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## Airline network design and adjustment in response to fluctuation in jet fuel prices



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### ABSTRACT

This study develops a series of models to determine aircraft types and flight frequencies on individual routes, and evaluate the reliability of proposed network planning during fluctuations in jet fuel prices. The reliability of individual routes is evaluated as to whether revenues from flights with initially proposed flight frequencies and aircraft types can accommodate variations in jet fuel expenditures. We define reliability as the probability that the proposed flight frequencies will operate in at least a break-even condition under future fuel price fluctuations. A case study is provided using an international airline in Taiwan to evaluate its network reliability in response to jet fuel price fluctuations in 2008. The results indicate that not only do routes with low load factors show low reliability, but long distance routes with high load factors also show low reliability during periods with high fuel prices. The results of the study provide effective ways to enhance commercial airline network designs in response to the uncertainty of jet fuel prices.

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### 1. Introduction

Jet fuel prices have fluctuated due to various environmental factors around the world, such as the global economy, resource shortages, wars, fluctuating exchange rates, etc. In recent years, tension in Middle Eastern countries and increasing fuel demands in China have also contributed to rising fuel prices [1]. For airline operators, the large proportion of overall operational expenses attributable to jet fuel has caused difficulties in cost control. In general, fuel costs for an airline account for about 30% of its operational costs but, in 2008, fuel prices increased so much that the proportion for jet fuel costs rose to 50%. The cost of fuel was also a factor that contributed to the decline in passengers. The International Civil Aviation Organization's annual report attributes the decline in passengers to slowing economic growth around the world and increases in airfares caused by soaring fuel prices in the first half of 2008; and this was especially true for tourism [2].

In response to the combination of increasing fuel expenses and a decline in passengers, airlines could take action to modify their operation strategies, such as rearranging schedules or adjusting operational routes. For example, during the first half of 2008, United Airlines removed 100 aircraft from its fleet, and Continental removed all its 737-300s and most of its 737-500s [3].

Hayashi and Trapani [4] examined how energy costs influence decisions regarding ticket prices and passenger service levels. The study emphasizes that fluctuations in energy prices influence the perceived benefit of some energy-intensive routes and change the capacity of aviation markets. Holloway [5] analyzed factors driving fuel costs in the aviation industry, which include age of fleets, fuel efficiency, jet fuel price in the global market, network design, and price pressure in local markets and airports. Changes in these forces have resulted in uncertainty regarding airline fuel costs, which has affected

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airline operations. Given this, airlines need to precisely analyze their supply and demand, and then plan appropriate routes, aircraft usage, and flight frequencies to increase the flexibility and performance of their networks. Swan and Adler [6] focused on aircraft operating cost related to operating distances for specific aircraft, addressing that fuel burn is almost linear in distance and weight across different aircraft types.

Within this domain, the network design problem has been applied to explore various issues. For example, in response to airport noise charges, Hsu and Lin [7] formulated an airline network design model to minimize operating costs in order to determine optimal air routes, flight frequencies, and types of aircraft used. The study indicated that airlines might reduce flight frequencies, reallocate aircraft and change routes under noise regulations. Reducing flight frequencies might reduce passenger demand, while using larger aircraft might increase passenger demand. Janic [8] modeled the operational, economic, and environmental performance of an air transport network, including airports and the air routes connecting them. The model is used under conditions of internalizing environmental externalities in order to maximize network profits for a given operational capacity and set of environmental constraints. Previous studies have also combined network design problems and reliability engineering to analyze network reliability. Hsu and Wen [9] considered passenger demand uncertainty in an airline network and evaluated the reliability of the network under conditions of fluctuating passenger demand. Lam et al. [10] simulated a road network to analyze the tradeoff between supply and demand under the uncertainty of weather conditions. Finally, Liu et al. [11] focused on the problem of allocating limited retrofit resources among transportation facilities, and modeled a two-stage stochastic programming problem that optimizes the mean-risk objective of transportation system loss. Few studies, however, have formulated models to analyze whether an airline network can accommodate variations in jet fuel prices and subsequent adjustment strategies.

This study develops a series of models to initially determine aircraft types and flight frequencies on individual routes, and then evaluates the reliability of the proposed network planning under conditions of fluctuating jet fuel prices. When airlines face a short term jet fuel price fluctuation, the priority consideration should be avoiding the increasing fuel cost and airfares thereby keeping the demand. In such a way, the demand can hold and the flight frequencies are further adjusted. However, in the long run, the travel demand and airfare fluctuations should be further considered since these factors are correlated with external environment such as the global economic growth. Furthermore, an adjustment and decision-making method is provided to determine whether to adjust flight frequencies or maintain the initial plan in response to fuel price fluctuation.

An airline network design model is used to determine types of aircraft and flight frequencies on individual routes, with the objective of minimizing airline network operation costs. To evaluate reliability in the airline network, the study assumes historic data on fuel prices can be classified into distinct intervals, and for each there exists a normal distribution. The fuel price intervals and distributions are calibrated and used as critical parameters for reliability evaluation. In this study, the reliability of individual routes is based on whether revenues from flights with initially proposed flight frequencies and aircraft types can accommodate variations in jet fuel expenditures. We define reliability as the probability that the proposed flight frequencies will operate in at least a break-even condition during future fuel price fluctuations. This study further examines the influence of fuel price and passenger demand on the reliability of an airline network using sensitivity analysis, where adjustments include changes in flight frequency and existing aircraft types.

A case study is presented to illustrate the feasibility of the models. The case uses current monthly travel demand and fuel prices in 2008 to evaluate the reliability of a network in response to uncertainty. The results provide adjustment suggestions to deal with a combination of passenger demand and fuel prices with varying fluctuation rates. The results show the critical load factors for unreliable routes, which enables airlines to easily identify those routes in order to adjust flight frequency and/or aircraft type, thereby reducing expected losses during fuel price fluctuation periods. In sum, the results of the study provide guidance to enhance commercial airline network design in response to future uncertainty of jet fuel prices and improve decision-making in network planning and adjustment.

The remainder of this study is organized as follows. In Section 2, we describe the airline network design problem, and Section 3 describes the framework for evaluating the reliability of the airline network. Section 4 provides methods for adjusting flight frequencies in response to jet fuel price fluctuation. A case study, using an international airline in Taiwan, is provided in Section 5 to illustrate the application of the models. Section 6 is devoted to overall conclusions and suggestions for future studies.

## 2. Airline network design model

The network design problem in this study is defined as determining flight frequencies, aircraft types, and air routes that satisfy travel demands and minimize total transportation costs for an airline network [7,12,13]. Consider an airline network  $G(N, A)$ , where  $N$  is the set of nodes and  $A$  is the set of links in graph  $G$ . We define the nodes as the airports in this network and  $i, j$  are the indices of the flight origin and destination (OD) airports, respectively ( $i, j \in N$ ). The route between airport  $i$  and airport  $j$  is designated as link  $a$  ( $a \in A$ ). Any OD pair  $i - j$  is connected by a set of routes  $P$  through the network. Let  $f_a$  represent the monthly frequency on link  $a$ , where  $a \in A$ . The decision variable in the airline network design model is  $f_{pk}$ , which represents the monthly frequency of flights served by aircraft  $k$  along route  $p$  ( $p \in P$ ). The link frequency is the sum of the frequencies on all routes going through that link, expressed as  $f_a = \sum_p \sum_k \delta_a^p f_{pk}$ , where  $\delta_a^p$  is the indicator variable. If  $\delta_a^p = 1$ , then link  $a$  is part of route  $p$ , otherwise  $\delta_a^p = 0$ .

The number of passengers represents those traveling from airport  $i$  to airport  $j$  through link  $a$ . Let  $Q_a$  be the monthly number of passengers on link  $a$  in the planning year, and  $Q_{pk}$  are the monthly numbers of passengers on the flights served

by aircraft  $k$  along route  $p$ . Use indicator  $\delta_a^p$  to express the relationship between the link flow and the route flow, then  $Q_a = \sum_p \sum_k \delta_a^p Q_{pk}$ .

Airline operating costs are normally divided into direct and indirect operating costs. Let  $\bar{O}^t$  be the average jet fuel price in a month of the planning year. In this study, we use the grey model [9] and Monthly Singapore Kerosene-Type Jet Fuel Price FOB (Free on Board) historical data to obtain a predicted value of  $\bar{O}^t$  in the planning year. The detail of the grey model can be referred to Hsu and Wen [9]. Let  $H_{ak}$  denote the number of gallons of fuel consumed for a one-way trip on link  $a$  served by aircraft  $k$ . Total jet fuel cost for a single one-way trip would be  $\bar{O}^t H_{ak}$ . Thus, the total jet fuel expense for all frequencies of flights in link  $a$ ,  $c_a^f(f_a)$ , can be expressed as  $c_a^f(f_a) = \sum_k f_a (\bar{O}^t H_{ak}) = \sum_p \sum_k \delta_a^p f_{pk} (\bar{O}^t H_{ak})$ .

Let  $b_{ak}$  denote the maintenance and flight crew costs for a one-way flight served by aircraft  $k$  on link  $a$ ;  $c_a^b(f_a)$  denotes the total maintenance and flight crew costs for all flights on link  $a$ , then,  $c_a^b(f_a) = \sum_k f_a b_{ak}$ .

Direct operating costs include the charges that airlines pay for airport usage, such as landing fees, noise surcharge, and emission charge. Let  $f_{ak}$  be the monthly frequency of flights served by aircraft  $k$  via link  $a$ . Then  $f_{ak}$  can be expressed as  $f_{ak} = \sum_p \delta_a^p f_{pk}$ .

Let  $c_{jk}^L$  and  $c_{jk}^E$  denote landing fees and environmental charges, respectively, for a flight served by aircraft  $k$  landing at airport  $j$ . The environmental charges ( $c_{jk}^E$ ) include both noise surcharge and emission charge for airport  $j$ . Thus, the airport charges for the flights served by aircraft  $k$  via link  $a$  and landing at airport  $j$ ,  $c_{jk}^\pi(f_{ak})$ , can be expressed as  $c_{jk}^\pi(f_{ak}) = f_{ak}(c_{jk}^L + c_{jk}^E)$ .

The formulation of total direct operating costs on link  $a$ ,  $C_a^D(f_a)$ , is as follows:

$$C_a^D(f_a) = \sum_k c_{jk}^\pi(f_{ak}) + c_a^b(f_a) + c_a^f(f_a) = \sum_p \sum_k \delta_a^p f_{pk} (c_{jk}^L + c_{jk}^E + \bar{O}^t H_{ak} + b_{ak}). \tag{1}$$

In line with Kanafani and Ghobrial [14] and Hsu and Lin [7], we define indirect operating costs as the expenses related to passengers. Let  $w_a$  denote the unit handling cost per passenger, in dollars, for those traveling on link  $a$ . The total indirect operating cost for link  $a$   $C_a^I(Q_a)$  is

$$C_a^I(Q_a) = w_a Q_a. \tag{2}$$

We assume the transportation capacities, provided in terms of number of seats, must satisfy the monthly travel demand in the airline network. Let  $\beta_a$  represent the load factor of aircraft  $k$  served by link  $a$ , and  $m_k$  is the number of available seats on aircraft  $k$ . The load factor can be then expressed as  $\beta_a = Q_a / \sum_k m_k f_{ak}$  [7].

Given this, the airline network design program can be formulated as follows.

$$\text{Min } C = \sum_{a \in A} [C_a^I(Q_a) + C_a^D(f_a)] \tag{3a}$$

$$\text{Subject to } \beta_a \sum_k m_k f_{ak} \geq Q_a, \quad \forall a \tag{3b}$$

$$\sum_p \tau_p f_{pk} \leq u_k Y_k, \quad \forall k \tag{3c}$$

$$f_a = \sum_p \sum_k \delta_a^p f_{pk}, \quad \forall a. \tag{3d}$$

$$\text{All } f_a, f_{pk}, Q_a \geq 0 \text{ and are integers.} \tag{3e}$$

As mentioned above,  $f_{pk}$  is the decision variable that represents the monthly frequency of flights served by aircraft  $k$  along route  $p$  ( $p \in P$ ). Eq. (3a) is the objective function that minimizes the total airline operating costs, including the direct and indirect operating costs of the air carrier. Eq. (3b) determines transportation capacities that, offered in terms of numbers of seats for each link, should be equal to or greater than the number of passengers on all links. Eq. (3c) indicates that total aircraft utilization has to be equal to or less than the maximum possible utilization, where  $u_k$  denotes the maximum possible monthly utilization of aircraft  $k$ ,  $\tau_p$  denotes the block time of aircraft  $k$  on route  $p$ , and  $Y_k$  is the number of aircraft  $k$  owned by the airline. In this case, the monthly maximum possible total hours flown by aircraft  $k$  are  $u_k Y_k$  (hours) based on the fleet fact. The maximum possible utilization implies the maximum possible monthly use of aircraft  $k$  for a period of time and it depends on the network structure and technical maintenance system [15]. This equation suggests that total aircraft utilization should be equal to or greater than the number of passenger seats on all routes. Eq. (3d) expresses the relationship between route frequency and link frequency. Eq. (3e) indicates that variables  $f_a$ ,  $f_{pk}$ , and  $Q_a$  are all non-negative integers.

### 3. Reliability evaluation model of an airline network

To evaluate the reliability of the airline network, we first assume historic data on fuel prices can be classified into distinct intervals, each of which is normally distributed. The fuel price intervals and distributions have been calibrated and are used as critical parameters for reliability evaluation. The reliability of individual routes is based on whether revenues from flights

with initially proposed flight frequencies and aircraft types can accommodate variations in jet fuel expenditures. In this case, reliability is defined as the probability that the proposed flight frequencies will operate in at least a break-even condition during future fuel price fluctuations.

We assume the price of jet fuel is a random variable for airline carriers when planning future networks. We define  $\tilde{O}^t$  as the random monthly jet fuel price variable where  $O^t$  is the realization of the monthly jet fuel price and superscript  $t$  denotes the month in the planning year,  $t = 1, 2, 3, \dots, 12$ ,  $t \in I$  where  $I$  is the set of 12 months in the planning year, that is  $I \equiv \{1, 2, \dots, 12\}$ .

To find the distribution characterizing jet fuel prices, we obtained historical data of Singapore Kerosene-Type Jet Fuel Prices from June 1986 to December 2008. We derived price density using Minitab 15.0 software by drawing histograms of the fluctuating jet fuel prices. If we only consider the density of jet fuel prices, we can detect the peaks and valleys in the price distributions. The peaks are where the prices occur with high frequency, while the valleys are where the prices occur with lower frequency. The shape of a distribution with a peak and two valleys is like a normal distribution. Therefore, the distribution of historical fuel data is analogous to several normal distributions. We assume the historical jet fuel prices can be categorized into different intervals according to price levels. The random jet fuel price  $\tilde{O}^t$  within a specific price interval would be normally distributed.

Interval analysis is used to demonstrate the distribution in fluctuating jet fuel prices. Moore and Bierbaum [16] defined an interval as a closed scope of a set of real numbers. Following their early work, in this study we let real number  $x$  denote the historical jet fuel price and the endpoint prices are  $\underline{x}^*$ ,  $\bar{x}^*$  respectively.  $\underline{x}^*$  is the lowest price and  $\bar{x}^*$  is the highest price. The relationship between  $x$  and endpoints can be expressed as  $[\underline{x}^*, \bar{x}^*] = \{x : \underline{x}^* \leq x \leq \bar{x}^*\}$ . Let  $X$  be an interval, and we denote its endpoints by  $\underline{X}$  and  $\bar{X}$ . Thus,  $X = [\underline{X}, \bar{X}]$ , ( $X \in R$ ) [16]. Jet fuel price  $x$  is contained in different intervals. If  $x$  is in the interval  $X$ , it can be expressed as  $\underline{X} \leq x \leq \bar{X}$ ,  $x \in X$ .

In this study, the jet fuel prices in an interval are assumed to be normally distributed with mean value  $\mu$  and standard deviation  $\sigma$ . The normal distribution can be expressed as  $x \sim N(\mu, \sigma)$ . Thus, we assume that when the random monthly jet fuel price  $\tilde{O}^t = x$ , then the price  $\tilde{O}^t$  is in the interval  $X$ . The normal distribution can be expressed as  $\tilde{O}^t \sim N(\mu, \sigma)$ . The probability space  $\Omega$  of  $x$  is defined as the probability of  $x$  in interval  $X$  and is  $P(\underline{X} \leq x \leq \bar{X})$ .

Assume  $n$  denotes intervals and the number of intervals is  $\gamma$ ,  $n = 1, 2, 3, \dots, \gamma$ . The historical fuel prices of different intervals can be indicated as  $X_n$ , which includes a set of jet fuel prices  $x_n$ . Interval analysis is used to examine the interval of the random jet fuel prices  $\tilde{O}^t$  during a planning period. When  $\tilde{O}^t$  are in the same interval as the initial proposed fuel price  $\tilde{O}^t$ , jet fuel prices are undergoing normal fluctuation.

The unreliability problem of the airline network model arises from the situation where profits made by proposed monthly flight frequencies cannot match the monthly fuel costs for the flights. To solve this problem, we first consider the operating cost of fuel consumption in link  $a$ , expressed as  $c_a^F = f_a(\tilde{O}^t H_{ak})$ .

Assume the airline can determine the revenue from all passengers in order to recover its jet fuel costs, taking into account possible variations in fuel prices during the planning period. Let  $c_a^{F, \text{pax}}$  denote the part of the operating costs allocated by the airline to recover the fuel costs. Let  $g_{ak}$  denote the part of air fares paid by passengers to recover the jet fuel costs of the airline serving aircraft  $k$  in link  $a$ . Then the part of the operating costs to account for the fuel costs can be expressed as  $c_a^{F, \text{pax}} = \sum_k Q_a g_{ak}$ . Our study focuses on the jet fuel price fluctuations and their impact on airline operating cost, especially on the fuel costs. Airlines may change flight frequencies and/or aircraft types to reduce the impact of rising fuel cost. Although the air fare could be adjusted in response to the fuel prices, nevertheless, it may lower passenger demand because passengers are unwilling to pay for relatively high fares. It is hard to conclude whether passenger demand is affected by the adjustment of flight frequencies, aircraft types or air fares, therefore, to simplify the problem, we assume that the passenger demand and the air fare are fixed, addressing only the impact of jet fuel prices fluctuation on operating costs.

We define the ratio of  $c_a^F$  to  $c_a^{F, \text{pax}}$  as  $C_a^*$ , and the formulation is as follows:

$$C_a^* = \frac{c_a^F}{c_a^{F, \text{pax}}} = \frac{f_a(\tilde{O}^t H_{ak})}{Q_a g_{ak}} = \frac{\sum_p \sum_k \delta_a^p f_{pk}(\tilde{O}^t H_{ak})}{\sum_p \sum_k \delta_a^p Q_{pk} g_{ak}} \tag{4}$$

The study initially solves the airline network design program, which determines aircraft types and flight frequencies to minimize operating costs. Then, the reliability evaluation follows. A basic criteria is set to evaluate the reliability of proposed monthly flight frequencies on link  $a$  across random monthly jet fuel prices,  $\tilde{O}^t$ . The ratio of  $c_a^F$  to  $c_a^{F, \text{pax}}$  under random monthly fuel prices can be further defined as a cost factor  $C_a^*(\tilde{O}^t)$ :

$$C_a^*(\tilde{O}^t) = \frac{\tilde{O}^t \sum_k \bar{f}_a H_{ak}}{\sum_k Q_a \bar{g}_{ak}} \tag{5}$$

where  $\bar{f}_a$  is the initial proposed monthly flight frequency. The initial part of air fares paid by passengers to recover the jet fuel costs of the airline serving aircraft  $k$  in link  $a$ , expressed as  $\bar{g}_{ak}$ , is set by the airline in the planning period under the break-

even assumption. To derive the value of  $\bar{g}_{ak}$ , let  $C_a^*(\bar{O}^t) = 1$ , then  $\bar{g}_{ak} = f_a(\bar{O}^t H_{ak})/Q_a$ . The cost factor  $C_a^*(\tilde{O}^t)$  is considered as a ratio, if  $C_a^*(\tilde{O}^t) = 1$ , it means the monthly operating jet fuel cost paid by the airline is equal to the part of air fares paid by passengers to recover the jet fuel costs of the airline serving aircraft  $k$  in link  $a$ . In this case, the jet fuel price can be recovered through the operating revenue; thus, the monthly fuel cost is break-even without extra loss for the airline. If  $C_a^*(\tilde{O}^t) > 1$ , the part of air fares paid by passengers could not cover the monthly jet fuel cost, thus the airline might be burdened with the overloading cost. If  $C_a^*(\tilde{O}^t) \leq 1$ , there are profits for the airline.

We assume there is a maximally acceptable cost factor  $\bar{C}_a^*$ . If  $\tilde{O}^t$  leads  $C_a^*(\tilde{O}^t)$  to  $C_a^*(\tilde{O}^t) \leq \bar{C}_a^*$ , then the proposed monthly flight frequencies on link  $a$  are defined as reliable under random jet fuel price  $\tilde{O}^t$  in month  $t$ . When  $\tilde{O}^t$  leads to  $C_a^*(\tilde{O}^t) > \bar{C}_a^*$ , then the proposed monthly flight frequencies are defined as unreliable under random jet fuel prices in month  $t$ . The reliability for link  $a$  in month  $t$  can be evaluated by using the cumulative distribution function for a normal distribution, which is

$$R_a(\tilde{O}^t) = \Pr \left[ \tilde{O}^t \leq \bar{C}_a^* \left( \frac{\sum_k Q_a \bar{g}_{ak}}{\sum_k \bar{f}_a H_{ak}} \right) \right] = \Phi \left( \frac{\bar{C}_a^* \left( \frac{\sum_k Q_a \bar{g}_{ak}}{\sum_k \bar{f}_a H_{ak}} \right) - \mu}{\sigma} \right) \tag{6}$$

where  $\Phi(z)$  is the cumulative distribution function of the standard normal distribution, that is  $\Phi(z) = \int_{-\infty}^z (1/\sqrt{2\pi}) e^{-\omega^2/2} d\omega$ .

Initially, the airline may plan for random monthly jet fuel prices  $\tilde{O}^t$  and proposed jet fuel prices  $\bar{O}^t$  being in the same price interval during the planning year. In this situation, the fluctuations in fuel prices are considered normal. The reliability in normal price fluctuation is defined as  $R_a(\bar{O}^t)$ . When the monthly fuel price is normal, the month is defined as  $t_0$ . If the number of normal months is  $h$ , then the average reliability under normal fuel price fluctuation  $\bar{R}_a$  can be defined as  $\bar{R}_a = \sum_{t_0=1}^h (1/h)R_a(\bar{O}^t)$ .

However, abnormal fluctuations may occur in fuel prices for the planning year. We define the situation of jet fuel price fluctuation as event  $s_y$  ( $y = 0, 1, \dots, Z$ ) where  $s_0$  represents the normal state, others represent the abnormal states, and  $Z$  gives the number of individual abnormal states. Let  $U$  represent the set of the whole normal and abnormal events that occur during the planning year,  $U \equiv \{s_0, s_1, s_2, s_3, \dots, s_Z\}$ , where  $Pr(s_y)$  is the probability that state  $s_y$  occurs in the planning year. Furthermore,  $Pr(s_y) \geq 0$  and  $\sum_{y=0}^Z Pr(s_y) = 1$ .

When the random jet fuel price is in an interval differs from that of the initially proposed price, it means an abnormal fluctuation occurred. The prices under normal and abnormal states are related to different distribution parameters.

In alignment with Hsu and Wen (2002) [9], we assume that an abnormal state  $s_y$  occurs at  $t_y^*$  with duration  $\tilde{v}_y$  with one month as the unit, where  $\tilde{v}_y$  is considered a random variable. Assume there is a finite discrete distribution of  $\tilde{v}_y$ , that is  $\{(v_y^q, p_q), q = 1, \dots, v\}$ , ( $p_q > 0, \forall q$ ) where  $p_q$  is the probability and  $v_y^q$  is a realization of  $\tilde{v}_y$ , respectively. Let  $I_q^y$  indicate the set of months during the time intervals with abnormal fuel prices  $s_y$ , and  $I_q^y$  can be expressed as  $I_q^y \equiv \left\{ t \mid \lfloor t_y^* \rfloor \leq t < \lceil t_y^* + v_y^q \rceil \right\}$ . Let  $I_q^0$  indicate the set of months with normal fuel prices  $s_0$ , that is  $I_q^0 \equiv I - I_q^y$ . In the abnormal state  $s_y$  with duration  $v_y^q$ , jet fuel price is considered another random variable  $\tilde{O}_{yq}^t, \forall t \in I_q^y$ . To evaluate the reliability in this situation, Eq. (6) can be used to calculate the reliability related to  $\tilde{O}_{yq}^t$ , as  $R_a(\tilde{O}_{yq}^t)$ . In this case, the conditional reliability over the planning year,  $\bar{R}_{a|s_y}$ , can be expressed as

$$\bar{R}_{a|s_y} = \sum_{q=1}^v \frac{1}{12} p_q \left[ \sum_{t \in I_q^y} R_a(\tilde{O}_{yq}^t) + \sum_{t \in I_q^0} R_a(\tilde{O}_{yq}^t) \right]. \tag{7}$$

We can further define the reliability of the proposed monthly flight frequencies on link  $a$ , under normal and abnormal jet fuel price fluctuations as

$$E[R_a] = \sum_{y=1}^Z \bar{R}_{a|s_y} Pr(s_y) + \bar{R}_a Pr(s_0). \tag{8}$$

#### 4. The adjustment of flight frequencies and aircraft types

In this study, we assume an airline might adjust the initially proposed flight frequencies and types of aircraft on some routes according to whether the route is assessed as having low reliability. We also assume that the flight plan of this network

**Table 1**  
The characteristics of available aircraft types.

Aircraft	Number of aircrafts	Maximum take-off weight (kg)	Number of seats	Fuel consumptions (liter/seat-km)
B747-400	13	396,890	397	0.035
A340-300	6	276,500	276	0.039
A330-300	17	230,000	313	0.035

design has been optimized. If the network consists of flights with low reliability, the airline could face great losses because of increased jet fuel prices. In this situation, the airline should make decisions about whether to adjust flight frequencies and/or aircraft types for individual routes, or do nothing.

Let  $\hat{t}$  ( $\hat{t} \in I_q^y$ ) represent the month in which low reliability, which needs adjustment, occurs in the network and is associated with abnormal state  $s_y$ . We can now use the abnormal monthly jet fuel price as an input for the model of airline network programming (3a)–(3e) to adjust the flight frequencies and types of aircraft in month  $\hat{t}$  by relaxing the load factor in the constraints (3b). The adjustment is intended to increase the load factor of an individual aircraft, thereby reducing vacant seats and operating costs. However, when flight frequency changes, it will cause a change in costs related to the flights. Let  $\bar{f}_{pk}^{\hat{t}}$  and  $f_{pk}^{\hat{t}}$  denote the initial and adjusted flight frequencies, respectively, served by type  $k$  aircraft along route  $p$  during the adjusted month. Then there is an adjustment cost in response to a change in the jet fuel price in month  $\hat{t}$  along route  $p$ ,  $C_p^{\hat{t}}$ , and the relationship between  $C_p^{\hat{t}}$  and flight frequency can be expressed as

$$C_p^{\hat{t}} = \sum_p \sum_k \theta_{pk}^{\hat{t}} \left| \bar{f}_{pk}^{\hat{t}} - f_{pk}^{\hat{t}} \right| \tag{9}$$

where  $\theta_{pk}^{\hat{t}}$  is the adjustment cost for a single flight served by type  $k$  aircraft along route  $p$  in month  $\hat{t}$ ; that is, the change in direct operating cost per flight. On the other hand, if the airline does not want to adjust its flight frequency in response to the fluctuation in fuel prices (i.e., maintain the initially proposed flight frequencies), then there may be a penalty cost for the jet fuel  $\hat{P}_1(O^{\hat{t}})$  as follows:

$$\hat{P}_1(O^{\hat{t}}) = \begin{cases} \sum_a \sum_p \sum_k \delta_{a, \bar{f}_{pk}^{\hat{t}}}^p (O^{\hat{t}} - \bar{O}^{\hat{t}}) H_{ak}, & \text{if } O^{\hat{t}} \geq \bar{O}^{\hat{t}} \\ 0, & \text{if } O^{\hat{t}} \leq \bar{O}^{\hat{t}}. \end{cases} \tag{10}$$

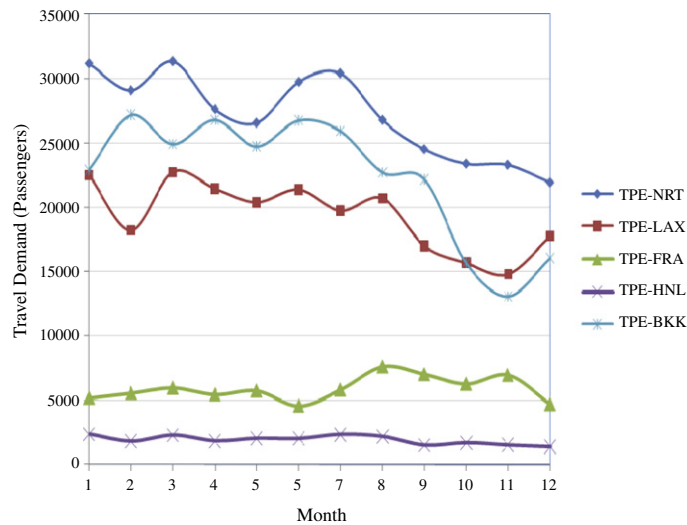
Eq. (10) defines the penalty cost due to the mounting fuel price; that is, when  $O^{\hat{t}}$ , the monthly jet fuel price in the adjusted month  $\hat{t}$ , is greater than the initially proposed fuel price  $\bar{O}^{\hat{t}}$ , and there is no adjustments in flight frequency or aircraft types in month  $\hat{t}$ , there will be a penalty cost. Since doing nothing is one of the alternatives, the evaluation of adjustment costs and the penalty costs without adjustment should be compared to determine whether to adjust the network.

By comparing the adjustment cost  $C_p^{\hat{t}}$  with the penalty cost, the airline can determine whether it is necessary to perform adjustments. If  $C_p^{\hat{t}}$  is lower than the penalty cost, flight frequencies for OD-pairs in month  $t$  may require adjustment. On the other hand, if  $C_p^{\hat{t}}$  is greater than the penalty cost, flight frequencies for OD-pairs in month  $t$  may not require adjustment.

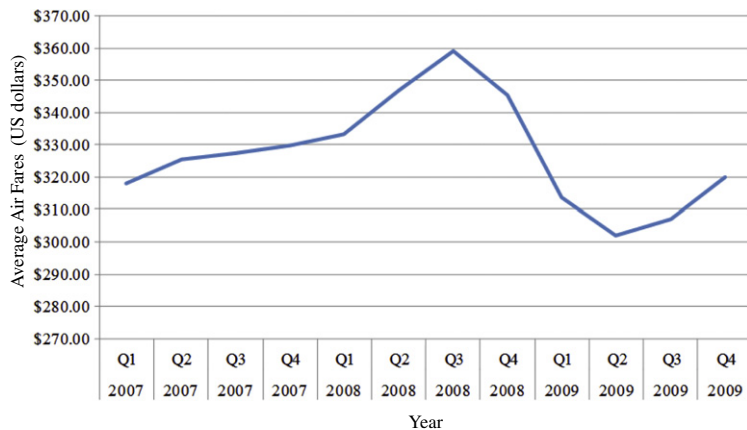
**5. Case study**

The application of the models discussed in Section 3 is demonstrated through a case study of an international airline in Taiwan. We collected aviation related data in 2008 and historical data of Singapore Kerosene-Type Jet Fuel Prices from June 1986 to December 2008 to apply the models. There are five selected routes in the international network of the airline. Let TPE (Taiwan Taoyuan International Airport) be the departure airport, and the arrival airports include NRT (Narita Airport), LAX (Los Angeles International Airport), FRA (Frankfurt Airport), HNL (Honolulu Airport), and BKK (Bangkok International Suvarnabhumi Airport). Fig. 1(a) shows the monthly number of passengers on individual routes in 2008. To understand the correlation between travel demand and air fares, Fig. 1(b) provides the average domestic air fare of each quarter in the United States from 2007 to 2009 (Bureau of Transportation Statistics, 2010). It shows the highest fare occurred in quarter 3 which is the peak travel season.

The aircraft characteristic data is from the international airline’s fleet facts as shown in Table 1. The types of aircraft include Boeing 747-400s, Airbus 340-300s, and Airbus 330-300s. The candidate aircrafts for each OD-pair are assigned considering travel demand, route distance, and flight block hours. Actual data for load factors (Table 2), travel demand and flight block hours from the Taiwan Civil Aeronautics Administration are used to determine the monthly frequency of each route. The charges that include landing fees, noise surcharges, and emission charges for each airport are estimated from the IATA Airport and Air Navigation Charges Manual and individual airport policies. The unavailable direct operating cost of individual link  $a$  is estimated using the passenger airline cost index of the Air Transport Association (ATA). We then use the airline network design model (Eqs. (3a)–(3e)) to determine flight frequencies and routings. The initial solution results are listed in Table 3. As shown in Table 3, the optimal aircraft assigned to short haul routes, such as TPE–NRT and TPE–BKK,



(a) Travel demand for individual routes in Taiwan, 2008.



(b) National-level average air fares in the US.

**Fig. 1.** Travel demand and average air fares.

**Table 2**  
Actual load factor data in 2008.

Route	Monthly load factors (%)											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
TPE–NRT	83.2	78.1	84.8	86.5	80.3	90.8	83.8	77.4	77	69	68	61
TPE–LAX	91.4	78.9	92.2	90	82.6	89.7	93.7	96.4	87.3	77	78	71
TPE–FRA	85.3	82.7	94	94	90	78.7	66.5	83.7	84.2	75	76	54
TPE–HNL	89.6	80.7	82.1	82	80.5	80	92.9	86.3	67	67	60	62
TPE–BKK	67.2	72.8	71.5	77.2	74.5	80.9	75.9	67.4	71.7	58	49	53

is the A330-300. As compared with A340-300s, A330-300s have more seats and better fuel-efficiency (i.e., 313 versus 276 seats, and 0.035 versus 0.039 l per seat-km fuel consumption). However, for routes over long distances, such as TPE–HNL, monthly flights using B747-400s are greater than those using A330-300s due to the long distance and relatively lower travel demand from TPE to HNL, and using B747-400s can reduce the monthly flight frequency. We further examine the results with the actual flight frequencies of the airline in 2008. The aircraft types assigned in each route are referred to the current assignment of airline operator as shown in Tables 3 and 4, the initial proposed flight frequency of each route is similar to the actual data. Furthermore, the flight assignment result shows a preference of using the larger aircraft to carry more passengers at one time, especially for long distance routes. This characteristic is also shown in the study of Hsu and Lin [7].

As mentioned above, in this case study we used historical data from Singapore Kerosene-Type Jet Fuel Prices from June 1986 to December 2008, and derived price densities by drawing histograms of optional fuel prices in order to clarify the

**Table 3**

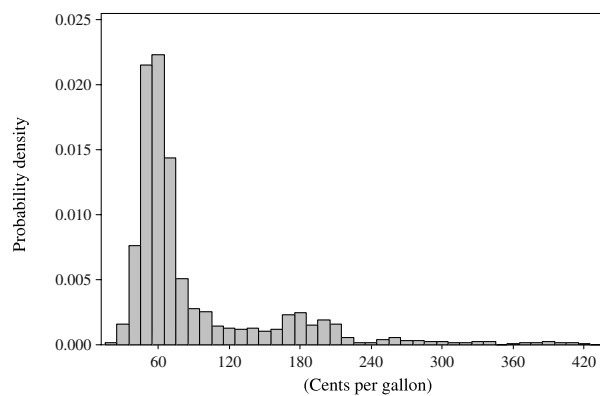
Initially proposed flight frequencies of an airline network in 2008.

Route (TPE-)	Aircraft	One-way monthly flight frequency											
		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
NRT	B747	0	0	0	0	0	0	0	0	0	0	0	0
	A340	0	0	0	0	0	0	0	0	0	0	0	0
	A330	120	120	121	102	106	105	116	111	102	109	110	115
LAX	B747	61	59	63	60	63	61	54	55	50	52	48	63
FRA	B747	19	17	18	15	17	15	23	23	22	22	24	22
HNL	B747	6	4	5	4	4	4	4	4	5	4	5	4
	A330	1	2	2	2	3	3	3	3	1	3	2	2
BKK	B747	0	0	0	0	0	0	0	0	0	0	0	0
	A340	1	6	2	1	0	2	6	3	1	4	4	1
	A330	108	114	117	110	107	104	104	105	98	83	82	96
Total operating cost (10 <sup>6</sup> US dollars)		25.5	25.1	27.8	27.4	30.6	30.5	31.6	28.3	23.9	20.6	18.7	18.9

**Table 4**

The actual assignment of flight frequencies.

Route	Monthly frequency											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
TPE–NRT	97	97	97	94	100	97	105	97	94	99	94	97
TPE–LAX	62	58	62	60	62	60	53	54	49	51	48	63
TPE–FRA	22	23	23	21	23	21	22	23	21	21	23	22
TPE–HNL	9	8	9	8	9	9	9	9	8	9	9	8
TPE–BKK	106	132	108	103	106	102	105	106	96	106	85	97

**Fig. 2.** Histogram of historical jet fuel prices.**Table 5**

Interval classifications of jet fuel prices.

$X_n$	Price level	Interval (cents per gallon)	$\tilde{O}^f \sim N(\mu, \sigma)$ (cents per gallon)	Probability	$p$ -value ( $p > 0.05$ )
1	Lowest	[27.40, 90.56]	$N(58.69, 12.31)$	0.746	0.072
2	Low	[90.60, 154.01]	$N(118.0, 20.44)$	0.096	0.108
3	Medium	[154.17, 230.25]	$N(187.3, 18.66)$	0.117	0.469
4	Medium high	[230.36, 307.52]	$N(266.70, 18.20)$	0.023	0.078
5	High	[310.02, 354.50]	$N(329.30, 11.30)$	0.008	0.155
6	Highest	[357.05, 432.50]	$N(392.42, 16.34)$	0.010	0.861

distributions characterizing those prices. Fig. 2 shows the histograms ordered by price level. We further divided the prices into six intervals and, in each interval, the price samples are tested as normal distributions. As shown in Table 5, at the 95% confidence level, the  $p$ -value for each interval is larger than 0.05. Table 5 shows intervals and fuel parameters of the normal distributions for performing reliability evaluations for the case study using the proposed model.

According to the trends in jet fuel prices in 2008, as shown in Fig. 3, the data show, for example, that from January 1 to April 2 jet fuel prices were in the interval of [230.36, 307.52], which falls within normal fluctuation values. However, fuel prices continuously increased and exceeded the normal price interval. The increasing trend in fuel prices was due to higher



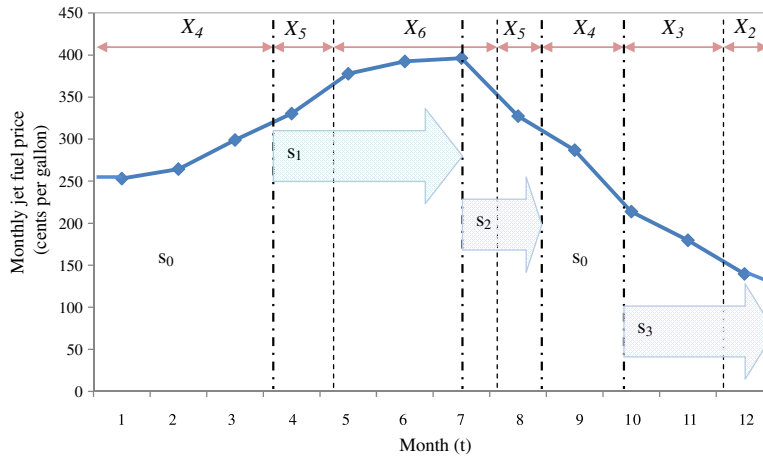


Fig. 3. The jet fuel price trend in 2008.

**Table 6**  
Reliability of proposed flight frequencies under normal fuel price fluctuations.

Route	$\bar{R}_a$		
	$\bar{C}_a^* = 1$	$\bar{C}_a^* = 0.95$	$\bar{C}_a^* = 0.90$
TPE–NRT	0.132	0.040	0.008
TPE–LAX	0.101	0.030	0.006
TPE–FRA	0.789	0.689	0.540
TPE–HNL	0.608	0.426	0.307
TPE–BKK	0.064	0.020	0.004

demand in Asia and supply restraints by OPEC during that period [17]. In state  $s_1$ , jet fuel prices increased abnormally; which included interval  $X_5$  from April 4 to May 8, and interval  $X_6$  from May 9 to July 16.

Starting around the middle of July 2008, the abnormal state  $s_2$  showed a sudden decline in fuel prices caused by the U.S. financial crisis and global economic recession. Although prices were decreasing, they were still in intervals  $X_5$  and  $X_6$ . The prices normalized when they decreased to interval  $X_4$  from August 30 to October 9. State  $s_3$  started on October 10 and lasted until the end of the year. In this state, jet fuel prices were abnormally decreasing due to the global economic recession.

Table 6 lists the estimated reliability values for the proposed monthly flight frequencies for individual OD pairs across normal monthly price fluctuations in 2008. The reliability values are estimated by considering three maximally acceptable cost factors  $\bar{C}_a^*$ , which are 1, 0.95, and 0.90, respectively. As shown in Table 6, OD pairs TPE–NRT, TPE–LAX and TPE–BKK show relatively low reliabilities compared with other routes. Furthermore, the results show the reliability values decrease with lower values of  $\bar{C}_a^*$ . The higher the expected profit, the lower the possibility for the airline company to realize a gain. Comparing the reliabilities  $\bar{R}_a$  among routes, when  $\bar{C}_a^* = 1$ , routes TPE–NRT, TPE–LAX and TPE–BKK have relatively lower reliabilities than routes TPE–FRA and TPE–HNL. According to Table 6 and Fig. 1, the common characteristics of routes with higher reliabilities are those with lower annual travel demand and relatively low travel demand fluctuation.

Data for the abnormal states, including occurrence durations, price distributions, and duration probabilities are listed in Table 7. We further calculated the reliabilities of the proposed monthly flight frequencies for individual OD pairs for abnormal monthly fluctuations in 2008, and the resulting values can be seen in Table 8. Since fuel prices increased abnormally, the reliabilities of routes decreased and were lower than those for the normal monthly price fluctuations, as shown in the table. For example, the reliability of the proposed monthly flight frequencies is 0.789 (Table 6) with the maximally acceptable cost factor  $\bar{C}_a^* = 1$  for TPE–FRA during normal monthly jet fuel price fluctuations. However, in states  $s_1$  and  $s_2$  (Table 8), due to jet fuel price increases, the reliabilities decrease to 0.250 and 0.236, respectively. The reliability of state  $s_3$ , in contrast, increases because fuel prices decline.

The reliabilities of the proposed monthly flight frequencies for all individual OD-pairs, considering both normal and abnormal jet fuel price fluctuations in 2008, were calculated using Eq. (8), and are listed in Table 9. The data show reliabilities over the planning year with different abnormal occurrence probabilities (i.e., 0.6, 0.7, 0.8, and 0.9). The results indicate that fluctuations in both fuel prices and travel demand affect reliability. Notably, route TPE–HNL shows a different pattern from other routes because of its stable passenger demand and lower flight frequency. Route TPE–FRA also shows a higher reliability and lower flight frequency. Since the flight frequencies of these two routes are low, the jet fuel costs are lower than other routes because the jet fuel cost is related to flight frequency. As a result, the jet fuel price has a less impact on this kind of routes.

**Table 7**  
Historical data regarding abnormal states in 2008.

Abnormal jet fuel price distributions		
Abnormal month $t_1^* = 4.1$	State occurrence duration ( $s_1$ )	
	$v_1^1 = 1.191$	$p_1 = 0.27$
	$v_1^2 = 4.405$	$p_2 = 0.73$
	Apr	$N(329.30, 11.30)$
	May	$N(392.42, 16.34)$
Abnormal month $t_2^* = 7.548$	State occurrence duration ( $s_2$ )	
	$v_2^1 = 0.612$	$p_1 = 0.44$
	$v_2^2 = 1.382$	$p_2 = 0.56$
	Jul	$N(392.42, 16.34)$
	Aug	$N(329.30, 11.30)$
Abnormal month $t_3^* = 10.333$	State occurrence duration ( $s_3$ )	
	$v_3^1 = 1.709$	$p_1 = 0.65$
	$v_3^2 = 2.644$	$p_2 = 0.35$
	Oct	$N(187.3, 18.66)$
	Nov	$N(187.3, 18.66)$
Dec	$N(118.0, 20.44)$	

**Table 8**  
Reliability of proposed flight frequencies under abnormal fuel price fluctuations.

Route	Maximally acceptable cost factor	$\bar{R}_{a s_1}$	$\bar{R}_{a s_2}$	$\bar{R}_{a s_3}$
TPE–NRT	$\bar{C}_a^* = 1$	0.045	0.045	0.153
	$\bar{C}_a^* = 0.95$	0.015	0.015	0.108
	$\bar{C}_a^* = 0.9$	0.003	0.003	0.078
TPE–LAX	$\bar{C}_a^* = 1$	0.030	0.030	0.141
	$\bar{C}_a^* = 0.95$	0.009	0.009	0.107
	$\bar{C}_a^* = 0.9$	0.002	0.002	0.081
TPE–FRA	$\bar{C}_a^* = 1$	0.250	0.236	0.356
	$\bar{C}_a^* = 0.95$	0.209	0.205	0.326
	$\bar{C}_a^* = 0.9$	0.161	0.160	0.281
TPE–HNL	$\bar{C}_a^* = 1$	0.154	0.146	0.249
	$\bar{C}_a^* = 0.95$	0.072	0.071	0.165
	$\bar{C}_a^* = 0.9$	0.024	0.024	0.106
TPE–BKK	$\bar{C}_a^* = 1$	0.024	0.024	0.086
	$\bar{C}_a^* = 0.95$	0.007	0.007	0.063
	$\bar{C}_a^* = 0.9$	0.001	0.001	0.048

**Table 9**  
Reliability of proposed flight frequencies under normal/abnormal jet fuel price fluctuations.

Route	Acceptable max. cost factor	Reliability over the planning year $E[R_a]$			
		$Pr(s_y) = 0.6$	$Pr(s_y) = 0.7$	$Pr(s_y) = 0.8$	$Pr(s_y) = 0.9$
TPE–NRT	$\bar{C}_a^e = 1$	0.198	0.209	0.220	0.231
	$\bar{C}_a^e = 0.95$	0.099	0.108	0.118	0.128
	$\bar{C}_a^e = 0.9$	0.054	0.061	0.069	0.077
TPE–LAX	$\bar{C}_a^e = 1$	0.161	0.171	0.181	0.191
	$\bar{C}_a^e = 0.95$	0.087	0.096	0.106	0.115
	$\bar{C}_a^e = 0.9$	0.053	0.061	0.069	0.077
TPE–FRA	$\bar{C}_a^e = 1$	0.821	0.826	0.831	0.836
	$\bar{C}_a^e = 0.95$	0.720	0.725	0.730	0.735
	$\bar{C}_a^e = 0.9$	0.578	0.584	0.590	0.596
TPE–HNL	$\bar{C}_a^e = 1$	0.572	0.566	0.560	0.555
	$\bar{C}_a^e = 0.95$	0.355	0.343	0.332	0.320
	$\bar{C}_a^e = 0.9$	0.214	0.199	0.184	0.168
TPE–BKK	$\bar{C}_a^e = 1$	0.105	0.112	0.119	0.127
	$\bar{C}_a^e = 0.95$	0.054	0.060	0.066	0.071
	$\bar{C}_a^e = 0.9$	0.032	0.037	0.042	0.046

**Table 10**

Comparisons of initial airline network designs with and without adjustment on routes TPE–NRT, TPE–LAX and TPE–BKK.

(a) Route TPE–NRT and TPE–LAX in April							
Route	Initially proposed aircraft types and flight frequencies		Adjustment of flight frequencies				
TPE–NRT	B747-400	0	0	0	0	0	0
	A330-300	102	182	175	165	160	157
TPE–LAX	B747-400	60	0	0	0	0	0
NRT–LAX	B747-400	0	60	60	60	60	60
Load factor		86.5%	86.5%	90%	95%	98%	100%
Penalty costs (\$)			\$148,037	\$408,702	\$781,080	\$967,269	\$1,078,982
Adjustment costs(\$)			\$12,69,760	\$1,158,656	\$999,936	\$920,576	\$872,960
Judgment				Do nothing	Do nothing	Adjust	Adjust
(b) Route TPE–NRT and TPE–LAX in June							
TPE–NRT	B747-400	0	0	0	0	0	0
	A330-300	105	180	172	160	164	164
TPE–LAX	B747-400	61	0	0	0	0	0
NRT–LAX	B747-400	0	61	61	61	61	61
Load factor		90.8%	90.8%	95%	98%	100%	100%
Penalty costs (\$)			\$373,618	\$703,623	\$909,876	\$1,033,628	\$936,448
Adjustment costs(\$)			\$1,190,400	\$1,063,424	\$984,064	\$936,448	\$936,448
Judgment			Do nothing	Do nothing	Do nothing	Adjust	Adjust
(c) Route TPE–NRT and TPE–LAX in July							
TPE–NRT	B747-400	0	0	0	0	0	0
	A330-300	116	189	178	169	161	161
TPE–LAX	B747-400	54	0	0	0	0	0
NRT–LAX	B747-400	0	54	54	54	54	54
Load factor		83.8%	85%	90%	95%	100%	100%
Penalty costs (\$)			\$68,286	\$524,816	\$898,341	\$1,230,364	\$1,230,364
Adjustment costs(\$)			\$1,158,656	\$984,064	\$841,216	\$714,240	\$714,240
Judgment			Do nothing	Do nothing	Adjust	Adjust	Adjust
(d) Route TPE–BKK in April							
TPE–BKK	B747-400	0	0	0	0	0	0
	A340-300	1	0	2	7	7	3
	A330-300	110	107	99	89	84	83
Load factor		77.2%	80%	85%	90%	95%	100%
Penalty costs(\$)			\$142,929	\$359,664	\$543,820	\$722,661	\$897,251
Adjustment costs(\$)			\$47,158	\$118,448	\$319,146	\$379,476	\$358,662
Judgment			Adjust	Adjust	Adjust	Adjust	Adjust

The months requiring adjustment during 2008 were identified from the results of the reliability evaluation mentioned above. Because jet fuel prices became abnormal at the beginning of April, we assumed the airline should adjust its network when observing the abnormally increasing prices. Three routes with relatively low reliabilities (TPE–LAX, TPE–NRT, and TPE–BKK) are identified as requiring flight frequency adjustments to decrease operating costs.

Considering the current state in Taiwan, many airlines provide connecting service for long haul routes. For example, United Airlines serves its route from Taipei to Chicago via Tokyo. The passengers might include people who travel from Taipei to Chicago, those whose destination is Tokyo, and people who travel from Tokyo to Chicago. In this way, airlines can improve the load factor by reducing vacant seats. Referring to actual practice, our strategy for adjusting direct routes TPE–NRT and TPE–LAX is to modify these two routes to become a single connecting route TPE–NRT–LAX; thereby improving the load factor of TPE–NRT and decreasing the operating cost of TPE–LAX. This adjustment strategy is based on the concern that the low reliability of TPE–LAX might be due to high fuel consumption, while that of TPE–NRT might be due to a low load factor. The new connected route might provide benefits by increasing both flight frequency and load factor.

The adjustment results are listed in Table 10(a)–(c), including monthly flight frequencies, related adjustment costs, expected penalty costs, and the judgments of adjusting or doing nothing for the unreliable OD pairs. Table 10(a)–(c) show the adjustment results in April, June, and July, respectively, for route TPE–NRT–LAX. Further adjustments in June and July are deemed necessary based on increasing travel demand during the peak period for summer vacations. For instance, as shown in Table 10(a), the initially proposed flight frequency of TPE–NRT is 102 flights per month, while the frequency of flights to LAX is 60 flights per month. When the load factor for TPE–NRT increases from 86.5% to 90.0% due to increases in travel demand, the airline can arrange 175 flights per month to satisfy that demand. However, the adjustment cost of an additional 73 monthly flights is larger than the penalty cost, so the judgment is to do nothing. For the same reason, we tested the flight frequency under load factors 95%, 98%, and 100%. When the load factor increases to 98%, the penalty cost for doing nothing is larger than the adjustment cost, so the airline saves by making the decision to increase flight frequencies by 58.

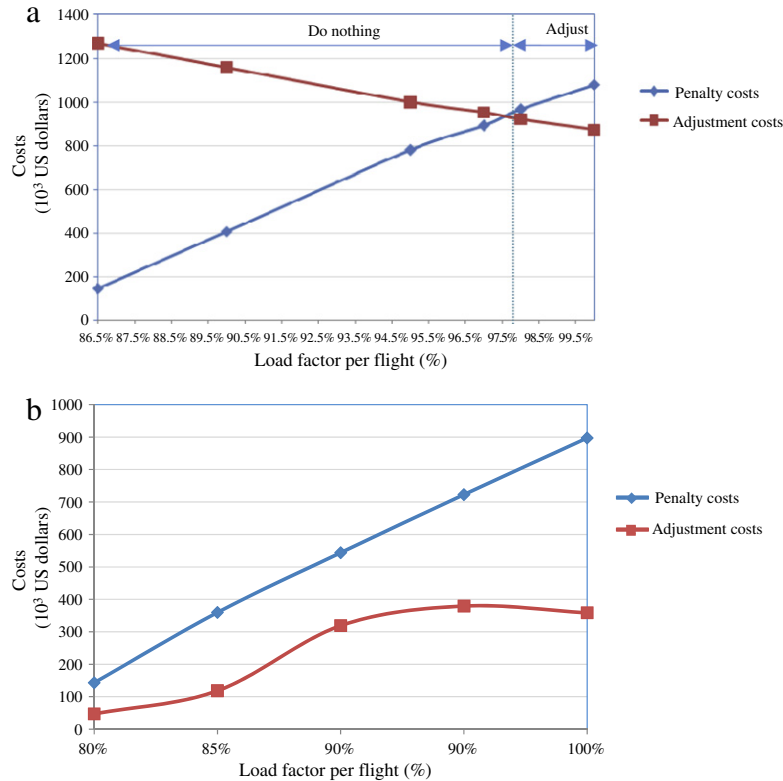


Fig. 4. Penalty and adjustment costs with different load factors: (a) route TPE-NRT-LAX (b) route TPE-BKK.

The relationship between penalty costs and adjustment costs under different load factors is shown in Fig. 4(a). The curves of the penalty and adjustment costs intersect when the load factor is about 97.5%, which means adjusting flight frequency is justified with load factors over 97.5%. The load factor of 97.5% can be referred to as a critical point to justify adjustment decision-making. To ensure the adjustment benefits the airline, flight frequency planning should be made under the load factor constraints.

Assume the airline provides 160 flights per month for route TPE-NRT since April as shown in Table 10(a), which means there are 50,080 seats available (313 seats per flight). The monthly supply is sufficient for travel demand in May; thus, in this case, no adjustment is needed based on flight frequency. However, travel demand will increase greatly during June and July because of the summer vacation peak as shown in Fig. 1. As a result, flight frequency for June and July should be adjusted further. From Table 10(b), the penalty cost is larger than the adjustment cost when the load factor is near 100%. The number of flights from TPE to NRT should be increased to 164 or 165 so the penalty cost is larger than the adjustment cost.

As shown in Table 10(b), the flight from NRT to LAX is adjusted to 61 monthly flights in June. However, because of decreasing travel demand for TPE-LAX in July and August, as shown in Fig. 1, the original 54 and 55 monthly flights, respectively, are lower than the planned flight frequency for June. The result causes a surplus of 10 flights between June and July since the load factors for July and August are above 90%, and higher than the 89.7% for June. Meanwhile, July is also the month with the highest fuel price of the year so the adjustment is deemed important for the airline in order to save operating costs. Considering the adjustment alternatives for July, if we keep the original load factor and only change the routes, the operating cost will be higher than the original planning cost, which results in a loss of \$56,223 and an extra adjustment cost of \$1,206,272. The main reason for this is that the fuel price in July is highest of all so increasing flight frequency could incur a large fuel cost. As shown in Table 10(c), if the airline intends to increase its TPE-NRT flights, the increased number should be equal to or lower than 53 flights (from 116 to 169 flights) to ensure the load factor can be above 95%. Thus, the penalty cost for no adjustment is higher than the adjustment cost, which justifies the adjustment for the airline. Furthermore, the planned seat supply for July is sufficient for the travel demand of August, though fuel prices start decreasing. As a result, the adjusted monthly flights for July can be maintained in August without other adjustments.

As shown in Table 10(d), for route TPE-BKK, the reason for the low reliability might be due to its low load factors combined with high frequency. A reasonable adjustment for a route with such characteristics is to decrease the monthly flights, especially in months with high fuel prices. We examine the frequency adjustments by decreasing the flights for TPE-BKK, and the results are shown in Table 10(d). According to the results, when the load factor is higher than 80%, the penalty cost is higher than the adjustment cost. For example, when monthly flights are decreased from 111 to 107, the load factor can be raised from 77.2% to 80%. The adjusted flight frequency is sufficient for the months following April, thus no

further adjustments are necessary in this case. Fig. 4(b) further demonstrates the penalty costs and adjustment costs for route TPE–BKK considering different load factors. Adjustment costs for flights on route TPE–BKK are less than the penalty costs, thus all flights were judged to benefit from adjustment.

## 6. Conclusions

This study focuses on reliability evaluation of airline network design in response to fluctuations in jet fuel prices. The network design model determined optimal solutions, including flight frequencies and aircraft types for the network. The results show that, to reach the objective of minimizing operating costs, more fuel-efficient aircraft are selected. The study further focused on the importance of fuel price and its influence on airline network design by analyzing historical jet fuel prices using the concept of interval analysis. A reliability model was applied to examine the possibility for airline companies to operate in at least a break-even condition during fuel price fluctuations.

The results show that an abnormal state with ever-increasing jet fuel prices results in relatively low reliabilities in the network, which confirms the impact of fuel price on operating costs and flight frequencies. The results indicate that not only do routes with low load factors exhibit low reliabilities, but also long distance routes with high load factors show low reliabilities during high fuel price periods. The critical points of penalty costs equaling adjustment costs are illustrated to determine whether adjustments are justified.

It is important for airline managers to decide which route has the priority to be adjusted by a given strategy in order to avoid losses under fuel price uncertainties. As presented in the case study, routes TPE–NRT, TPE–LAX, and TPE–BKK should be adjusted based on the results of reliability evaluation. For route TPE–BKK, initial flight frequency is high but the load factor is low, which causes a relatively low reliability. With a reduction in flight frequency, the penalty costs for no adjustment are larger than the adjustment costs with different load factors, indicating the necessity of adjusting flight frequency on such a route. Moreover, the results suggest that consolidation of two routes with low load factors can improve the resulting load factor and reduce operating costs (e.g., combining two TPE–LAX and TPE–NRT direct routes into a TPE–NRT–LAX connected route during high fuel price and low demand periods).

The models developed in this study provide an effective way to measure the reliability of an airline network. In sum, this study provides guidelines for post-design evaluation of airline network designs in response to uncertain fluctuation in jet fuel prices. Several ways to adjust the airline network are provided in order to examine the benefits of adjustment. Our developed models and adjustment methods might improve airline management for operational planning, as well as provide directions to cope with the uncertainty of fuel prices and operating costs. Future studies may incorporate issues related to air fares adjustment or possible aircraft design in response to fuel prices and the impacts on airline operation revenue. Currently, airlines apply other control strategies in response to fuel price fluctuations such as hedge management and fuel surcharge. For long-term planning, the replacement of new aircraft types must be also emphasized. The effects above may be considered in future studies.

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