An Iterative Weighted Reliability Decoding Algorithm for Two-Step Majority-Logic Decodable Cyclic Codes

Hsiu-Chi Chang, Chih-Lung Chen, and Hsie-Chia Chang

***Abstract*—An iterative weighted reliability two-step majority logic decoding (IWRTS-MLGD) algorithm for two-step majority-logic (TS-MLG)-decodable cyclic codes is presented. In contrast to other message passing decoding algorithms that utilize real number operations, our proposed decoding algorithm requires only logical operations and integer additions. Therefore, large computational complexities can be reduced. For moderate-length TS-MLG-decodable cyclic codes, the proposed algorithm aided with soft information and a scaling factor outperforms the hard-decision TS-MLGD algorithm and hard-decision BCH codes with similar length by 1.2- and 1.0-dB, respectively.**

***Index Terms*—Finite geometry code, reliability-based message passing algorithm, two-fold EG code, cyclic code.**

I. Introduction

FINITE geometry (FG) codes received great attention in the late 1960s and the early 1970s [1]–[3]. FG codes form a class of cyclic codes with reasonable minimum distance which can be decoded with simple majority logic (MLG). There are two types of MLG-decodable cyclic codes: one-step and multi-step. One-step MLG-decodable cyclic codes are rediscovered in [4] as FG-LDPC codes whose Tanner graphs are free of cycles of length 4. Long FG-LDPC codes decoded by the *sum-product algorithm* (SPA) [5] and *min-sum algorithm* (MSA) [6] can nearly achieve Shannon's theoretical limit. However, multi-step MLG-decodable cyclic codes contain many short cycles of length 4. Thus, standard SPA or MSA is not effective for decoding multi-step MLG-decodable cyclic codes [7] and [8]. In [7] and [8], a two-step soft decision decoding algorithm based on SPA and MSA was introduced for decoding TS-MLG-decodable cyclic codes, which is called the *two-step iterative decoding algorithm* (TS-IDA). TS-IDA uses a five level *message passing tree* (MPT), which is constructed based on the *orthogonal* structure of the relation between frames and lines of the finite geometries. Therefore, it can avoid the degrading effects of short cycles. The simulation results in [8] show that TS-IDA outperforms other hard decision decoding methods by 2dB. However, the computational complexity is very large since a large portion of the computation involves real number.

Recently, [9] has introduced an efficient iterative algorithm called the *soft reliability based iterative majority logic decoding algorithm* (SRBI-MLGD) for decoding one-step MLG-decodable LDPC codes constructed by finite geometries. SRBI-MLGD is a binary message passing algorithm which utilizes only logical operations and integer additions. Thus, it greatly reduces the computational complexities compared to SPA or MSA. In contrast to other binary decoding methods such as *differential binary message passing algorithm* (DBMPA) [10], the SRBI-MLGD requires less computational complexity and memory.

In this letter, we propose an *iterative weighted reliability two-step majority logic decoding* (IWRTS-MLGD) algorithm for decoding TS-MLG-decodable cyclic codes. The idea of the algorithm is derived from SRBI-MLGD which greatly reduces the computational complexity compared to the previous TS-IDA [8]. We use a subclass of TS-MLG-decodable cyclic FG codes called two-fold Euclidean geometry (EG) codes to demonstrate the effectiveness of the proposed algorithm. The letter is organized as follows: Section II introduces the background of the two-fold EG codes, and the corresponding two-step (TS)-MLGD algorithm. Section III provides the proposed IWRTS-MLGD algorithm in detail. The computational complexity analysis is also included. Section IV shows the simulation results. Finally, section V presents the conclusion.

II. Construction of Two-Fold EG Codes and TS-MLGD Algorithm

Consider a d-dimensional Euclidean geometry EG(d,q) over the field GF(q). The field GF(q^d) as an extension field of the field GF(q) is a realization of EG(d,q). Let α be a primitive element of GF(q^d). Then the powers of α, $\alpha^{-\infty} = 0, \alpha^0 = 1, \alpha, ...\alpha^{q^d-2}$, represent the q^d points of EG(d,q) and $\alpha^{-\infty} = 0$ represents the origin of EG(d,q). Let EG$*(d,q)$ be a subgeometry obtained from EG(d,q) by removing the origin and all the lines passing through the origin. Let $n = q^d - 1$. There are n nonorigin points and $J_0 = n(q^{d-1}-1)/(q-1)$ lines not passing through origin in EG$*(d,q)$. A line has q points. If a point is on a line in EG$*(d,q)$, we say that the line passes through the point(or orthogonal on the point). Every point in EG$*(d,q)$ is intersected by $J_1 = n/(q-1) - 1$ lines. For the i-th line L_i in EG$*(d,q)$, where $0 \le i < J_0$, there are $J_2 = q^{d-1} - 2$ lines parallel to it denoted as $L_{t,i}$, where $0 \le t < J_2$. Let $\{L_i, L_{t,i}\}$ be a $(1,2)$-frame in EG$*(d,q)$. There are J_2 $(1,2)$-frames orthogonal on line L_i denoted as $\{L_i, L_{0,i}\}, \{L_i, L_{1,i}\}, \ldots, \{L_i, L_{J_2-1,i}\}$. There are a total of $r = n(q^{d-1}-1)(q^{d-1}-2)/2(q-1)$ $(1,2)$-frames in EG$*(d,q)$. We consider an $r \times n$ matrix $\mathbf{H}$ over GF(q^d) with each row as a binary incidence vector of the $(1,2)$-frames in EG$*(d,q)$. Then the null space of $\mathbf{H}$ gives a cyclic code of length n, called as two-fold EG [11] code.

Manuscript received May 23, 2013. The associate editor coordinating the review of this letter and approving it for publication was G. Liva.

This work was supported by the NSC, Taiwan, R.O.C., under Contract NSC 101-2628-E-009-013-MY3.

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Digital Object Identifier 10.1109/LCOMM.2013.082413.131191

The generator polynomial $\mathbf{g}(X)$ of a two-fold EG code can be found in [8].

Let $\mathbf{u} = (u_0, u_1, u_2, \cdots, u_{n-1})$ be a codeword of two-fold EG code. Suppose $\mathbf{u}$ is transmitted via BPSK with unit energy over the binary AWGN with two-sided power spectral density $N_0/2$. Let $\mathbf{y}= (y_0, y_1, y_2, \cdots, y_{n-1})$ be the sequence of *samples* at the output of the sampler in the receiver. Let $\mathbf{z}= (z_0, z_1, z_2, \cdots, z_{n-1})$ be the hard decision sequence of $\mathbf{y}$. The hard decision received sequence will be a two-fold EG codeword if $\mathbf{H}\mathbf{z}^T = 0$ or if the polynomial representation $\mathbf{z}(X)$ of $\mathbf{z}$ is divisible by the generator polynomial $\mathbf{g}(X)$. The parity-check matrix $\mathbf{H}$ can be built by using the $(1, 2)$-frames with J_0 lines in EG$*(d, q)$. Consider the i-th line L_i in EG$*(d, q)$, where $0 \leq i < J_0$. Let $\mathbf{v}_{L_i}$ be the incidence vector of line L_i denoted as $\mathbf{v}_{L_i} = \{v_{i,0}, v_{i,1}, \cdots, v_{i,n-1}\}$ whose components are the n-tuples over GF(2) that correspond to the n non-origin points of EG$*(d, q)$ with $v_{i,j} = 1$ if α^j is a point on L_i, otherwise $v_{i,j} = 0$, where $0 \leq j < n$. Based on the J_0 lines in EG$*(d, q)$, we form a $J_0 \times n$ matrix denoted as $\mathbf{L}$, with J_0 incidence vectors of lines as rows and n points as columns. Let $\mathbf{v}_{L_0}$,$\mathbf{v}_{L_1}$,$\cdots$,$\mathbf{v}_{L_{J_0-1}}$ be the rows of $\mathbf{L}$. For $0 \leq i < J_0$ and $0 \leq j < n$, we define $N_i = \{j : 0 \leq j < n, v_{i,j} = 1\}$ and $M_j = \{i : 0 \leq i < J_0, v_{i,j} = 1\}$. The indices in N_i denote the location of 1-component in the i-th row of $\mathbf{L}$. The indices in M_j denote the location of 1-component in the j-th column of $\mathbf{L}$. Let $S(L_i)$ be the line-sum of L_i, which can be calculated by the inner product of $\mathbf{z}$ and the incidence vector $\mathbf{v}_{L_i}$ as follows:

$$S(L_i) = \sum_{j \in N_i} z_j. \tag{1}$$

Consider a $(1, 2)$-frame $F = \{L_i, L_{t,i}\}$ in EG$*(d, q)$. The frame-sum of F can be derived by XOR two line-sums: $S(L_i)$ and $S(L_{t,i})$, denoted as $S(F) = S(L_i) \oplus S(L_{t,i})$, where $\oplus$ is the XOR operation. Because $L_{t,i}$ is also a line in EG$*(d, q)$, both $S(L_i)$ and $S(L_{t,i})$ can be calculated by (1), which equal to either 0 or 1. There are a total of r frame-sums in the parity-check matrix $\mathbf{H}$. Each frame-sum is the inner product of $\mathbf{z}$ and the binary incidence vector of $(1, 2)$ frame F composed of two lines L_i and $L_{t,i}$ in EG$*(d, q)$.

Consider updating a received bit z_j in a two-fold EG code [11], where $0 \leq j < n$. The received bit z_j corresponds to a point α^j in EG$*(d, q)$. There are J_1 lines in EG$*(d, q)$ passing through α^j denoted as L_u^j, where $0 \leq u < J_1$. The line-sum of L_u^j is denoted as $S(L_u^j)$. These line-sums are orthogonal on z_j. For $0 \leq t < J_2$, there are J_2 lines in EG$*(d, q)$ parallel to L_u^j, denoted as $L_{t,u}^j$. The line-sum of $L_{t,u}^j$ is denoted as $S(L_{t,u}^j)$. The first step of decoding is to decode $S(L_u^j)$ using the J_2 $(1, 2)$-frames of EG$*(d, q)$ that are orthogonal on L_u^j. Let $F^{j,u,t} = \{L_u^j, L_{t,u}^j\}$ be a $(1, 2)$-frame in EG$*(d, q)$ that is orthogonal on L_u^j. The frame-sum of $F^{j,u,t}$ is denoted as $S(F^{j,u,t}) = S(L_u^j) \oplus S(L_{t,u}^j)$. The line-sum $S(L_{t,u}^j)$ of $S(F^{j,u,t})$ is the *extrinsic information* for decoding $S(L_u^j)$. A received bit in $\mathbf{z}$ that is not contained in L_u^j can appear in at most one $L_{t,u}^j$. With the above concept, we can decode the value of $S(L_u^j)$ correctly with MLGD using the J_2 $S(F^{j,u,t})$ that are orthogonal on $S(L_u^j)$ provided there are no more than $\lfloor J_2/2 \rfloor$ errors in $\mathbf{z}$. The second step of decoding is to decode z_j by J_1 $S(L_u^j)$ that are orthogonal on z_j. Any received bit of $\mathbf{z}$ other than z_j can appear in at most one of these J_1 lines. These bits orthogonal on z_j are the *extrinsic information* for z_j. The *intrinsic information* of z_j comes from the hard-decision of itself. Since $J_1 > J_2$, with MLGD based on these J_1 lines, the value of z_j can be correctly decoded with no more than $\lfloor J_2/2 \rfloor$ errors in received bits [11]. The above decoding of a two-fold EG code is called two-step (TS)-MLGD.

III. Iterative Weighted Reliability Decoding Algorithm for Two-Fold EG Codes

TS-MLGD is a *one-pass* decoding algorithm with only hard-decision values from the received bits. TS-MLGD has low computational complexity, yet its performance can be greatly improved. In [8], TS-IDA algorithm improved the performance of two-fold EG codes by employing the soft information from the channel along with an iterative decoding process. The computational complexity of TS-IDA is very large because much of its computation involves real number. In the following, we propose an iterative decoding algorithm called *iterative weighted reliability two-step*-MLGD (IWRTS-MLGD) algorithm which utilizes only logical operations and integer additions. The unweighted algorithm is also included.

Let r_j be the quantized value of the sample y_j, where $0 \leq j < n$. The quantized value is an integer representation of the $2^b - 1$ quantized intervals symmetric to the origin. Each interval has a length $\triangle$ and is represented by b bits. Therefore, r_j is in the range of $\left[-(2^{(b-1)} - 1), +2^{(b-1)} - 1\right]$. The magnitude $|r_j|$ of r_j gives the *soft measure* of the reliability of the hard decision received bit z_j. Next, we need some notations to employ the iterative decoding process. Let l be the iteration number. Let l_{max} be the maximum iterations to be performed in the decoding process. For $0 \leq l < l_{max}$, let $\mathbf{z}^{(l)} = \{z_0^{(l)}, z_1^{(l)}, \cdots, z_{n-1}^{(l)}\}$ be the received vector generated in the l-th iteration decoding, where $z_j^{(l)}$ is the j-th hard-decision received bit at the l-th iteration. In each iteration, we first update the line-sum of all the lines by

$$S^{(l)}(L_i) = \sum_{j \in N_i} z_j^{(l)}. \tag{2}$$

Let $R_j^{(l)}$ be the reliability measure of the j-th bit at the l-th iteration. Moreover, let $\psi^{(l)}(L_i)$ be the reliability measure of L_i at the l-th iteration, which is determined by the least reliable bit in L_i as

$$\psi^{(l)}(L_i) = \min_{j \in N_i} |R_j^{(l)}|. \tag{3}$$

We set $R_j^{(0)}$ equal to the reliability r_j as an initial reliability measure of a received bit z_j. Note that $S^{(l)}(L_i)$ and $\psi^{(l)}(L_i)$ can be shared during the decoding process for the received bits of $\mathbf{z}^{(l)}$ in an iteration. The received bit z_j participates in $F^{j,u,t}$ consisting of two parallel lines, L_u^j and $L_{t,u}^j$. The line-sum of L_u^j and $L_{t,u}^j$ at the l-th iteration are denoted as $S^{(l)}(L_u^j)$ and $S^{(l)}(L_{t,u}^j)$, respectively. Both $S^{(l)}(L_u^j)$ and $S^{(l)}(L_{t,u}^j)$ can be derived from (2). Besides, $\psi^{(l)}(L_{t,u}^j)$ can be derived from (3). Suppose we update $z_j^{(l)}$ in L_u^j, where $0 \leq u < J_1$. The *extrinsic information* of $z_j^{(l)}$ comes from the following two steps. In the first step, the *extrinsic information* contributed by the J_2 parallel lines for the u-th line is calculated by

$$\phi_u^{(l)} = \sum_{t=0}^{J_2-1} \psi^{(l)}(L_{t,u}^j)\left\{2S^{(l)}(L_{t,u}^j)-1\right\}. \quad (4)$$

Let $\varphi_u^{(l)}$ be the hard-decision of $\phi_u^{(l)}$, which can be expressed by an indicator function $\varphi_u^{(l)} = I(\phi_u^{(l)} > 0)$. In the second step, the *extrinsic information* is contributed by the received bits of $\mathbf{z}^{(l)}$ that participate in L_u^j except $z_j^{(l)}$ which is given by

$$\sigma_{u,j}^{(l)} = S^{(l)}(L_u^j) \oplus {z_j}^{(l)}. \quad (5)$$

The *total extrinsic information* can be derived from the aforementioned two steps by J_1 $\sigma_{u,j}^{(l)}$ XOR $\varphi_u^{(l)}$ along with a scaling factor α as

$$E_j^{(l)} = \alpha \sum_{u=0}^{J_1-1}\left\{2(\sigma_{u,j}^{(l)} \oplus \varphi_u^{(l)})-1\right\}, \quad (6)$$

where α can be optimized by simulation. The reliability measure for the j-th received bit $z_j^{(l+1)}$ at the $(l+1)$-th iteration is calculated by

$$R_j^{(l+1)} = R_j^{(l)} + E_j^{(l)}. \quad (7)$$

In addition, in updating the soft reliability value $R_j^{(l)}$ for a received bit z_j, if the magnitude $|R_j^{(l)} + E_j^{(l)}|$ is larger than the quantization range $(2^{(b-1)}-1)$, the reliability value is set to $(2^{(b-1)}-1)$. Thus, the soft reliability value is always clipped under the maximal value of the quantization. If an unweighted decoding algorithm is desired, the scaling factor should be set to 1. With the concept and notations defined above, the proposed algorithm is formulated as **Algorithm 1**.

Algorithm 1 IWRTS-MLGD algorithm for two-fold EG codes

1: **Initialization**:
Set $\mathbf{z}^{(0)} = \mathbf{z}$, $l=0$, and the maximum number of iterations to l_{max}. For $0 \le j < n$, set $R_j^{(0)} = r_j$.
2: Let $\mathbf{s}^{(l)}(X)$ be the syndrome derived by dividing the received polynomial $\mathbf{z}^{(l)}(X)$ by the generator polynomial $\mathbf{g}(X)$ of the codes. If $\mathbf{s}^{(l)}(X)$=0, stop decoding and output $\mathbf{z}^{(l)}$ as the decoded codeword. If l=l_{max} and $\mathbf{s}^{(l)}(X) \neq$0, stop decoding and declare a decoding failure.
3: For $0 \le i < J_0$, compute all $S^{(l)}(L_i)$ and $\psi^{(l)}(L_i)$ by (2) and (3).
4: Update $E_j^{(l)}$ by (4), $\varphi_u^{(l)}$, (5), and (6).
5: Update the reliability measure of $R_j^{(l+1)}$ by (7).
6: $l \leftarrow l+1$. Make the following hard-decision:
1) if $R_j^{(l)} \le 0$, then ${z_j}^{(l)} = 0$, and 2) if $R_j^{(l)} > 0$, then ${z_j}^{(l)} = 1$. Form a new received vector $\mathbf{z}^{(l)}$. Go to Step 2.

We analyze the computational complexity as follows. For step 3, there are $J_0 = n(q^{(d-1)}-1)/(q-1)$ lines in EG$*(d,q)$. The line-sum of a line needs $(q-1)$ logical XOR operations. Thus, it needs $n(q^{(d-1)}-1)$ XOR operations to compute all the line-sums in EG$*(d,q)$. In addition, it needs $q-1$ comparisons to determine the minimum value for the reliability of a line. Therefore, a total of $n(q^{(d-1)}-1)$ comparisons is needed. In step 4, $S^{(l)}(L_u^j)$ and $S^{(l)}(L_{t,u}^j)$ are derived by assigning their corresponding line-sums in $S^{(l)}(L_i)$. Meanwhile, $\psi^{(l)}(L_{t,u}^j)$ is derived by assigning its corresponding reliability of line in $\psi^{(l)}(L_i)$. In (4), it needs J_2-1 integer additions and J_2 logical operations for $\phi_u^{(l)}$. In addition, a logical operation is needed to determine the hard-decision of $\varphi_u^{(l)}$. (5) needs a logical operation and (6) needs J_1-1 integer additions with J_1-1 logical operations. The scaling factor α is set to be an integer for simplicity. If the scaling factor is an even number, the scaling can be accomplished with a simple shift operation. Otherwise, the scaling operation needs to add the original value to the shifted value, this will require another $J_1 n$ integer additions. Step 5 requires n integer additions to update the reliability of the n received bits. Finally, step 6 needs n logical operation to test the sign of an integer. To perform one iteration, we need $n(J_1+J_2+3)+J_0(q-1) \approx 3q^{d-1}(q^d-1)$ logical operations, $J_0(q-1)+n\{J_1J_2-(J_1+J_2)+2\} \approx q^d(q^d-1)$ integer additions. Since the code length is $n = q^d-1$, by taking $d=2$ for two-fold EG codes constructed by two dimensional Euclidean geometry, the number of logical operations is of $O(n^{3/2})$, while the number of integer additions is of $O(n^2)$.

TABLE I
COMPUTATION COMPLEXITY REQUIRED PER ITERATION OF SPA, DBMPA, TS-IDA, AND PROPOSED IWRTS-MLGD ALGORITHM.

Decoding algorithm	Computation Cost per Iteration				
	BO	**IA**	**RA**	**RM**	**Log**
SPA [5]				$6n^2$	n
DBMPA [10]	$4n^2+n$	$4n^2+2n$			
TS-IDA [8]	$3n^{3/2}$		n^2+n		$7n^{3/2}$
IWRTS-MLGD	$3n^{3/2}$	n^2+n			

BO:Binary Operation; **IA**:Integer Addition; **RA**:Real Addition; **RM**:Real Multiplication; **Log**:Logarithm;

In Table I we compare the numbers of operations per iteration required by SPA [5], DBMPA [10], TS-IDA [8], and the proposed IWRTS-MLGD. The numbers of operations for SPA and DBMPA are obtained from Table I in [9]. The number of operations for TS-IDA is derived from the analysis in [8]. In [9], δ is the number of 1-entries in the parity-check matrix $\mathbf{H}$. There are $n(q^{d-1}-1)(q^{d-1}-2)/2(q-1)$ rows in $\mathbf{H}$ with each row containing $2q$ 1's. Therefore, $\delta = 2q \times n(q^{d-1}-1)(q^{d-1}-2)/2(q-1) \approx n^2$. With some translations, we demonstrate the complexity comparison in terms of n. Table I shows that the computation of the proposed IWRTS-MLGD with binary operations and integer additions employs the least operations compared to SPA and TS-IDA with real number operations. In addition, the proposed algorithm reduces computational complexity by at least 75 percent compared to DBMPA, which is also a binary message passing algorithm.

IV. Simulation Results

In the following two examples, we use 8-bit uniform quantization with 255 levels and an interval length $\triangle = 0.015$ for $(255,191)$ code, and 10-bit uniform quantization with 1024 levels and an interval length $\triangle = 0.0075$ for $(1023,813)$ code.

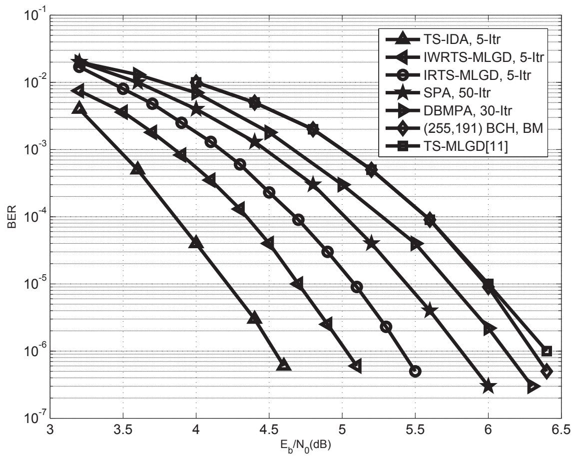

Fig. 1. Error performance of the two-fold EG code $(255, 191)$ in Ex. 1 with various decoding algorithms over the AWGN channel.

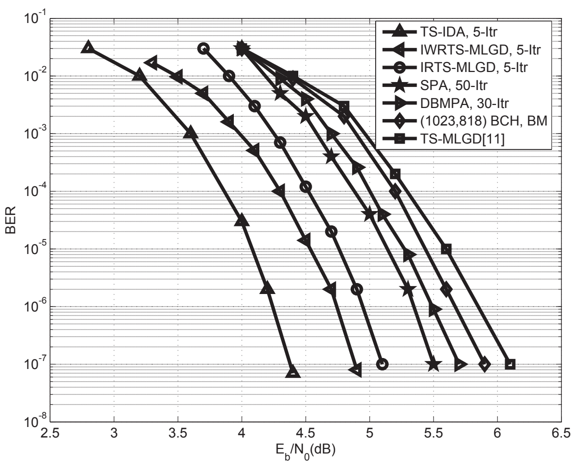

Fig. 2. Error performance of the two-fold EG code $(1023, 813)$ in Ex. 2 with various decoding algorithms over the AWGN channel.

Both codes have scaling factor $\alpha = 3$. The interval length and the scaling factor are obtained by simulation. The proposed unweighted decoding algorithm is denoted as IRTS-MLGD.

Example 1: Regarding $GF(2^8)$ as the geometry for the two-dimensional EG$(2, 2^4)$, the two-fold EG code $(255, 191)$ can be constructed. Fig. 1 shows the bit error performance of the proposed algorithms and TS-IDA with 5 iterations, DBMPA with 30 iterations, SPA with 50 iterations and TS-MLGD. The performance of a $(255, 191)$ BCH code with minimum distance 17 decoded by the hard-decision Berlekamp-Massey(BM) algorithm [11] is also included. At BER of 10^{-6}, the proposed IWRTS-MLGD degrades 0.5dB from TS-IDA, but outperforms IRTS-MLGD, SPA, DBMPA, BCH code, and TS-MLGD by 0.4-, 0.8-, 1.2-, 1.3- and 1.4-dB, respectively.

Example 2: Regarding $GF(2^{10})$ as the geometry for the two-dimensional EG$(2, 2^5)$, the two-fold EG code $(1023, 813)$ can be constructed. Fig. 2 depicts the bit error performance of the proposed algorithms, TS-IDA with 5 iterations, DBMPA with 30 iterations, SPA with 50 iterations and TS-MLGD. The performance of a $(1023, 818)$ BCH code with minimum distance 43 decoded by the hard-decision BM algorithm is also included. At BER of 10^{-7}, IWRTS-MLGD degrades 0.5dB from TS-IDA, but outperforms IRTS-MLGD, SPA, DBMPA, BCH code and TS-MLGD by 0.2-, 0.6-, 0.8-, 1- and 1.2-dB, respectively.

V. Conclusion and Remarks

In this letter, we develop an iterative weighted reliability decoding algorithm for TS-MLG-decodable cyclic codes that utilizes the orthogonal structure of the parity-check matrices. Although the algorithm is developed for a special subclass of TS-MLG-decodable cyclic codes, called two-fold EG codes, it can be applied to any TS-MLG-decodable cyclic code. Unlike TS-IDA using real number operations, our proposed algorithm utilizes only logical operations and integer additions, thus reducing computational complexity. From simulation results, though the two-fold EG codes decoded with the proposed IWRTS-MLGD algorithm are 0.5dB away from TS-IDA, the proposed algorithm outperforms standard SPA and binary message passing DBMPA at least 0.6dB. Besides, it achieves 1dB gain over similar length BCH codes decoded with the hard-decision BM algorithm, and 1.2dB gain over hard-decision TS-MLGD.

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