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Heterotic massive Einstein-Yang-Mills-type symmetry and Ward identity

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Abstract

We show that there exist spontaneously broken symmetries for massive modes with transformation parameters $\theta^{(ab)}{}_{[\mu\nu]}$, $\theta^{(ab)}{}_{\mu}$ etc. containing both Einstein and $E_8 \otimes E_8$ (or SO(32)) Yang-Mills indices in the 10D Heterotic string. The corresponding on-shell Ward identities are also constructed.

It was believed that the 10D fundamental string theory possesses an infinite space-time symmetry structure at high energy [1]. The recently developed toy 2D string model reveals that this is indeed the case [2]. This is also suggested from, for example, the string field theory formalism [3] although we still have no satisfactory nonperturbative second quantized closed string formulation. In this paper, we would like to address this issue from the *first* quantized supersymmetric *massive* σ-model point of view. We choose to work on the 10D Heterotic string etc. [4] as it is still the one of most phenomenological interest. There have been very few articles in the literature discussing the bosonic massive σ-model [5], not to mention the supersymmetric one. This is mainly because it contains nonrenormalizable terms, and thus makes the perturbative renormalization group β-function calculation uncontrollable. In this short note, however, we will use the nonperturbative weak field approximation (WFA) [1] instead of the usual perturbative loop (α') expansion scheme. This will avoid the difficulty of the nonrenormalizability of the massive σ-model (at least, to the first order WFA). Moreover, the results we obtain from WFA are valid even at high energy ($\alpha' \to \infty$), thus WFA is the appropriate approximation scheme to study the high energy behavior of string. We will first discuss symmetry of the massless mode in the Yang-Mills sector aiming to relate the $E_8 \otimes E_8$ or SO(32) gauge symmetry to the right-moving gauge state of the spectrum. This will include both the S-matrix approach and the *modified weak field* σ-model approach. We then generalize both approaches to the first (with appropriate GSO-like projection) massive case. After deriving the massive Ward identities corresponding to the massive right-moving gauge states of the spectrum, we will also construct the explicit form of the weak field symmetry transformation laws. *We find that the symmetry transformation parameters contain* $\theta^{(ab)}{}_{[\mu\nu]}$, $\theta^{(ab)}{}_{\mu}$ *etc. with mixed Einstein and* $E_8 \otimes E_8$ (*or* SO(32)) *Yang-Mills indices, which point field theories would never have.* This reveals the peculiar structure of Heterotic string theory at high energy in contrast to the renormalizable point field theories.

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We first discuss the massless Ward identity in the Yang-Mills sector of the Heterotic string. The vertex operator for the massless gauge bosons in the (0) and (− 1) ghost pictures are

$$V_0 = \epsilon_\mu{}^a V_0'^\mu(z)\bar{V}^a(\bar{z}) = \epsilon_\mu{}^a(\partial X^\mu + ik\cdot\psi\psi^\mu)\bar{J}^a(\bar{z})e^{ikX(z,\bar{z})}\,, \tag{1}$$

$$V_{-1} = \epsilon_\mu{}^a V_{-1}^\mu(z)\bar{V}^a(\bar{z}) = \epsilon_\mu{}^a\psi^\mu e^{-\phi}\bar{J}^a(\bar{z})e^{ikX(z,\bar{z})}\,, \tag{2}$$

with $k^\mu\epsilon_\mu{}^a=0$, $k^2=0$. In Eqs. (1) and (2), $\bar{J}^a$ form the adjoint representation of either SO(32) or $E_8\otimes E_8$ Kac-Moody algebra and ϕ is the bosonized superconformal ghost. The following Ward identity can be easily verified after calculating the four point correlator [6]:

$$k_\mu T^{\lambda a\mu b\nu c\alpha d} = k_\mu\int\prod_{i=1}^{4}d^2x_i\langle V_0'^\lambda(x_1)\bar{V}^a(x_1)V_0'^\mu(x_2)\bar{V}^b(x_2)V_{-1}^\nu(x_3)\bar{V}^c(x_3)V_{-1}^\alpha(x_4)\bar{V}^d(x_4)\rangle = 0. \tag{3}$$

Note that we have chosen the ghost pictures for the vertex operators such that the total superconformal ghost charge adds up to -2. The momentum k_μ is arbitrary chosen to be the momentum of the second vertex operator. Eq. (3) can be interpreted as the result of a gauge state in the spectrum. In fact, there are two types of gauge states in the right-moving NS-sector of the spectrum, they are (we use the notation in [7])

$$\text{Type I:}\quad G_{-1/2}|\chi\rangle,\quad\text{where } G_{1/2}|\chi\rangle = G_{3/2}|\chi\rangle = L_0|\chi\rangle = 0\,, \tag{4}$$

$$\text{Type II:}\quad (G_{-3/2}+2G_{-1/2}L_{-1})|\xi\rangle,\quad\text{where } G_{1/2}|\xi\rangle = G_{3/2}|\xi\rangle = (L_0+1)|\xi\rangle = 0\,. \tag{5}$$

Note that type I states have zero-norm at any space-time dimension while type II states have zero-norm *only* at $D=10$. The following massless gauge state solution of Eq. (4),

$$k_\mu b_{-1/2}^\mu\bar{J}^a|0,k\rangle\,, \tag{6}$$

is the origin of the Ward identity (3). We now discuss the symmetry corresponding to the Ward identity in (3) from modified supersymmetric σ-model point of view [1]. Instead of using the usual σ-model approach starting from a worldsheet action, we will write down directly the weak field energy-momentum tensor T and the supercurrent T_F based on the consideration of vertex operators. Assume that T and T_F have the following weak field expansion:

$$T = T^{(0)} + T^{(1)} + \ldots;\quad T_F = T_F^{(0)} + T_F^{(1)} + \ldots \tag{7}$$

and the same for the left-moving pieces. In Eq. (7), each $T^{(0)}$ represents the corresponding free piece and each $T^{(1)}$ contains the weak background fields. An *on-fixed-point superconformal deformation* $(T^{(1)}, \bar{T}^{(1)}, T_F^{(1)}, \bar{T}_F^{(1)})$ is defined to be a deformation such that after deforming, the superconformal charge algebra for T's is kept the same as that of $T^{(0)}$'s [1]. It can be proved that a convenient choice to satisfy the above conditions are $T^{(1)} = \bar{T}^{(1)}$, $\bar{T}_F^{(0)} = \bar{T}_F^{(1)} = 0$ (for the gauge sector) and $T_F^{(1)}$ has conformal dimension (1/2, 1) w.r.t. $T^{(0)}$ and $\bar{T}^{(0)}$ [1]. Now the superconformal deformation constructed from Eq. (1) is [1]

$$T^{(1)} = A_\mu{}^a\partial X^\mu\bar{J}^a + \partial_{[\nu}A_{\mu]}{}^a\psi^\nu\psi^\mu\bar{J}^a\,,$$
$$T_F^{(1)} = \tfrac{1}{2}A_\mu{}^a\psi^\mu\bar{J}^a\,, \tag{8}$$

with conditions

$$\Box A_\mu{}^a = 0,\quad \partial^\mu A_\mu{}^a = 0. \tag{9}$$

Eq. (9) is the conditions of vanishing renormalization group β-function of supersymmetric σ-model in the first order WFA and can be interpreted as the linearized Yang-Mills equation in the covariant gauge. On the other hand, the superconformal deformation constructed from gauge state (6) is

$$\delta T = \partial_\mu\theta^a\partial X^\mu\bar{J}^a,\quad \delta T_F = \tfrac{1}{2}\partial_\mu\theta^a\psi^\mu\bar{J}^a\,, \tag{10}$$

with condition $\Box\theta^a=0$. These induce the linearized form of the $E_8\otimes E_8$ or SO(32) gauge transformation in the covariant gauge

$$\delta A_\mu{}^a=\partial_\mu\theta^a\,. \tag{11}$$

We have thus related the massless Yang-Mills gauge symmetry of the Heterotic string to the right-moving gauge state in the spectrum. One can get a similar result for the gravitational sector. Note that the gauge conditions in Eq. (9) and θ^a in Eq. (10) are due to the on-shell formulation of the first quantized string. However, it was shown how to dispense with these gauge conditions in Ref. [8]. Eq. (11) is valid to all orders in α' in contrast to the usual α' four-loop calculation [9]. It does not contain the usual homogeneous piece $C^{abc}A_\mu{}^b\theta^c$. However, this should not be thought of as a drawback of the formalism, as has been pointed out in [1]. In fact, there are infinitely many terms *in string theory* in contrast to the usual Yang-Mills theory if one wants to include them [10]. Moreover, the advantage of the WFA formalism above is that one can generalize the calculation to massive particles, as we now turn to discuss.

We first discuss the massive on-shell Ward identities. We will arbitrarily choose two particles in the first massive Yang-Mills sector to illustrate the identities [7]. Its vertex operators in the (o) ghost picture read

$$\begin{aligned} V_0^{(1)} &= [\epsilon^{(ab)}{}_{[\mu\nu\lambda]}(3\partial X^\mu+ik\psi\psi^\mu)\psi^\nu\psi^\lambda-i\epsilon^{(ab)}{}_{[\mu\nu]}(-\psi^\mu\partial\psi^\nu+ik\psi\psi^\mu\partial X^\nu)]\bar J^{(a}\bar J^{b)}e^{ikX(z,\bar z)}\\ &=\epsilon^{(ab)}{}_{[\mu\nu\lambda]}V_0^{[\mu\nu\lambda]}\bar V^{(ab)}-i\epsilon^{(ab)}{}_{[\mu\nu]}V_0^{[\mu\nu]}\bar V^{(ab)}\,;\\ \epsilon^{(ab)}{}_{[\mu\nu]}&=k^\lambda\epsilon^{(ab)}{}_{[\lambda\mu\nu]}\,, \end{aligned} \tag{12}$$

$$\begin{aligned} V_0^{(2)} &= [\epsilon^{(ab)}{}_{(\mu\nu)}(\partial X^\mu\partial X^\nu-\psi^\mu\partial\psi^\nu+ik\psi\psi^\mu\partial X^\nu)-i/2\,\epsilon^{(ab)}{}_\mu(\partial^2X^\mu+ik\psi\partial\psi^\mu)]\bar J^{(a}\bar J^{b)}e^{ikX(z,\bar z)}\\ &= \epsilon^{(ab)}{}_{(\mu\nu)}V_0^{(\mu\nu)}\bar V^{(ab)}-i/2\,\epsilon^{(ab)}{}_\mu V_0^\mu\bar V^{(ab)}\,;\\ \epsilon^{(ab)}{}_\mu&=-k^\nu\epsilon^{(ab)}{}_{\nu\mu},\quad \epsilon^{(ab)}{}_\mu{}^\mu-k^\mu k^\nu\epsilon^{(ab)}{}_{\mu\nu}=0\,. \end{aligned} \tag{13}$$

We have made the most general gauge choice for these two states. We are now ready to calculate the correlators. For simplicity, we choose to calculate three point correlators with two massless gauge vectors and one massive state. The correlator of state (12) decay into two vectors is defined by the doublet $\{S^{\alpha a,[\mu\nu\lambda]bc,\beta d},\,S^{\alpha a,[\mu\nu]bc,\beta d}\}$ where

$$S^{\alpha a,[\mu\nu\lambda](bc),\beta d}=\int\prod_{i=1}^{3}d^2x_i\langle V^\alpha_{-1}(x_1)\bar V^a(\bar x_1)V_0^{[\mu\nu\lambda]}(x_2)\bar V^{(bc)}(\bar x_2)V^\beta_{-1}(x_3)\bar V^d(\bar x_3)\rangle\,, \tag{14}$$

$$S^{\alpha a,[\mu\nu](bc),\beta d}=-i\int\prod_{i=1}^{3}d^2x_i\langle V^\mu_{-1}(x_1)\bar V^a(\bar x_1)V_0^{[\mu\nu]}(x_2)\bar V^{(bc)}(\bar x_2)V^\beta_{-1}(x_3)\bar V^d(\bar x_3)\rangle\,. \tag{15}$$

Similarly, the correlator of state (13) decay into two vectors is defined by the doublet $\{S^{\alpha a,(\mu\nu)(bc),\beta d},\,S^{\alpha a,\mu(bc),\beta d}\}$. The evaluation of Eqs. (14) and (15) is straightforward. One can, for example, use the fermionic representation of Heterotic string for the compatified left-moving sector

$$\bar J^a=1/2:\bar\lambda^i\bar\lambda^j:(T^a)^{ij};\quad \mathrm{Tr}(T^aT^b)=2\delta^{ab}\,, \tag{16}$$

where $\bar\lambda^i$'s are 2D Majorana-Weyl Fermions and T^a are the 496 generators of either $E_8\otimes E_8$ or SO(32). The final results for Eqs. (14) and (15) are

$$\begin{aligned} S^{\alpha a,[\mu\nu\lambda](bc),\beta d}&=24x_{12}x_{23}/x_{13}\,[\mathrm{Tr}(T^a\{T^b,T^c\}T^d)+\mathrm{Tr}(T^aT^{(b})\mathrm{Tr}(T^{c)}T^d)]\\ &\times\{\eta^{\alpha[\nu}\eta^{\lambda|\beta}k_1^{|\mu]}/x_{12}-\eta^{\alpha[\nu}\eta^{\lambda|\beta}k_3^{|\mu]}/x_{23}\}\,, \end{aligned} \tag{17}$$

$$\begin{aligned} S^{\alpha a,[\mu\nu](bc),\beta d}&=16x_{12}x_{23}/x_{13}\,\{1/x_{12}(\eta^{\alpha[\nu}\eta^{\mu]\beta}+k_2{}^\alpha k_3{}^{[\nu}\eta^{\mu]\beta}-k_2{}^\beta k_3{}^{[\nu}\eta^{\mu]\alpha})\\ &+1/x_{23}\,(\eta^{\alpha[\mu}\eta^{\nu]\beta}+k_2{}^\beta k_3{}^{[\nu}\eta^{\mu]\alpha}-k_2{}^\alpha k_3{}^{[\nu}\eta^{\mu]\beta})\}\\ &\times[\mathrm{Tr}(T^a\{T^b,T^c\}T^d)+\mathrm{Tr}(T^aT^{(b})\mathrm{Tr}(T^{c)}T^d)] \end{aligned} \tag{18}$$

where $x_{ij}=|x_i-x_j|$. In deriving (17) and (18), we have done the SL(2, C) gauge fixing and have kept three positions of vertex operators at arbitrary but fixed values. Now one of the first massive even G-parity gauge state solution of Eq. (4) is

$$\{k_\lambda \theta^{(ac)}{}_{\mu\nu} b^\lambda{}_{-1/2} b^\mu{}_{-1/2} b^\nu{}_{-1/2} + 2\theta^{(ac)}{}_{\mu\nu}\alpha^\mu{}_{-1} b^\nu{}_{-1/2}\} \bar{J}^{(a}\bar{J}^{c)}|0,k\rangle\,,$$
$$\theta^{(ac)}{}_{\mu\nu} = -\theta^{(ac)}{}_{\nu\mu},\quad k^\mu\theta^{(ac)}{}_{\mu\nu}=0\,. \tag{19}$$

One can thus verify the following Ward identity after some algebra

$$k_{2\lambda}\theta^{(bd)}{}_{\mu\nu} S^{\alpha a,[\mu\nu\lambda](bd),\beta c} + 2\theta^{(bd)}{}_{\mu\nu} S^{\alpha a,[\mu\nu](bd),\beta c} = 0\,. \tag{20}$$

We are now in a position to discuss the background field symmetry transformation as we did for the massless mode. The on-fixed-point superconformal deformations constructed from Eq. (12) are

$$T^{(1)} = [M^{(ab)}{}_{[\mu\nu\lambda]}(3\partial X^\mu + \overleftarrow{\partial}_\alpha \psi^\alpha\psi^\mu)\psi^\nu\psi^\lambda + E^{(ab)}{}_{[\mu\nu]}(-\psi^\mu\partial\psi^\nu + \overleftarrow{\partial}_\lambda \psi^\lambda\psi^\mu\partial X^\nu)]\bar{J}^{(a}\bar{J}^{b)}\,,$$
$$T_F^{(1)} = \{3/2\, M^{(ab)}{}_{[\mu\nu\lambda]} + 2\partial^\alpha\partial_\lambda M^{(ab)}{}_{[\mu\nu\alpha]} + 1/2\,\partial_\lambda E^{(ab)}{}_{[\mu\nu]}\}\psi^\mu\psi^\nu\psi^\lambda \bar{J}^{(a}\bar{J}^{b)}$$
$$+ (3\partial^\lambda M^{(ab)}{}_{[\mu\nu\lambda]} + \partial^\lambda\partial_\nu E^{(ab)}{}_{[\lambda\mu]} + E^{(ab)}{}_{[\mu\nu]})\partial X^\mu\psi^\nu\bar{J}^{(a}\bar{J}^{b)}\,;$$
$$E^{(ab)}{}_{[\mu\nu]} = \partial^\lambda M^{(ab)}{}_{[\lambda\mu\nu]},\quad (\Box-2)M^{(ab)}{}_{[\lambda\mu\nu]}=0\,. \tag{21}$$

$T_F^{(1)}$ in Eq. (21) is a (1/2, 1) superconformal deformation as in Eq. (8). The symbol $\leftarrow$ in Eq. (21) means that $\overleftarrow{\partial}_\alpha$ acts backward on the background fields. Note that $E^{(ab)}{}_{[\mu\nu]}$ is the gauge artifact of the propagating background field $M^{(ab)}{}_{[\lambda\mu\nu]}$. If we compare Eq. (21) with the superconformal deformation constructed from Eq. (19), we get the following symmetry transformation:

$$\delta M^{(ab)}{}_{[\mu\nu\lambda]} = \partial_{[\mu}\theta^{(ab)}{}_{\nu\lambda]}\,, \tag{22}$$

with conditions $(\Box-2)\theta^{(ab)}{}_{[\mu\nu]} = \partial^\mu\theta^{(ab)}_{[\mu\nu]} = 0$. In deriving Eq. (22) we have used the gauge condition in Eq. (21). Eq. (22) is the symmetry transformation corresponding to the Ward identity in Eq. (20). It is valid to all orders in α'. Other symmetries and Ward identities can be similarly constructed. For example, there exists a type I *inter particle gauge state*

$$\{k_{[\mu}\theta^{(ac)}{}_{\nu]}\alpha^\mu{}_{-1}b^\nu{}_{-1/2} - \theta^{(ac)}\cdot b_{-3/2}\}\bar{J}^{(a}\bar{J}^{c)}|0,k\rangle;\quad k\cdot\theta^{(ac)}=0\,. \tag{23}$$

The inter particle Ward identity and symmetry constructed from (23) are

$$k_{2[\mu}\theta^{(bd)}{}_{\nu]} S^{\alpha a,[\mu\nu](bd),\beta c} - \theta^{(bd)}{}_\mu S^{\alpha a,\mu(bd),\beta c} = 0 \tag{24}$$

and

$$\delta M^{(ab)}{}_{[\mu\nu\lambda]} = \partial_{[\mu}\partial_\nu\theta^{(ab)}{}_{\lambda]},\quad \delta G^{(ab)}{}_{(\mu\nu)} = \partial_{(\mu}\theta^{(ab)}{}_{\nu)} \tag{25}$$

where $(\Box-2)\theta_\mu{}^{(ab)} = \partial^\mu\theta^{(ab)}{}_\mu = 0$, and $G^{(ab)}{}_{(\mu\nu)}$ is the physical propagating background field of state (13). Another symmetry transformation containing a nonderivative piece is

$$\delta G^{(ab)}{}_{(\mu\nu)} = 2\partial_\mu\partial_\nu\theta^{(ab)} - \eta_{\mu\nu}\theta^{(ab)}\,, \tag{26}$$

where $(\Box-2)\theta^{(ab)}=0$. We have used the appropriate gauge conditions similar to Eq. (21) to derive (25) and (26). Eq. (25) means that the decoupling of $\theta^{(ab)}{}_\mu$ gauge state induces simultaneously changes of both background fields, thus they form a large multiplet. Those symmetries which are generated by *massive* gauge states, like Eqs. (22), (25) and (26), were shown to be broken spontaneously by the space-time metric $\langle\eta_{\mu\nu}\rangle\neq 0$ [10].

It is possible to exhaust all solutions of Eqs. (4) and (5) level by level. One then expects the existence of infinite number of mixed higher spin symmetry transformation parameters in the theory. Unfortunately, the algebraic structure of this symmetry is still difficult to identify. The situation is, however, much easier to handle in the case of toy 2D string model where w_∞ symmetry [2] is believed to be related to the discrete gauge states in the spectrum. Works in this direction is in progress.

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