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Spontaneous emission dynamics in an omnidirectional waveguide made of photonic crystals

Chih-Hsien Huang^{1,2}, Szu-Cheng Cheng³, Jing-Nuo Wu^{3,4} and Wen-Feng Hsieh^{1,2,4}

¹ Department of Photonics and Institute of Electro-Optical Engineering, National Chiao Tung University, Hsinchu 300, Taiwan

² Institute of Electro-Optical Science and Engineering, National Cheng Kung University, Tainan 701, Taiwan

³ Department of Physics, Chinese Culture University, Taipei 111, Taiwan

E-mail: jingnuowu@gmail.com and wfhsieh@mail.nctu.edu.tw

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Abstract

The spontaneous emission dynamics of atoms embedded in an omnidirectional waveguide (ODWG), a novel optical waveguide, is studied on the basis of the complete reflection of one-dimensional photonic crystals. With the dispersion curve of the single waveguide mode within the photonic band gap and various extents of background dissipation, we characterize the photon–atom interaction in the ODWG. The photon emitter of the system is a two-level atom embedded in the low-index medium of the multilayer-film ODWG or the atom–ODWG system. Fractional calculus, an innovative mathematical method in optical systems, is applied to solve the equation of motion for this atom–ODWG system. Two kinds of states with different group velocities exhibit totally distinctive dynamical behavior. The high frequency waveguide mode with a fast group velocity shows fast exponential decay in propagation while the band-edge mode with a slow group velocity displays non-Markovian dynamics with non-exponential oscillating time evolution. We therefore suggest different functions of this atom–ODWG system for these two kinds of states. The richness of the physical content of the system is also revealed through investigating the dynamical behavior of the band-edge mode. These results aid in further application and fundamental understanding of the atom–ODWG system.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

An omnidirectional waveguide (ODWG), surrounded by dielectric omnidirectional (OD) reflectors [1], is also a waveguide (WG) structure based on the complete reflection of one-dimensional (1D) photonic crystals (PCs) [2]. It is acclaimed as a flexible WG due to its ability to guide electromagnetic (EM) waves with high transmission of up to 0.99 at arbitrary and random bending angles as long as the radius of curvature R is large enough ($R \geq 26a$ with a being the lattice constant) [3]. The WG structure, shown in figure 1(a), consists of a dielectric layer (or air) of lower index

n_a sandwiched by a 1D PC with refractive indices $n_2 > n_1 > n_a$ and variable thicknesses h_2 , h_1 , and h_a . The dispersion curve of the single WG mode is sketched in figure 1(b) using the plane wave expansion method [4]. This dispersion curve and OD frequency range can be tailored by proper design of the geometrical structure of the system. This structure can exhibit complete reflection of radiation in a given frequency range for all incident angles and polarizations which has been proven both theoretically and experimentally [1, 5, 6]. It combines the features of both metallic-like OD reflectivity and frequency selectivity with the typical low-loss behavior of a PC. Within the frequency range of OD reflection, i.e., the dispersion curve lies within the photonic band gap (PBG)

⁴ Authors to whom any correspondence should be addressed.

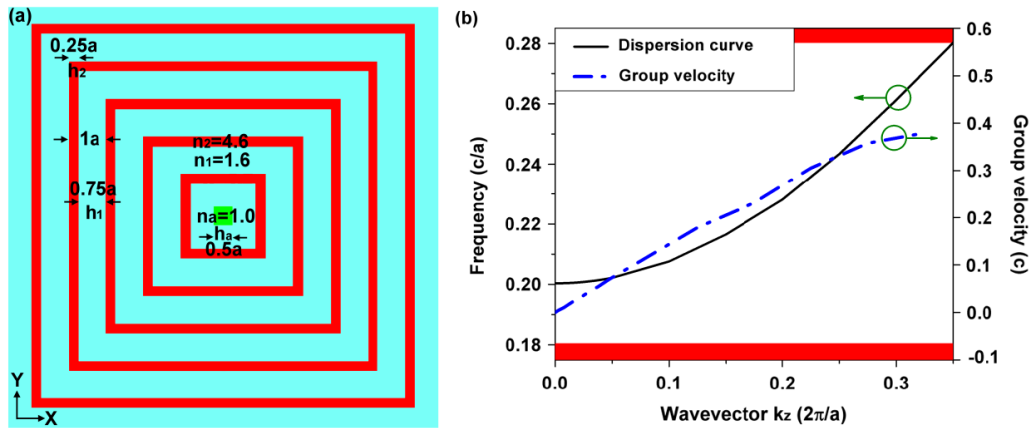


Figure 1. (a) Scheme of an omnidirectional waveguide and (b) its dispersion curve and the group velocity of a single guide mode within the photonic band gap made of the 1D photonic crystal. These parameters are referred to the actual polystyrene–tellurium film [1]. Here a is the lattice constant of the structure and c is the light speed in vacuum.

and above the light line of the guiding medium, the EM wave is allowed to propagate in this medium but is forbidden in (or completely reflected by) the PC that functions as an OD mirror. The reason for the high transmission in this ODWG is the single mode guiding within the waveguide.

Spontaneous emission (SE) from an excited atom in free space decays exponentially with time, which can be described under the Markovian approximation. This behavior will change if the emitter is put into environments with different densities of states (DOS) of photons and this is called the Purcell effect [7]. In this case, the atoms strongly couple with the ‘structured’ reservoir that leads to the invalidity of the Markovian approximation. Especially, in the presence of a threshold or singularities in the photon DOS [8–14], the non-Markovian effect becomes of major relevance. Typical features of non-Markovian dynamics include non-exponential decay, fractional decay, population trapping, atom–photon bound states and damped Rabi oscillation [12, 15]. Kleppner predicted in the early 1980s [16] that the SE rate can be changed when the emission frequency lies near the band-edge mode of an optical WG. Near the band-edge mode, the photon DOS increases and the optical mode possesses a slow group velocity (see figure 1(b)). An optical mode with slow group velocity implies greatly enhanced light–matter coupling strength and explicit non-Markovian effect. This enhanced coupling allows the efficient channeling of interaction between photons and emitters fabricated in the WG.

Study of the SE dynamics of atoms embedded in an ODWG (atom–ODWG) is a timely requirement for both practical application and fundamental understanding. It was indicated by Joannopoulos *et al* [17] that when a light source is embedded close to the interface of an ODWG, the emitted light will couple to the extended modes propagating in the 1D PC and thus the reflective property of the OD mirror will be changed. That is, embedding a light emitter within an ODWG would induce distortion of the PC mirror. This distortion should lead to a change of the SE dynamics. Therefore, study of the dynamical behavior of an atom–ODWG system is important due to its unexplored properties and potential applications.

Fractional calculus, integration and differentiation of an arbitrary order, has been applied widely in science, such as in fractional kinetics [18], thermodynamics [19–21], fractal media [22], and chaos dynamics [23, 24]. This mathematical method has been proven to be better for studying the dynamical behavior of optical systems near threshold [15]. Traditionally, the dynamical behavior of these optical systems is studied through the Laplace transform method [9, 10, 25] and/or the smoothing method [11, 26] to solve the time evolution equations. The Laplace transform method had given many important results such as photon–atom bound states but would lead to a multiple-valued problem and inconsistency with experimental results, which was pointed out in our previous study [15]. Fractional calculus gives a rigorous solution and correct physical results for this kind of optical system in PCs without dealing with the multiple-valued problem and artificial smoothing parameters.

In this paper, we applied fractional calculus to study the SE dynamics of a two-level atom embedded in the lower-index medium of the ODWG with its transition frequency being nearly resonant with the single WG mode within the PBG. We found two distinctive dynamical behaviors governed by two regimes with a fast group velocity for the high frequency modes and a slow group velocity for the band-edge mode. These two kinds of regimes are discussed under different extents of background dissipation, which are associated with fabrication defects of the PC mirrors. Functions of this atom–ODWG system are suggested based on these two distinctive modes. We describe the atom–ODWG system from the quantum point of view in section 2. The characteristic of the atom–ODWG is depicted by the specific memory kernels associated with the dispersion curve of the single WG mode and the background dissipation property of the PC mirror. In section 3, we introduce the fractional calculus to this optical system and obtain the corresponding fractional kinetic equation and dynamical solution. The SE dynamics of the system with various atomic detuning frequencies and background dissipations is discussed. Finally, we summarize our results in section 4.

2. Quantum description of the atom-waveguide system

When the system of a two-level atom coupled to a WG reservoir is considered, the total Hamiltonian can be written as

$$\begin{aligned}
 H = & \omega_a |1\rangle\langle 1| + \sum_{\mathbf{k}} \omega_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + \sum_{\mathbf{k}} \omega_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} \\
 & + \sum_{\mathbf{k}} g_c(\omega_{\mathbf{k}}) (c_{\mathbf{k}}^{\dagger} |0\rangle\langle 1| + |1\rangle\langle 0| c_{\mathbf{k}}) \\
 & + \sum_{\mathbf{k}} g_b(\omega_{\mathbf{k}}) (b_{\mathbf{k}}^{\dagger} |0\rangle\langle 1| + |1\rangle\langle 0| b_{\mathbf{k}}), \quad (1)
 \end{aligned}$$

where ω_a denotes the atomic transition frequency; $|0\rangle$ and $|1\rangle$ are the atomic ground and excited states; $g_c(\omega_{\mathbf{k}})$ and $g_b(\omega_{\mathbf{k}})$ specifying the frequency-dependent coupling between the atom and photon reservoirs are defined as $g = \frac{\omega_a d}{\hbar} \left(\frac{\hbar}{2\varepsilon_0 \omega_{\mathbf{k}} L} \right)^{1/2} \mathbf{e}_{\mathbf{k}} \cdot \mathbf{u}_d$ with d and \mathbf{u}_d being the absolute value and the unit vector of the atomic dipole moment; L is the length of the WG. There are two kinds of reservoirs in this system. The pair of creation and annihilation operators $c_{\mathbf{k}}^{\dagger}$ and $c_{\mathbf{k}}$ correspond to the photon associated with the WG resonance while the other pair $b_{\mathbf{k}}^{\dagger}$ and $b_{\mathbf{k}}$ stand for the photons referred to the background-dissipating modes due to imperfection in the periodic structure. The corresponding wavefunction of the system can be expressed as

$$\begin{aligned}
 |\psi(t)\rangle = & e^{-i\omega_c t} [A(t)|2, \{0\}\rangle + \sum_{\mathbf{k}} B_{\mathbf{k}}(t)|1, \{1_{\mathbf{k}_b}\}\rangle \\
 & + \sum_{\mathbf{k}} C_{\mathbf{k}}(t)|1, \{1_{\mathbf{k}_c}\}\rangle] \quad (2)
 \end{aligned}$$

with the cutoff frequency ω_c (e.g., ~ 0.2 in figure 1(b)) of the WG. The probability amplitude $A(t)$ indicates the atom in its excited state $|2\rangle$ with no photon or in the vacuum state while $B_{\mathbf{k}}(t)$ and $C_{\mathbf{k}}(t)$ indicate that the atom is in its ground state $|1\rangle$ with one photon in mode \mathbf{k}_b and one in mode \mathbf{k}_c , respectively. The initial condition of $A(t=0) = 1$ and $B_{\mathbf{k}}(t=0) = C_{\mathbf{k}}(t=0) = 0$ expresses the atom being in the excited state $|2\rangle$ with no photon in either reservoir initially.

The equations of motion for these probability amplitudes can be derived from the time-dependent Schrödinger equation as

$$\begin{aligned}
 \frac{d}{dt} A(t) = & -i(\omega_a - \omega_c) A(t) - i \sum_{\mathbf{k}} g_b(\omega_{\mathbf{k}}) B_{\mathbf{k}}(t) \\
 & - i \sum_{\mathbf{k}} g_c(\omega_{\mathbf{k}}) C_{\mathbf{k}}(t), \quad (3)
 \end{aligned}$$

$$\frac{d}{dt} B_{\mathbf{k}}(t) = -i(\omega_{\mathbf{k}} - \omega_c) B_{\mathbf{k}}(t) - i g_b(\omega_{\mathbf{k}}) A(t), \quad (4)$$

$$\frac{d}{dt} C_{\mathbf{k}}(t) = -i(\omega_{\mathbf{k}} - \omega_c) C_{\mathbf{k}}(t) - i g_c(\omega_{\mathbf{k}}) A(t). \quad (5)$$

These equations can be further simplified after making the following transformations:

$$\begin{aligned}
 A(t) = & e^{-i(\omega_a - \omega_c)t} \alpha(t); & B_{\mathbf{k}}(t) = & e^{-i(\omega_{\mathbf{k}} - \omega_c)t} \beta_{\mathbf{k}}(t); \\
 C_{\mathbf{k}}(t) = & e^{-i(\omega_{\mathbf{k}} - \omega_c)t} \gamma_{\mathbf{k}}(t). \quad (6)
 \end{aligned}$$

This gives

$$\begin{aligned}
 \frac{d}{dt} \alpha(t) = & -i \sum_{\mathbf{k}} g_b(\omega_{\mathbf{k}}) e^{-i(\omega_{\mathbf{k}} - \omega_a)t} \beta_{\mathbf{k}}(t) \\
 & - i \sum_{\mathbf{k}} g_c(\omega_{\mathbf{k}}) e^{-i(\omega_{\mathbf{k}} - \omega_a)t} \gamma_{\mathbf{k}}(t), \quad (7)
 \end{aligned}$$

$$\frac{d}{dt} \beta_{\mathbf{k}}(t) = -i g_b(\omega_{\mathbf{k}}) e^{-i(\omega_{\mathbf{k}} - \omega_a)t} \alpha(t), \quad (8)$$

$$\frac{d}{dt} \gamma_{\mathbf{k}}(t) = -i g_c(\omega_{\mathbf{k}}) e^{-i(\omega_{\mathbf{k}} - \omega_a)t} \alpha(t). \quad (9)$$

After integrating equations (8) and (9) and substituting these results into equation (7), we obtain the integral-differential equation

$$\begin{aligned}
 \frac{d}{dt} \alpha(t) = & -i \sum_{\mathbf{k}} g_b(\omega_{\mathbf{k}})^2 \int_0^t e^{-i(\omega_{\mathbf{k}} - \omega_a)(t-\tau)} \alpha(\tau) d\tau \\
 & - \sum_{\mathbf{k}} g_c(\omega_{\mathbf{k}})^2 \int_0^t e^{-i(\omega_{\mathbf{k}} - \omega_a)(t-\tau)} \alpha(\tau) d\tau. \quad (10)
 \end{aligned}$$

A neat form of this equation reads

$$\frac{d}{dt} \alpha(t) = - \int_0^t K_b(t-\tau) \alpha(\tau) d\tau - \int_0^t K_c(t-\tau) \alpha(\tau) d\tau, \quad (11)$$

where the memory kernels are defined as

$$\begin{aligned}
 K_b(t-\tau) \equiv & \sum_{\mathbf{k}} g_b(\omega_{\mathbf{k}})^2 e^{-i(\omega_{\mathbf{k}} - \omega_a)(t-\tau)}, \\
 K_c(t-\tau) \equiv & \sum_{\mathbf{k}} g_c(\omega_{\mathbf{k}})^2 e^{-i(\omega_{\mathbf{k}} - \omega_a)(t-\tau)}. \quad (12)
 \end{aligned}$$

The characteristics of the WG reservoir are determined by these memory kernels. For an ODWG, the kernel $K_b(t-\tau)$ referred to the background dissipative modes may be assumed to be

$$K_b(t-\tau) = \gamma_b \delta(t-\tau) \quad (13)$$

with the Dirac δ function and the decay rate γ_b indicating the reflective property and scattering traits of the OD PC mirror. When the atom is embedded inside this ODWG as a light emitter, its transition dipole would induce distortion of the PC mirror [17]. This decay rate could also express the extent of the distortion.

On the basis of the dispersion curve of the single WG mode in this optical system, the other kernel $K_c(t-\tau)$ connected to the WG resonance can be expressed in terms of the frequency-dependent mode density $\rho(\omega)$ as

$$K_c(t-\tau) = \left(\frac{\omega_a^2 d^2}{2\varepsilon_0 \omega_c \hbar} \right) \int_0^\infty \rho(\omega) e^{-i(\omega - \omega_a)(t-\tau)} d\omega. \quad (14)$$

Here we have assumed that the energy of dipole transition is close to the PBG and the dipole is perpendicular to the propagation direction of the WG. For an ODWG with the structure given in figure 1(a), the dispersion relation of the single WG mode shown in figure 1(b) can be approximated by a parabolic curve as $\omega = \omega_c + M k^2$ with cutoff frequency ω_c and curvature M . The corresponding photon DOS exhibits a threshold-like behavior with frequency dependence as

$$\rho(\omega) = \frac{1}{2\pi \sqrt{M} \sqrt{\omega - \omega_c}} \Theta(\omega - \omega_c), \quad (15)$$

where $\Theta(x)$ is a Heaviside step function. This photon DOS is singular for a frequency ω near the threshold (cutoff) frequency ω_c . For a frequency below threshold, $\omega < \omega_c$, the photon DOS is zero and no propagation mode exists in the WG. In this case, the WG-resonance kernel K_c vanishes and the decaying behavior comes from the background-dissipating kernel K_b . Substituting this photon DOS into the memory kernel $K_c(t-\tau)$ and applying the complex Fresnel integral $\int_0^\infty x^{p-1} e^{-ax} dx = \Gamma(p)/a^p$, we obtain

$$K_c(t-\tau) = \beta^{3/2} \frac{e^{-i[\pi/4 - \Delta_c(t-\tau)]}}{\sqrt{t-\tau}} \quad (16)$$

with the coupling constant $\beta^{3/2} = \omega_a^2 d^2 / 4\sqrt{M\pi} \varepsilon_0 \omega_c \hbar$; $\Gamma(x)$ is the gamma function and the atomic detuning frequency $\Delta_c = \omega_a - \omega_c$ under the long time limit $t > \tau$. Substituting these two memory kernels into equation (11), we obtain the equation for this atom-ODWG system as

$$\frac{d}{dt} \alpha(t) = -\gamma_b \alpha(t) - \beta^{3/2} e^{-i\pi/4} e^{i\Delta_c t} \int_0^t \frac{e^{-i\Delta_c \tau}}{\sqrt{t-\tau}} \alpha(\tau) d\tau. \quad (17)$$

3. Fractional kinetic equation and solutions

In this section, we apply the fractional calculus to deal with the dynamical behavior of the system. The integral on the right hand side of equation (17) can be expressed as the Riemann-Liouville fractional differential operator, which is defined as

$$\frac{d^v}{dt^v} f(t) = \frac{1}{\Gamma(-v)} \int_0^t (t-\tau)^{-v-1} f(\tau) d\tau. \quad (18)$$

After transforming the probability amplitude $\alpha(t)$ back to $A(t)$ by $\alpha(t) = e^{i\Delta_c t} A(t)$, equation (17) can be written in a fractional form as

$$\frac{d}{dt} A(t) + (\gamma_b + i\Delta_c) A(t) + \beta^{3/2} e^{-i\pi/4} \sqrt{\pi} \frac{d^{-1/2}}{dt^{-1/2}} A(t) = 0. \quad (19)$$

This is the fractional kinetic equation of this atom-ODWG system. It can be solved through manipulating the fractional operators including the integral operator d^{-1}/dt^{-1} followed by the fractional differentiation operator $d^{3/2}/dt^{3/2}$. The first step yields

$$A(t) - A(0) + (\gamma_b + i\Delta_c) \frac{d^{-1}}{dt^{-1}} A(t) + \beta^{3/2} e^{-i\pi/4} \sqrt{\pi} \frac{d^{-3/2}}{dt^{-3/2}} A(t) = 0, \quad (20)$$

and the second step gives

$$\frac{d^{3/2}}{dt^{3/2}} A(t) + i(\Delta_c - i\gamma_b) \frac{d^{1/2}}{dt^{1/2}} A(t) + \beta^{3/2} e^{-i\pi/4} \sqrt{\pi} A(t) = -\frac{1}{2\sqrt{\pi}} t^{-3/2}, \quad (21)$$

where we have applied the initial condition $A(0) = 1$. By taking the Laplace transform with the fractional derivative of $A(t)$, we obtain

$$\tilde{A}(s) = \frac{\sqrt{s}}{s^{3/2} + i(\Delta_c - i\gamma_b)s^{1/2} - e^{i3\pi/4}\beta_W^{3/2}}. \quad (22)$$

Here we have defined a new coupling constant $\beta_W^{3/2} = \beta^{3/2} \sqrt{\pi}$, which denotes the coupling strength between the two-level atom and the ODWG mode depending on the orientation and distance of the atomic dipole relative to the interface of the multilayer structure.

The standard solving procedures for the dynamical behavior of the atom-ODWG system involve expressing $\tilde{A}(s)$ as a sum of partial fractions and inversely Laplace transforming the partial fractions. The partial-fraction form of $\tilde{A}(s)$ is

$$\tilde{A}(s) = \frac{C_1}{\sqrt{s-Z_1}} + \frac{C_2}{\sqrt{s-Z_2}} + \frac{C_3}{\sqrt{s-Z_3}}, \quad (23)$$

where Z_n are the roots of the indicial equation

$$Z^3 + i(\Delta_c - i\gamma_b)Z - e^{i3\pi/4}\beta_W^{3/2} = 0, \quad (24)$$

and C_n are the coefficients related to Z_n . The roots Z_n and the coefficients C_n are solved as

$$Z_1 = \beta_W^{1/2} [-(2/3)^{1/3} \Delta/\zeta + \zeta/18^{1/3}] e^{i\pi/4}, \quad (25)$$

$$Z_2 = \beta_W^{1/2} [(2/3)^{1/3} \Delta e^{i\pi/3}/\zeta - \zeta e^{-i\pi/3}/18^{1/3}] e^{i\pi/4}, \quad (26)$$

$$Z_3 = \beta_W^{1/2} [(2/3)^{1/3} \Delta e^{-i\pi/3}/\zeta - \zeta e^{i\pi/3}/18^{1/3}] e^{i\pi/4}, \quad (27)$$

with

$$\zeta = 3^{2/3} \left[1 + \left(1 + \frac{4}{27} \Delta^3 \right)^{1/2} \right]^{1/3}, \quad \Delta = \frac{\Delta_c}{\beta_W} - i \frac{\gamma_b}{\beta_W}, \quad (28)$$

and

$$C_n = \frac{Z_n}{(Z_n - Z_j)(Z_n - Z_m)} (n \neq j \neq m; n, j, m = 1, 2, 3). \quad (29)$$

Applying the fractional inverse Laplace transform, $L^{-1}\{\frac{1}{\sqrt{s-a}}\} = E_t(-1/2, a^2) + a E_t(0, a^2)$, to the partial fractions in equation (23), we get the excited-state probability amplitude $A(t)$ as

$$A(t) = \sum_{n=1}^3 C_n [E_t(-1/2, Z_n^2) + Z_n E_t(0, Z_n^2)] = \sum_{n=1}^3 C_n [Z_n^2 E_t(1/2, Z_n^2) + Z_n e^{Z_n^2 t}]. \quad (30)$$

Here $E_t(v, a) = t^v \sum_{n=0}^\infty \frac{(at)^n}{\Gamma(v+n+1)}$ is the fractional exponential function of order v with variable t and constant a . Equation (30) can be further written as

$$A(t) = \sum_{n=1}^3 C_n [Y_n \text{erf}(Y_n \sqrt{t}) + Z_n] e^{Z_n^2 t} \quad (31)$$

with Y_n being the square roots of Z_n^2 in the first or the second quadrant because the fractional exponential function $E_t(1/2, Z)$ can be expressed as the error function through $E_t(1/2, Z) = e^{Zt} \text{erf}(\sqrt{Zt})/\sqrt{Z}$.

The dynamical behavior of this excited-state population $P(t) = |A(t)|^2$ is strongly related to the number Z_n^2 . When Z_n^2 is a complex number, the excited-state population contributed from this dressed state (DS) will exhibit decaying behavior.

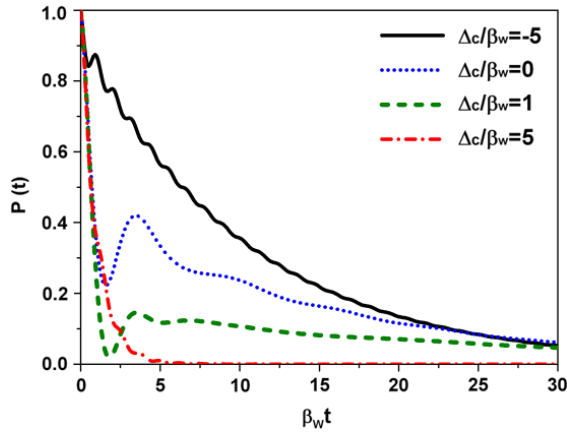


Figure 2. The spontaneous emission dynamics of the atom–ODWG system with $\gamma_b = 0.05\beta_w$. It is plotted as a function of $\beta_w t$ for various values of the atomic detuning frequency $\Delta_c = \omega_a - \omega_c$.

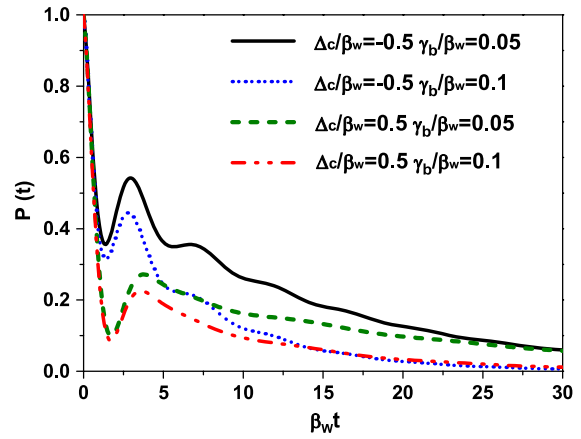


Figure 3. Spontaneous emission dynamics of the omnidirectional waveguide system in the slow-light states with different detuning frequencies $\Delta_c = 0.5\beta_w$ and $-0.5\beta_w$ and different background decaying rates ($\gamma_b = 0.05\beta_w$ and $0.1\beta_w$).

As Z_n^2 is a pure positive imaginary number, the population amplitude contributed from this DS equals $2C_n Z_n \exp(Z_n^2 t)$ at $t = \infty$ with $Z_n = \alpha_n \exp(i\pi/4)$ and $\alpha_n > 0$ because $\text{erf}(\sqrt{Z_n^2 t}) = 1$ at $t = \infty$. On the other hand, the population amplitude equals 0 at $t = \infty$ as $\alpha_n < 0$ because $Y_n = -Z_n$. Therefore, only the DS with Z_n having amplitude $\alpha_n \exp(i\pi/4)$ with $\alpha_n > 0$ contributes to a bound state. When analyzing the indicial equation of equation (24), we find that there always exists a bound DS when $\gamma_b = 0$. The DS with larger atomic detuning frequency from the photonic band edge contributes less to the bound state. This contribution even becomes zero when the detuning frequency is high enough, i.e., $\omega_a \gg \omega_c$. Under this circumstance, the existence of background dissipation accelerates the decay of the excited state. On the other hand, as the DS has an atomic frequency close to the PC band edge, the background dissipation provides a channel to release the bound state energy through a photon decaying into the reservoir.

We plot the SE dynamics of this atom–ODWG system in terms of the excited-state probability, $P(t) = |A(t)|^2$ in figure 2, for different atomic detuning frequencies $\Delta_c = \omega_a - \omega_c$ with background dissipation $\gamma_b = 0.05\beta_w$. It can be seen that distinct dynamical behaviors exist between the states with frequencies far away from the guiding threshold frequency ($\Delta_c/\beta_w \geq 5$) and those with frequency near the threshold frequency $\Delta_c/\beta_w \approx 0$.

The excited-state population or the SE dynamics exhibits fast exponential decay for the case of $\Delta_c/\beta_w \geq 5$ where the high frequency WG mode has a fast group velocity. Thus this atom–ODWG at high detuning is named the high frequency WG (HFWG) mode hereafter. It reveals that the excited atom favors dissipation of its stored energy by emitting a photon to the WG mode. Chen *et al* [3] indicated numerically that this WG can achieve a high transmission rate of up to 0.99 within the ODWG corresponding to the HFWG here. On the other hand, the band-edge mode at near-zero detuning having slow group velocity displays non-exponential decay, which is one of the typical features of non-Markovian dynamics. This non-Markovian effect results from strong light–matter interaction

enhanced by the singularity of the photon DOS near threshold. The enlarged photon DOS or the slow group velocity enhances the light–matter interaction. This band-edge mode reflects the strong coupling between the excited-state atoms and the WG reservoir results in non-exponential dynamics with damped Rabi oscillation. When the atomic transition frequency is below the WG propagating mode, i.e., $\Delta_c/\beta_w = -5$ in figure 2, the energy of the atom in the excited state will almost release via the background dissipation. The oscillation of the population at the beginning is caused by the interaction or energy transfer between the WG and the atom.

It is worth investigating the behavior of the system at the band-edge modes where the atomic transition frequency (ω_a) lies near the cutoff frequency (ω_c) of the WG, i.e., $\omega_a - \omega_c = \Delta_c \cong 0$. In figure 3, the SE dynamics is plotted with two different values of the background decaying rate. Typical features of non-Markovian dynamics with non-exponential decay and damped Rabi oscillation are observed in figure 3 for both dissipation cases. For large background dissipation, the SE dynamics with the atomic detuning frequency positive ($\Delta_c > 0$) displays more seriously propagating behavior, while that of the negative-detuning states ($\Delta_c < 0$) manifests bound behavior in the strong atom–WG interaction region, e.g., $\beta_w t = 0-5$; then the decaying behaviors are dominated by the background dissipation rate γ_b at the long time scale. Therefore, in the band-edge mode, the dynamics of the population is dominated by the atom–WG interaction initially but is dissipated at the long time scale. In the HFWG region, the emitted photon energy is quickly transferred to the WG mode so the influence of the background dissipation will be dramatically reduced.

The ODWG in this band-edge mode is not suitable for guiding waves because of the binding feature from the strong light–matter interaction. With this binding feature, the atom–ODWG system can act as an active optical device such as a single-photon emitter or a qubit. The enhanced coupling allows the channeling of photons from the emitter into a bound propagating mode and thus results in the efficient extraction

and manipulation of the emitted photons, which are the criteria for producing single-photon sources [27].

4. Conclusion

We have studied the SE dynamics of an omnidirectional waveguide with a two-level atom embedded in the low-index medium of the multilayer-film structure. The characteristic of the waveguide is denoted by the dispersion curve and background dissipations of the system through the memory kernels. The relative extent of the background dissipation is expressed in terms of the background dissipation rate γ_b with respect to the coupling constant between the atom and the waveguide mode. By applying fractional calculus to derive and solve the dynamical behavior of this optical system, a fractional kinetic equation and a corresponding solution of the system are obtained through manipulating the fractional operators. The dynamics of this system shows that there are two kinds of regimes behaving very differently. The population of the excited state in the ODWG with a fast group velocity decays exponentially implying propagating characteristics. The existence of background dissipation enhances the decay rate. The population of the excited state in the ODWG with a slow group velocity near the WG cutoff, however, exhibits non-exponential decay with damped Rabi oscillation, which is a typical feature of the non-Markovian effect. The strong light-matter interaction leads to bound propagating behavior of the fractional decay with a large value of the excited-state probability density, while the dissipation provides the channel to release the bound state photon due to the finite size or imperfect manufacture of the photonic crystals. The physical content of the atom-ODWG system is rich and worth investigating further both experimentally and theoretically.

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