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Supplier Selection Critical Decision Values for Processes with Multiple Independent Lines

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The process yield is the most common criterion considered for decision making in supplier selection problem. For normally distributed processes with multiple independent lines, the S_{pk}^M index provides an exact measurement for the overall yield. Therefore, the S_{pk}^M index can be implemented to deal with the supplier selection problem with processes having multiple independent lines. In this article, a test statistic obtained by a division method is employed to establish a hypothesis testing procedure, with two phases, which is developed to determine whether two suppliers are equally capable or not. The sampling distribution and the probability density function of the test statistic are derived. For various minimum requirements of process capability, number of lines, sample sizes, magnitudes of the difference between the two suppliers and the type I error, the critical values for decision making are presented. The required sample sizes for various designated powers at given type I error are tabulated. A thin-film transistor type liquid-crystal display application example is provided to demonstrate the testing procedure. Copyright © 2012 John Wiley & Sons, Ltd.

Keywords: critical value; multiple independent lines; supplier selection problem

1. Introduction

Supplier selection is a problem of comparing two or even more suppliers and selecting the one that has a significantly higher process capability. Process capability indices (PCI) have been widely used to be a criterion for dealing with supplier selection problem in manufacturing industries. Process yield is an important factor that needs to be considered in supplier selection problem. Boyles[1] proposed the S_{pk} index to provide an exact measurement for the process yield. However, processes with multiple characteristics or multiple independent lines often occur in practice. For processes with multiple independent characteristics, Chen *et al.*[2] firstly introduced the S_{pk}^T index to evaluate the process performance. Pearn and Cheng[3] investigated the relationship between process parameters and the sampling distribution of natural estimator of S_{pk}^T. Pearn *et al.*[4] derived the asymptotic distribution for the natural estimator of S_{pk}^T index under multiple samples. Recently, more investigations for processes with multiple characteristics include Pearn *et al.*[5–7]

For normally distributed processes with multiple independent lines, Tai *et al.*[8] proposed the overall yield index S_{pk}^M to establish the relationship between the actual overall process yield and the manufacturing specifications. Thus, the S_{pk}^M index can be used as a benchmark for evaluating process performance. Tai *et al.*[8] developed an effective method to measure the manufacturing yield for photolithography processes with multiple independent manufacturing lines by S_{pk}^M index.

For the supplier selection problem, Tai *et al.*[9] investigated the glass substrate processes selection problem in thin-film transistor type liquid-crystal display (TFT-LCD) manufacturing industries. Lin and Pearn[10] developed an analytical approach based on the yield index S_{pk} to compare two processes. Lin and Pearn[11] extended the results of Lin and Pearn[10] to cases with multiple independent manufacturing lines. Yum and Kim[12] and Wu *et al.*[13] provided some reviews and overviews for PCI.

2. The S_{pk}^M index for multiple independent lines

For a multiple independent lines process with k identical lines (flows), an overall capability index was proposed by Tai *et al.*[8] designed as follows:

$$S_{pk}^M = \frac{1}{3}\Phi^{-1}\left\{\left[\frac{1}{k}\sum_{j=1}^{k}\left(2\Phi\left(3S_{pkj}\right)-1\right)+1\right]/2\right\}, \tag{1}$$

where S_{pkj}, $j=1,\ldots,k$ is the S_{pk} index value of the jth line. $\Phi(\cdot)$ means the cumulative distribution function of the standard normal distribution. For normally distributed processes, the yield of the jth line P_j, $j=1,2,\ldots,k$ can be obtained by

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$$\text{Yield} = P_j = 2\Phi(3S_{pkj}) - 1. \tag{2}$$

Consequently, from Equation (1), a one-to-one relationship between the index S_{pk}^M and the overall process yield P can be presented as follows:

$$P = \frac{1}{k}\sum_{j=1}^{k} P_j = \frac{1}{k}\sum_{j=1}^{k}\left[2\Phi(3S_{pkj}) - 1\right] = 2\Phi\left(3S_{pk}^M\right) - 1. \tag{3}$$

Hence, the S_{pk}^M index provides an exact measurement of the yield for normally distributed processes with multiple independent lines. That is, the S_{pk}^M can be used to deal with the supplier selection problem on the basis of the overall process yield. Because the process parameters such as means and variances are unknown, in general, S_{pk}^M sample data should be collected to estimate the S_{pk}^M index. The natural estimator of S_{pk}^M, $\hat{S}_{pk}^M$, can be expressed as

$$\hat{S}_{pk}^M = \frac{1}{3}\Phi^{-1}\left\{\left[\frac{1}{k}\sum_{j=1}^{k}\left(2\Phi(3\hat{S}_{pkj}) - 1\right) + 1\right]/2\right\}, \tag{4}$$

where

$$\hat{S}_{pkj} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\left[\Phi\left(\frac{USL_j - \bar{X}_j}{S_j}\right) + \Phi\left(\frac{\bar{X}_j - LSL_j}{S_j}\right)\right]\right\}, \quad j = 1, 2, \ldots, k, \tag{5}$$

is the natural estimator of S_{pk} index value of the jth line[1]. The exact sampling distribution of $\hat{S}_{pk}^M$ is mathematically intractable. Tai *et al.*[8] used the first-order Taylor expansion for multiple variables to derive the asymptotic distribution of $\hat{S}_{pk}^M$ as

$$\hat{S}_{pk}^M \approx N\left(S_{pk}^M, \frac{D^2\phi(3D)}{2k^2 n\phi^2\left(3S_{pk}^M\right)}\right), \tag{6}$$

where

$$D = \frac{1}{3}\Phi^{-1}\left\{\left[k\left(2\Phi\left(3S_{pk}^M\right) - 1\right) - (k-2)\right]/2\right\}. \tag{7}$$

The results mentioned earlier can be implemented to compare two suppliers with multiple independent lines and normally distributed processes.

3. Supplier selection for processes with multiple independent lines

Consider two suppliers, suppliers I and II, supplier II claims that it has a significantly higher capability than supplier I. Our main object is to compare two suppliers and make a reliable decision at a given significance level α. Based on given data from the two suppliers with multiple independent lines, $\hat{S}_{pk1}^M$ and $\hat{S}_{pk2}^M$ would be first calculated. The quotient $R = \hat{S}_{pk2}^M/\hat{S}_{pk1}^M$ would then be considered. If the quotient $R = \hat{S}_{pk2}^M/\hat{S}_{pk1}^M$ is sufficiently large, then it is clear that supplier II is better than supplier I, and supplier II in this case would be selected. The critical decision values, however, must be determined by statistical hypothesis testing. When the suppliers have S_{pk}^M index values S_{pk1}^M and S_{pk2}^M, the testing of the hypothesis

$$\begin{aligned} H_0 &: S_{pk1}^M \geqslant S_{pk2}^M, \\ H_1 &: S_{pk1}^M < S_{pk2}^M. \end{aligned} \tag{8}$$

is considered to handle the supplier selection problem. Next, the probability density function of the test statistic R is derived explicitly.

4. Test statistic quotient *R*

In this section, we implement a test statistic R to investigate the hypothesis testing mentioned earlier. Firstly, Equation (8) can be represented as

$$\begin{aligned} H_0 &: \frac{S_{pk2}^M}{S_{pk1}^M} \leqslant 1, \\ H_1 &: \frac{S_{pk2}^M}{S_{pk1}^M} > 1. \end{aligned} \tag{9}$$

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The ratio of the two estimators, $R=\hat{S}^{M}_{pk2}/\hat{S}^{M}_{pk1}$, is applied to deal with the hypothesis testing of Equation (9). From Equation (6), the natural estimator $\hat{S}^{M}_{pk}$ is an asymptotic, normally distributed random variable. Consequently, the sampling distribution of the test statistic R is as follows:

$$R=\frac{\hat{S}^{M}_{pk2}}{\hat{S}^{M}_{pk1}}\approx\frac{N\left(S^{M}_{pk1},\ \frac{D_1{}^2\phi(3D_1)}{2k_1{}^2n_1\phi^2\left(3S^{M}_{pk1}\right)}\right)}{N\left(S^{M}_{pk2},\ \frac{D_2{}^2\phi(3D_2)}{2k_2{}^2n_2\phi^2\left(3S^{M}_{pk2}\right)}\right)}. \tag{10}$$

Thus, the distribution of the test statistic R is the quotient of two independent, normally distributed random variables and is related to the Cauchy distribution. Let $X\sim N\left(\mu_1,\sigma_1^2\right)$ and $Y\sim N\left(\mu_2,\sigma_2^2\right)$ be two independent random variables with normal distribution. Using the Jocobian transformation technique, the probability density function of $R=Y/X$ can be represented as

$$f_R(r)=\frac{1}{2\pi\sigma_1\sigma_2}\left\{2\sigma_3^2\exp\left(-\frac{\mu_3^2}{2\sigma_3^2}\right)+\mu_3\sigma_3\sqrt{2\pi}\left[1-2\Phi\left(\frac{\mu_3}{\sigma_3}\right)\right]\right\}\times\exp\left[-\frac{1}{2}\left(\frac{\mu_1^2}{\sigma_1^2}+\frac{\mu_2^2}{\sigma_2^2}-\frac{\mu_3^2}{\sigma_3^2}\right)\right], \tag{11}$$

where

$$\mu_3=\frac{\mu_1/\sigma_1^2+r\mu_2/\sigma_2^2}{1/\sigma_1^2+r^2/\sigma_2^2}=\frac{r\mu_2\sigma_1^2+\mu_1\sigma_2^2}{r^2\sigma_1^2+\sigma_2^2},\ \text{and}\ \sigma_3^2=\left[\frac{1}{\sigma_1^2}+\frac{r^2}{\sigma_2^2}\right]^{-1}=\frac{\sigma_1^2\sigma_2^2}{r^2\sigma_1^2+\sigma_2^2}. \tag{12}$$

Finally, the probability distribution of $R=\hat{S}^{M}_{pk2}/\hat{S}^{M}_{pk1}$, can be established by substituting the parameters as

$$\mu_1=S^{M}_{pk1},\ \mu_2=S^{M}_{pk2},\ \sigma_1^2=\frac{D_1{}^2\phi(3D_1)}{2k_1{}^2n_1\phi^2\left(3S^{M}_{pk1}\right)},\ \text{and}\ \sigma_2^2=\frac{D_2{}^2\phi(3D_2)}{2k_2{}^2n_2\phi^2\left(3S^{M}_{pk2}\right)}. \tag{13}$$

In the next section, on the basis of the sampling distribution of R developed in Equation (11), a procedure having two phases is proposed to deal with the supplier selection problem.

5. Supplier selection procedure

Let C denote the minimum requirement of S^{M}_{pk} values for all suppliers. When the existing supplier, supplier I, has achieved the process requirement (i.e., $S^{M}_{pk1}\geqslant C$), a new supplier, supplier II claims that its capability is better than suppler I. Our object is to compare two suppliers and make a reliable decision at a given significance level α risk.

5.1. *Phase I: selecting supplier with higher capability*

In the first phase, the hypothesis testing: $H_0: S^{M}_{pk2}\leqslant S^{M}_{pk1}$ versus $H_1: S^{M}_{pk2}>S^{M}_{pk1}$ is considered to test whether supplier II has a better process capability than supplier I or not. On the basis of the testing statistic $R=\hat{S}^{M}_{pk2}/\hat{S}^{M}_{pk1}$, and a given significance level α, the decision rule is to reject H_0 if $R\geqslant c_0$. The critical value c_0 satisfies the following equation:

$$\text{Type I Error}=P\left(R\geqslant c_0|H_0: S^{M}_{pk2}\leqslant S^{M}_{pk1}, n_1, n_2, k_1, k_2\ \text{and}\ S^{M}_{pk1}\geqslant C\right)\leqslant\alpha. \tag{14}$$

That is, the probability that falsely rejects H_0 is no more than α. Because the smaller the value of S^{M}_{pk2}/S^{M}_{pk1}, the larger the type I error is, then, we calculate the critical value c_0 under the conditions $S^{M}_{pk1}=S^{M}_{pk2}=C$. Therefore, the critical value c_0 can be obtained by solving the following equation

$$P\left(R\geqslant c_0|S^{M}_{pk1}=S^{M}_{pk2}=C, n_1, n_2, k_1, k_2\right)=\alpha. \tag{15}$$

Table I shows the critical values to test $H_1: S^{M}_{pk2}>S^{M}_{pk1}$ for various values of $k_1=k_2=k$ and $n_1=n_2=n=30(10)200$ at $\alpha=0.05$. It is to be noted that the critical value is the same as the result in Lin and Pearn.[10]

5.2. *Phase II: magnitude outperformed measurement*

In Phase I, the decision is based solely on the two S^{M}_{pk} values without further regard to the magnitude of the difference between the two suppliers. In practice, owing to the high cost of the process replacement, the supplier change is considered only if the new supplier significantly outperforms the existing supplier's capability by a given magnitude $h>0$. In this case, the proposed approach can be modified to test the corresponding hypothesis:

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$$H_0: S^M_{pk2} \leqslant S^M_{pk1} + h,$$
$$H_1: S^M_{pk2} > S^M_{pk1} + h. \quad (16)$$

The decision rule is similar to Phase I. We will reject H_0 and accept that $S^M_{pk2} > S^M_{pk1} + h$ when the test statistic R is greater than or equal to the critical value c_0, where c_0 satisfies the following:

$$\text{Type I Error} = P(R \geqslant c_0 | H_0 : S^M_{pk2} \leqslant S^M_{pk1} + h, n_1, n_2, k_1, k_2 \text{ and } S^M_{pk1} \geqslant C) \leqslant \alpha. \quad (17)$$

Similarly, the critical value c_0 is obtained by keeping the type I error less than α under the conditions $S^M_{pk1} = C$ and $S^M_{pk2} = C + h$. That is, c_0 is obtained by solving the following equations

$$P(R \geqslant c_0 | S^M_{pk1} = C, S^M_{pk2} = C + h, n_1, n_2, k_1, k_2) \leqslant \alpha. \quad (18)$$

If the decision is rejecting the null hypothesis (16), then we have sufficient evidence to conclude that supplier II is significantly better than supplier I by a magnitude of h. Table II shows the critical values for given numbers of lines $k_1 = k_2 = k = 2(1)5$, sample sizes $n_1 = n_2 = n = 30(10)200$, the magnitude of the difference between the two suppliers $h = 0.1(0.1)0.5$, and minimum requirements of suppliers $C = 1.00, 1.33, 1.50$.

5.3. Required sample size

The decision making in Phases I and II solely depends on the given type I error α risk, but does not consider the type II error β risk (or power), which is the probability of falsely accepting H_0. It is an unfavorable risk for the competing supplier. To decrease the β risk, then increasing the decision power at a given α risk, sample sizes need to be increased. Obviously, the larger the sample size, the smaller the β risk (the larger the power of test) is. By calculating the power for a specific combination of $\left(S^M_{pk1}, S^M_{pk2}\right)$, the minimal sample size required for various given power (or β risk) and α risk can be established. The required sample size can be obtained by solving the following two constraints

$$\text{Type I Error} = P(R \geqslant c_0 | H_0 : S^M_{pk2} \leqslant S^M_{pk1}, n_1, n_2, k_1, k_2 \text{ and } S^M_{pk1} \geqslant C) \leqslant \alpha, \quad (19)$$

$$\text{Power} = P(R \leqslant c_0 | H_1 : S^M_{pk2} > S^M_{pk1}, n_1, n_2, k_1, k_2 \text{ and } S^M_{pk1} \geqslant C) \geqslant 1 - \beta. \quad (20)$$

For application, Table III tabulates the required sample sizes for various minimal capability requirements $C = 1.00, 1.30, 1.50, 1.67$ and magnitude of difference $h = 0.15(0.05)1.00$ with given power $= 0.90, 0.95, 0.975, 0.99$ when the sample size and the number of lines of two suppliers are the same (i.e., $n_1 = n_2$, $k_1 = k_2$). For example, if two suppliers both have $k = 3$ lines, the minimal capability requirement $C = 1.30$, the designated α risk is 0.05, and the expected magnitude of difference $h = S^M_{pk2} - S^M_{pk1} = 0.3$, then it requires 183 samples from both suppliers to reach a testing power of 0.95 (i.e., β risk $= 0.05$).

Table I. Critical values for rejecting $S^M_{pk1} \leqslant S^M_{pk2}$ with $n_1 = n_2 = n = 30(10)200$ at $\alpha = 0.05$

	k									
n	1	2	3	4	5	6	7	8	9	10
30	1.3581	1.3037	1.2734	1.2526	1.2368	1.2242	1.2137	1.2046	1.1968	1.1899
40	1.3019	1.2571	1.2321	1.2148	1.2016	1.1910	1.1822	1.1747	1.1680	1.1622
50	1.2653	1.2266	1.2048	1.1897	1.1782	1.1690	1.1613	1.1547	1.1489	1.1438
60	1.2391	1.2046	1.1852	1.1717	1.1613	1.1531	1.1461	1.1402	1.1350	1.1304
70	1.2192	1.1879	1.1701	1.1578	1.1484	1.1409	1.1345	1.1291	1.1243	1.1201
80	1.2034	1.1746	1.1582	1.1468	1.1381	1.1311	1.1252	1.1202	1.1158	1.1118
90	1.1906	1.1637	1.1485	1.1378	1.1297	1.1231	1.1176	1.1129	1.1088	1.1051
100	1.1799	1.1546	1.1403	1.1303	1.1226	1.1164	1.1112	1.1068	1.1029	1.0994
110	1.1707	1.1469	1.1333	1.1238	1.1166	1.1107	1.1058	1.1016	1.0979	1.0946
120	1.1628	1.1402	1.1272	1.1182	1.1113	1.1057	1.1011	1.0970	1.0935	1.0904
130	1.1559	1.1343	1.1219	1.1133	1.1067	1.1014	1.0969	1.0930	1.0897	1.0867
140	1.1498	1.1290	1.1172	1.1090	1.1026	1.0975	1.0932	1.0895	1.0862	1.0833
150	1.1443	1.1244	1.1130	1.1050	1.0990	1.0940	1.0899	1.0863	1.0832	1.0804
160	1.1394	1.1202	1.1092	1.1015	1.0956	1.0909	1.0869	1.0835	1.0804	1.0778
170	1.1349	1.1164	1.1057	1.0983	1.0926	1.0881	1.0842	1.0809	1.0779	1.0753
180	1.1309	1.1129	1.1026	1.0954	1.0899	1.0854	1.0817	1.0785	1.0756	1.0732
190	1.1271	1.1097	1.0998	1.0928	1.0874	1.0831	1.0794	1.0763	1.0736	1.0711
200	1.1237	1.1068	1.0971	1.0903	1.0851	1.0809	1.0774	1.0743	1.0717	1.0693

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Table II. Critical values for rejecting $S_{pk1}^{M} \leqslant S_{pk2}^{M} + h$ with $\alpha = 0.05$ and $h = 0.1(0.1)0.5$

	h									
	0.10	0.20	0.30	0.40	0.50	0.10	0.20	0.30	0.40	0.50
$C = 1.00$										
n			$k = 2$					$k = 3$		
30	1.4374	1.5709	1.7044	1.8377	1.9710	1.4060	1.5384	1.6707	1.8028	1.9348
40	1.3857	1.5142	1.6426	1.7709	1.8990	1.3598	1.4873	1.6148	1.7421	1.8694
50	1.3518	1.4769	1.6020	1.7269	1.8518	1.3293	1.4537	1.5780	1.7022	1.8263
60	1.3274	1.4501	1.5728	1.6953	1.8178	1.3073	1.4294	1.5514	1.6733	1.7951
70	1.3089	1.4297	1.5505	1.6712	1.7919	1.2906	1.4109	1.5311	1.6513	1.7713
80	1.2941	1.4135	1.5328	1.6521	1.7713	1.2772	1.3962	1.5150	1.6337	1.7524
90	1.2820	1.4002	1.5184	1.6365	1.7545	1.2663	1.3841	1.5018	1.6194	1.7369
100	1.2719	1.3891	1.5063	1.6234	1.7404	1.2571	1.3740	1.4907	1.6073	1.7238
110	1.2633	1.3796	1.4959	1.6122	1.7283	1.2493	1.3653	1.4812	1.5970	1.7127
120	1.2558	1.3714	1.4870	1.6025	1.7179	1.2426	1.3578	1.4730	1.5881	1.7031
130	1.2493	1.3642	1.4791	1.5940	1.7088	1.2366	1.3512	1.4658	1.5802	1.6946
140	1.2435	1.3578	1.4722	1.5864	1.7007	1.2313	1.3454	1.4594	1.5733	1.6871
150	1.2383	1.3521	1.4660	1.5797	1.6935	1.2266	1.3402	1.4537	1.5671	1.6804
160	1.2336	1.3470	1.4604	1.5737	1.6869	1.2224	1.3355	1.4485	1.5615	1.6744
170	1.2294	1.3424	1.4553	1.5682	1.6810	1.2185	1.3312	1.4438	1.5564	1.6689
180	1.2255	1.3381	1.4507	1.5632	1.6756	1.2150	1.3273	1.4396	1.5518	1.6640
190	1.2220	1.3342	1.4464	1.5586	1.6706	1.2118	1.3238	1.4357	1.5475	1.6594
200	1.2187	1.3306	1.4425	1.5543	1.6661	1.2088	1.3205	1.4321	1.5436	1.6551
$C = 1.00$										
n			$k = 4$					$k = 5$		
30	1.3844	1.5161	1.6476	1.7790	1.9101	1.3681	1.4993	1.6303	1.7611	1.8917
40	1.3419	1.4689	1.5958	1.7225	1.8491	1.3283	1.4550	1.5814	1.7077	1.8339
50	1.3138	1.4377	1.5615	1.6852	1.8087	1.3019	1.4256	1.5490	1.6723	1.7955
60	1.2935	1.4152	1.5367	1.6582	1.7795	1.2829	1.4043	1.5255	1.6467	1.7677
70	1.2779	1.3979	1.5177	1.6375	1.7572	1.2682	1.3880	1.5076	1.6271	1.7464
80	1.2655	1.3842	1.5026	1.6211	1.7393	1.2566	1.3750	1.4932	1.6114	1.7294
90	1.2554	1.3728	1.4902	1.6075	1.7247	1.2470	1.3643	1.4815	1.5985	1.7155
100	1.2469	1.3634	1.4798	1.5962	1.7124	1.2390	1.3554	1.4716	1.5878	1.7038
110	1.2396	1.3553	1.4710	1.5865	1.7019	1.2322	1.3477	1.4632	1.5785	1.6938
120	1.2333	1.3483	1.4633	1.5781	1.6929	1.2262	1.3411	1.4558	1.5705	1.6851
130	1.2278	1.3422	1.4565	1.5707	1.6849	1.2210	1.3352	1.4494	1.5635	1.6774
140	1.2229	1.3367	1.4505	1.5642	1.6778	1.2164	1.3301	1.4437	1.5572	1.6707
150	1.2185	1.3318	1.4451	1.5583	1.6714	1.2122	1.3254	1.4386	1.5517	1.6647
160	1.2145	1.3274	1.4403	1.5531	1.6658	1.2085	1.3213	1.4340	1.5466	1.6592
170	1.2109	1.3234	1.4359	1.5483	1.6606	1.2051	1.3175	1.4298	1.5420	1.6542
180	1.2076	1.3198	1.4319	1.5439	1.6559	1.2020	1.3140	1.4260	1.5379	1.6497
190	1.2046	1.3165	1.4282	1.5399	1.6515	1.1991	1.3109	1.4225	1.5340	1.6455
200	1.2019	1.3134	1.4248	1.5362	1.6475	1.1965	1.3079	1.4193	1.5305	1.6417
$C = 1.33$										
n			$k = 2$					$k = 3$		
30	1.4261	1.5272	1.6283	1.7294	1.8304	1.4067	1.5073	1.6079	1.7083	1.8087
40	1.3718	1.4690	1.5661	1.6631	1.7602	1.3559	1.4526	1.5493	1.6459	1.7425
50	1.3363	1.4309	1.5253	1.6198	1.7142	1.3226	1.4167	1.5109	1.6050	1.6990
60	1.3108	1.4035	1.4961	1.5887	1.6812	1.2986	1.3910	1.4833	1.5755	1.6678
70	1.2915	1.3827	1.4739	1.5650	1.6562	1.2803	1.3713	1.4623	1.5531	1.6440
80	1.2761	1.3662	1.4562	1.5463	1.6363	1.2658	1.3557	1.4455	1.5353	1.6250
90	1.2635	1.3527	1.4418	1.5309	1.6200	1.2539	1.3429	1.4318	1.5207	1.6096
100	1.2530	1.3414	1.4297	1.5181	1.6064	1.2440	1.3322	1.4204	1.5085	1.5966
110	1.2440	1.3317	1.4194	1.5071	1.5948	1.2355	1.3231	1.4106	1.4981	1.5855
120	1.2363	1.3234	1.4106	1.4976	1.5847	1.2282	1.3152	1.4022	1.4891	1.5760
130	1.2295	1.3161	1.4027	1.4893	1.5759	1.2218	1.3083	1.3947	1.4811	1.5676
140	1.2234	1.3097	1.3958	1.4820	1.5681	1.2161	1.3022	1.3882	1.4742	1.5601
150	1.2181	1.3039	1.3897	1.4754	1.5611	1.2110	1.2967	1.3823	1.4679	1.5535

(*Continues*)

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Table II. *Continued.*

	h									
	0.10	0.20	0.30	0.40	0.50	0.10	0.20	0.30	0.40	0.50
160	1.2132	1.2987	1.3841	1.4695	1.5549	1.2064	1.2917	1.3770	1.4623	1.5475
170	1.2088	1.2940	1.3791	1.4641	1.5492	1.2022	1.2872	1.3722	1.4571	1.5421
180	1.2048	1.2897	1.3745	1.4592	1.5440	1.1984	1.2831	1.3678	1.4525	1.5371
190	1.2011	1.2857	1.3703	1.4548	1.5392	1.1950	1.2794	1.3638	1.4482	1.5326
200	1.1978	1.2821	1.3664	1.4506	1.5349	1.1918	1.2760	1.3601	1.4442	1.5284
C = 1.33										
n			*k* = 4					*k* = 5		
30	1.3933	1.4935	1.5937	1.6937	1.7937	1.3829	1.4829	1.5828	1.6826	1.7823
40	1.3448	1.4413	1.5377	1.6340	1.7302	1.3363	1.4325	1.5287	1.6248	1.7208
50	1.3129	1.4069	1.5008	1.5947	1.6885	1.3055	1.3993	1.4931	1.5868	1.6803
60	1.2900	1.3822	1.4743	1.5663	1.6584	1.2833	1.3754	1.4674	1.5593	1.6512
70	1.2725	1.3633	1.4541	1.5448	1.6354	1.2665	1.3572	1.4478	1.5384	1.6289
80	1.2586	1.3483	1.4380	1.5276	1.6172	1.2530	1.3427	1.4322	1.5217	1.6112
90	1.2472	1.3361	1.4248	1.5136	1.6023	1.2420	1.3308	1.4195	1.5081	1.5967
100	1.2377	1.3258	1.4138	1.5018	1.5897	1.2328	1.3209	1.4088	1.4967	1.5845
110	1.2296	1.3170	1.4044	1.4918	1.5791	1.2250	1.3124	1.3997	1.4869	1.5741
120	1.2225	1.3094	1.3962	1.4831	1.5698	1.2182	1.3050	1.3917	1.4784	1.5651
130	1.2164	1.3028	1.3891	1.4754	1.5617	1.2122	1.2985	1.3848	1.4710	1.5572
140	1.2109	1.2969	1.3828	1.4687	1.5545	1.2068	1.2928	1.3787	1.4644	1.5502
150	1.2060	1.2916	1.3771	1.4626	1.5481	1.2021	1.2877	1.3731	1.4586	1.5439
160	1.2016	1.2868	1.3720	1.4572	1.5423	1.1979	1.2830	1.3682	1.4533	1.5383
170	1.1976	1.2825	1.3674	1.4522	1.5370	1.1940	1.2789	1.3637	1.4485	1.5332
180	1.1939	1.2786	1.3632	1.4478	1.5323	1.1904	1.2750	1.3596	1.4441	1.5285
190	1.1906	1.2750	1.3593	1.4436	1.5279	1.1872	1.2715	1.3558	1.4400	1.5243
200	1.1875	1.2716	1.3558	1.4398	1.5238	1.1842	1.2683	1.3523	1.4363	1.5203
C = 1.50										
n			*k* = 2					*k* = 3		
30	1.4213	1.5112	1.6010	1.6908	1.7805	1.4057	1.4951	1.5846	1.6740	1.7633
40	1.3663	1.4525	1.5388	1.6250	1.7112	1.3534	1.4394	1.5254	1.6113	1.6972
50	1.3303	1.4142	1.4982	1.5820	1.6659	1.3192	1.4029	1.4866	1.5702	1.6538
60	1.3045	1.3868	1.4690	1.5512	1.6334	1.2947	1.3767	1.4587	1.5407	1.6227
70	1.2849	1.3659	1.4468	1.5278	1.6087	1.2760	1.3568	1.4375	1.5183	1.5990
80	1.2694	1.3493	1.4293	1.5092	1.5891	1.2611	1.3409	1.4207	1.5004	1.5802
90	1.2567	1.3358	1.4149	1.4940	1.5731	1.2490	1.3280	1.4069	1.4859	1.5648
100	1.2460	1.3245	1.4029	1.4813	1.5597	1.2388	1.3171	1.3954	1.4737	1.5519
110	1.2370	1.3148	1.3926	1.4705	1.5483	1.2301	1.3079	1.3856	1.4632	1.5409
120	1.2291	1.3064	1.3838	1.4611	1.5384	1.2226	1.2999	1.3771	1.4542	1.5314
130	1.2223	1.2991	1.3760	1.4528	1.5297	1.2161	1.2928	1.3696	1.4463	1.5231
140	1.2162	1.2927	1.3691	1.4456	1.5220	1.2103	1.2867	1.3630	1.4393	1.5157
150	1.2108	1.2869	1.3630	1.4391	1.5152	1.2051	1.2811	1.3571	1.4331	1.5091
160	1.2059	1.2817	1.3575	1.4332	1.5090	1.2004	1.2761	1.3518	1.4275	1.5031
170	1.2015	1.2770	1.3525	1.4279	1.5034	1.1961	1.2716	1.3470	1.4224	1.4978
180	1.1974	1.2727	1.3479	1.4231	1.4983	1.1922	1.2675	1.3426	1.4177	1.4928
190	1.1937	1.2687	1.3437	1.4187	1.4936	1.1887	1.2636	1.3386	1.4135	1.4884
200	1.1903	1.2650	1.3398	1.4146	1.4894	1.1854	1.2601	1.3349	1.4095	1.4842
C = 1.50										
n			*k* = 4					*k* = 5		
30	1.3947	1.4840	1.5731	1.6622	1.7512	1.3863	1.4754	1.5643	1.6532	1.7420
40	1.3444	1.4302	1.5160	1.6017	1.6873	1.3375	1.4232	1.5088	1.5943	1.6798
50	1.3114	1.3950	1.4785	1.5619	1.6453	1.3054	1.3889	1.4723	1.5556	1.6388
60	1.2877	1.3696	1.4515	1.5334	1.6152	1.2824	1.3642	1.4460	1.5277	1.6094
70	1.2697	1.3503	1.4310	1.5116	1.5922	1.2648	1.3454	1.4259	1.5065	1.5869
80	1.2553	1.3350	1.4147	1.4943	1.5739	1.2508	1.3304	1.4100	1.4896	1.5691
90	1.2435	1.3224	1.4013	1.4802	1.5590	1.2394	1.3182	1.3970	1.4758	1.5545
100	1.2337	1.3119	1.3901	1.4683	1.5464	1.2298	1.3079	1.3861	1.4642	1.5422

(*Continues*)

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Table II. *Continued.*

	h									
	0.10	0.20	0.30	0.40	0.50	0.10	0.20	0.30	0.40	0.50
110	1.2253	1.3030	1.3806	1.4582	1.5357	1.2216	1.2992	1.3768	1.4543	1.5318
120	1.2181	1.2952	1.3723	1.4494	1.5265	1.2146	1.2917	1.3687	1.4457	1.5227
130	1.2117	1.2884	1.3651	1.4418	1.5184	1.2084	1.2850	1.3616	1.4382	1.5148
140	1.2061	1.2824	1.3587	1.4350	1.5112	1.2029	1.2791	1.3554	1.4316	1.5078
150	1.2011	1.2770	1.3530	1.4289	1.5048	1.1979	1.2739	1.3498	1.4256	1.5015
160	1.1965	1.2722	1.3478	1.4234	1.4990	1.1935	1.2692	1.3447	1.4203	1.4958
170	1.1924	1.2678	1.3431	1.4185	1.4937	1.1895	1.2648	1.3401	1.4154	1.4907
180	1.1886	1.2637	1.3389	1.4139	1.4890	1.1858	1.2609	1.3360	1.4110	1.4860
190	1.1852	1.2601	1.3349	1.4098	1.4846	1.1825	1.2573	1.3322	1.4069	1.4817
200	1.1820	1.2567	1.3313	1.4060	1.4806	1.1794	1.2540	1.3286	1.4032	1.4778

Table III. Sample size required for testing $H_0: S_{pk2}^M \leqslant S_{pk1}^M$ versus $H_1: S_{pk2}^M \leqslant S_{pk1}^M$ at $\alpha = 0.05$

		Power						Power			
S_{pk1}^M	S_{pk2}^M	0.90	0.95	0.975	0.99	S_{pk1}^M	S_{pk2}^M	0.90	0.95	0.975	0.99
		$k=2$						$k=2$			
1.00	1.15	338	429	515	626	1.30	1.45	612	774	930	1129
	1.20	200	254	305	371		1.50	357	452	544	660
	1.25	135	171	205	250		1.55	238	301	361	439
	1.30	98	124	150	182		1.60	171	217	260	316
	1.35	76	96	115	140		1.65	130	165	198	241
	1.40	61	77	92	113		1.70	103	131	157	191
	1.45	50	63	76	93		1.75	85	107	129	157
	1.50	42	54	65	79		1.80	71	90	108	131
	1.55	36	46	56	68		1.85	61	77	92	112
	1.60	32	41	49	60		1.90	53	67	80	97
	1.65	28	36	43	53		1.95	46	59	71	86
	1.70	25	32	39	48		2.00	41	52	63	76
	1.75	23	29	35	43		2.05	37	47	56	69
	1.80	21	27	32	39		2.10	34	42	51	62
	1.85	19	25	30	36		2.15	31	39	47	57
	1.90	18	23	27	34		2.20	28	36	43	52
	1.95	17	21	26	31		2.25	26	33	40	48
	2.00	16	20	24	29		2.30	24	31	37	45
		$k=2$						$k=2$			
1.50	1.65	834	1054	1267	1537	1.67	1.82	1047	1324	1589	1929
	1.70	485	613	736	894		1.87	607	767	921	1118
	1.75	320	405	487	591		1.92	400	505	607	737
	1.80	230	291	349	424		1.97	286	361	434	527
	1.85	174	220	265	322		2.02	216	273	328	398
	1.90	138	174	209	254		2.07	170	215	258	314
	1.95	112	142	170	207		2.12	138	174	210	255
	2.00	93	118	142	173		2.17	115	145	174	212
	2.05	80	101	121	147		2.22	97	123	148	180
	2.10	69	87	105	127		2.27	84	106	128	155
	2.15	60	76	92	112		2.32	74	93	112	136
	2.20	54	68	81	99		2.37	65	82	99	120
	2.25	48	61	73	89		2.42	58	73	88	107
	2.30	43	55	66	80		2.47	52	66	80	97
	2.35	39	50	60	73		2.52	48	60	72	88
	2.40	36	46	55	67		2.57	44	55	66	81
	2.45	33	42	51	61		2.62	40	50	60	73

(*Continues*)

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Table III. *Continued.*

S_{pk1}^M	S_{pk2}^M	Power 0.90	0.95	0.975	0.99	S_{pk1}^M	S_{pk2}^M	Power 0.90	0.95	0.975	0.99
	2.50	31	39	47	57		2.67	36	46	55	67
		$k=3$						$k=3$			
1.00	1.15	286	362	436	531	1.30	1.45	553	700	841	1022
	1.20	170	215	259	316		1.50	323	410	493	599
	1.25	114	145	175	213		1.55	215	273	328	399
	1.30	84	106	128	156		1.60	155	197	237	288
	1.35	65	82	99	121		1.65	118	150	181	220
	1.40	52	66	80	97		1.70	94	119	144	175
	1.45	43	55	66	81		1.75	77	98	118	143
	1.50	36	46	56	69		1.80	65	82	99	120
	1.55	31	40	49	59		1.85	55	70	85	103
	1.60	28	35	43	52		1.90	48	61	74	90
	1.65	25	31	38	46		1.95	42	54	65	79
	1.70	22	28	34	42		2.00	38	48	58	70
	1.75	20	26	31	38		2.05	34	43	52	63
	1.80	18	23	28	35		2.10	31	39	47	58
	1.85	17	22	26	32		2.15	28	36	43	53
	1.90	16	20	24	30		2.20	26	33	40	48
	1.95	15	19	23	28		2.25	24	30	37	45
	2.00	14	17	21	26		2.30	22	28	34	42
		$k=3$						$k=3$			
1.50	1.65	772	977	1174	1425	1.67	1.82	984	1244	1495	1815
	1.70	450	569	683	831		1.87	571	721	867	1053
	1.75	297	376	453	550		1.92	376	476	572	695
	1.80	213	270	325	395		1.97	269	340	409	497
	1.85	162	205	247	300		2.02	203	257	309	376
	1.90	128	162	195	237		2.07	160	203	244	296
	1.95	104	132	159	193		2.12	130	165	198	241
	2.00	87	110	133	162		2.17	108	137	165	201
	2.05	74	94	113	138		2.22	92	117	140	170
	2.10	64	81	98	119		2.27	79	101	121	147
	2.15	56	71	86	105		2.32	70	88	106	129
	2.20	50	63	76	93		2.37	62	78	94	114
	2.25	45	57	68	83		2.42	55	70	84	102
	2.30	41	51	62	75		2.47	50	63	76	92
	2.35	37	47	56	69		2.52	45	57	69	84
	2.40	34	43	52	63		2.57	41	53	63	77
	2.45	31	39	48	58		2.62	38	48	58	70
	2.50	29	37	44	54		2.67	36	45	55	67
		$k=4$						$k=4$			
1.00	1.15	251	319	384	468	1.30	1.45	513	650	781	949
	1.20	149	190	229	279		1.50	301	381	458	557
	1.25	101	129	155	190		1.55	200	254	306	372
	1.30	74	94	114	139		1.60	145	183	221	269
	1.35	57	73	88	108		1.65	110	140	169	206
	1.40	46	59	71	87		1.70	88	111	134	164
	1.45	38	49	59	73		1.75	72	91	110	134
	1.50	33	42	50	62		1.80	60	77	93	113
	1.55	28	36	44	54		1.85	52	66	79	97
	1.60	25	32	39	47		1.90	45	57	69	84
	1.65	22	28	34	42		1.95	40	50	61	74
	1.70	20	25	31	38		2.00	35	45	54	66
	1.75	18	23	28	35		2.05	32	41	49	60
	1.80	17	21	26	32		2.10	29	37	44	54
	1.85	15	20	24	29		2.15	26	34	41	50

(*Continues*)

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Table III. *Continued.*

S_{pk1}^{M}	S_{pk2}^{M}	Power 0.90	0.95	0.975	0.99	S_{pk1}^{M}	S_{pk2}^{M}	Power 0.90	0.95	0.975	0.99
	1.90	14	18	22	27		2.20	24	31	37	46
	1.95	13	17	21	26		2.25	23	29	35	42
	2.00	12	16	19	24		2.30	21	27	32	39
		$k=4$						$k=4$			
1.50	1.65	730	924	1111	1349	1.67	1.82	941	1190	1429	1735
	1.70	425	538	647	787		1.87	546	690	830	1008
	1.75	282	356	429	521		1.92	360	456	548	665
	1.80	202	256	308	375		1.97	258	326	392	476
	1.85	154	195	234	285		2.02	195	247	297	361
	1.90	121	154	185	225		2.07	154	194	234	284
	1.95	99	126	151	184		2.12	125	158	190	231
	2.00	83	105	126	154		2.17	104	132	158	193
	2.05	71	89	108	131		2.22	88	112	135	164
	2.10	61	77	93	114		2.27	76	97	116	142
	2.15	54	68	82	100		2.32	67	85	102	124
	2.20	48	60	73	89		2.37	59	75	90	110
	2.25	43	54	65	80		2.42	53	67	81	98
	2.30	39	49	59	72		2.47	48	60	73	89
	2.35	35	45	54	66		2.52	43	55	66	81
	2.40	32	41	49	60		2.57	40	50	61	74
	2.45	30	38	45	56		2.62	36	46	55	67
	2.50	28	35	42	51		2.67	33	42	50	61
		$k=5$						$k=5$			
1.00	1.15	226	287	346	422	1.30	1.45	483	612	736	895
	1.20	135	172	207	253		1.50	283	359	432	526
	1.25	91	117	141	172		1.55	189	240	289	351
	1.30	67	86	104	127		1.60	137	173	209	254
	1.35	52	67	81	99		1.65	104	133	160	195
	1.40	42	54	65	80		1.70	83	105	127	155
	1.45	35	45	54	67		1.75	68	87	104	127
	1.50	30	38	46	57		1.80	57	73	88	107
	1.55	26	33	40	49		1.85	49	62	75	92
	1.60	23	29	35	44		1.90	43	54	66	80
	1.65	20	26	32	39		1.95	38	48	58	71
	1.70	18	24	29	35		2.00	34	43	52	63
	1.75	17	21	26	32		2.05	30	39	47	57
	1.80	15	20	24	30		2.10	28	35	42	52
	1.85	14	18	22	27		2.15	25	32	39	48
	1.90	13	17	21	25		2.20	23	30	36	44
	1.95	12	16	19	24		2.25	21	27	33	41
	2.00	11	15	18	22		2.30	20	25	31	38
		$k=5$						$k=5$			
1.50	1.65	698	884	1063	1291	1.67	1.82	908	1148	1380	1675
	1.70	407	515	620	754		1.87	527	666	802	974
	1.75	270	342	411	500		1.92	348	440	529	643
	1.80	194	246	295	360		1.97	249	315	379	461
	1.85	147	187	225	273		2.02	188	239	287	349
	1.90	116	148	178	217		2.07	148	188	226	275
	1.95	95	121	145	177		2.12	121	153	184	224
	2.00	79	101	121	148		2.17	101	128	154	187
	2.05	68	86	104	126		2.22	86	108	131	159
	2.10	59	75	90	109		2.27	74	94	113	137
	2.15	52	65	79	96		2.32	65	82	99	120
	2.20	46	58	70	86		2.37	57	73	87	107
	2.25	41	52	63	77		2.42	51	65	78	95

(*Continues*)

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Table III. *Continued.*

S_{pk1}^M	S_{pk2}^M	Power 0.90	0.95	0.975	0.99	S_{pk1}^M	S_{pk2}^M	Power 0.90	0.95	0.975	0.99
	2.30	37	47	57	69		2.47	46	59	71	86
	2.35	34	43	52	63		2.52	42	53	64	79
	2.40	31	39	48	58		2.57	38	49	59	72
	2.45	29	36	44	54		2.62	35	44	53	64
	2.50	27	34	41	50		2.67	31	40	48	58

6. Supplier selection for thin-film transistor type liquid-crystal display

Manufacturing yield has been the most basic common criterion used in the manufacturing industry for measuring process performance. Because of fiercer competition in the global TFT-LCD industry, the supplier must have very low fraction of defectives, normally measured by parts per million (ppm) or parts per billion (ppb). Therefore, the multiple independent lines yield index S_{pk}^M can be used as a criterion to select the suppliers. For the investigated model of TFT-LCD[10], the target thickness value of a glass substrate is set to $T = 0.70$ mm with upper specification limit $USL = 0.77$ mm and lower specification limit $LSL = 0.63$ mm. When the minimum requirement of the supplier is $S_{pk}^M = 1.00$, and two suppliers both have $k = 4$ lines, $n_1 = n_2 = 150$ data are collected for suppliers I and II. The calculated sample means, sample standard deviations and the estimated S_{pki} index values for each line are summarized in Table IV.

6.1. *Phase I: select a supplier with higher capability*

To determine whether supplier II has a better process capability than supplier I or not, the hypothesis testing: $H_0 : S_{pk2}^M \leqslant S_{pk1}^M$ versus $H_1 : S_{pk2}^M > S_{pk1}^M$ is considered. From Table IV, we have $S_{pk1}^M = 1.055755$, $S_{pk2}^M = 1.407204$ and thus $R = 1.332889$. At $\alpha = 0.05$, $k = 4$ and $n_1 = n_2 = 150$, from Table I, the critical value is $c_0 = 1.1050$. Because the test statistic $R = 1.332889 > 1.1050$, we conclude that supplier II is better than supplier I with a 95% confidence level. Next, the second-phase testing would investigate the magnitude of the capability difference between the two suppliers.

6.2. *Phase II: magnitude outperformed*

The hypothesis testing $H_0 : S_{pk2}^M \leqslant S_{pk1}^M + h$ versus $H_1 : S_{pk2}^M > S_{pk1}^M + h$ is performed. For various values of the magnitude h, the decisions of the hypotheses are shown in Table V ($\alpha = 0.05$). The decision maker would replace the existing supplier when supplier II (competition) significantly outperforms supplier I by a magnitude of 0.20. On the basis of the testing result in Table V, we conclude that supplier II (competition) has a manufacturing capability that is significantly better than Supplier I by a magnitude of 0.20, that is, $S_{pk2}^M > S_{pk1}^M + 0.2$. Consequently, the supplier replacement would be suggested.

Table IV. Estimated values of capability indices for suppliers I and II

Suppliers	Lines	$\bar{x}$	s	$\hat{S}_{pki}$
I	1	0.7098303	0.0192028	1.108760
	2	0.7104621	0.0215073	0.992385
	3	0.7104065	0.0192131	1.099091
	4	0.7140126	0.0187125	1.065612
II	1	0.7001798	0.0142802	1.633835
	2	0.6969854	0.0166799	1.378086
	3	0.6976766	0.0172959	1.337498
	4	0.7001785	0.0137853	1.692482

Table V. Critical values and decisions of testing the two suppliers ($\alpha = 0.05$, $k = 4$, $n = 150$)

Test cases	I	II	III	IV	V
S_{pk1}^M	1.00	1.00	1.00	1.00	1.00
S_{pk2}^M	1.10	1.20	1.21	1.22	1.23
h	0.10	0.20	0.21	0.22	0.23
c_0	1.21847	1.33183	1.343152	1.354492	1.365816
Decision	Reject H_0	Reject H_0	Non-Reject H_0	Non-Reject H_0	Non-Reject H_0

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6.3. Sample size required for designated power

For the cases in which the minimal requirement $C = 1.00$ and number of lines $k = 4$, the decision maker would replace the existing supplier with designated power 0.95 when the new supplier has an S_{pk}^{M} index value significantly higher than the existing process by a scale of 0.20. The required sample size is 190 as shown in Table III. In the application example mentioned earlier, because the sample sizes of two suppliers are smaller than the required sample size ($150 < 190$), the power would be less than 0.95. In fact, the power of test for $S_{pk2}^{M} = 1.20$ is 90.18%. That is, the β risk is up to 9.82%. To reduce the β risk and increase the decision power, we would suggest the decision maker to collect more samples as recommended in Table III.

7. Conclusions

In this article, the supplier selection problem for normal processes with multiple independent lines was investigated; the overall yield index S_{pk}^{M} provided a one-to-one relationship between the specification limits and the overall process yield. A two-phase procedure on the basis of the quotient test statistic was proposed to deal with the supplier selection problem. The probability density function of the test statistic was also established. For applications, some tables of the critical values for decision making were presented under various minimal capability requirements, magnitudes of difference of two suppliers, number of lines, and sample sizes. The required sample sizes to make a reliable decision were also provided for various given power. A TFT-LCD application was presented.

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