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The equilibrium quantity and production strategy in a fuzzy random decision environment: Game approach and case study in glass substrates industries

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ABSTRACT

This paper develops a two-stage Cournot production game that integrates strategic and operational planning under the fuzzy random environment, which to our best knowledge has not appeared in the literature. At the strategic level, two competing decision-makers determine the upper bound of a production quantity under a high-production strategy and the lower bound of the production quantity under a low-production strategy. Then at the operational level, the two competitors determine the range-type production quantity that is assumed to be a triangular fuzzy number represented by the apex and the entropies rather than a crisp value. The apex of a fuzzy equilibrium quantity can be obtained by the conventional Cournot game as the membership value is equal to one. A fuzzy random decision can be represented by entropies derived from the fuzzy random profit function of each firm in a specific production strategy. A case study of two leading firms in the glass substrates industry demonstrates the applicability of the proposed model. The finding that both firms would tend to adopt the common strategy coincides with observed real-world behavior. We conclude that our proposed method can provide decision-makers with a simple mathematical foundation for determining production quantity under a production strategy in a fuzzy random environment.

1. Introduction

Decision-making in complicated and competitive environments can be a difficult task because of uncertainty in the form of *ambiguity* or *randomness*. After the notion of fuzzy sets theory was introduced by Zadeh (1965) to manage ambiguity, the theory underwent extensive development and today is routinely applied to solve a variety of real-world problems, (Kunsch and Fortemps, 2002; Tanaka, 1987; Wong and Lai, 2011). Problems of randomness can be properly modeled by probability theory; applications to real problems appear in (Pastor et al., 1999; Valadares Tavares et al., 1998; Zhang et al., 2004). However, decision-makers often work in a hybrid (uncertain) environment where ambiguity and randomness exist simultaneously. Given such environments, a fuzzy random variable as introduced by Kwakernaak (1978) is a useful tool for solving these two aspects of uncertainty. Other studies (Colubi et al., 2001; Krätschmer, 2001) also extend several theories (Wang et al., 2007; Wang et al., 2008) to environments with these two aspects of uncertainty.

The typical Cournot game (see Cournot, 1838) models a duopoly in which two competing firms choose their production quantity. In the equilibrium quantity, no firm can be better off by a unilateral change in its solution. The exact values of parameters are required information when the Cournot game is applied to decision-making models, but exact values are often unobtainable in a business environment. Yao and Wu (1999) probably initiated a non-cooperative game involving fuzzy data by applying the ranking method to defuzzify the fuzzy demand and fuzzy supply functions into crisp values such that both consumer surplus and producer surplus can be calculated in a conventional manner. Their method of transforming fuzzy numbers to crisp values is also utilized to construct the monopoly model in Chang and Yao (2000). Liang et al. (2008) propose a duopoly model with fuzzy costs to obtain the optimal quantity of each firm. Recently Dang and Hong (2010) propose a fuzzy Cournot game with rigorous definitions ensuring a positive equilibrium quantity and with a flexible controlling mechanism that adjusts the parameters of associated objective functions. As mentioned, the resulting crisp values derived by previous studies are counter-intuitive outcomes of the problem in the fuzzy sense. A crisp decision is too precise

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to be believed in a business environment with a fuzzy sense. To bridge this gap, we solve for a range-type solution rather than a crisp-value one. In addition, decision-makers may prefer a decision with more information in a range they themselves can adjust. In this case study, we empirically demonstrate our proposed approach to investigate the behaviors of the equilibrium quantity and production strategy in the glass substrates industry, which is highly volatile and ambiguous in the market demand and production costs.

In reality, the parameters of the Cournot game may behave with the characteristic of randomness in nature. In a business environment, decision-makers may predict demand behavior in a market, i.e. a functional form describes the relationship between price and quantity. The parameters of the functional form of market demand typically can be estimated by the technique of econometrics. Regression is one of the most popular statistical approaches employed and leads to game-theoretical models with random parameters. Hosomatsu (1969) indicates that the Cournot solution to an oligopolistic market is based upon the implicit assumptions given the estimated market demand function. Sato and Nagatani (1967) propose a model to relax the Cournot assumption and substitute firms' subjective evaluation of a market with a more general form with randomness.

Unpredictable events may drive the price fluctuation over a short period. It is appropriate to apply the fuzzy regression method (Chen and Dang, 2008; González-Rodríguez et al., 2009) to manage the data with ambiguity sense. The resulting estimators derived by econometrics involve in ambiguity and randomness, which occur in many fields (Guo and Lu, 2009; Xu and Zhao, 2010). In fact, Guo and Lu (2009) state that the coexistence of ambiguity and randomness becomes an intrinsic characteristic in the real world. Thus, there is a strong motivation to develop the Cournot production game with parameters of ambiguity and randomness described by fuzzy random parameters. To our best knowledge, none of or very little research involves in such a Cournot production game.

The Cournot production game proposed in this paper incorporates a production strategy which refers to the pattern of production quantities chosen by a decision-maker. Facing uncertainty with ambiguity and randomness, the choice of production strategy has important consequences for the selection, deployment and management of production resources. In general, there are two major stages of decision sequence: strategic and operational levels (see Ballou (1992)). Many studies focus only on the strategic level or operational level and ignore the importance of the interaction between them. In this paper, we propose a model considering both strategic and operational levels. Our model is similar to other two-stage games (see (Bae et al., 2011; Dhaene and Bouckaert, 2010)) where a player's decision in the first stage affects the action taken in the second stage. We consider two specific types of production strategies – high and low – by which the decision-maker's profit function depends on the highest production quantity or the lowest production quantity, respectively. In the long term, a high- or low-production strategy may be employed because the decision-maker desires to gain market share or to enhance the quality of products (Stout, 1969; Walters, 1991). Furthermore, Yang and Wee (2010) indicate that the production strategy is needed to respond to the market demand because of the rapid technology change (see (Droge et al., 2012; Kenne et al., 2012; Xu et al., 2012) for other models adopting strategy perspectives). The aim of this paper is to solve for the fuzzy equilibrium quantity of each decision-maker and to provide an appropriate production strategy under the fuzzy random business environment.

The remainder of this paper is organized as follows. Section 2 presents the preliminary knowledge of the fuzzy sets theory, entropy, and fuzzy random variable. Section 3 addresses the Cournot game in the fuzzy random environment and solves for the fuzzy equilibrium quantity of each firm. Section 4 investigates the insights of the proposed method including the extension and discussion. Section 5 illustrates the applicability of the proposed model in the real-world situation. Section 6 discusses our conclusions and gives suggestions for future research.

2. Definitions and concepts

This section introduces the fuzzy sets theory, entropy and expected operator which are integral to this paper.

2.1. Fuzzy sets theory

The fuzzy sets theory initiated by Zadeh (1965) attempts to analyze and solve problems with a source of ambiguity called fuzziness. In the following, we introduce the definitions and notations of triangular fuzzy numbers, α-level cut and the extension principle.

2.1.1. Triangular fuzzy number

For practical purposes, one of the most commonly used fuzzy numbers is the triangular type because it is easy to handle arithmetically and has an intuitive interpretation (Dağdeviren and Yüksel, 2008; Şen et al., 2010). Giannoccaro et al. (2003) and Petrovic et al. (1999) show that triangular fuzzy numbers are the most suitable for modeling market demand in the fuzzy sense (see (Ayağ and Özdemir, 2012; Vijay et al., 2005) for other applications of triangular fuzzy numbers). The membership function $\mu_{\tilde{A}}(x)$ of a triangular fuzzy number $\tilde{A}$ can be defined by

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-m_A+l_A}{l_A}, & m_A-l_A\le x\le m_A\\ \frac{m_A+r_A-x}{r_A}, & m_A\le x\le m_A+r_A\\ 0, & \text{otherwise},\end{cases} \quad (1)$$

where $\tilde{A}$ is represented as a triplet (m_A-l_A, m_A, m_A+r_A) and m_A, l_A and r_A are the apex, left and right spreads of the fuzzy number $\tilde{A}$, respectively. Furthermore, a triangular fuzzy number $\tilde{A}$ can be shown in Fig. 1.

2.1.2. α-Level cut

One of the most important concepts of fuzzy sets is the α−level cut given by

$$\tilde{B}_\alpha := \{x\in\mathbb{R}|\tilde{B}(x)\ge\alpha\}$$

where $\alpha\in[0,1]$, which means for a fuzzy number $\tilde{B}$, those elements whose membership values are greater than or equal to α.

2.1.3. Extension principle

Let "⊙" be any binary operation ⊕ and ⊗ between two fuzzy numbers $\tilde{A}$ and $\tilde{B}$. Based on the extension principle, the membership function of $\tilde{A}\odot\tilde{B}$ is defined by

$$\mu_{\tilde{A}\odot\tilde{B}}(z)=\sup_{x\circ y}\min\{\mu_{\tilde{A}}(x),\mu_{\tilde{B}}(y)\}$$

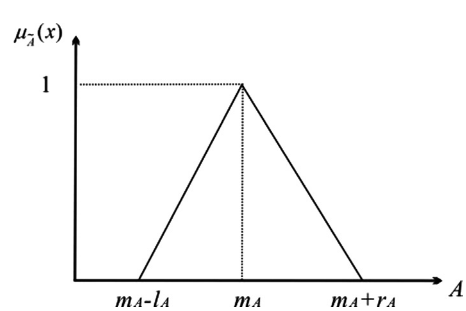

Fig. 1. A triangular fuzzy number $\tilde{A}$.

where "$\odot = \oplus$ or $\otimes$" corresponds to the operation "$\circ = +$ or $\times$". This result helps us to derive the membership function of a fuzzy number.

2.2. Entropy

The fuzziness of a fuzzy number can be described by a membership function applied in many aspects (Petrovic et al., 1999; Shu and Wu, 2010). However, Kao and Lin (2005) mention a preference for using a simple index to show the fuzziness rather than using a membership function. They apply entropy, a simple index method, to interpret the fuzziness as a crisp value and consider the randomness of the fuzzy number.

Denote $h(\mu_{\tilde{A}}(x))$, $h:[0,1]\to[0,1]$, as the entropy function that is monotonically increasing in $[0,1/2]$ and monotonically decreasing in $[1/2,1]$. The most well-known entropy function is the Shannon function (Zimmermann, 1996) as described in (2).

$$h(u) = -u\ln u - (1-u)\ln(1-u) \tag{2}$$

where u is the membership function of the fuzzy number. Integrating the entropy over all elements $x \in X$ leads to a global entropy measure $H(\tilde{A})$:

$$H(\tilde{A}) = \int_{x\in X} h(\mu_{\tilde{A}}(x))p(x)dx \tag{3}$$

where $p(x)$ denotes the probability density function of the available data set defined over X. It is common to assume a uniform distribution $p(x)=k$ following Kao and Lin (2005)) and Pedrycz (1994). According to (3), the entropy of the triangular fuzzy number $\tilde{A}$ is calculated by decomposing it into $H_L(\tilde{A})$ and $H_R(\tilde{A})$:

$$H(\tilde{A}) = H_L(\tilde{A}) + H_R(\tilde{A}) = \int_{m_A-l_A}^{m_A} h(\mu_{\tilde{A}}(x))p(x)dx + \int_{m_A}^{m_A+r_A} h(\mu_{\tilde{A}}(x))p(x)dx \tag{4}$$

The three resulting entropies are the left entropy $H_L(\tilde{A}) = k\times l_A/\ln 4$, the right entropy $H_R(\tilde{A}) = k\times r_A/\ln 4$, and the entropy $H(\tilde{A}) = k\times(l_A+r_A)/\ln 4$ (Kao and Lin, 2005). Furthermore, Kao and Lin (2005) show that by ignoring the constant k a triangular fuzzy number can be determined by the unique apex, left and right entropies without complicated membership functions. In other words, the left (right) entropy can be regarded as the left (right) spread of a triangular fuzzy number. The concept of entropy can be extended to other types of fuzzy numbers such as trapezoid, exponential, etc (Kao and Lin, 2005).

2.3. Expected value of the fuzzy random variable

The mathematical notations of fuzzy random variables are given in this section. Let (Ω,Σ,P) be a probability space where Σ is a $\sigma-$ field and P is a probability measure.

Definition 1. (Liu and Liu, 2003) Let (Ω,Σ,P) be a probability space. A fuzzy random variable is mapping $\xi:\Omega\to F_v(\mathfrak{R})$ such that for any closed C of $\mathfrak{R}$, and the function

$$\xi^*(C)(\omega) = \sup_{x\in C}\mu_{\xi(\omega)}(x)$$

is a measurable function of ω, where $\mu_{\xi(\omega)}$ is the possibility distribution function of a fuzzy variable $\xi(\omega)$ and $F_v(\mathfrak{R})$ is a collection of fuzzy variables defined on a possibility space.

A triangular fuzzy random variable can be described by a triplet composed of the apex and the two crisp values of the left and right spreads. A detailed representation of a triangular fuzzy random variable ξ is defined as

$$\xi = (X-l, X, X+r)$$

where X is the apex distributed with the normal distribution, $N(\mu,\sigma^2)$, having mean μ and variance σ^2.

Definition 2. (Liu and Liu, 2003) Let ξ be a triangular fuzzy random variable. The expected value $E(\xi)$ of ξ is calculated as:

$$E(\xi) = \mu + \frac{1}{4}(r-l)$$

Next, we introduce the linearity of the expected operator of fuzzy random variables.

Theorem 1. (Liu and Liu, 2003) Assume that ξ and η are fuzzy random variables, then for any real numbers a and b, we have

$$E(a\xi + b\eta) = aE(\xi) + bE(\eta) \tag{5}$$

3. Model and methodology

The Cournot game is a situation where each firm independently chooses its production quantity in order to maximize the respective profit function. In the real world, ambiguity and randomness may appear simultaneously. If the parameters are fuzzy random variables, the profit function with these parameters is possibly a fuzzy random variable (Liu and Liu, 2003). This section proposes the theoretical model as a coherent, rigorous and novel philosophy position that not only substantiates the case study of glass substrates industries, but also provides helpful implications for both theoretical development and real-world applications. Furthermore, the proposed method solves for the closed-form equilibrium quantity with entropy spreads under a given production strategy of each firm.

3.1. The Cournot production game under the fuzzy random environment

At the strategic level of the game, it is essential to consider a high- or low-production strategy in response to the market because of the rapid change resulting in increasing or decreasing market demand. Under a high-production, firm i, $i=1,2$, supplies a highest quantity (the highest in the range-type solution) to the market, but a lowest quantity (the lowest in the range-type solution) under a low-production strategy. The high- or low-production strategy assists us to derive a range-type production quantity of each firm. Furthermore, this leads to four (2×2) strategic scenarios in a duopoly where each firm adopts a high- or low-production strategy, respectively, in the strategic level. At the operational level, each firm determines its production quantity so as to maximize its *Fuzzy Random Profit Function* (FRPF). Fig. 2 represents the decision sequence of our model. Assuming it behaves rationally, firms anticipate their best response in the operational level to the chosen production strategy in the strategic level. This allows us to solve this two-stage sequential game by moving from the operational level to the strategic level based on the chosen production strategy.

We assume the production quantity of each firm to be a triangular fuzzy number represented by the apex and the entropies. We develop the Cournot production game including the parameters with triangular fuzzy random variables, a linear

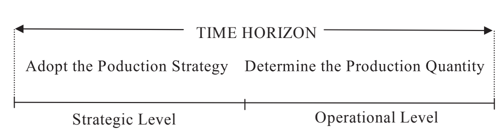

Fig. 2. Decision timeline of the Cournot production game under the fuzzy random environment.

inverse demand and cost functions as shown below. Consider the fuzzy random inverse demand function as

$$p(\xi) = \xi_a - \xi_b \tilde{Q} \tag{6}$$

where $\boldsymbol{\xi} = (\xi_a, \xi_b)$ is a fuzzy random vector representing the parameters of market demand. The total market quantity is that $\tilde{Q} = \tilde{q}_1 + \tilde{q}_2$; ξ_a and ξ_b are triangular fuzzy random variables defined as

$$\xi_a = (X_a - l_a, X_a, X_a + r_a),$$
$$\xi_b = (X_b - l_b, X_b, X_b + r_b). \tag{7}$$

In (7), ξ_a is with left spread l_a, right spread r_a and apex X_a where X_a follows the normal distribution with mean μ_a and variance σ_a^2. Parameter ξ_b can be explained in a similar manner. In addition, the total cost function of firm i with fuzzy random parameters denoted by $TC_i(\xi)$ is

$$TC_i(\xi) = \xi_{d_i} \tilde{q}_i \tag{8}$$

where ξ_{d_i} is a triangular fuzzy random variable defined by

$$\xi_{d_i} = (X_{d_i} - l_{d_i}, X_{d_i}, X_{d_i} + r_{d_i}) \tag{9}$$

In (9), the fuzzy random variable cost of firm i, ξ_{d_i}, is with left spread l_{d_i}, right spread r_{d_i} and apex X_{d_i} where X_{d_i} follows the normal distribution with mean μ_{d_i} and variance $\sigma_{d_i}^2$. Thus, the FRPF of firm i is

$$\pi_i = (\xi_a - \xi_b \tilde{Q}) \tilde{q}_i - \xi_{d_i} \tilde{q}_i \tag{10}$$

In (10), we can recognize that the problem involves in the ambiguous uncertainty according to Liu and Liu (2003). For simplicity, we take the expected value of (10) as:

$$E(\pi_i) = E[(\xi_a - \xi_b \tilde{Q}) \tilde{q}_i] - E[\xi_{d_i} \tilde{q}_i] \tag{11}$$

Similar approaches in different applications with fuzzy random parameters appear in (Dutta et al., 2005; Kwakernaak, 1978). Because the production quantity of each firm is assumed to be a triangular fuzzy number, this allows us to characterize the production quantity by the apex and the entropies. When the membership value, α, is equal to 1, the values of the production quantity and the fuzzy random parameters are apexes based on the extension principle (see Zadeh, 1965). Thus, the case involving an ambiguous uncertainty can be treated in a crisp manner.

Furthermore, the right and left entropies are decision variables to characterize the highest and lowest production levels. It follows that the expected FRPF of firm i is with the highest entropies under a high-production strategy, and vice versa. Therefore, we can decompose the original problem into (i) the *center problem* that solves for the apex of the fuzzy equilibrium quantity by the conventional Cournot game given that each parameter has a membership value equal to 1, and (ii) the *spreads problem* that maximizes each firm's expected FRPF over its entropies under a production strategy. The solutions of these two problems give a triangular fuzzy equilibrium quantity, which provides decision-makers with a range-type solution instead of a crisp-value solution. In the following section, we first solve for the apex of the fuzzy equilibrium quantity of firm i followed by the spreads problem.

3.2. The operational-level decision: The center problem

As mentioned, when the membership value is equal to 1, the expected FRPF of firm i can be represented by the apex of each parameter as follows:

$$E(\pi_i) = E[(\xi_a - \xi_b Q) q_i] - E(\xi_{d_i}) q_i \tag{12}$$

We derive the apex of the fuzzy equilibrium quantity q_i according to the conventional Cournot game (see Rasmusen, 2001). The best response function is obtained by maximizing firm i's expected FRPF over the apex of the production quantity, q_i. This generates the first-order condition returning firm i's best response function to firm j's production quantity, q_j, as:

$$q_i = \frac{\mu_a - q_j - \mu_{d_i}}{2} \tag{13}$$

Similarly, we derive firm j's best response function to firm i's production quantity as

$$q_j = \frac{\mu_a - q_i - \mu_{d_j}}{2} \tag{14}$$

The apex of the fuzzy equilibrium quantity of firm i, shown in (15), is obtained by simultaneously solving (13) and (14).

$$q_i = \frac{\mu_a + \mu_{d_j} - 2\mu_{d_i}}{3\mu_b} \tag{15}$$

Similarly,

$$q_j = \frac{\mu_a + \mu_{d_i} - 2\mu_{d_j}}{3\mu_b} \tag{16}$$

It is clear that q_i is the solution of the center problem given that the membership value is 1. Utilizing the resulting outcome we can now solve the spreads problem.

3.3. The operational-level decision: The spreads problem

We note that the upper bound of the total production quantity can be achieved since the lower bound of the market price occurs because of the law of demand. Substituting the highest production quantity of each firm into the market demand, we define the expected FRPF of firm i under a high-production strategy as:

$$E(\pi_i^H) = E(\xi_a - \xi_b Q^H) q_i^H - E(\xi_{d_i}) q_i^H \tag{17}$$

where

$$Q^H = q_i + e_i^R + q_j + e_j^R$$

In (17), we note that the upper bound of market demand, Q^H, is the production quantity determined by each firm plus the right entropy. The right entropy of the fuzzy equilibrium quantity can be interpreted as the increasing quantity of one firm.

Similarly, the expected FRPF of firm i under a low-production strategy is

$$E(\pi_i^L) = E(\xi_a - \xi_b Q^L) q_i^L - E(\xi_{d_i}) q_i^L \tag{18}$$

where

$$Q^L = q_i - e_i^L + q_j - e_j^L$$

The lower bound of the market demand, Q^L, is the production quantity determined by each firm minus the left entropy of each firm. In the real word, firms may adjust their capacities to produce products in peak and off-peak seasons so that the relation between the designed capacities in peak and off-peak seasons practically behaves in a fixed ratio manner. This allows us to assume that the ratio of the right entropy of firm i to its left entropy is a given parameter, λ_i; that is, $e_i^R = \lambda_i e_i^L$ where $\lambda_i > 0$. This assumption assists in obtaining the qualitative managerial insights with less analytical complexity. Substituting $e_i^R = \lambda_i e_i^L$ into (17), the expected FRPF under a high-production strategy is concave in e_i^L since $(\partial^2 E(\pi_i^H)/\partial(e_i^L)^2) = -2\lambda_i^2 E(\xi_b) < 0$. Similarly, in (18), the expected FRPF under a low-production strategy is concave in e_i^L. In the following, we derive the resulting entropies of each firm for our four strategic scenarios where each firm maximizes its expected FRPF. Furthermore, we adopt the concept of the production strategy to construct the fuzzy equilibrium quantity of each firm in this paper.

Strategic Scenario 1. Both firms i and j adopt the high-production strategy.

Firm i maximizes its expected FRPF over e_i^L. The first-order condition returns firm i's best response function to firm j's decision variable, e_j^L; that is

$$-\lambda_i E(\xi_b)(q_i+\lambda_i e_i^L) + \lambda_i[E(\xi_a)-E(\xi_b)(q_i+\lambda_i e_i^L+q_j+\lambda_j e_j^L)]-\lambda_i E(\xi_{d_i})=0 \quad (19)$$

Similarly, the first-order condition returning firm j's best response function to firm i's decision variable, e_i^L, is

$$-\lambda_j E(\xi_b)(q_j+\lambda_j e_j^L) + \lambda_j[E(\xi_a)-E(\xi_b)(q_i+\lambda_i e_i^L+q_j+\lambda_j e_j^L)]-\lambda_j E(\xi_{d_j})=0. \quad (20)$$

Let e_{i1}^L be the left equilibrium entropy of firm i in Strategic Scenario 1 derived by simultaneously solving (19) and (20).

$$e_{i1}^L = \frac{E(\xi_a)-3E(\xi_b)q_i-2E(\xi_{d_i})+E(\xi_{d_j})}{3\lambda_i E(\xi_b)}, i, j=1,2,\ i\neq j. \quad (21)$$

To ensure a non-negative left equilibrium entropy of firm i, we impose the condition such that $e_{i1}^L \geq 0$, $i=1,2$. Assumption 1 follows from the condition, where q_i is derived in (15).

Assumption 1. $E(\xi_a)-3E(\xi_b)q_i-2E(\xi_{d_i})+E(\xi_{d_j})\geq 0$.

Combining (15) and (21), the fuzzy equilibrium quantity of firm i in Strategic Scenario 1 becomes

$$(q_i-e_{i1}^L, q_i, q_i+\lambda_i e_{i1}^L), i=1,2$$

Strategic Scenario 2. Firm i adopts the low-production strategy and firm j adopts the high-production strategy.

Under a low-production strategy, firm i solves the spreads problem by maximizing its expected FRPF over e_i^L. Eq. (18) is maximized when the first-order condition holds. Using the first-order condition to derive firm i's best response function to firm j's decision variable, e_j^L, gives

$$E(\xi_b)(q_i-e_i^L)-[E(\xi_a)-E(\xi_b)(q_i-e_i^L+q_j-e_j^L)]+E(\xi_{d_i})=0 \quad (22)$$

Firm j maximizes its expected FRPF, as shown in (17), under a high-production strategy. As mentioned, $E(\pi_j^H)$ is concave in e_j^L so the first-order condition of (17) gives

$$-\lambda_j E(\xi_b)(q_j+\lambda_j e_j^L) + \lambda_j[E(\xi_a)-E(\xi_b)(q_i+\lambda_i e_i^L+q_j+\lambda_j e_j^L)]-\lambda_j E(\xi_{d_j})=0 \quad (23)$$

Let e_{i2}^L be the left equilibrium entropy of firm i in Strategic Scenario 2 derived by solving (22) and (23). The final results of e_{i2}^L and e_{j2}^L are

$$e_{i2}^L = \frac{-E(\xi_a)(1+2\lambda_j)+E(\xi_b)(q_i(1+4\lambda_j)+2q_j(1+\lambda_j))+2\lambda_j E(\xi_{d_i})+E(\xi_{d_j})}{E(\xi_b)(4\lambda_j-\lambda_i)} \quad (24)$$

and

$$e_{j2}^L = \frac{E(\xi_a)(2+\lambda_i)-E(\xi_b)2q_i(1+\lambda_i)-E(\xi_b)q_j(4+\lambda_i)-\lambda_i E(\xi_{d_i})-2E(\xi_{d_j})}{E(\xi_b)(4\lambda_j-\lambda_i)}. \quad (25)$$

To ensure non-negative left equilibrium entropies of firms i and j, we impose the condition such that $e_{i2}^L\geq 0$ and $e_{j2}^L \geq 0$. Assumption 2 follows the condition, where q_i and q_j are derived in (15) and (16).

Assumption 2.

$$(4\lambda_j-\lambda_i)[-E(\xi_a)(1+2\lambda_j)+E(\xi_b)(q_i(1+4\lambda_j)+2q_j(1+\lambda_j))+2\lambda_j E(\xi_{d_i}) + E(\xi_{d_j})]\geq 0$$

and

$$(4\lambda_j-\lambda_i)[E(\xi_a)(2+\lambda_i)-E(\xi_b)2q_i(1+\lambda_i)-E(\xi_b)q_j(4 + \lambda_i)-\lambda_i E(\xi_{d_i})-2E(\xi_{d_j})]\geq 0.$$

The fuzzy equilibrium quantity of firm i can be constructed by (15) and (24) as:

$$(q_i-e_{i2}^L, q_i, q_i+\lambda_i e_{i2}^L), i=1,2$$

Strategic Scenario 3. Firm i adopts the high-production strategy and firm j adopts the low-production strategy.

Here, the solution procedure to derive the entropies of each firm is similar to Strategic Scenario 2. Let e_{i3}^L be the left equilibrium entropy of firm i in Strategic Scenario 3. The resulting outcomes of e_{i3}^L and e_{j3}^L can be obtained as

$$e_{i3}^L = \frac{E(\xi_a)(2+\lambda_j)-E(\xi_b)q_i(4+\lambda_j)-E(\xi_b)2q_j(1+\lambda_j)-2E(\xi_{d_i})-\lambda_j E(\xi_{d_j})}{E(\xi_b)(4\lambda_i-\lambda_j)} \quad (26)$$

and

$$e_{j3}^L = \frac{-E(\xi_a)(1+2\lambda_i)+E(\xi_b)(2q_i(1+\lambda_i)+q_j(1+4\lambda_i))+E(\xi_{d_i})+2\lambda_i E(\xi_{d_j})}{E(\xi_b)(4\lambda_i-\lambda_j)} \quad (27)$$

Similarly, to ensure non-negative left equilibrium entropies of firms i and j, we impose Assumption 3, where q_i and q_j are derived in (15) and (16).

Assumption 3.

$$(4\lambda_i-\lambda_j)[E(\xi_a)(2+\lambda_j)-E(\xi_b)q_i(4+\lambda_j)-E(\xi_b)2q_j(1+\lambda_j)-2E(\xi_{d_i})-\lambda_j E(\xi_{d_j})]\geq 0$$

and

$$(4\lambda_i-\lambda_j)[-E(\xi_a)(1+2\lambda_i)+E(\xi_b)(2q_i(1+\lambda_i)+q_j(1+4\lambda_i))+E(\xi_{d_i}) + 2\lambda_i E(\xi_{d_j})]\geq 0.$$

The fuzzy equilibrium quantity of firm i can be constructed by (15) and (26) as

$$(q_i-e_{i3}^L, q_i, q_i+\lambda_i e_{i3}^L), i=1,2$$

Strategic Scenario 4. Both firms i and j adopt the low-production strategy.

As mentioned, the expected FRPF under a low-production strategy is concave in e_i^L, so (18) is maximized when the first-order condition holds. From the first-order condition, we have

$$E(\xi_b)(q_i-e_i^L)-[E(\xi_a)-E(\xi_b)(q_i-e_i^L+q_j-e_j^L)]+E(\xi_{d_i})=0 \quad (28)$$

Similarly, we can obtain the first-order condition of firm j as

$$E(\xi_b)(q_j-e_j^L)-[E(\xi_a)-E(\xi_b)(q_i-e_i^L+q_j-e_j^L)]+E(\xi_{d_j})=0 \quad (29)$$

Let e_{i4}^L be the left equilibrium entropy of firm i in Strategic Scenario 4 derived by solving (28) and (29):

$$e_{i4}^L = \frac{-E(\xi_a)+3E(\xi_b)q_i+2E(\xi_{d_i})-E(\xi_{d_j})}{3E(\xi_b)}, i, j=1,2,\ i\neq j. \quad (30)$$

To ensure a non-negative left equilibrium entropy of firm i, we impose the condition such that $e_{i4}^L\geq 0$, $i=1,2$. Assumption 4 follows from this condition, where q_i is derived in (15).

Assumption 4. $-E(\xi_a)+3E(\xi_b)q_i+2E(\xi_{d_i})-E(\xi_{d_j})\geq 0$.

Due to (15) and (30), the fuzzy equilibrium quantity of firm i in Strategic Scenario 4 is

$(q_i - e_{i4}^L, q_i, q_i + \lambda_i e_{i4}^L), i = 1, 2$

4. Analysis at the strategic level

In this section, we derive the conditions such that one of the four strategy combinations is the Nash equilibrium outcome in the strategic level. The Nash equilibrium, where no player has an incentive to deviate from its strategy given that the other players do not change their strategies, allows us to analyze the relationship between each firm's production strategies.

Proposition 1. The expected FRPF of firm i, $i=1$, 2 under a high-production strategy is equal to the FRPF of firm i, $i=1$, 2 under a low-production strategy if both firms adopt the common strategy; that is, $E(\pi_{i1}^H) = E(\pi_{i4}^L)$, where $E(\pi_{i1}^H)$ and $E(\pi_{i4}^L)$ are the expected FRPF of firm i under a high-production strategy in Strategic Scenario 1 and a low-production strategy in Strategic Scenario 4, respectively.

Proof. Substituting e_{i1}^L in (21) into (17), we have

$$E(\pi_{i1}^H) = \left(\frac{E(\xi_a) - 2E(\xi_{d_i}) + E(\xi_{d_j})}{3}\right)\left(\frac{E(\xi_a) - 2E(\xi_{d_i}) + E(\xi_{d_j})}{3E(\xi_b)}\right)$$

Similarly $E(\pi_{i4}^L)$ can be obtained

$$E(\pi_{i4}^L) = \left(\frac{E(\xi_a) - 2E(\xi_{d_i}) + E(\xi_{d_j})}{3}\right)\left(\frac{E(\xi_a) - 2E(\xi_{d_i}) + E(\xi_{d_j})}{3E(\xi_b)}\right)$$

It is clear that $E(\pi_{i1}^H)$ is equal to $E(\pi_{i4}^L)$ and this completes the proof.

We can calculate the expected FRPF of each firm under a high- or low-production strategy by substituting the resulting entropies derived in Section 3.3. As mentioned, four strategic scenarios are considered in our model. We let the first and second attributes of $(\cdot, \cdot)$ denote the production strategy adopted by firm i and firm j, respectively. Each firm chooses the optimal strategy for the long term to maximize its expected FRPF in the short term. Table 1 represents the expected FRPF of firms i and j under a specific combination of production strategies chosen by the two firms. Next, we utilize the results in Table 1 to derive the conditions such that a production strategy combination is the Nash equilibrium outcome.

Table 1
The expected FRPF under a high- or low-production strategy in four strategic scenarios.

		Firm j's production strategy	
		High-production	Low-production
Firm i's production strategy	High-production	$(E(\pi_{i1}^H), E(\pi_{j1}^H))$	$(E(\pi_{i3}^H), E(\pi_{j3}^L))$
	Low-production	$(E(\pi_{i2}^L), E(\pi_{j2}^H))$	$(E(\pi_{i4}^L), E(\pi_{j4}^L))$

Proposition 2. The conditions for the four possible Nash equilibrium outcomes are given in Table 2.

Proof. (i) Based on the definition of the Nash equilibrium, if the strategy combination (high-production, high-production) is the Nash equilibrium outcome, it means that $E(\pi_{i1}^H) \geq E(\pi_{i2}^L)$ and $E(\pi_{j1}^H) \geq E(\pi_{j3}^L)$. First, we have

$$E(\pi_{i1}^H) - E(\pi_{i2}^L) = [E(\xi_a) - E(\xi_b)(q_i + \lambda_i e_{i1}^L + q_j + \lambda_j e_{j1}^L) - E(\xi_{d_i})](q_i + \lambda_i e_{i1}^L) - [E(\xi_a) - E(\xi_b)(q_i - e_{i2}^L + q_j - e_{j2}^L) - E(\xi_{d_i})](q_i - e_{i2}^L) \geq 0. \quad (31)$$

For notational simplicity, let $\Delta_1 = E(\xi_a) - E(\xi_b)(q_i + \lambda_i e_{i1}^L + q_j + \lambda_j e_{j1}^L) - E(\xi_{d_i})$ and $\Delta_2 = E(\xi_a) - E(\xi_b)(q_i - e_{i2}^L + q_j - e_{j2}^L) - E(\xi_{d_i})$. Note that Δ_1 and Δ_2 are firm i's expected unit profits, which are reasonably assumed non-negative. Substituting Δ_1 and Δ_2 into (31), we have

$$E(\pi_{i1}^H) - E(\pi_{i2}^L) = (\Delta_1 - \Delta_2) q_i + \Delta_1 \lambda_i e_{i1}^L + \Delta_2 e_{i2}^L \geq 0 \quad (32)$$

Let $\Delta^* = \min\{\Delta_1, \Delta_2 - \Delta_1\}$. Since $\Delta_2 > \Delta_2 - \Delta_1, \Delta_1$, the terms of Δ_2 and $\Delta_2 - \Delta_1$ in (32) can be replaced by the smaller term Δ^*, we have

$$q_i \leq e_{i2}^L + \lambda_i e_{i1}^L. \quad (33)$$

Next, to satisfy the Nash equilibrium requirement, we have

$$E(\pi_{j1}^H) - E(\pi_{j3}^L) = [E(\xi_a) - E(\xi_b)(q_i + \lambda_i e_{i1}^L + q_j + \lambda_j e_{j1}^L) - E(\xi_{d_j})](q_j + \lambda_j e_{j1}^L) - [E(\xi_a) - E(\xi_b)(q_i - e_{i3}^L + q_j - e_{j3}^L) - E(\xi_{d_j})](q_j - e_{j3}^L) \geq 0. \quad (34)$$

Similarly, let $\Delta_3 = E(\xi_a) - E(\xi_b)(q_i + \lambda_i e_{i1}^L + q_j + \lambda_j e_{j1}^L) - E(\xi_{d_j})$ and $\Delta_4 = E(\xi_a) - E(\xi_b)(q_i - e_{i3}^L + q_j - e_{j3}^L) - E(\xi_{d_j})$. Because of non-negative expected unit profits, $\Delta_3 \geq 0$ and $\Delta_4 \geq 0$. Now (34) can be rewritten as

$$E(\pi_{j1}^H) - E(\pi_{j3}^L) = (\Delta_3 - \Delta_4) q_j + \Delta_3 \lambda_j e_{j1}^L + \Delta_4 e_{j3}^L \geq 0 \quad (35)$$

Let $\Delta^{**} = \min\{\Delta_3, \Delta_4 - \Delta_3\}$. Since $\Delta_4 \geq \Delta_4 - \Delta_3$, the terms of Δ_3, Δ_4 and $\Delta_4 - \Delta_3$ in (35) can be replaced by the smaller term Δ^{**}, we have

$$q_j \leq e_{j3}^L + \lambda_j e_{j1}^L \quad (36)$$

Combining (33) with (36) results in the strategy combination (high-production, high-production) being the Nash equilibrium outcome.

Table 2
Conditions for the four possible Nash equilibrium outcomes.

		Firm j's production strategy	
		High-production	Low-production
Firm i's production strategy	High-production	$q_i \leq e_{i2}^L + \lambda_i e_{i1}^L$ $q_j \leq e_{j3}^L + \lambda_j e_{j1}^L$	$q_i \leq e_{i4}^L + \lambda_i e_{i3}^L$ $q_j \geq e_{j3}^L + \lambda_j e_{j1}^L$
Low-production	$q_i \geq e_{i2}^L + \lambda_i e_{i1}^L$ $q_j \leq e_{j4}^L + \lambda_j e_{j2}^L$	$q_i \geq e_{i4}^L + \lambda_i e_{i3}^L$ $q_j \geq e_{j4}^L + \lambda_j e_{j2}^L$	

(ii) The strategy combination (low-production, low-production) satisfying the condition $E(\pi_{i4}^L) \geq E(\pi_{i3}^H)$ and $E(\pi_{j4}^L) \geq E(\pi_{j2}^H)$ is the Nash equilibrium outcome. We first discuss

$$E(\pi_{i4}^L) - E(\pi_{i3}^H) = \Delta_5(q_i - e_{i4}^L) - \Delta_6(q_i + \lambda_i e_{i3}^L) \geq 0 \tag{37}$$

where

$$\Delta_5 = E(\xi_a) - E(\xi_b)(q_i - e_{i4}^L + q_j - e_{j4}^L) - E(\xi_{d_i})$$

and

$$\Delta_6 = E(\xi_a) - E(\xi_b)(q_i + \lambda_i e_{i3}^L + q_j + \lambda_j e_{j3}^L) - E(\xi_{d_i})$$

The terms, Δ_5 and Δ_6, are greater than 0 due to non-negative expected unit profits. Based on the solution procedure of (i), we have

$$q_i \geq e_{i4}^L + \lambda_i e_{i3}^L \tag{38}$$

Similarly,

$$q_j \geq e_{j4}^L + \lambda_j e_{j2}^L \tag{39}$$

(iii) The strategy combination (low-production, high-production) is the Nash equilibrium outcome if $E(\pi_{i2}^L) \geq E(\pi_{i1}^H)$ and $E(\pi_{j2}^H) \geq E(\pi_{j4}^L)$. By changing "≤" to "≥" in (33) and "≥" to "≤" in (39), we have $E(\pi_{i2}^L) \geq E(\pi_{i1}^H)$ and $E(\pi_{j2}^H) \geq E(\pi_{j4}^L)$. Therefore, the strategy combination (low-production, high-production) is the Nash equilibrium outcome if $q_i \geq e_{i2}^L + \lambda_i e_{i1}^L$ and $q_j \leq e_{j4}^L + \lambda_j e_{j2}^L$.

(iv) The strategy combination (high-production, low-production) is the Nash equilibrium outcome if $E(\pi_{i3}^H) \geq E(\pi_{i4}^L)$ and $E(\pi_{j3}^L) \geq E(\pi_{j1}^H)$. By changing "≥" to "≤" in (38) and changing "≤" to "≥" in (36), we have $E(\pi_{i3}^H) \geq E(\pi_{i4}^L)$ and $E(\pi_{j3}^L) \geq E(\pi_{j1}^H)$. Therefore, the strategy combination (high-production, low-production) is the Nash equilibrium outcome if $q_i \leq e_{i4}^L + \lambda_i e_{i3}^L$ and $q_j \geq e_{j3}^L + \lambda_j e_{j1}^L$. This completes the proof. ■

Proposition 2. shows that the strategy combination becoming the Nash equilibrium outcome is based on both the apex and the entropies. In other words, our model provides decision-makers with both the fuzzy equilibrium quantity in the short term as well as the equilibrium production strategy in the long term.

5. Case study

In this section, we utilize the model presented in Section 3 as a planning tool to demonstrate how the two competing firms determine the equilibrium quantity against ambiguity in the glass substrates industry.

5.1. Industry background

During the last decade, aggressive marketing strategies coupled with low-cost thin-film transistor liquid crystal display (TFT–LCD) production have induced increasing numbers of consumers to favor flat screens over conventional cathode ray tube (CRT) products. The physical sizes of glass substrates required for various TFT–LCD products play a key role in the growing demand. As mentioned earlier, there are two prohibitive barriers to entry into the glass substrates industry: capital outlay and the materials. Since the market share of the two major firms in our case study totals approximately 90% in Taiwan (Hwang and Lin, 2008), we consider the glass substrates industry a duopoly market.

A recent report by DisplaySearch[1] indicates that the production of TFT–LCD glass substrates reached a peak of 14.2 million square meters in second quarter 2010 and then dropped to 12.2 million square meters in the third quarter, a reduction of 14% from last season. Thus, despite apparent consumer demand, global flat screen manufacturers still need to adjust their production strategies. It implies that because of the TFT–LCD panel prices and weak demand, the manufactures have to adjust their production strategy to meet the market demand. This results in twofold production strategies: high- and low-production strategy.

Research on the Cournot game applied in the real world includes the world oil, electricity and petroleum products markets (see (Ruiz et al., 2008; Slade, 1986; Salant, 1976)). The previous studies have proposed to assist decision makers to determine the equilibrium quantity or analyze the market efficiency. Acknowledging the need for improved decision-making, the model proposed in Section 3 depicts the behavior of two competing glass substrates manufacturers in a hybrid uncertain environment, and constructs each firm's fuzzy equilibrium quantity. After determining their production strategies, we obtain the apex of the fuzzy equilibrium quantity by the center problem. We then define each firm's profit function considering the production strategy by utilizing the resulting apexes. Due to the special characteristics of the glass substrates industry, we apply the model to demonstrate how to obtain the fuzzy equilibrium quantity and the production strategy of each firm.

Table 3
Market price of the glass substrates corresponding to the size type.

		Size type					
		1	2	3	4	5	6
Price (USD)	High	57	55	61	63	76	81
	Apex	56	53	60	60	73	78
	Low	54	50	58	59	71	76

Table 4
Market demand of the glass substrates in each period of the case study (Shao and Lin, 2009).

		Size type					
		1	2	3	4	5	6
Period	1	1800	5000	20,000	7000	20,000	19,000
	2	1800	3000	12,600	8600	16,000	14,000
	3	1500	3000	16,000	5500	20,000	16,000
	4	1680	4700	17,000	6000	22,000	16,000

5.2. Insights from the Cournot production game case study

5.2.1. Case study overview and input data

Our case study is based upon timely representative data for the glass substrates industry. We note that the data will differ for other industry sectors, geographic regions, and/or time epochs.

As mentioned earlier, the demand function of the market is given by $p(\xi) = \xi_a - \xi_b \tilde{Q}$. The intercept, ξ_a, represents the amount of glass substrates sold by the glass substrates manufacturers where the price, p, is zero and the slope, ξ_b, is the price sensitivity to the increase in the amount of glass substrates per unit of the price added. Based on the information published by DisplaySearch[2], we arrange the unit price of the glass substrates as shown in Table 3.

[1] http://www.newso.org/ITNews/Trade/DisplaySearch-LCD-substrate-in to-the-third-quarter-will-be-reduced/dc242f45-3d93-48b1-b410-0c2b114a0da1

[2] http://www.displaysearch.com/cps/rde/xchg/displaysearch/hs.xsl/resource s_pricewise.asp

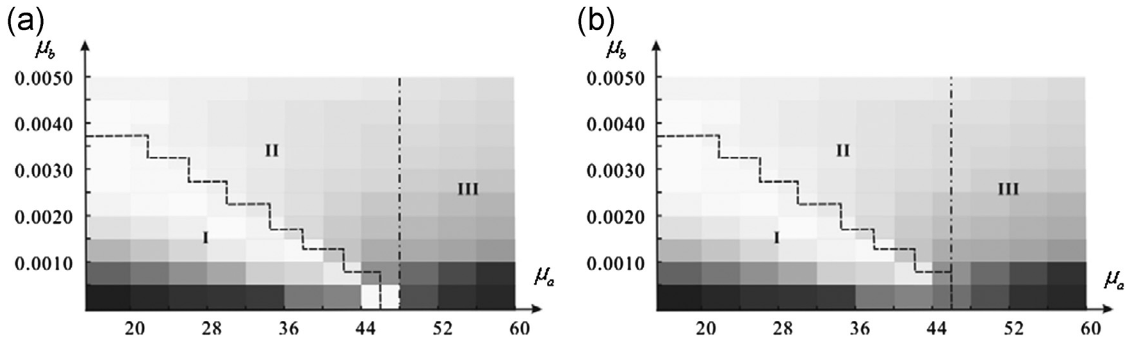

Fig. 3. Impacts of μ_a and μ_b on two firms' strategic scenarios. (a): The impacts of μ_a and μ_b on the strategic scenarios when $\mu_{d_1} < \mu_{d_2}$. (b): The impacts of μ_a and μ_b on the strategic scenarios when $\mu_{d_1} = \mu_{d_2}$.

Table 3 shows the six types of glass substrates (denoted by 1 to 6), and the three prices in USD (high, apex and low) for each size type. Table 4 shows the sold quantity in the market for each size type after Shao and Lin (2009). From these tables, we can derive the parameters ξ_a and ξ_b by the fuzzy regression method Chen and Dang (2008). In the case study, ξ_a and ξ_b can be estimated as $\xi_a = (X_a - 3.2300, X_a, X_a + 2.8040)$ with $X_a \sim N(52.7089, 4.9773)$ and $\xi_b = (X_b - 0.000248, X_b, X_b + 0.000303)$ with $\xi_b = (0.000987\ \ , 3 \cdot 10^{-8})$. In Table 4, we observe that the peak-season is period 1 and the off-season is period 2 because the total sold quantity in period 1, 89,000, is the highest, and in period 2, 56,000, is the lowest. Therefore, we estimate the ratio of production quantity in the peak-season to the off-season as 1.59 (89,000/56,000), which can be viewed as is λ in our model. We approximately estimate each firm's variable cost based on firm 1's consolidated financial report[3] which indicates a net income of around 22.6%, i.e. firm 1's total cost is about 77.4% (100%−22.6% = 77.4%). We know that the unit net income can be simply derived by market price minus the variable cost. Our Table 3 shows an average market price of 63.33 USD. Therefore, the ballpark estimate of the variable cost of firm 1 is 63.33·77.4% = 49.02 and similarly the variable cost of firm 2 is 50.03 (63.33·79%), as a result of firm 2's 21%[4] net income.

5.2.2. *Case study results and sensitivity analysis*

According to the proposed method in Section 3, we can derive the apex of the fuzzy equilibrium of firm 1 by substituting the parameters estimated in the case study into (18). Then we have $q_1 = 1{,}587$. Similarly, the apex of the fuzzy equilibrium quantity of firm 2 is $q_2 = 570$. It is obvious that the apex of the fuzzy equilibrium quantity of firm 1 is higher than of firm 2 due to firm 1's low variable cost. Next, to solve for the entropy of each firm, we consider four strategic scenarios in the spread problem with assumptions. With the available data, we find that the resulting solutions only satisfy Assumption 4, in other words, the entropy of each firm can be obtained in Strategic Scenario 4 where both firms adopt low-production strategies. As a result, the left entropy is 57 for firm 1 and 43 for firm 2. Then we have the fuzzy equilibrium quantity of firm 1 $\tilde{q}_1 = (1{,}530,\ 1{,}587,\ 1{,}677)$ and the fuzzy equilibrium quantity of firm 2 $\tilde{q}_2 = (527,\ 570,\ 638)$. In addition, we know that the production quantity ranges from 1530 to 1677 for firm 1 and 527 to 638 for firm 2. The report by DisplaySearch[5] indicates that the glass substrates industry tends to decrease production quantities, which coincides with the behaviors predicted in our model.

Next, we investigate the impacts of market demand, μ_a and μ_b, on each firm's choice of strategic scenarios. Obviously, two zones exist where both firms adopt the high- or low-production strategies shown in Fig. 3. Knowing that μ_a can be interpreted as the potential demand in the market and given a specific value of μ_b, we find that each firm adopts the low-production strategy (Strategic Scenario 4) as an increase in μ_a as shown in Fig. 3(a). Therefore, if the potential demand is high enough, both firms will determine the lower production quantities in order to maximize their profits and vice versa. Similarly, given a specific value of μ_a, an increase in μ_b results in a scenario whereby both firms employ the low-production strategies (Strategic Scenario 4). In other words, both firms adopt the low-production strategy once market demand becomes sensitive.

Fig. 3(b) shows how market demand affects the choice of strategic scenarios, given the variable cost of firm 1 being equal to firm 2 and all other parameters remaining the same. We observe that Fig. 3(a) is similar to Fig. 3(b), i.e. market demand heavily impacts each firm's production strategy rather than each firm's cost structure. Thus, in our case study both firms tend to simultaneously adopt low- or high-production strategies.

6. Conclusions

Decision-making in a complicated and competitive environment is often made more difficult due to uncertainty, e.g. customer demand, production fluctuations, etc. Furthermore, real-world problems frequently involve ambiguity and randomness. This paper has described a new version of a two-stage Cournot production game, which embeds an operational-level decision in the short term within a strategic-level decision in the long term. In our model, two firms determined a high or low production strategy at the strategic level, followed by determining their production quantities at the operational level under the specific production strategy. The concept of the production strategy was utilized to construct each firm's the range-type production quantity. At the operational level, the production quantity of each firm was assumed to be a triangular fuzzy number, which allowed the production quantity to be represented by an apex and entropies.

At the operational level, the game was divided into the center and spreads problems and the fuzzy equilibrium quantity of each firm constructed from the outcomes of the two problems. Unlike previous studies, the equilibrium fuzzy production quantity gave each firm a production interval when obtaining accurate parameters is impossible. At the strategic level, the Nash equilibrium concept was applied to derive the conditions such that a strategy combination became the Nash equilibrium outcome. Applying the proposed model to the case study derived the fuzzy equilibrium quantity of each firm in the glass substrates industry. The results showed that both firms tended to

[3] http://www.agc.com/english/news/2012/0208e_1.pdf
[4] http://www.corning.com/tw/tc/news_center/news_releases/2012/2012012501.aspx
[5] http://www.honghaiglass.com/en/nshow.aspx?id=31

adopt the common strategy, a finding which coincides with the real-world situation. In addition, sensitivity analysis revealed that the potential market demand, μ_a, plays a key role in determining a firm's production strategy. We suggest that further research should explore the issue of spreads with probability distributions by refining our proposed model. Another interesting extension is to investigate combinations of production strategies, where the market demand depends on the considered combination of strategies. Detailed technical explanations can be found in (Dang, 2012).

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