WDM Coding System with Single Parity-Check Channel

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Abstract-We propose a modified WDM coding system with a single parity-check channel to achieve an efficient and flexible WDM coding system. The system provides most of the advantages of the original WDM coding system with just one parity-check channel. The bit rate and the data sequences are unaffected by the coding processes, making it very attractive for practical lightwave systems with standard rates. Compared to serial coding systems, it is able to reduce the number of encoding/decoding pairs in an m-channel WDM transmission system operated at STS-N from m x *N* **to one. We have evaluated the system performance and analyzed the performance of frame synchronization needed for the modified WDM coding system. An example shows that the WDM coding can reduce the error floor of a four-channel dispersion**limited WDM system from 10^{-10} to 10^{-19} with reliable frame **synchronization.**

I. INTRODUCTION

ORWARD error corrected (FEC) codes are widely used **F** in various communication systems to achieve a reliable transmission. Many FEC codes with very good error correction capability have been adopted in numerous practical systems, for example, the well-known Reed-Solomon codes having *2t* redundant bits in a codeword block are able to correct *t* errors simultaneously. In lightwave systems, bit error probability can be reduced if FEC are suitably applied. However, FEC is rarely used in high-speed lightwave systems because coding in general changes system bit rate, which is often unacceptable in practical systems with a well-specified bit rate. Also the required signal processing in encoding/decoding is difficult in high-speed lightwave systems. Moreover, the coding gain is estimated to be small in power-limited lightwave systems [l]. Nevertheless, FEC had been implemented for DS-3 and STS-I systems by putting the parity-check bits in some unused stuffed bits of DS-3 or unassigned overhead fields of STS-1 signals in order not to affect system bit rate. In addition, it is found that FEC could provide infinite coding gain in dispersion-limited lightwave systems [11, [2]. However, owing to the complicated encoding/decoding processes, the FEC can just be implemented at the DS3 or STS-I level. Therefore, *N* encoder/decoder pairs are necessary for an STS-N system, and $m \times N$ encoder/decoder pairs are needed for an m-channel high-speed WDM system with each wavelength channel operating at the STS-N rate.

In our previous work, a novel coding system as shown in Fig. **1** was proposed *[3].* It is named as the WDM coding

Manuscript received June 21, 1993; revised March 29, 1994.

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IEEE Log Number 9403816.

system since coding is performed among different wavelength channels. In the system, a set of *k* independent data sequences $(D_1 \cdots D_k)$, each operating at the STS-N rate, are encoded in parallel by an FEC encoder to output codeword sequences $(D_1, D_2 \cdots D_k, P_1 \cdots P_r)$, where $P_1 \cdots P_r$ are parity-check sequences. The original data sequences are unaffected by the encoder, whereas parity-check bits are carried by the *^T* panty-check sequences, which have the same rate as the data sequences. The encoded codeword then modulates an LD array consisting of *n* different wavelength lasers $n = k + r$, to form an n-channel WDM transmission. At the receiving end, the *n* wavelength channels are separately received, and the codeword sequences are decoded by an FEC decoder to retrieve original data sequences.

In this paper, we consider a modified WDM coding system with only one parity-check channel. That is, we just use one parity-check channel to accommodate all the parity-check bits. The idea stems from the fact that the ratio of the block size (n) to the number of parity-check bits (r) in a block code in general increases with *T.* For instance, an *(n, k)* Hamming code with r parity-check bits, then $n = 2^r - 1$. Thus, the ratio $n/r = (2^r - 1)/r$ increases almost exponentially with *r*. Based on this observation, we can group a number of data channels in a WDM system to be a large data block and put the paritycheck bits to one parity-check channel only. The resulting modified WDM coding system will have only one parity-check channel instead of *T* in the original WDM coding system. Also, unlike the original WDM coding systems in which the number of parity-check channels varied with the number of data channels, the number of parity-check channels in the modified WDM coding system is always one, independent of the number of data channels.

This paper is organized as follows. We will first describe the architecture of the modified WDM coding systems in Section 11. Next, we analyze the system performance in Section 111. Finally, we summarize the paper in Section IV.

11. SYSTEM DESCRIPTION

The architecture of the modified WDM coding system is shown in Fig. 2, where *rn* STS-N data channels are encoded in parallel by an FEC encoder. The produced parity-check bits are transmitted by a parity-check channel *P.* The *m* data channels plus the parity-check channel modulate an LD array at wavelengths $\lambda_1 \cdots \lambda_{m+1}$, respectively, to form an $(m+1)$ channel WDM transmission. At the receiving end, those $m+1$ channels are separately received by a PD array and decoded

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Fig. 2 Block diagram of the modified WDM coding system,

by an FEC decoder to correct possible errors and retrieve the original data.

One of the limitations in WDM coding systems is the bitskew among wavelength channels. Normally, bit skew caused by group velocity mismatch among different wavelength channels is dominant at long-distance multichannel transmission. Since the number of wavelength channels of the modified WDM coding system is less than that of the original WDM coding system, the limitation caused by bit-skew will be less for the modified WDM coding system. Bit-skew can be compensated by using short fiber delay lines after the WDM demultiplexer at the receiving end *[3].* For example, to compensate a bit-skew of 1 ns, a fiber delay line of about 0.2 m is needed. For simplicity, we assume bit-skew have been compensated and ignore its effect hereafter.

A. Coding Scheme

To make the encoding/decoding circuits compact and feasible, the modified WDM coding system adopts one of the parallel single-error-correction, double-error-detection (SEC-DED) codes---the Hsiao code---which is widely used in computer systems [4], *[5].* Hsiao codes are a class of SEC-DED codes whose minimum Hamming distance is 4. The interested

readers can refer to **[6]** for detailed description of Hsiao codes. Practically, Hsiao codes can be implemented easily with simple hardware for encoding/decoding. Also, the undetectable error probabilities of triple errors and quadruple errors are lower than those of a conventional modified Hamming code. Therefore, we adopt Hsiao code as the coding scheme for the modified WDM coding system.

B. Encoder Configuration

The encoded data block of the modified WDM coding system is given in Fig. 3. We group the *m* data channels, each with *b* bits, to form a data block $k = mb$ bits. The *k* data bits are encoded in parallel by an (n, k) FEC encoder, $n = k+r$, to generate *r* parity-check bits. The frame alignment word (FAW), consisting of f bits $(F_1 \cdots F_f)$, and the r paritycheck bits constitute the parity-check channel, and $r + f = b$. The FAW is necessary for the encoded block to be aligned and decoded correctly at the receiving end **[7],** [8]. Thus the entire encoded frame has $(m + 1)b$ bits.

Fig. 4 shows the block diagram of the FEC encoder. To encode the *m* data channels, we first pass each data channel to a *b*-stage shift register. A *b*-mode clock loads mb data bits $d_1 \cdots d_{mb}$ from *m* data channels to an *mb*-stage buffer.

Fig. 3. Frame structure of the modified WDM coding system.

These mb bits are then sent to a Hsiao code encoder in parallel to generate r parity-check bits $P_1 \cdots P_r$. The clock is suitably delayed and then loads the r parity-check bits and an f -bit FAW to a b -stage shift register to form the parity-check channel. Note that the m data channels are completely unchanged except that they are delayed suitably to be synchronized with the parity-check channel. The $m + 1$ output sequences, each operating at STS-N, are going to modulate $m + 1$ lasers to form an $(m + 1)$ -channel WDM transmission.

C. Decoder Configuration

The FEC decoder at the receiving end is shown in Fig. 5. Bit sequences from the m received data channels are fed to m b -stage shift registers, separately. The parity-check channel enters another b-stage shift register where frame alignment is executed. The bit sequence in the parity-check channel is compared with a stored FAW pattern and the number of mismatched bits is summed. If the number of mismatched bits is less than a preset threshold d_H , the detection of FAW is declared and the FAW detector outputs an enable pulse to load the m b-stage data bits $V_1 \cdots V_{mb}$ into an mbstage buffer. The mb data bits and the receive parity-check bits $V_{mb+1} \cdots V_{mb+r}$ enter an error indicator simultaneously. The error indicator calculates the syndrome and indicates possible error location by the error-indicating bits $e_1 \cdots e_{mb}$. We download these error-indicating bits into m b-stage shift registers corresponding to the m data channels to correct any possible error that has occurred in the m data channels. For example, if an error happened at the third position of channel 1, then e_3 will be "1" to indicate the error position and the other error-indicating bits are "0". After two clocks, the bstage shift register corresponding to channel 1 will output an "1" to the "EXOR" input of channel 1 to correct the error bit at the third position, while the other error-indication outputs are all "0", which have no affect on the bits in each channel.

A. BER Performance

As the original WDM coding system, the BER performance of each wavelength channel, suffering from different characteristics of LD's and a different attenuation and dispersion of the transmission fiber, is therefore not the same and is independent [3]. For simplicity, here we use the error probability of the worst channel to estimate system performance conservatively.

 $D3$ the code can correct single error only, P_w can be expressed straightforwardly as

$$
P_w = \sum_{i=2}^{n} {n \choose i} \cdot P_e^i \cdot (1 - P_e)^{n-1} \tag{1}
$$

where P_e is the BER prior to decoding. The BER after decoding depends on the error extension characteristics of the code. The average number of decoded errors $E[(X(e))]$ resulting from **e** primary errors for an SEC-DED code can be described as [2]

$$
E[X(e)]\begin{cases} 0 & e = 0 \text{ or } 1\\ \frac{k-e}{n} \cdot (e+1) + \frac{r}{n} \cdot e & e = 2\\ +\frac{e}{n} \cdot (e-1), & e > 2 \end{cases}
$$
 (2)

Since the error extension resulting from miscorrection is uniformly distributed over the whole encoded data bits, the average decoded errors of each channel are $E[X(e)]/m$. The decoded BER for each channel P_{e} , FEC, under the condition that statistical independence among channels is valid, can be expressed as

$$
P_{e, \text{FEC}} = \frac{1}{b} \sum_{e=2}^{n} \frac{E[X(e)]}{m} \cdot \frac{n!}{e!(n-e)!} \cdot P_e^e \cdot (1 - P_e)^{n-e}
$$

$$
= \frac{1}{mb} \sum_{e=2}^{n} E[X(e)] \cdot \frac{n!}{e!(n-e)!} \cdot P_e^e \cdot (1 - P_e)^{n-e}.
$$
(3)

Fig. 6 shows the BER of three different (n, k) modified WDM coding systems and the uncoded system. When P_e < 10^{-4} , the decoded BER is dominated by the case $e=2$. In other words, FEC reduces the original BER to the order of $O(P_e^2)$. In ordinary lightwave systems with $P_e < 10^{-9}$, FEC could approach nearly error-free transmission. The figure also indicates that the decoded errors for different WDM Hsiao codes are small. This is a direct consequence of the parallel coding scheme, which is different from serial coding systems. Since the complexity of encoder/decoder increases with block size, for a given number of data channels, the Hsiao code with a minimum block size should be chosen in practice.

FEC could provide infinite coding gain in dispersion-limited lightwave systems. [Fig. 7](#page-5-0) shows the BER floors induced by relative intensity noise (RIN), mode partition noise (MPN), laser chirping, and optical reflection noise, and so on. These results are calculated based on formulas in [3]: assuming an average APD gain $\langle M \rangle = 9.1$, transmission length $L = 69$ km, and at the transmission rate of STS-12 (622 mb/s). The **111.** PERFORMANCE EVALUATION coding gain for the WDM system at $P_e = 10^{-9}$ is about 4 dB, and it may have infinite coding gain if the desired bit error rate is below the error floor. Moreover, the error floor is declined from 10^{-10} to 10^{-19} , indicating that WDM coding can improve performance of a dispersion-limited lightwave system drastically.

Fig. 4. Encoder block diagram of the modified WDM coding system.

Fig. *5.* Decoder block diagram of the modified WDM coding system.

B. Analysis of Frame Synchronization

A reliable frame synchronization is crucial to the modified WDM coding system. Loss of synchronization results in loss of frame. The frame alignment algorithm of the modified WDM coding system is similar to the frame alignment procedure in normal time-division-multiplexed PCM systems. Usually, frame synchronization depends on the available synchroniza-

[Fig.](#page-2-0) 6. Decoded BER **versus** uncoded BER for **three** different *(n. k)* modified WDM **Hsiao** codes.

tion bits, structure of the synchronizing character, and frame length [9]. In the modified WDM coding system, the paritycheck channel is used to transmit parity-check bits and the FAW. Referring to Fig. 3, for an (n, k) Hsiao code we have

$$
2^{r-1} - r \ge k. \tag{4}
$$

Hence,

$$
2^{r-1} - r \ge m(f+r)
$$
 (5)

where 2^{r-1} is the maximum codeword length given r paritycheck bits for Hsiao coding. A valid FAW could be missed by the FAW detector due to error bits or be simulated by random data. When the FAW is simulated, the frame alignment circuit declares a spurious frame alignment. The probability of such a spurious frame alignment is called simulation probability. In the modified WDM coding system, we would like to minimize the simulation probability to achieve a reliable frame synchronization. Therefore, we take $f \geq r$ to reduce the simulation probability. Under this constraint, *(5)* becomes

$$
\frac{2^{r-1}-r}{2r} \ge m.
$$
 (6)

Equation (6) indicates the relationship between the channel number and the number of parity-check bits. For a given number of channels, there exists a minimum number of paritycheck bits satisfying (6).

Reliable frame synchronization is obtained through an efficient frame alignment procedure [101 and selection of optimal frame structure [7], [8]. For the modified WDM coding system,

as shown in Fig. 8, the frame structure restricts the frame alignment process operating in the overlap region (a case that the *f* bits frame alignment window contains a combination of parity-check bits and FAW bits). Following **an** efficient frame alignment algorithm, the frame alignment are locked after "c-successive" confirmation processes and then the FAW are examined for every frame length duration [111. **If** the contents of the alignment window are unacceptable, a bit-by-bit search must be initiated to find the correct FAW position. The frame alignment procedure goes on to retain correct frame alignment state.

To achieve good frame alignment, we adopt those FAW patterns given in [8] for frame synchronization. The miss probability is simply given as

$$
P_m = 1 - \sum_{i=0}^{d_H} \binom{f}{i} P_e^i (1 - P_e)^{f-i} \tag{7}
$$

where d_H is the acceptable distance between the received word and the stored FAW, that is, the minimum distance.

Assume that the probability of simulation during the search processes is constant and is negligible, the average probability of simulation $P_{a, \text{sim}}$ in the modified WDM coding system can be expressed as [7], [8]

$$
P_{a,\text{sim}} = \frac{1}{b-1} \sum_{i=1}^{b-1} P_{\text{sim}}(i) \tag{8}
$$

where $P_{sim}(i)$ is the probability of simulation when the FAW pattern is *i*-bit displacement away from the frame

Fig. 7. Illustration of the uncoded and WDM coded BER as a function of average received power for an OOK system.

Fig. 8. Illustration of frame alignment in the modified WDM coding system.

alignment window. Following the standard approach derived in **[8],** the average probability of simulation can be calculated accordingly.

Table **I** lists several Hsiao codes and the parameters involved with channel number $m = 4$ and $d_H = 0, 1, 2$, respectively. In frame synchronization, it is desirable to minimize both $P_{a, \text{sim}}$

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TABLE I ENCODED BLOCK LENGTH VERSUS SIMULATION AND MISS PROBABILITIES

(n,k)	f	r	d_H	P_{sim}	P_m
(72, 64)	8	8	0	2.6×10^{-4}	8×10^{-9}
			1	3.39×10^{-3}	2.8×10^{-17}
			$\overline{2}$	3.82×10^{-2}	5.6×10^{-26}
(80, 72)	10	8	0	1.61×10^{-21}	1×10^{-8}
			$\mathbf{1}$	3.22×10^{-12}	4.5×10^{-17}
			$\boldsymbol{2}$	1.61×10^{-3}	1.2×10^{-25}
(88, 80)	12	8	0	6.58×10^{-30}	1.2×10^{-8}
			1	1.97×10^{-20}	6.6×10^{-17}
			2	1.97×10^{-11}	2.2×10^{-25}
(96, 88)	14	8	0	1.86×10^{-30}	1.4×10^{-8}
			1	5.58×10^{-21}	9.1×10^{-17}
			\overline{c}	5.58×10^{-12}	3.64×10^{-25}
(104, 96)	16	8	0	6.79×10^{-40}	1.6×10^{-8}
			1	2.72×10^{-30}	1.2×10^{-16}
			$\boldsymbol{2}$	4.08×10^{-21}	5.6×10^{-25}
(112, 104)	18	8	0	6.25×10^{-49}	1.8×10^{-8}
			1	3.13×10^{-39}	1.53×10^{-16}
			$\overline{2}$	6.25×10^{-30}	8.16×10^{-25}
(120, 112)	20	8	0	4.05×10^{-57}	2×10^{-8}
			1	2.43×10^{-47}	1.9×10^{-16}
			$\boldsymbol{2}$	6.08×10^{-38}	1.14×10^{-24}
(128, 120)	22	8	0	9.43×10^{-66}	2.2×10^{-8}
			1	6.6×10^{-56}	2.31×10^{-16}
			$\overline{2}$	1.98×10^{-46}	1.54×10^{-24}
(133, 124)	22	8	0	9.24×10^{-66}	2.2×10^{-8}
			1	6.47×10^{-56}	2.31×10^{-16}
			$\overline{2}$	1.94×10^{-46}	1.54×10^{-24}
(137, 128)	23	8	0	1.89×10^{-66}	2.3×10^{-8}
			1	1.32×10^{-56}	2.53×10^{-16}
			$\overline{2}$	3.97×10^{-47}	1.77×10^{-24}

and P_m . For a given d_H , P_a , sim decreases as f increases, but P_m increases instead. In contrast, for a given f, P_a sim increases with d_H but P_m decreases. In designing the modified WDM coding system, it is crucial to choose a Hsiao code with a minimum block length to simplify encoding/decoding while keeping acceptable $P_{a, \text{sim}}$, P_m . As shown in Table I, the (72, 64) code with $f = r = 8$ has an unacceptable high simulation probability since the FAW is easily simulated by the parity-check bits. For the (88, 80) code with $d_H = 1$, both P_a sim and P_m are less than 10^{-16} and therefore the frame synchronization will be very reliable.

A good frame synchronization should have the following attributes [7]: I) rapid initial frame acquisition, 2) rapid detection of timing anomalies and frame recovery, 3) reliable

lock indication, and 4) a simple synchronization algorithm. These attributes can be measured by four key parameters: the maximum average reframe time (T_{RF}) , false in-frame time (T_{FF}) , out-of frame detection time (T_{OF}) , and misframe time (T_{MR}) [11]. The objective of frame alignment is to have short T_{RF} (attributes 1, 2) and T_{OF} (attribute 2) and very long T_{FF} and T_{MR} (attribute 3). Table II shows these parameters with corresponding block length and simulation and miss probabilities given in Table **1.** Here, the number of successive FAW detection in the "c-successive'' frame detection algorithm is $C_l = 2$ and the number of successive out-of-frame declaration is $C_s = 4$ [12]. We note that $T_{\rm RF}$ is equal to 2 frame lengths and $T_{\rm OF}$ is equal to 4 frame

lengths because $C_l = 2$ and $C_s = 4$. T_{FF} increases as $P_{a, \text{sim}}$ decreases, and T_{MF} increases as P_m decreases. From Table 11, we can select the desirable encoded block length with very short T_{RF} and T_{OF} and very long T_{FF} and T_{MF} . For the $(88, 80)$ code with $d_H = 1$, both T_{FF} and T_{MF} are very long, which can result in reliable frame synchronization.

Iv. DISCUSSION AND CONCLUSION

Serial FEC codes are rarely used in high-speed lightwave systems because of the change of system bit rate, complicated encoding/decoding processes, and small coding gain in power-limited lightwave systems. WDM coding systems provide reliable high-capacity WDM systems by employing an abundant fiber bandwidth for coding purposes. The key idea of WDM coding is to use the tremendous fiber bandwidth to carry high-capacity information and to perform interchannel coding. Specifically, the WDM coding system could offer an unchanged data rate, a single system clock, the saving of encoder/decoder pairs, and simple encoding/decoding processes.

In this paper we propose a modified WDM coding system with only one parity-check channel. The serial coding system proposed in [2] can be operated only at the **STS-I** level because of the complicated signal processing involved. If it is implemented in an m -channel WDM system operated at STS-N line rate, the number of codecs needed is $m \times N$ [2], for example, a four-channel WDM transmission system operated at STS-12 needs $4 \times 12 = 48$ codecs. The modified WDM coding system, however, puts the parity-check bits in one additional parity-check channel, and needs only one codec. This saves a number of codecs. In addition, the codec of WDM coding is even simpler than that of serial coding. The major part of serial and WDM codecs are the shift registers for elastic storage. In the serial coding proposed in [2] (at the STS-1 rate), a codec needs 7200 shift registers with a delay around **139** us. In comparison, the codec for a four-channel (88; 80) WDM coding (at STS-12 rate) needs only about 500 shift registers with a delay less than 0.1 us. Thus, the WDM coding not only saves circuits, but also has very little delay.

Compared to the WDM coding system with *T* parity-check channels **[3],** the modified WDM coding system maintains the same advantages, such as an unchanged data rate, a single publication.

system clock, and the saving of encoderldecoder pairs, but needs only one additional parity-check channel. It is more flexible since the number of parity-check channels is always one, independent of the number of data channels. It is a promising coding scheme to improve the performance of WDM systems with many high-speed wavelength channels.

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Ming-Seng Kao, (S'89-M'90) photograph and biography not available at the time of publication.

Shou-Kuo Shao, photograph and biography not available at the time of publication.

Hsin-Yuan Chen, photograph and biography not available at the time of