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A transportation programming model considering project interdependency and regional balance ☆

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ABSTRACT

Transportation Programming (TP) plays an important role in the development of the infrastructure of a country. Given the limited budget, it is a challenging decision to select the projects to be funded and implemented from the numerous options. The problem is complicated by the fact that some of the potential projects are interdependent. The benefit (and/or the cost) of the joint project combining multiple projects can be different from the sum of the benefits (and/or the costs) if the associated projects are implemented separately. Besides, some projects cannot be selected at the same time as they are incompatible or exclusive to each other by nature. The typical examples are the projects utilizing the same resource, such as a piece of land. In addition, much more attention nowadays is paid to the fairness of budget allocation and the balance of regional development as the society becomes more democratic and diversified. Thus, in order to address the equity issue and the political feasibility, a new integer programming (IP) model based on the set covering problem (SCP) has been proposed to ensure that the regional balance issue is addressed. This SCP-based model, with the constraints taking into account the budget limitation and the projects' mutual exclusivity, is transformed into a linear programming (LP) model by Lagrangian Relaxation (LR). The key theme of this study is then to design the solution algorithm that can efficiently adjust the LP multipliers and find the feasible solutions so as to achieve a high-quality approximate solution within an acceptable computation time. Finally, a numerical experiment that can reflect the practical situations is performed to validate the applicability of the developed model and solution algorithm.

1. Introduction

Transportation Programming (TP) is a core decision problem for the development of the infrastructure of a country. Given the limited budget, it is a challenge to select the projects to be funded and implemented from the numerous potential projects. The problem is complicated by the fact that some of the potential projects are interdependent. The benefit (and/or the cost) of a joint project, which combines multiple projects, can be different from the sum of the benefits (and/or the costs) if the associated projects are implemented separately. Besides, some projects cannot be selected at the same time as they are exclusive to each other by nature. The typical examples are the various versions (or scales) of one project or the multiple projects utilizing the same resource, such as a piece of land. In the literature, it is common for the TP problem

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to be modeled by a formulation based on the knapsack problem (KP), which is not particularly suitable for dealing with project interdependency.

Another key concern for transportation programming in this study is the fairness of budget allocation. The relationship between the transport infrastructure provision and the regional equity has drawn the attention of many researchers. For example, de Almeida and Haddad (2010) recently conducted a case study for Minas Gerais, which is Brazil's third richest state and second most populous state, but with a strong regional inequality within its territory. As societies in general become more democratic and diversified, much more attention nowadays is being paid to the balance of regional development. It is believed that the traditional TP models need to be modified or enhanced to take into account the issue of regional balance so as to find an acceptable trade-off during the decision process of budget allocation.

In this study, the area being examined is thus divided into multiple regions, and the relevance (significance) of the projects to the regions is assumed to be evaluated in advance. With the aim being to raise the equality and feasibility for transportation programming, this study develops a new integer programming (IP) model based on the set covering problem (SCP), in which the significance constraint ensures that no region is ignored during the planning process. For the solution algorithm of this SCP-type IP problem, this study employs the approach of Lagrangian Relaxation (LR), which has been proved to be an effective method for the classic SCPs. The significance constraints and the budget constraint are chosen to be relaxed after attaching the associated penalties to the objective function. A heuristic procedure has been designed to handle the relaxed problem and generate the feasible solution and the lower bound.

This paper is organized as follows. The next section provides a literature review of the TP problem, and in particular elaborates upon the special features of project interdependence. In Section 3, the MIP model of the concerned TP is presented and the development of the solution algorithm is explained in detail, including the transformation of the original problem into a Lagrangian-relaxed set covering problem, the procedures in the recursive algorithm to derive the lower bound and the feasible solution. The design of the numerical experiment and the associated results are described in Section 4. Finally, the findings of this study are concluded in Section 5.

2. Literature review

In addition to its wide application in problems such as loading, packing, and material cutting, the models based on the Knapsack Problem are the major techniques used for solving the TP problems (e.g., Sinha and Muthusubramanyam, 1981). The budget is viewed as the capacity of the knapsack, and the transportation projects are thought of as the items to be selected. The KP formulation is simple, but it is nonetheless an NP-complete problem. In general, a heuristic algorithm is required to deal with large-scale problems. Regarding the KP formulations as well as the solution algorithms and applications, the survey paper by Wilbaut et al. (2008) serves as an excellence source for further information.

As for the frameworks and the mathematical models for transportation programming, the book chapter by Sinha and Labi (2007) provides the basic introduction and the related discussion. Due to the intrinsic complexity of transportation programming, many researchers have begun to extend the TP models to the version of multiple objectives from a practical point of view. For example, both Teng and Tzeng (1998) and Avineri et al. (2000) make use of the fuzzy theory to deal with the TP problems under the multi-objective context. Iniestra and Gutiérrez (2009) develop a TP model based on a variant of the multi-objective 0–1 Knapsack Problem. On the other hand, Zhong and Young (2010) transform the multi-objective problem into the single-objective problem by using the Analytic Hierarchy Process (AHP) to determine the weights. As the problem size is small, the IP solver of a software package (LINGO) is used to find the optimal solution.

The projects to be selected in most of the prior research works for transportation programming have been assumed to independent. One of the exceptions is Gomes (1990), who models the interdependencies among urban transportation system alternatives under the multi-criteria framework by developing a ranking method. Teng and Tzeng (1996) further categorize the projects into four kinds: independent, complementary, substitutive, and both complementary and substitutive. The formulation is based on a binary multi-objective multidimensional Knapsack Problem, given that the constraints for multiple types of resources are considered. They further develop a solution algorithm called the Spatial Efficiency Algorithm to find the approximate solution. Similar to the context of multiple criteria and the consideration of project interdependency in Teng and Tzeng (1996), Iniestra and Gutiérrez (2009) believe that the overall effect of a portfolio of infrastructure investment is different from the sum of all individual investments and use an evolutionary-based framework to identify the Pareto solutions. Finally, one specific feature of project interdependency, the mutually exclusive relationship, has been considered in Zhong and Young (2010).

Although some features of project interdependency have been addressed in the prior research works described in the previous paragraph, we believe that not all of the possible relationships between two (or even multiple) projects have been considered simultaneously in a model. Due to the possible synergy between two projects, the extra benefit from implementing them at the same time should be considered. On the other hand, there is a chance, though not desirable, for the simultaneous implementation of projects to result in a reduction in the overall benefit. In addition, when multiple projects are implemented as a joint project, the cost change should be modeled explicitly as well. Thus, in order to provide a higher degree of flexibility for modeling project interdependency, the combination of multiple projects is treated as a different project whenever its benefit (and/or cost) is different from the sum of the benefits (and/or the costs) of the associated projects under a separate implementation. The basic projects and the joint projects are later referred to as the *options*, for which each one is represented by a distinct binary decision variable from the modeling viewpoint.

The other important modeling contribution of this study is that the regional balance issue is addressed by introducing the concept of coverage from the set covering formulation. In particular, the traditional 0–1 coverage has been extended to a continuous value, referred to as the *significance level*, to allow a more flexible and practical way to quantify the relevance or contribution associated with an option and a region. We believe that, by introducing the constraints regarding the aggregate significance level for each region, the model developed in this study can provide better decision support during the transportation planning process.

3. Mathematical models and solution algorithm

3.1. Classic set covering problem

The classic Set Covering Problem has a wide range of applications, given its simple and flexible formulation as in (1)–(3). The objective function (1) seeks to minimize the total cost by combining the costs of the selected sets. Constraint (2) ensures that each item is covered by at least one of the selected sets. Finally, in Constraint (3), the binary variable x_j represents the decision to select a set.

$$\text{Minimize} \sum_{j \in J} m_j x_j \tag{1}$$

subject to

$$\sum_{j \in J} a_{ij} x_j \geqslant 1, \quad \forall i \in I \tag{2}$$

$$x_j \in \{0,1\}, \quad \forall j \in J \tag{3}$$

- i: indices of items (assuming I is the collection of all items),
- j: indices of sets (assuming J is the collection of all sets),
- m_j: cost of set j,
- a_{ij}: binary constant; $a_{ij} = 1$ if item i is covered by set j, and $a_{ij} = 0$ otherwise,
- x_j: binary decision variable indicating that set j is selected.

This SCP formulation serves as the basis to model the concerned transportation planning problem, which takes into account the issue of regional balance and the feature of project interdependency. In this study, a region can be thought of as an item to be covered, and the selection of an option is represented by the corresponding binary decision variable.

3.2. SCP-based problem formulation

This study makes the following four modifications to the classic SCP formulation so as to take into account the special features of the concerned transportation planning problem. First, the coefficients in the objective function are made to be negative numbers by adding a negative sign to the benefits of options. Thus, the problem remains a minimization problem, but the goal is in fact to maximize the overall benefit. Second, the binary constant a_{ij} is changed to be a real number within the range $[0,1]$ and is referred to as the significance level. This modification provides a higher degree of flexibility to model the relevance or contribution of an option to a region. Third, a budget constraint is added. Lastly, one type of constraint is incorporated to consider the mutually exclusive relationships between options. This SCP-based formulation is presented as

$$\text{Minimize} \sum_{j \in J} b_j x_j \tag{4}$$

subject to

$$\sum_{j \in J} a_{ij} x_j \geqslant 1, \quad \forall i \in I \tag{5}$$

$$\sum_{j \in J} m_j x_j \leqslant G \tag{6}$$

$$\sum_{j \in J} f_{rj} x_j \leqslant 1, \quad \forall r \in R \tag{7}$$

$$x_j \in \{0,1\}, \quad \forall j \in J \tag{8}$$

- i: indices of regions (assuming I is the collection of all regions),
- j: indices of options (assuming J is the collection of all options),
- b_j: benefit (with the negative sign) of option j,
- a_{ij}: significance level of option j for region i, a real constant within the range of $[0,1]$,
- m_j: cost of option j,
- G: budget limit,
- r: indices of the mutually exclusive relationships (assuming that R is the collection of all relationships),

- f_{rj}: binary constant; $f_{rj} = 1$ if option j is included in the mutually exclusive relationship r, and $f_{rj} = 0$ otherwise,
- x_j: binary decision variable indicating that option j is selected.

The binary decision variable x_j in the formulation represents the decision of selecting an option, which could be a basic project or a joint project combining multiple basic projects. A joint project, as well as the corresponding decision variable, is created if its overall benefit (and/or the cost) is different from the sum of the benefits (and/or the costs) for the case where the associated basic projects are implemented separately. In this way, the project interdependence caused by the interdependent effect of the projects can be precisely modeled.

Given the definition of the decision variable, the objective function (4) minimizes the negative values of the benefits for the selected options and thus maximizes the overall benefit. In Constraint (5), for each region, the sum of the significance levels of the selected options is required to be larger than one. This constraint ensures that each region collectively obtains the deserved significance level (relevance or contribution) from the selected option(s). The determination of the value of the constant a_{ij} is surely a challenge and may become an arguable issue in practice. However, the extension from the binary constant in the classic SCP to the constant with a continuous value provides a great modeling advantage to deal with the various real-world situations. For example, a highway connecting two regions does not necessarily mean the same thing, *i.e.*, the same level of significance, to each of them, if the different regional characteristics, such as population, tax contribution and political/economical importance are further taken into account. Constraint (6) imposes the maximum limit on the total cost of the selected options based on the given budget. Constraint (7) guarantees that the options are not simultaneously selected if they are included in a mutually exclusive relationship, which can arise from the following situations:

- For one single initiative, the various versions (e.g., a one-lane or two-lane expansion of a highway section) are treated as the different options and cannot be selected at the same time.
- For multiple initiatives, the corresponding options (a public bus terminal vs. a commuter parking facility) cannot be selected simultaneously if they utilize the same resource, such as a piece of land.
- Multiple basic projects cannot be selected simultaneously if there is a corresponding joint project. In addition, the basic projects and the associated joint project cannot be selected simultaneously for a similar reason.

The number of the mutually exclusive relationships can be large, but these relationships can be determined in advance and are thus assumed to be given. This type of exclusivity constraint further accomplishes the work of modeling project interdependence. The classic SCP is an NP-complete problem, although the formulation appears to be simple. Solving the problem of (4)–(8) is surely an even more challenging task. In general, the classic Lagrangian Relaxation has been found to be a successful approach to the SCP-based applications in prior research works (e.g., Beasley, 1990; Haddadi, 1997; Ceria et al., 1998; Caprara et al., 1999), and this study thus uses it as the backbone to develop a recursive heuristic solution algorithm, which is presented in the following two sub-sections.

3.3. Lagrangian Relaxation problem and linearization

This study chooses to relax Constraints (5) and (6) by moving them to the objective function with the penalty represented by the LR multipliers to develop the relaxed problem, which is much easier to solve. The corresponding Lagrangian multipliers for the significance level constraints and the budget constraint are denoted by Λ_i's and π, respectively. Based on the set of Lagrangian multipliers derived in the previous iteration, the resulting relaxed problem is as shown in (9)–(12). In particular, the second summation of the objective function can be removed, as it is a constant given the fixed set of Lagrangian multipliers in each iteration

$$L(\Lambda,\pi) = Min\sum_{j\in J} c_j(\Lambda,\pi)x_j + \left(\sum_{i\in I}\lambda_i - \pi G\right) \tag{9}$$

$$\text{subject to} \sum_{j\in J} f_{rj}x_j \leqslant 1, \quad \forall r \in R \tag{10}$$

$$x_j \in \{0,1\}, \quad \forall j \in J \tag{11}$$

where

$$c_j(\Lambda,\pi) = b_j - \sum_{i\in I}\lambda_i a_{ij} + \pi m_j \tag{12}$$

- Λ_i: Lagrangian multiplier of region i for the significance level constraint (5) (assuming Λ is the collection of all Λ_i's).
- π: Lagrangian multiplier for the budget constraint (6).
- c_j: Lagrangian cost of option j (determined by the Lagrangian multipliers).

For the case of the classic SCP problem, the Lagrangian-relaxed problem can easily be solved by inspection as the binary constraint, similar to (11), is the only remaining constraint. The binary decision variable is set to one if the corresponding Lagrangian cost is negative; otherwise, it is set as zero. However, the relaxed problem of (9)–(12) cannot be easily dealt with due to the extra constraint of (10). Of course, for the variables not existing in (10), they can be determined by the above simple rule for the relaxed classic SCP problem. However, for the rest of the variables, at most one variable can be set as one due to the limitation of (10). Conceptually, the Lagrangian cost c_j can be viewed as how attractive option j is. The more negative the value is, the more desirable the option is. The determination of the variables to be set as one under the mutually exclusive relationships (R) is a problem possessing some similarity to the classic assignment problem (AP), in which, with the objective of minimizing the overall assignment cost, each task must have one agent assigned, and each agent can at most be assigned to a job. For this analogy, an option can be thought as an agent, and a mutually exclusive relationship is thus a task. Unfortunately, unlike the AP, the problem shown in (9)–(12) does not exhibit the nice Total Unimodularity property, which guarantees an integer solution for an IP problem under the linearization of the binary constraints. Although some values of the coefficients of the f_{rj}'s may result in a totally unimodular matrix, a solution with some fractional numbers is in general expected.

Nonetheless, due to the similarity between the classic AP and the LR-relaxed problem of (9)–(12), this study has decided to replace constraint (11) by a linear constraint as in (13). This linearly relaxed problem can easily be solved by any linear programming (LP) solver, but there are two disadvantages associated with the short-cut. First, some decision variables may end up with fractional values. Second, the objective function value from (9) becomes a lower bound that is less strong when compared with the original Lagrangian bound. However, based on the numerical experiment in the next section, it is found that the heuristic procedure presented in the next sub-section can generate a fairly good approximate solution by modifying the solution from the linearized LR-relaxed problem, in which usually only few decision variables are found to be fractional. As for the second disadvantage, it is also found in the numerical experiment that the lower bound after the linearization remains a strong one.

$$0 \leqslant x_j \leqslant 1, \quad \forall j \in J \tag{13}$$

3.4. Determination of feasible solution

Once the linearly relaxed problem is solved, an option is thought of as being selected if its corresponding decision variable is set as one. The rest of the options, whose corresponding variables are found to be zero or fractional, are treated as non-selected. Based on this initial solution, the approximate solution to the Lagrangian-relaxed problem is determined according to the following sequential steps:

- STEP 1: If the significance constraint (5) is violated, based on the Lagrangian cost $c_j(\Lambda, \pi)$, rank in ascending order the unselected options that can provide some degree of significance level to the regions with an un-met significance requirement. Add the options accordingly until constraint (5) is satisfied.
- STEP 2: If the budget constraint (6) is violated, based on the Lagrangian cost $c_j(\Lambda, \pi)$, rank the selected options in descending order. Under the condition that the significance constraint (5) is not violated, remove the selected options accordingly until the overall budget is under the limit.
- STEP 3: If there is a slack for the budget constraint (6), further add the option with the most negative Lagrangian cost $c_j(\Lambda, \pi)$ until the budget is fully utilized.
- Note: For STEPS 1 and 3, an option can be added only if adding the option does not violate the constraint of the mutually exclusive relationship represented by the constraint of (7).

The above procedure does not guarantee a feasible solution, as removing the selected options according to the ranking list may lead to the violation of the significance constraint, while seeking to satisfy the budget constraint in STEP 2. However, this kind of situation does not occur frequently, for example, about six times out of 100 iterations for the hypothetical test problems in Section 4.2. No matter whether a feasible solution is found or not, the Lagrangian multipliers are updated by the traditional sub-gradient method (Held and Karp, 1970) as in (14)–(17) based on the solution of the relaxed problem of (9), (10), (12), and (13).

$$\lambda_i^{t+1} = \max\left\{\lambda_i^t + \alpha \frac{UB^t - L(\lambda^t, \pi^t)}{\|s(\lambda^t)\|^2} s_i(\Lambda^t), 0\right\}, \quad \forall i \in I \tag{14}$$

$$s_i(\Lambda^t) = 1 - \sum_{j \in J_i} x_j(\Lambda^t, \pi^t), \quad \forall i \in I \tag{15}$$

$$\pi^{t+1} = \max\left\{\pi^t + \beta \frac{UB^t - L(\Lambda^t, \pi^t)}{\|s(\pi^t)\|^2} s(\pi^t), 0\right\} \tag{16}$$

$$s(\pi^t) = -G + \sum_{j \in J_i} c_j x_j(\Lambda^t, \pi^t), \quad \forall i \in I \tag{17}$$

- Λ_i^t: Lagrangian multiplier of region i for the significance level constraint at iteration t.
- Λ^t: Vector of Lagrangian multipliers for the significance level constraints at iteration t.
- π^t: Lagrangian multiplier for the budget at iteration t.

- UB^t: Objective function value for the feasible solution to the original problem at iteration t.
- J_i: Set of options that have a non-zero a_{ij} for a given region i.
- α: Step parameter of the multiplier for the significance level constraints.
- β: Step parameter of the multiplier for the budget constraint.

The whole solution algorithm can be summarized and illustrated by the flowchart in Fig. 1. As for the initial values of the Lagrangian multipliers, they can be set as zero. Regarding the termination criteria, the study limits the number of iterations (denoted by N in the flowchart) to be performed if the iterative procedure is not terminated when the gap between the upper bound based on the current incumbent solution and the lower bound derived from solving the linearized Lagrangian-relaxed problem is less than a pre-defined parameter (denoted by δ in the flowchart).

4. Numerical experiment

The numerical experiment consists of two parts: the illustrative practical example, which is based on some real projects in Taiwan and the hypothetical test problems, which are randomly generated based on some guidelines. These two parts are presented in the following two sub-sections.

4.1. Illustrative practical example

In order to demonstrate how the transportation programming problem with which we are concerned is modeled by the IP formulation developed in this study, the related project data from a recent MS thesis (Niou, 2011) are used to design the illustrative example. There are 25 basic projects, which are the transportation projects in Taiwan with a budget larger than 1 billion NTD in 2010. The basic information regarding these 25 basic projects is listed in Table 1.

In addition, the 25 basic projects together with the 18 counties or cities, which are thought of as the regions in the example, are illustrated in the map of Taiwan in Fig. 2. Based on the figure, the significance level between a region and a basic project can be linked. In particular, for this illustrative example, the value of a_{ij} is set as 1 if part of the basic project j falls

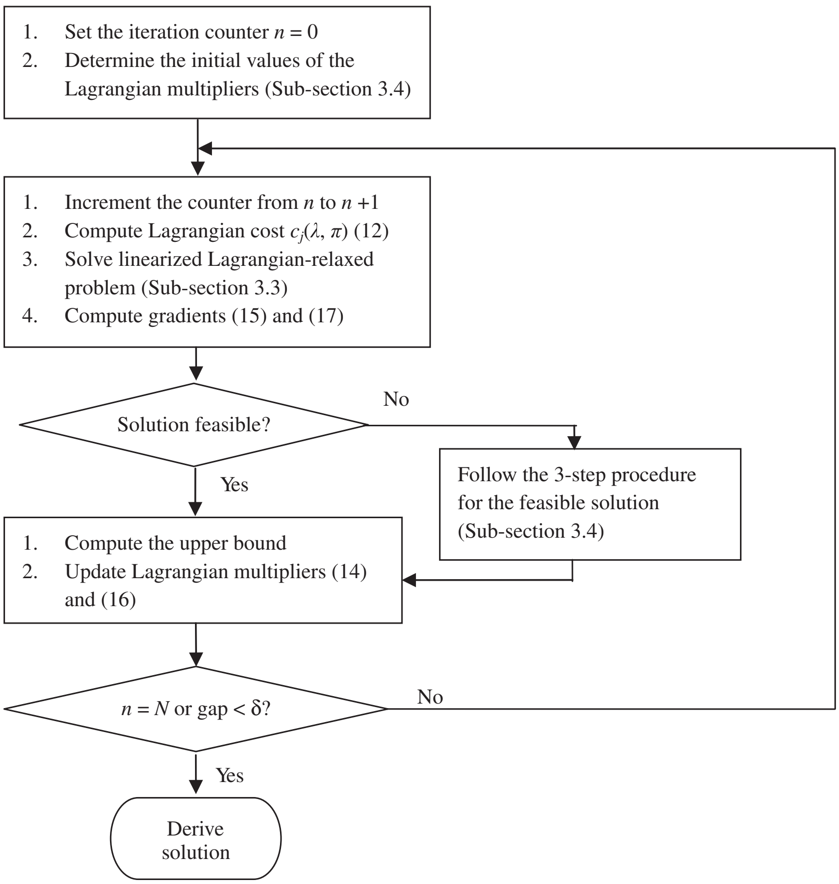

Fig. 1. Flowchart of the solution algorithm.

Table 1
Costs and benefits of basic projects (Unit: 100,000,000 NTD).

Project ID	Project name	Cost	Benefit
1	National freeway No.2 expansion	128.7	164.8
2	National freeway No.6 Nantou section construction	375.6	495.8
3	National freeway No.4 Fengyuan-Takeng section construction and Taichung metropolis highway No.4 construction	317.8	409.9
4	Taichung metropolis highway No.4 north section construction and highway No.2 east section construction	207.0	262.8
5	National freeway No.1 Wugu-Yangmei section expansion	882.6	1103.2
6	Taoyuan International Airport access MRT system construction	1138.5	1525.6
7	Taiwan Railway transformation for rapid transit system (Keelung–Miaoli section)	84.8	114.5
8	Taiwan Railway Taipei City underground transformation (Nankang section)	830.7	938.7
9	Taiwan Railway transformation for rapid transit system (Taichung metropolis)	288.3	322.9
10	Taiwan Railway transformation for rapid transit system (Pingtung–Chaochou section)	152.4	201.1
11	Taiwan Railway Kaohsiung City underground transformation	715.8	909.1
12	Taiwan Railway East Coast line electrification and speed improvement (Hualien–Taitung section)	150.0	201.0
13	Taiwan Railway underground transformation (Zuoying section extension)	106.6	146.1
14	Taiwan Railway transformation for rapid transit system (Taoyuan metropolis)	308.4	364.0
15	Taiwan Railway Tainan City underground transformation	293.6	372.9
16	East Coast highway No.11 improvement	110.4	151.2
17	Express highway No.84 construction (Freeway No. 1–Highway No. 1 section)	115.3	148.8
18	Hualien-Taitung highway No.9 improvement (Phase III)	35.6	45.6
19	West coast express highway follow-up construction	775.1	1030.9
20	Taoyuan International Airport runway pavement rehabilitation and navigation facilities upgrade	107.4	149.3
21	Taoyuan International Airport terminal one re-development	19.9	25.5
22	Nation freeway No.7 construction for Port Kaohsiung access	659.9	864.5
23	Taiwan Railway Keelung station relocation and reconstruction	26.3	37.0
24	Suao-Hualien Highway No.9 mountain section improvement project	423.3	584.2
25	Taiwan Railway Chiayi City overheadization transformation	139.6	181.5

in the territory of region i; otherwise, it is set as zero. Finally, it is assumed that there is no mutually exclusive relationship for any pair of basic projects.

The possible joint projects as well as the interdependent relationship between the projects are also derived based on Niou (2011). The information on the total options, including the basic projects and the joint projects, is summarized in Table 2. The significance level of an option to a region is set as 1, if part of the option is within the territory of the region. In addition, the exclusivity constraint is imposed on the options originally from the basic projects with a corresponding joint project or on the options consisting of the basic project(s) and the associated joint project.

The size of the illustrative example is small, and the original IP problem of (4)–(8) can be solved by any IP solver within a short period of time. The instances of five different budget levels are tested to show how the decision of selecting the options is affected by the available budget. The results are recorded as the second and third columns (SCP solution and objective, respectively) of Table 3. In addition, the problem without constraint (5) for the significance level is also solved, and the results are shown as the fourth and fifth columns (KP solution and objective, respectively) of Table 3. When the budget level is high, the difference between the two models is insignificant. The significance constraint does not have a strong impact on selecting the options when maximizing the benefit for a given budget. However, when the budget is tight, the price paid to cover all regions is critical, as some low-benefit options must be selected so as to provide the required full coverage. For example, in the case of the budget level of 2000, there is no overlapping for the options selected, and the gap between the two objectives is as high as 41.36%.

4.2. Hypothetical test problems

A square is assumed to be the study area, and is divided into smaller squares to serve as the regions in the test problems. For the small illustrative example in Fig. 3(a), there are nine regions within the study area of size 30×30, and the center of the small square (marked as the district center) is used to determine its relationship with a project. The center of a basic project is randomly generated in the study area. In addition, the radius of a project, denoted by r, is randomly generated based on the uniform distribution of $[3,10]$. The cost of a project is assumed to be related to the radius and is set as $10 \times r^2$, and its benefit is randomly generated by assuming that the benefit-to-cost ratio follows the uniform distribution of $[1.0,1.3]$. Suppose that the distance between the center of a region and that of a project is denoted by d. The significance level of a project to a region is set by the following rule, which normally can lead to the feasibility of test problems with respect to Constraint (5).

- $a_{ij} = 1$ if $d \leqslant 0.5r$
- $a_{ij} = 3/4$ if $0.5r < d \leqslant r$
- $a_{ij} = 1/2$ if $r < d \leqslant 1.5r$
- $a_{ij} = 1/4$ if $1.5r < d \leqslant 2r$
- $a_{ij} = 0$ if $2r < d$

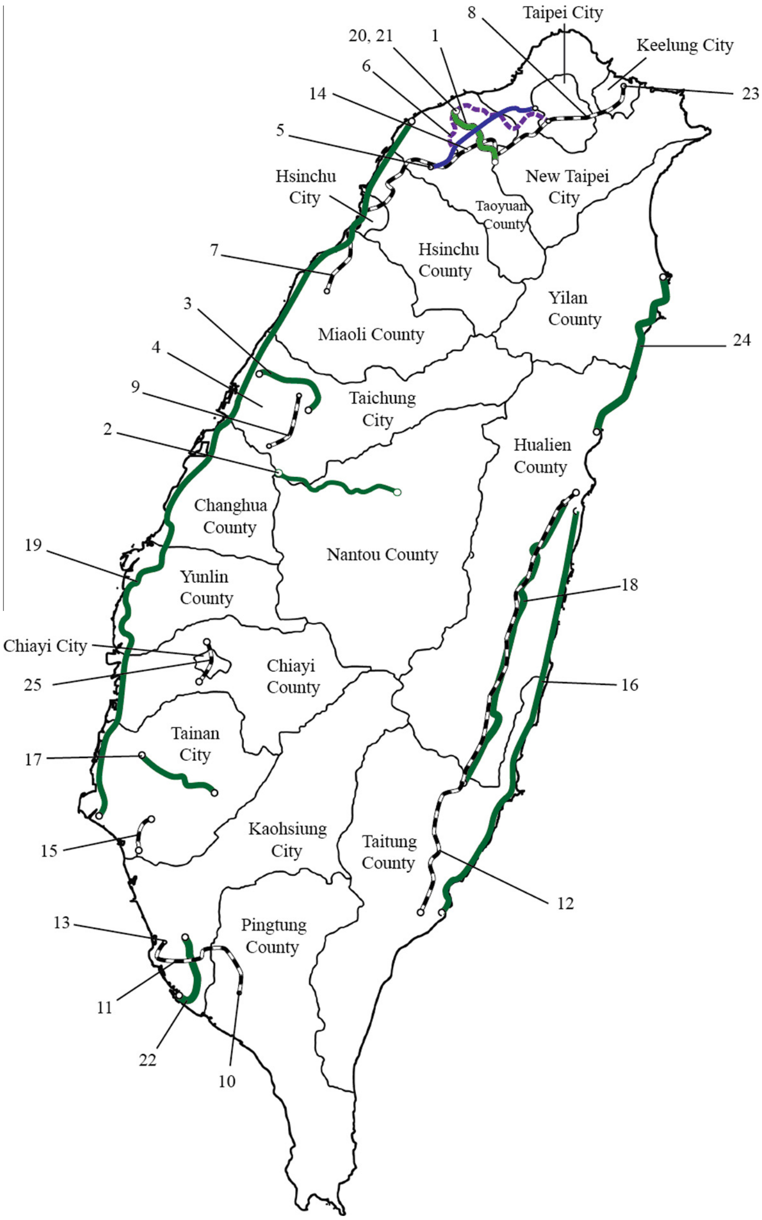

Fig. 2. Locations of basic projects and boundaries of regions.

Regarding the interdependency between the basic projects, it is assumed that there is no mutually exclusive relationship for any pair of basic projects. On the other hand, the radii of two basic projects and the distance between their centers are used to determine whether they are dependent. Suppose that, for two projects indexed by g and h, the radii are denoted by r_g and r_h respectively, as shown in Fig. 3(b). If the distance $f(g,h) \leqslant r_g + r_h$, the two projects are assumed to be dependent. A new variable is then used to represent the joint project. The cost of the joint project is assumed to be the sum of the costs of the two projects multiplied by a random number following the uniform distribution of [0.7, 1.1]. In addition, the benefit of the joint project is assumed to be the sum of the benefits of the two projects multiplied by a random number following the uniform distribution of [0.9, 1.2]. In addition, it is assumed that the significance level of the joint project to a region is the sum of the individual significance levels.

The test problems are designed with four scales of 50, 100, 150, and 200 basic projects, the size of the study area, the number of regions within the study area, and the available budget is adjusted accordingly, as shown in Table 4. The center, radius, benefit, and cost of the basic projects are generated randomly. The significance level of the basic projects to the regions can be determined accordingly; in addition, whether two basic projects are dependent on each other can be decided. For any pair of dependent basic projects, the benefits and costs of the joint project are generated randomly based on the rule

Table 2
Options in the illustrative example (including all basic projects and joint projects).

Corresponding variable	Basic project(s) included	Option cost	Benefit to cost ratio	Corresponding variable	Basic project(s) included	Option cost	Benefit to cost ratio
x_1	1, 5	1011.3	1.63	x_{33}	8, 11, 13, 15	1946.7	1.62
x_2	1	128.7	1.28	x_{34}	8, 11, 13	1653.1	1.53
x_3	5	882.6	1.25	x_{35}	8, 11, 15	1840.1	1.59
x_4	2, 3, 4	900.3	1.92	x_{36}	8, 13, 15	1230.9	1.44
x_5	2, 3	693.3	1.79	x_{37}	11, 13, 15	1116.0	1.54
x_6	2, 4	582.5	1.71	x_{38}	8, 11	1546.5	1.50
x_7	3, 4	524.7	1.64	x_{39}	8, 13	937.3	1.34
x_8	2	375.6	1.32	x_{40}	8, 15	1124.3	1.40
x_9	3	317.8	1.29	x_{41}	11, 13	822.4	1.46
x_{10}	4	207.0	1.27	x_{42}	11, 15	1009.4	1.51
x_{11}	6, 20, 21	1265.8	2.00	x_{43}	13, 15	400.2	1.44
x_{12}	6, 20	1245.9	1.35	x_{44}	8	830.7	1.13
x_{13}	6, 21	1158.4	1.94	x_{45}	11	715.8	1.27
x_{14}	20, 21	127.3	1.44	x_{46}	13	106.6	1.37
x_{15}	6	1138.5	1.34	x_{47}	15	293.6	1.34
x_{16}	20	107.4	1.39	x_{48}	12, 16, 18	296.0	2.05
x_{17}	21	19.9	1.28	x_{49}	12, 16	260.4	1.98
x_{18}	7, 9, 10, 14	834.0	1.42	x_{50}	12, 18	185.6	1.77
x_{19}	7, 9, 10	525.5	1.36	x_{51}	16, 18	146.0	1.70
x_{20}	7, 9, 14	681.6	1.35	x_{52}	12	150.0	1.34
x_{21}	7, 10, 14	545.7	1.39	x_{53}	16	110.4	1.37
x_{22}	9, 10, 14	749.1	1.38	x_{54}	18	35.6	1.28
x_{23}	7, 9	373.1	1.27	x_{55}	7, 23	111.1	1.47
x_{24}	7, 10	237.2	1.40	x_{56}	23	26.3	1.41
x_{25}	7, 14	393.3	1.32	x_{57}	18, 24	458.9	1.44
x_{26}	9, 10	440.7	1.30	x_{58}	24	423.3	1.38
x_{27}	9, 14	596.8	1.30	x_{59}	17	115.3	1.29
x_{28}	10, 14	460.8	1.35	x_{60}	19	775.1	1.33
x_{29}	7	84.8	1.35	x_{61}	22	659.9	1.31
x_{30}	9	288.3	1.12	x_{62}	25	139.6	1.30
x_{31}	10	152.4	1.32				
x_{32}	14	308.4	1.18				

Table 3
Results in the illustrative example.

Budget	SCP solution	SCP objective	KP solution	KP objective	Gap (%)
2000	8, 24, 57, 60, 62	−2701.11	5, 11, 54	−3818.33	41.36
3000	4, 16, 24, 48, 58, 59, 60, 62	−4762.09	4, 11, 48, 55, 58	−5614.52	17.90
4000	4, 11, 24, 46, 51, 58, 60, 62	−6783.16	1, 4, 11, 43, 48, 55	−7255.08	6.96
5000	1, 4, 11, 24, 50, 56, 58, 60, 62	−8402.87	4, 11, 33, 48, 56, 58, 62	−8823.46	5.01
6000	4, 11, 24, 33, 48, 58, 60 62	−10149.40	1, 4, 11, 21, 33, 48, 56	−10464.60	3.11

described above. Moreover, the associated constraints for the mutually exclusive relationships between the two basic projects and between the basic project and the joint projects are added. For each problem scale, 30 problems are designed. The average number of options, equivalent to the number of binary decision variables, and the average number of constraints in the IP model of (4)–(8) are also reported in Table 4.

The problem scales of the test problems are close to the practical problems in the real world, but the IP problem of (4)–(8) for all problem scales is still solvable by the IP solver of most software packages. Particularly for this study, the test problems are solved by the GLPKMEX toolbox in MATLAB R2009a, and the derived optional solution serves as the basis for the evaluation of solution quality and bound quality. With the number of iterations (N) for the solution algorithm set as 500, the objective function values of the approximate solutions and the lower bound based on the linearized Lagrangian-relaxed problem of (9), (10), (12), and (13) are recorded for all 30 test problems of each problem scale. The information for the solution quality and the bound quality is provided in Table 4, in terms of the gap in percentage with respect to the optimal solution from the IP model. Moreover, the computation times of the IP solver and the heuristic algorithm, denoted as IP and LR times respectively, are provided in Table 4 to highlight the differences from the computational aspect. Regarding the environment of the numerical experiment, the operating system is Windows XP SP3, and the hardware is a PC with Intel Core 2 Duo E8400 3.00 GHz CPU and 2G RAM.

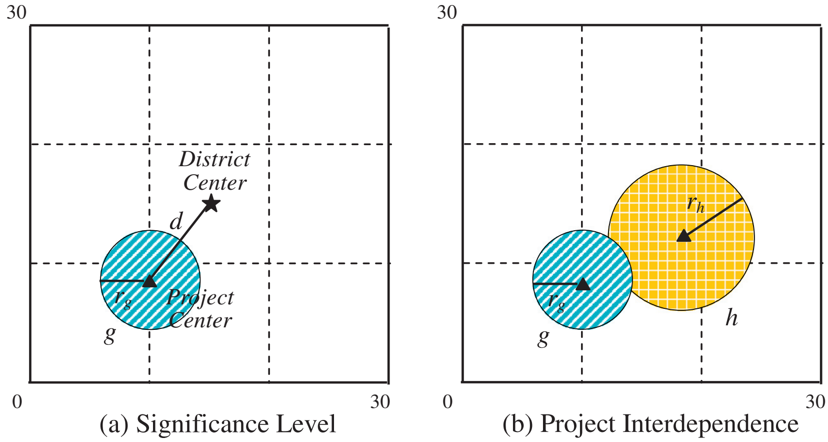

Fig. 3. Illustration of project-region significance level and project interdependency.

Table 4
Information and results for the hypothetical test problems.

Number of basic projects	Number of regions	Budget	Average number of options	Average number of constraints	IP time (s)	LR time (s)	Solution quality		Bound quality	
							Mean (%)	Std. dev. (%)	Mean (%)	Std. dev. (%)
50	25	10,000	265	455	0.9	3.4	0.53	0.63	1.76	1.27
100	49	20,000	570	993	11.5	5.9	0.20	0.15	1.07	0.65
150	81	30,000	821	1424	66.5	12.4	0.32	0.17	1.67	0.54
200	121	40,000	1012	1745	225.0	14.7	0.63	0.60	1.90	0.80

Based on the results of the numerical experiment, the solution algorithm generates an approximate solution that is very close to the optimal solution for all problem scales, with the average gap being less than 1%. In addition, the lower bound based on the linearized Lagrangian-relaxed problem appears to be an effective one, as the average gap is less than 2%. For small and medium-sized problems, we believe the developed IP model can generate the optimal solution within an acceptable time, given the computational power of today's computers, although the computation time is increased exponentially as shown in Table 4. For large-sized problems, the developed heuristic algorithm should be able to provide a very good solution within a modest computation time.

5. Conclusions

This study develops a new integer programming model based on the set covering problem to address two important practical features of the transportation programming problem: project interdependency and regional balance. A solution algorithm is developed based on the technique of Lagrangian Relaxation. According to the results of the numerical experiment, the solution algorithm can generate an approximate solution that is very close to the optimal solution and the lower bound based on the linearized Lagrangian-relaxed problem is strong. The applicability of the integer programming model and the solution algorithm developed in this study is basically verified. The directions for future research can be summarized as follows.

The Lagrangian-relaxed problem is not directly solved, as it is still an integer programming problem, which cannot be solved easily. Thus, its binary constraint is further relaxed, so as to be solved by a general LP solver. Although the quality of the approximate solution and the quality of the bound appear to be good based on the numerical experiment, it is still possible to generate an even better solution to the relaxed problem if it is dealt with by a more sophisticated approach. In particular, the relaxed problem has a very special feature: the coefficients are either zero or one in all constraints, and their right-hand sides are one. By making use of this special feature, an efficient procedure (e.g., a modified version of the Hungarian method originally for in the assignment problem) can probably lead to a high-quality solution to the relaxed problem without too much computational effort.

The transportation programming problem plays an important role as the decision support for the planning of the transportation infrastructure. Although two important features, namely, project interdependency and regional balance, have been included in the model in this study, there are still quite a few issues that are potentially important to the quality of policy making and resource planning. First of all, the continuity of the planning is critical to the development of transportation infrastructure. A multi-stage model can better address the issue of consistency. Second, there exists some uncertainty regarding the benefits and costs of a project (e.g., Li et al., 2010). This issue is made more complicated by taking project interdependency into account. However, the importance of making a robust decision in today's ever-changing environment cannot be overlooked. Finally, given the diversified interests of all stakeholders, a multiple-objective context is probably more appropriate than the current single objective of benefit maximization.

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