ground water

Discussion of Papers/ Christopher Neuzil, Discussion Editor

"*Approximate Discharge for Constant Head Test with Recharging Boundary*," by Philippe Renard, May–June 2005 issue, v. 43, no. 3: 439–442.

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Renard (2005) studied discharge in a constant head test with a recharging boundary in a radial confined aquifer. He proposed Laplace-domain solutions for the drawdown in an aquifer and the discharge for an aquifer with one discharging well and a recharge boundary represented by one recharging well. In this comment, we wish to point out problems that exist with the unit step response function $\bar{s}_{Du}$ and the drawdown $\bar{s}_D$, as given in the equations 12 and 15 in Renard (2005). In addition, we derive a time-domain solution of the discharge for the same problem and suggest a numerical approach to evaluate the solution with accuracy to five decimal places.

The definitions of the symbols used herein are identical to those given by Renard (2005). In the case of one discharging well and one recharging well separated by a distance $2l_D$, the observation well is at distance r_D and $(2l_D - r_D)$ from the real well (discharging well) and imaginary well (recharging well), respectively. By applying the superposition principle, the unit step response function can be obtained as:

$$\bar{s}_{Du}(r_D, p) = \frac{K_0(r_D\sqrt{p})}{p\sqrt{p}K_1(\sqrt{p})} - \frac{K_0[(2l_D - r_D)\sqrt{p}]}{p\sqrt{p}K_1(\sqrt{p})} \quad (1)$$

where p is the Laplace variable and $K_0(\cdot)$ and $K_1(\cdot)$ are the Bessel functions of the second kind of order zero and one, respectively. The first term on the right-hand side of Equation 1 represents the effect of discharge and the second term represents that of recharge. Equation 1 is valid only when the real, observation, and imaginary wells are along a straight line. Renard (2005) gave a distance between the observation and the imaginary wells as $(2l_D - 1)$, which was incorrect. Therefore, equation 15 of Renard (2005) should read as:

$$\bar{s}_D(r_D, p) = \frac{K_0(r_D\sqrt{p}) - K_0[(2l_D - r_D)\sqrt{p}]}{p\{K_0[\sqrt{p}] - K_0[(2l_D - 1)\sqrt{p}]\}} \quad (2)$$

Renard (2005) presented the Laplace-domain solution of the discharge from the well in his equation 13 as:

$$\bar{q}_D(p) = \frac{K_1(\sqrt{p})}{\sqrt{p}\{K_0[\sqrt{p}] - K_0[(2l_D - 1)\sqrt{p}]\}} \quad (3)$$

In addition, he also gave a simple approximate solution of the discharge rate into a well using a weighted average of the two asymptotes plus a correction term as:

$$q_D(t_D) = \frac{A}{\ln(1 + \sqrt{\pi t_D})} + \frac{B}{\ln(2l_D - 1)} + C \quad (4)$$

In fact, the analytical solution in the time domain for Equation 3 can be derived using the Bromwich integral method (Peng et al. 2002; Yang and Yeh 2002), and the final result is:

$$q_D(t_D) = \frac{2}{\pi}\int_0^\infty e^{-t_D u^2} \frac{J_1(u)B_2(u) - Y_1(u)B_1(u)}{B_1^2(u) + B_2^2(u)} du \quad (5)$$

where $J_0(\cdot)$ and $Y_0(\cdot)$ are, respectively, the Bessel functions of the first and second kinds of order zero, and $J_1(\cdot)$ and $Y_1(\cdot)$ are, respectively, the Bessel functions of the first and second kinds of order one. In addition, $B_1(u) = J_0(u) - J_0((2l_D - 1)u)$ and $B_2(u) = Y_0(u) - Y_0((2l_D - 1)u)$. A numerical approach, including the singularity removal scheme, the Gaussian quadrature, and Shanks' method (Peng et al. 2002; Yeh et al. 2003), can be used to evaluate Equation 5 with accuracy to five decimal places for a very wide range of dimensionless time.

References

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doi: 10.1111/j.1745-6584.2007.00386.x