

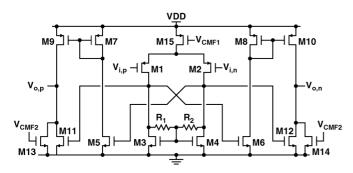


Fig. 7. Operational amplifier schematic

area of the spikes. The AAR in CP3 eliminates the perturbation caused by  $e_1$ ,  $e_2$ , and x. The AAR has a BPD threshold of  $N_{\rm th3}$ . The output  $T_3$  adjusts G as expressed in (12). For our design, the duty ratio of  $g_3$  is  $D_{g3}=0.25$ . The BPD threshold for the AAR is  $N_{\rm th3}=96$ . The G control step size is  $\Delta G=0.25$ . The above design parameters result in a calibration time constant of  $\tau_3=6.514\times 10^5$  or  $\tau_3\times T_s=651.4~\mu{\rm s}$  with a 1-GHz clock. This time constant is larger than the time constants for the integrator leakage calibration,  $\tau_1$  and  $\tau_2$ .

During the power-on phase,  $CP_3$  is reset and  $T_3=0$  and  $G=G_0$ . According to (11), when the calibration converges,  $G\approx 1/(\alpha_1\alpha_2)$ . To reduce the initial calibration convergence time, we choose a  $G_0$  that is close to  $1/(\alpha_1\alpha_2)$ , which can be calculated as

$$G_0 = \frac{C_{i1}}{C_{s1}} \frac{C_{i2}}{C_{s2}} \left( 1 + \frac{C_{f1}}{C_{s1} + C_{f1}} + \frac{C_{f1}}{C_{i1}} \right) \left( 1 + \frac{C_{f2}}{C_{s2} + C_{f2}} + \frac{C_{f2}}{C_{i2}} \right)$$
(13)

where  $C_{i1}$ ,  $C_{s1}$ , and  $C_{f1}$  are the capacitors in the integrator  $H_1$ , and  $C_{i2}$ ,  $C_{s2}$ , and  $C_{f2}$  are the capacitors in the integrator  $H_2$ . Capacitors  $C_{f1}$  and  $C_{f2}$  are functions of control signals  $T_1$  and  $T_2$  respectively. The power-on sequence for calibration is described as follows. The DSM first executes the  $H_1$  calibration for  $4\tau_1T_s\approx 1$  ms, and then the  $H_2$  calibration for  $4\tau_2T_s\approx 0.4$  ms. Afterward, the DSM uses the values of  $T_1$  and  $T_2$  from the above calibrations to estimate  $C_{f1}$  and  $C_{f2}$ , and applies (13) to calculate  $G_0$ . The DSM then activates CP3 every time when the  $H_2$  calibration is in progress.

### V. CIRCUIT DESIGN

Here, we describe the circuit design to implement the DSM of Fig. 1. The DSM is to be fabricated using a 65-nm CMOS technology. The supply voltage is 1 V. The DSM is expected to achieve a sampling rate higher than 1 GS/s and a DR larger than 80 dB

Fig. 7 shows the schematic of the opamps used in the integrators. It is a two-stage class-AB amplifier without frequency compensation [19]. All MOSFETs are sized with the minimum channel length of 60 nm. The amplifier has a dc voltage gain of 10. Its dominant poles are located at the output nodes  $V_{o,p}$  and  $V_{o,n}$ . When configured as an integrator, the opamp achieves an unity-gain frequency of 6 GHz and a phase margin of 63 degrees. The voltages  $V_{\rm CMF1}$  and  $V_{\rm CMF2}$  are generated by two separate continuous-time common-mode feedback (CMFB) circuits. Each CMFB contains additional voltage amplification to increase the loop gain and improve the common-mode rejection

against power-line fluctuation. The push-pull output stage also provides additional common-mode rejection.

The opamp is used to realize the fully differential version of the integrator shown in Fig. 3. The input common-mode voltage of the opamp is set to  $V_{DD}/4$ . The analog switches connected to the opamp's inputs are nMOSFETs with boosting gate control [19]. Consider the integrator  $H_1$ . Its ideal value of the gain factor  $\alpha_1$  is determined by the capacitor ratio  $C_{s1}/C_{i1}$ .

In our design, the thermal noise from the first integrator  $H_1$  is the dominant source of thermal noise. Its total noise power showing up in the signal band of the DSM output y is about  $(14/3)(kT/C_{s1})$  [8]. We choose  $C_{s1}=1.9~\mathrm{pF}$  so that the power of this thermal noise in the signal band,  $P_{\theta}$ , is 90 dB below the power of a full-scale sine-wave input,  $P_S$ . By using the periodic noise analysis of the circuit simulator, we estimate that the total  $P_{\theta}$  of the entire DSM is 86 dB below  $P_S$ .

The control signal  $T_1$  adjusts the capacitor  $C_{f1}$  in the integrator  $H_1$  as described by (4), thus varying  $\beta_1$  as described by (5). The capacitor step size  $\Delta C_{f1}$  determines the step size  $\Delta \beta_1$ . From Section III, we want  $\Delta \beta_1 = 1.126 \times 10^{-3}$ . In our design,  $T_1$  is a 6-bit control signal, and  $C_{f1}$  can be varied from 0 to 378 fF with a step size  $\Delta C_{f1} = 6$  fF. As a result,  $\beta_1$  can be varied from 0.964 to 1.028 with a step size  $\Delta \beta_1 \approx 1.13 \times 10^{-3}$ . Similarly, the  $\mathrm{CP}_2$  output  $T_2$  controls the capacitor  $C_{f2}$  in the integrator  $H_2$ . The signal  $T_2$  is a 5-bit control signal, and  $C_{f2}$  can be varied from 0 to 186 fF with a step size  $\Delta C_{f2} = 6$  fF. As a result,  $\beta_2$  can be varied from 0.957 to 1.027 with a step size  $\Delta \beta_2 \approx 2.24 \times 10^{-3}$ .

Fig. 8 shows the schematic of DAC<sub>1</sub> and the adder preceding  $ADC_1$  in the first modulation loop of the DSM. Only one side of the fully-differential circuit is depicted. The adder is a passive switched-capacitor circuit [20], comprising three capacitors  $C_{a1}$ to  $C_{a3}$ . The capacitors have an identical capacitance of 60 fF. During  $\phi_2 = 1$ , the preamplifier in the comparator is reset with its input shorted to its output. Meanwhile, the inverse of  $u_1$  from the integrator  $H_1$ , the output  $u_3$  from the integrator  $H_3$ , and a comparator threshold voltage  $V_{
m th1}$  are sampled onto capacitors  $C_{a1}$ ,  $C_{a2}$ , and  $C_{a3}$ , respectively. The input offset of the preamplifier is also stored on the capacitors. During  $\phi_1 = 1$ , the output  $u_1$  from  $H_1$ , the output  $u_2$  from  $H_2$ , and the modulator input xare connected to the capacitors respectively, yielding a differential voltage  $x + 2u1 + u2 - u3 - V_{th1}$  at the input of the preamplifier. Upon the falling edge of  $\phi_1$ , the latch in the comparator compares  $V_a = x + 2u1 + u2 - u3$  with the threshold  $V_{\text{th1}}$ , generating a single-bit digital output  $D_1$ . There are 3 comparators in  $ADC_1$ . They compare  $V_a$  with three different thresholds, which are  $V_{\text{th}1} = -(2/3)V_R$ ,  $V_{\text{th}2} = 0$ , and  $V_{\text{th}3} = +(2/3)V_R$ .

All of the integrators in this DSM sample their input during  $\phi_1=1$  and perform the integration during  $\phi_2=1$ . As shown in Fig. 8, the input sampling capacitor  $C_{s1}$  of the integrator  $H_1$  is divided into three identical capacitors  $C_{s1a}$ ,  $C_{s1b}$ , and  $C_{s1c}$  to implement the 2-bit equally weighted DAC<sub>1</sub>. During  $\phi_1=1$ , the input x is sampled onto all three capacitors. During  $\phi_2=1$ , the capacitors are connected to either  $V_R$  or 0, depending on the three binary signals  $B_1$ ,  $B_2$ , and  $B_3$ , respectively. For capacitor  $C_{s1a}$ , it is connected to  $V_R$  if  $B_1=1$  and is connected to ground if  $B_1=0$ .

The analog input of the DSM, x(t), is sampled onto the capacitor  $C_{s1}$  in the first integrator  $H_1$  and the capacitor  $C_{a3}$  in the

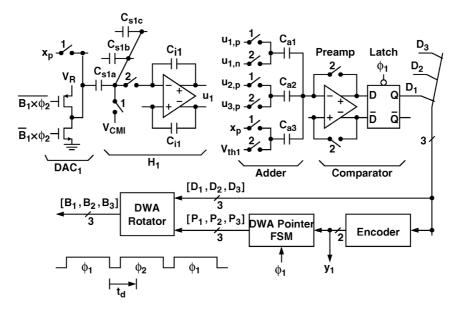


Fig. 8. Schematic of the first loop in the DSM and its timing scheme.

passive adder simultaneously. The sampling switches connected to  $C_{s1a}$ ,  $C_{s1b}$ , and  $C_{s1c}$  are boostrapped nMOSFET switches [21]. These switches need to meet the linearity requirement the DSM. The sampling switches connected to the  $C_{a3}$  capacitors are regular CMOS switches. The linearity requirement of these switches are less critical.

In Fig. 8, the comparator output set  $\mathbf{D} = [D_1, D_2, D_3]$  is a thermometer code. An encoder converts  $\mathbf{D}$  into a 2-bit Gray code  $y_1$ , which serves as the  $\mathrm{ADC}_1$  output. A data-weighted-averaging (DWA) rotator also receives  $\mathbf{D}$  and generates the  $\mathrm{DAC}_1$  input set  $\mathbf{B} = [B_1, B_2, B_3]$  to mitigate the mismatches among  $C_{s1a}$ ,  $C_{s1b}$ , and  $C_{s1c}$ . The rotator is a passive switch matrix controlled by a DWA pointer, which is a finite-state machine (FSM) [22].

In Fig. 8, the critical path is from the latch in the comparator to the first integrator  $H_1$ . Upon the falling edge of  $\phi_1$ , the latches in all three comparators begin the regeneration process to update  $\mathbf{D} = [D_1, D_2, D_3]$ . The change must propagate to the DAC<sub>1</sub> switches in time so that the integrator  $H_1$  receives the correct DAC<sub>1</sub> output during  $\phi_2 = 1$ . The duration from the  $\phi_1$  falling edge that triggers the latches in the comparators to the  $\phi_2$  falling edge that ends the  $H_1$  integration operation is only half of the clock period. If the sampling frequency of the DSM is 1 GHz, half of the clock period is 500 ps. We define the propagation delay from the latch in the comparator to the DAC<sub>1</sub> switches as  $t_d$ . It is critical to minimize  $t_d$  so that the integrator  $H_1$  is given enough time to settle during  $\phi_2 = 1$ .

As illustrated in Fig. 8, a comparator consists of a preamplifier followed by a latch. The preamplifier is a two-path amplifier that combines the high gain of a two-stage amplifier with the high-speed of a single-stage amplifier [23]. The latch is a cascade of a dynamic sense amplifier [24] and a static S-R latch [25]. The preamplifier has a dc gain of 22 dB. It achieves a unit-gain frequency of 16.5 GHz with a phase margin of 68 degree. The propagation delay  $t_d$  defined in Fig. 8 depends on the signal magnitude  $|V_a-V_{\rm th1}|$ , where  $V_a=x+2u1+u2-u3$  is the comparator input and  $V_{\rm th1}$  is the threshold of the com-

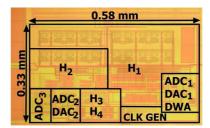


Fig. 9. Modulator chip micrograph.

TABLE II POWER AND AREA OF CIRCUIT BLOCKS

Block	Power (mW)	Power (%)	Area (%)
Integrator $H_1$	56.4	60	50
Integrator H <sub>2</sub>	17.7	19	22
Integrator $H_3$	3.75	4	4.5
Integrator $H_4$	3.75	4	4.5
ADCs, DACs	12.4	13	19
Total	94	100	100

parator. As  $|V_a - V_{\rm th1}|$  decreases, it takes longer time for the latch to regenerate a valid output. In our design,  $t_d = 125$  ps if  $|V_a - V_{\rm th1}| = 1$  mV. That leaves about 300 ps for the first integrator  $H_1$  to settle during  $\phi_2 = 1$ , if the rise time, fall time, and nonoverlapping time of the clocks are taken into account.

## VI. EXPERIMENTAL RESULTS

The DSM shown in Fig. 1 was fabricated using a 65-nm CMOS technology. Fig. 9 shows the chip micrograph. Its active area is  $0.58 \times 0.33 \ \mathrm{mm^2}$ . The calibrations processors  $\mathrm{CP_1}$ ,  $\mathrm{CP_2}$ , and  $\mathrm{CP_3}$  and the noise cancellation filters  $\mathrm{NCF_1}$  and  $\mathrm{NCF_2}$  are realized off-chip. Operating at  $f_s=1.1 \ \mathrm{GHz}$  sampling rate, the chip consumes a total of 94 mW from a 1-V supply. The power and area of the circuit blocks are listed in Table II.

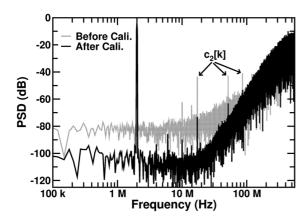


Fig. 10. Measured output spectra of the DSM.

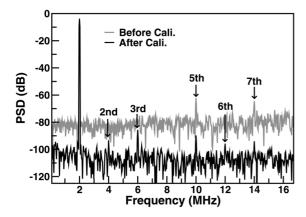


Fig. 11. Measured in-band output spectra of the DSM

The DSM chip is mounted directly on a printed circuit board for measurement. The ADC outputs, including  $y_1$ ,  $y_2$ , and  $y_3$ , are taken off-chip through current-steering buffers with low-voltage-swing differential outputs. The  $y_1$ ,  $y_2$ , and  $y_3$  data were collected using a logic analyzer and analyzed subsequently. The calibration signals  $c_1$  and  $c_2$  are generated on chip. They can be enabled externally. The control signals  $T_1$  and  $T_2$  are generated externally and are fed to the chip to adjust integrators  $H_1$  and  $H_2$ . The reference  $V_R=1$  V, the input common-mode voltage  $V_{\rm CMI}=(1/4)V_{\rm DD}$ , and the output common-mode voltage  $V_{\rm CMO}=(1/2)V_{\rm DD}$  are supplied externally. The differential full-scale input range is  $\pm V_R$ .

The DSM has a sampling rate of  $f_s=1.1~{\rm GHz}$ . Its signal bandwidth is 16.67 MHz if the OSR is 33. Fig. 10 is the measured DSM output spectra before and after the calibration. The input signal is a -3-dBFS 2-MHz sine wave. Without the calibration, the quantization noise of the first stage  $e_1$  dominates the low-frequency band and is shaped by a 40-dB/decade slope for frequencies above 10 MHz. The calibration signal  $c_2$  is visible. It has a frequency of  $f_s/64=17.19$  MHz. The measured SNR is 57 dB. The SNDR is 54 dB. After the calibration, the quantization noise  $e_1$  is minimized. The noise floor drops, and the noise at high frequency is shaped by a 80 dB/decade slope. The calibration signal  $c_2$  is attenuated by 30 dB. The measured SNR and SNDR become 76 and 74 dB, respectively. If the calibration signal  $c_1$  is injected instead of  $c_2$ , the  $c_1$  tones are -34 dB

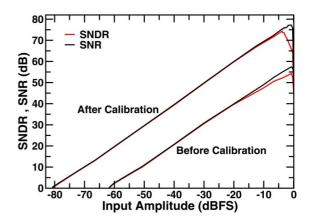


Fig. 12. Measured dynamic performance of the DSM.

in the DSM output spectra. Fig. 11 shows the passband details of the output spectra. Before the calibration, the fifth and seventh harmonics are the dominant tones. The spurious-free dynamic range (SFDR) is 58 dB. After the calibration, the third harmonic becomes the dominant tone. The SFDR increases to 83 dB.

Fig. 12 shows the measured SNR and SNDR versus the input signal level. The input is a 2-MHz sine wave. Before the calibration, the peak SNR and the peak SNDR are 57.42 and 54.23 dB, respectively, at -0.71-dBFS input level. The DR is 62 dB. After the calibration, the peak SNR is 77.22 dB at -0.93-dBFS input level, and the peak SNDR is 74.32 dB at -3.85-dBFS input level. The DR increases to 81 dB. Note that, before the calibration, there is a visible difference between the SNR and the SNDR for the input level higher than -20 dBFS. This is because the quantization noise  $e_1$  leaking to the DSM output contains input harmonics. After the calibration, the  $e_1$  leakage is reduced, and the difference between SNR and SNDR is also decreased.

In our design, the opamps employ the constant-current biasing scheme. Simulations show that the dc gain of the opamps varies from 10.74 to 8.76 if the temperature changes from 0 °C to 80 °C. Thus, after calibration, the SNR of the DSM will degrade by -3 dB if the temperature changes by 10 °C. However, this degradation will not occur if the time for the temperature change is much longer than the calibration time constant. On the other hand, the dc gain of the opamp varies from 9.77 to 10.87 if the supply voltage changes from 0.9 to 1.1 V. Thus, after calibration, the SNR degradation is less than -3 dB if the supply voltage variation is less than  $\pm 50$  mV.

Table III summarizes the performance of this DSM chip and compares it with other wide-bandwidth DSMs. The figure of merit (FOM) are defined as follows [26]:

$$FOMs = DR_{dB} + 10log_{10} \frac{BW}{Power}$$

$$FOMw = \frac{Power}{2BW \times 2^{\frac{(SNDR-1.76)}{6.02}}}.$$
(14)

The FOMs of this chip is 163.487 dB, and the FOMw is 660 fJ/level. Comparing with other discrete-time (DT) DSMs, this DSM has the highest sampling rate, operates under the lowest supply voltage, and achieves competitive FOMs and FOMw. In Table III, the continuous-time (CT) DSMs show better FOMw

Publication	This work	[27]	[28]	[4]	[5]	[6]	[7]	[29]	[30]	[31]	[32]	[33]
Туре	DT	DT	DT	DT	DT	DT	DT	CT	CT	CT	CT	CT
Process (nm)	65	65	180	32	180	90	90	90	90	65	130	90
Supply (V)	1.0	1.25	1.8	1.05	1.8	1.2	1.4	1.2	1.2	1.2	1.5	1.4
Clock (MHz)	1100	240	160	400	80	420	330	3600	500	2400	900	600
OSR	33	8	8	10	8	10.5	8	50	10	64	22.5	30
BW (MHz)	16.67	15	20	20	10	20	20	36	25	18.8	20	10
DR (dB)	81	-	64	66	75.5	-	-	83	72	78	-	-
Peak SNR (dB)	77.22	-	63	-	74	72	67	76	69	76	81	83
Peak SNDR (dB)	74.32	67	63	63	74	70	63	71	68	74	78	78
Power (mW)	94	37	16	28	22	28	73	15	8.5	39	87	16
Die Size (mm <sup>2</sup> )	0.19	0.28	0.36	0.13	0.62	1	1.3	0.12	0.23	0.075	0.45	0.36
FOMs (dB)	163.5	153*	155	155	162	161*	151*	176.8	167	165	165*	171*
FOMw (pJ/conv.)	0.66	0.67	0.35	0.61	0.27	0.27	1.5	0.07	0.09	0.25	0.33	0.12

TABLE III COMPARISON OF WIDE-BANDWIDTH DSMS

performance. However, they require clocks of more stringent jitter performance.

#### VII. CONCLUSION

A fourth-order discrete-time DSM was fabricated using a 65-nm CMOS technology. It has a sampling rate of 1.1 GHz and an input bandwidth of 16.67 MHz with an OSR of 33. We use low-complexity circuits to achieve high speed. The opamps have a unity-frequency of 6 GHz in the integrator configuration, but have a dc gain of only 10. The low-gain opamps result in lossy integrators. We apply the integrator leakage calibration to adjust the leakage-compensating capacitors of the integrators to move their poles  $\beta$  back to 1. We use the noise leakage calibration to minimize the quantization noise of the first loop leaking to the DSM's combined output. The noise leakage calibration relaxes the matching requirement for the MASH structure. The calibration is simplified since it does not need to correct the  $\beta$  coefficients. Both digital calibrations can operate in the background without interrupting the normal DSM operation. The calibration processors are simple digital circuits. They do not have complex filters. The chip's measured SNDR and DR are 74.32 and 81 dB, respectively. The chip consumes 94 mW from a 1-V supply. The active area is  $0.33 \times 0.58 \text{ mm}^2$ .

We demonstrate that the combination of low-complexity circuits and digital calibration can yield a high-speed high-performance DSM. The design technique is specially suitable for advanced nanoscale CMOS technologies.

## $\begin{array}{c} \text{Appendix A} \\ \text{Averaged Transient Behavior of } \beta_1 \end{array}$

As shown in Fig. 1,  $\mathrm{CP}_1$  receives the  $\mathrm{ADC}_1$  output  $y_1$ .  $\mathrm{ADC}_1$ 's input thresholds are at  $\{0,\pm(2/3)V_R\}$ , and its corresponding digital outputs are  $y_1 \in \{\pm 1/3,\pm 1\}$ .  $\mathrm{ADC}_1$  has a conversion gain of  $G_{A1}=1/V_R$ . From Fig. 4, the averaged variation of s for one clock cycle is  $\Delta s=(D_{g1}V_{cf1})\times G_{A1}$ . The voltage  $V_{cf1}$  is expressed as (6). As illustrated in Fig. 5, s takes an average of  $N_{\mathrm{th}1}/\Delta s$  cycles to accumulate from 0 to  $+N_{\mathrm{th}1}$  (or  $-N_{\mathrm{th}1}$ ). Once s reaches the threshold,  $T_1$  is

changed by 1 and  $\beta_1$  is changed by  $\Delta\beta_1$ . Thus, the averaged  $\beta_1$  variation rate is

$$\frac{d\beta_1}{dk} = \frac{\Delta\beta_1}{N_{\text{th}1}/\Delta s} \approx \frac{\Delta\beta_1 D_{g1} V_{c1}}{\alpha_1 N_{\text{th}1} V_R} (1 - \beta_1). \tag{15}$$

The above equation leads to (7) and (8).

## APPENDIX B AVERAGED STANDARD DEVIATION OF $eta_1$

When the  $H_1$  calibration process converges, the behavior of  $\beta_1$  becomes a discrete random fluctuation around 1. If  $N_{th1}=\infty$ ,  $\beta_1$  alternates between only two values, which are  $\beta_a=1-x\Delta\beta_1$  and  $\beta_b=1+(1-x)\Delta\beta_1$ , where  $0\leq x\leq 1$  depending on  $\beta_{1,0}$ . Define the probability for  $\beta_1=\beta_a$  as  $P_a$ , and the probability for  $\beta_1=\beta_b$  as  $P_b$ . Since the expected value of  $\beta_1$  is 1, i.e.,  $\beta_a P_a + \beta_b P_b = 1$ , we have  $P_a=1-x$  and  $P_b=x$ . If x is given, define the standard deviation of  $\beta_1$  as

$$\sigma(\beta_1[x]) = \sqrt{(\beta_a - 1)^2 P_a + (\beta_b - 1)^2 P_b} = \Delta \beta_1 \sqrt{x(1 - x)}.$$
(16)

If x is uniformly distributed from 0 to 1, the standard deviation of  $\beta_1$  is

$$\sigma(\beta_1) = \sqrt{\int_0^1 \sigma(\beta_1[x])^2 dx} = \frac{\Delta \beta_1}{\sqrt{6}}.$$
 (17)

The above equation is the same as (9).

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<sup>\*</sup> Assume DR equals to peak SNDR or peak SNR.

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Su-Hao Wu (S'10) was born in Tainan, Taiwan. He received the B.S. degree in electrical engineering from National Tsing-Hua University, Hsin-Chu, Taiwan, in 2002, and the M.S. degree in communication engineering from National Chiao-Tung University, Hsin-Chu, Taiwan, in 2004, where he is currently working toward the Ph.D. degree in electronics engineering.

His current research interests include high-resolution data converters and low-power mixed-signal integrated circuits.



**Jieh-Tsorng Wu** (S'83–M'87–SM'06) was born in Taipei, Taiwan. He received the B.S. degree in electronics engineering from National Chiao-Tung University, Hsin-Chu, Taiwan, in 1980, and the M.S. and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, USA, in 1983 and 1988, respectively.

From 1980 to 1982, he served in the Chinese Army as a Radar Technical Officer. From 1982 to 1988, at Stanford University, he focused his research on high-speed analog-to-digital conversion in CMOS VLSI.

From 1988 to 1992, he was a Member of Technical Staff with Hewlett-Packard Microwave Semiconductor Division, San Jose, CA, USA, where he was responsible for several linear and digital gigahertz IC designs. Since 1992, he has been with the Department of Electronics Engineering, National Chiao-Tung University, Hsin-Chu, Taiwan, where he is now a Professor. His current research interests are high-performance mixed-signal integrated circuits.

Dr. Wu is a member of Phi Tau Phi. He has served as an associate editor of the IEEE JOURNAL OF SOLID-STATE CIRCUITS. Since 2012, he has served on the Technical Program Committee of the International Solid-State Circuits Conference (ISSCC).