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Reprint of “Modeling two-vehicle crash severity by a bivariate generalized ordered probit approach”☆

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ABSTRACT

This study simultaneously models crash severity of both parties in two-vehicle accidents at signalized intersections in Taipei City, Taiwan, using a novel bivariate generalized ordered probit (BGOP) model. Estimation results show that the BGOP model performs better than the conventional bivariate ordered probit (BOP) model in terms of goodness-of-fit indices and prediction accuracy and provides a better approach to identify the factors contributing to different severity levels. According to estimated parameters in latent propensity functions and elasticity effects, several key risk factors are identified—driver type (age > 65), vehicle type (motorcycle), violation type (alcohol use), intersection type (three-leg and multiple-leg), collision type (rear ended), and lighting conditions (night and night without illumination). Corresponding countermeasures for these risk factors are proposed.

1. Introduction

Two-vehicle accidents are the most common accidents on urban streets, especially at intersections. Without knowing the factors contributing to crash severity, effective countermeasures cannot be proposed. Numerous studies have been conducted to identify the contributing factors to crash severity. However, the severity of two parties (drivers or riders) involved in the same two-vehicle accident may significantly differ due to different driving behaviors, vehicle characteristics, traffic environments, and other risk factors. For instance, speeding drivers and/or impaired drivers may cause serious injury to other drivers when an accident occurs, but not to themselves. Thus, the crash severity levels of both parties should be simultaneously considered so as to correctly identify associated factors.

For crash severity modeling from an individual accident perspective, various methodological approaches have been applied, such as logistic regression (e.g., Sze and Wong, 2007; Al-Ghamdi, 2002), bivariate models (e.g., Yamamoto and Shankar, 2004; de Lapparent, 2008; Lee and Abdel-Aty, 2008), multinomial and nested logit structures to evaluate accident-injury severities (e.g., Shankar and Mannering, 1996; Chang and Mannering, 1999; Carson and Mannering, 2001; Lee and Mannering, 2002; Abdel-Aty, 2003; Ulfarsson and Mannering, 2004; Holdridge et al., 2005; Savolainen and Mannering, 2007) and mixed logit models (Milton et al., 2008; Gkritza and Mannering, 2008; Pai et al., 2009).

Notably, the discrete ordered probability model is one of the most common approaches used in recent accident severity studies (Shibata and Fukuda, 1994; O'Donnell and Connor, 1996; Duncan et al., 1998; Renski et al., 1999; Khattak, 2001; Kockelman and Kweon, 2002; Abdel-Aty, 2003; Zajac and Ivan, 2003; Abdel-Aty and Keller, 2005; Lee and Abdel-Aty, 2005; Williams, 2006; Eluru and Bhat, 2007; Pai and Saleh, 2007; Eluru et al., 2008; Gray et al., 2008; Pai and Saleh, 2008; Wang and Abdel-Aty, 2008; Yamamoto et al., 2008). This approach has considerable appeal because severity outcomes are discrete and ordered from low severity to high severity (e.g., property damage only, possible injury, evident injury, and disabling injury and fatality). The injury-severity categories are ordered in categories that are in some cases closely related (e.g., levels of no injury and possible injury); additionally, injury levels may be closely related (Savolainen et al., 2011). These crash severity studies have applied ordered response modeling to accommodate the natural order of crash severity levels. In the same vein, most of these studies applied the univariate ordered probit model to analyze two-vehicle accidents (e.g., Shibata and Fukuda, 1994; Shankar et al., 1996; Chang and Mannering, 1999; Carson and Mannering, 2001; Khattak, 2001; Kockelman and Kweon, 2002; Lee and Mannering, 2002; Abdel-Aty, 2003; Abdel-Aty and Keller, 2005; Pai and Saleh, 2008). However, due to a restriction on the number of dependent variables—only one variable is allowed—those studies simply determined the severity level of two-vehicle accidents by adopting the crash severity level of party injured most.

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As mentioned, the injury severity levels of the two parties involved in the same accident may differ markedly. Undoubtedly, considering the severity levels of two parties along with the corresponding factors is necessary to obtain insights from crash data and to propose effective safety strategies. Only considering the most-injured party may result in loss of valuable information. Moreover, the severity levels of the two parties along with contributing factors cannot be modeling separately, as these factors are typically closely related, resulting from the interrelationships among potential risk factors such as driver behavior, vehicle type, and collision type. The interaction between risk aversion behaviors of both drivers may also affect resulting severity levels. Thus, the severity levels of the two parties along with the corresponding contributing factors cannot be modeling separately, as observed or unobserved factors are usually correlated to some degree. Neglecting these potential correlations may lead to parameter over-estimation in crash severity modeling, thereby endogeneity problems (Winston et al., 2006; de Lapparent, 2008; Savolainen et al., 2011). Providing a relatively more efficient estimation by considering common unobserved factors for all involved parties, such as passengers (Hutchinson, 1986; Yamamoto and Shankar, 2004; Lee and Abdel-Aty, 2008) and other involved drivers (Rana, 2009) in one accident is essential. From the estimated correlation parameter (ρ) of the abovementioned studies indicates that the error terms of drivers or passengers' injury severities are positively correlated with each other. Hence, the modeling of the severity of only one driver may lead to the overestimation of coefficients. The similar endogeneity problem also exists for aggregate frequency modeling approaches (e.g., Maher, 1990; Tunaru, 2002; Bijleveld, 2005; N'Guessan et al., 2006; Geedipally and Lord, 2010).

To avoid the endogeneity and overestimation problems, this study uses the bivariate generalized ordered probit (BGOP) model to simultaneously model crash severity of both parties without losing the important information of both parties. The BGOP model, a flexible and comprehensive analytical approach, can analyze two ordered target variables simultaneously, i.e., crash severity levels of two parties, on contributing factors. Additionally, the threshold functions of the BGOP model can be calibrated during the model estimation process to condense model heterogeneity and to provide more insights for crash severity classification than the BOP model.

The remainder of this paper is organized as follows. Section 2 briefly introduces the bivariate ordered probit (BOP) and BGOP models. Section 3 presents the descriptive statistics of data used to develop the models. Section 4 compares and discusses estimation results by the BOP and BGOP models. Finally, Section 5 gives conclusions and recommendations for future research.

2. Model

The BGOP model is an extension of the BOP model. A common assumption of ordered discrete outcome models is that parameter estimates are constant across severity levels. However, the BGOP model allows the thresholds of the BOP model to vary according to both observed characteristics of two parties to minimize model heterogeneity and to provide additional insights for crash severity classification. To facilitate estimation in a traditional closed log-likelihood function form, the BGOP model is derived based on previous studies of the BOP model (Yamamoto and Shankar, 2004) and generalized ordered response logit (GORL) models (Eluru et al., 2008); this differs from the previous derivation by de Lapparent (2008).

2.1. Bivariate ordered probit

A BOP model is a hierarchical system of two equations that can be used to model a simultaneous relationship of two response variables, and addresses possible endogeneity problems, such that the severity levels of injuries to two or more participants involved in the same accident are typically correlated (Savolainen et al., 2011).

Let q_n (n = 1, 2) be an index representing two drivers involved in the same accident q (q = 1, 2, ..., Q). Suppose y_{qn} is the observed injury severity representing the latent (unobserved) injury severity propensity of drivers. Moreover, u_{q1} and u_{q2} are thresholds or cut-off values used to determine observed injury severity levels of both drivers relative to their corresponding injury propensities in crash q. Additionally, k (k = 1, 2, ..., K) and l (l = 1, 2, ..., L) are the indices representing ordinal categories of injury severity sustained by each driver. Thus, the latent injury severity propensities of the two drivers match their actual injury severity, as in the following equations:

$$y^*_{q,n=1} = k, \quad \text{if } \mu_{n=1,k-1} < y^*_{q,n=1} < \mu_{n=1,k} \tag{1}$$

$$y^*_{q,n=2} = l, \quad \text{if } \mu_{n=2,l-1} < y^*_{q,n=2} < \mu_{n=2,l} \tag{2}$$

Based on the above notations, the joint equation system from modeling injury severity of the two drivers involved in a two-vehicle accident is given by Eqs. (3) and (4), respectively:

$$y^*_{q,n=1} = \beta'_1 X_{q1} + \varepsilon_{q1} \tag{3}$$

$$y^*_{q,n=2} = \beta'_2 X_{q2} + \varepsilon_{q2} \tag{4}$$

where β'_n is a parameter vector, and ε_{qn} represents the random components that capture all unobserved factors associated with all involved parties. Under the assumption of a bivariate normal distribution of random components, the joint probability of the two drivers involved in the same accident can be expressed as follows (Yamamoto and Shankar, 2004):

$$\begin{aligned}
&Pr(\mu_{n=1,k-1} < y^*_{q,n=1} < \mu_{n=1,k}; \mu_{n=2,\ell-1} < y^*_{q,n=2} < \mu_{n=2,\ell}) \\
&= Pr(\mu_{n=1,k-1} < \beta'_1 X_{q1} + \varepsilon_{q,n=1} < \mu_{n=1,k}; \mu_{n=2,\ell-1} < \beta'_2 X_{q2} + \varepsilon_{q,n=2} < \mu_{n=2,\ell}) \\
&= Pr(\mu_{n=1,k-1} - \beta'_1 X_{q1} < \varepsilon_{q,n=1} < \mu_{n=1,k} - \beta'_1 X_{q1}; \mu_{n=2,\ell-1} - \beta'_2 X_{q2} < \varepsilon_{q,n=2} < \mu_{n=2,\ell} - \beta'_2 X_{q2}) \\
&= \Phi_2(\mu_{n=1,k} - \beta'_1 X_{q1}, \mu_{n=2,\ell} - \beta'_2 X_{q2}; \rho) - \Phi_2(\mu_{n=1,k-1} - \beta'_1 X_{q1}, \mu_{n=2,\ell} - \beta' X_{q2}; \rho) \\
&\quad - \Phi_2(\mu_{n=1,k} - \beta'_1 X_{q1}, \mu_{n=2,\ell-1} - \beta'_2 X_{q2}; \rho) + \Phi_2(\mu_{n=1,k-1} - \beta'_1 X_{q1}, \mu_{n=2,\ell-1} - \beta'_2 X_{q2}; \rho)
\end{aligned} \tag{5}$$

where $\Phi_2(\cdot)$ is the standard bivariate normal cumulative distribution function. ρ is an estimated correlation parameter between ε_{q1} and ε_{q2}.

2.2. Bivariate generalized ordered probit

As abovementioned, the BGOP model is based on the BOP (Yamamoto and Shankar, 2004) and the GORL models (Eluru et al., 2008). In Eqs. (1) and (2), thresholds u_{q1} and u_{q2} are now subscripted by index q to show that these cutoffs can vary across accidents involving different individuals to account for individual observed risk features.

$$y^*_{q,n=1} = k, \quad \text{if } \tilde{\mu}_{q,n=1,k-1} < y^*_{q,n=1} < \tilde{\mu}_{q,n=1,k} \tag{6}$$

$$y^*_{q,n=2} = l, \quad \text{if } \tilde{\mu}_{q,n=2,l-1} < y^*_{q,n=2} < \tilde{\mu}_{q,n=2,l} \tag{7}$$

In what follows, this study adopts a specific parametric function for each threshold to satisfy the following two ordering conditions: $(-\infty < \tilde{u}_{q,n=1,1} < \tilde{u}_{q,n=1,2} < \cdots < \tilde{u}_{q,n=1,K-1} < \infty)$ and $(-\infty < \tilde{u}_{q,n=2,1} < \tilde{u}_{q,n=2,2} < \cdots < \tilde{u}_{q,n=2,L-1} < \infty)$ for crash q. Thus, this study specifies them as:

$$\tilde{u}_{q,n=1,k} = \tilde{u}_{q,n=1,k-1} + \exp(\alpha_k + \gamma'_k \cdot Z_{q,k}) \tag{8}$$

$$\tilde{u}_{q,n=2,\ell} = \tilde{u}_{q,n=2,\ell-1} + \exp(\theta_\ell + \varsigma'_l \cdot Z_{q,l}) \tag{9}$$

where $Z_{q,k}$ and $Z_{q,l}$ are two exogenous variable vectors. Each is linked to its associated thresholds $\tilde{u}_{q,n=1,k}$ and $\tilde{u}_{q,n=2,\ell}$. Additionally, $\gamma'_{q,k}$ and $\varsigma'_{q,\ell}$ are coefficients associated with crash vectors.

Parameters $\alpha_{q,k}$ and $\theta_{q,l}$ are included in each party's specific injury severity level k (k = 1, 2, . . .,K−1) and l (l = 1, 2, . . .,L−1). In consideration of model identification, this study employs normalization, such that $\tilde{u}_{q,n=1,l=1}$ and $\tilde{u}_{q,n=2,l=1}$ equal zero for all q. Since the BGOP model is an extension of the BOP model, which restricts all non-constant parameters in the threshold function to zero. One can evaluate the validity of restrictions imposed by the restrictive BOP model using the likelihood ratio index (ρ^2), model information criteria: Akaike information criterion (AIC), Bayesian information criterion (BIC) and Chi-squared test.

2.3. Model estimation and validation

The log-likelihood of the BGOP model while considering both parties is given by:

$$\begin{aligned} LL = \sum Ln\{ & \Phi_2(\tilde{\mu}_{q,n=1,k} - \beta'_1 X_{q1}, \tilde{\mu}_{q,n=2,\ell} - \beta'_2 X_{q2}; \rho) \\ & - \Phi_2(\tilde{\mu}_{q,n=1,k-1} - \beta'_1 X_{q1}, \tilde{\mu}_{q,n=2,\ell} - \beta' X_{q2}; \rho) \\ & - \Phi_2(\tilde{\mu}_{q,n=1,k} - \beta'_1 X_{q1}, \tilde{\mu}_{q,n=2,\ell-1} - \beta'_2 X_{q2}; \rho) \\ & + \Phi_2(\tilde{\mu}_{q,n=1,k-1} - \beta'_1 X_{q1}, \tilde{\mu}_{q,n=2,\ell-1} - \beta'_2 X_{q2}; \rho)\} \end{aligned} \tag{10}$$

The corresponding parameters $\beta'_1, \beta'_2, \tilde{\mu}_{qk}(\gamma'_{q,k}, \alpha_{q,k}), \tilde{\mu}_{qk}$ $(\gamma'_{q,k}, \alpha_{q,k})$ and ρ are estimated simultaneously using the maximum likelihood method. The positive (negative) value of the coefficient estimate means that the probability of higher severity levels increase (decrease).

The study utilizes GAUSS software (Aptech Systems, 1995) and estimates the BOP and BGOP models using the maximum likelihood method. Estimation results for the BOP model can identify important explanatory variables and provide initial values for BGOP model estimations.

To ensure that the proposed model is applicable, this study applies mean absolute percentage error (MAPE) and root mean square error (RMSE) to compare different models. In this comparison, two datasets are separately used to estimate and validate the BOP and BGOP models. Additionally, MAPE is utilized as the decisive performance index because it is expressed as a generic percentage term with a straightforward and comprehensive meaning (Lewis, 1982).

3. Data

In total, 2661 two-vehicle accidents (5332 drivers) that occurred at signalized intersections during 2006–2007 in Taipei City are collected. Each of accident data contains a variety of crash information, including the severity levels of the two parties as well as potential factors, including driver type (for both parties), vehicle type (for both parties), violation type (for both parties), roadway type, collision type, intersection type, and lighting conditions factors (Table 1). The target variables are the injury severities of drivers/riders of two parties. The injury severities of passengers are not modeled in this study. Notably, four violation types are considered in this study including liability, alcohol use, speeding, use of safety equipments, where liability (a dummy variable) indicates whether or not the party should be responsible for the crash, gauged by the police officer at the accident site. Usually, the first party of an accident is termed as the one who has to take greater responsibility for the crash than the other, which is also classified by the police officer. Given the liability and classification, this study attempts to examine the effect of liability to crash severity and the differences in the injury severities of two parties. However, the proposed model is still applicable for those accident data without classification of involved parties.

For model estimation and validation, this dataset is randomly divided into two sets: one set for model estimation (2050 cases and 4100 drivers) and the other for model validation (611 cases and 1222 drivers). Most potential explanatory variables (Table 1)

Table 1
Sample distribution by explanatory variables.

	Variable	First party	Second party
Driver type			
Gender (Male)		79.9%	76.0%
Age	Age ≤ 20	8.1%	11.4%
	20 < Age ≤ 40	44.0%	54.2%
	40 < Age ≤ 65	44.6%	32.4%
	Age > 65	3.4%	2.0%
	Average (Std.)	39.9 (14.1)	35.8 (13.6)
Violation type			
Liability	Yes	99.9%	66.2%
	No	0.01%	33.8%
Alcoholic use	Yes	3.3%	0.6%
	No	96.7%	99.4%
Speeding	Yes	8.5%	4.5%
	No	91.5%	95.5%
Use of safety equipments (seatbelt/helmet)	Yes	98.8%	99.1%
	No	1.2%	0.9%
Vehicle type			
Bus		1.8%	1.1%
Truck		4.3%	1.6%
Car		41.1%	22.8%
Taxi		16.7%	10.2%
Motorcycle		36.1%	64.3%
Collision type			
Head-on			0.5%
Rear-end			2.1%
Sideswipe			52.7%
Angle			44.7%
Intersection type			
Four-leg intersection			76.9%
Three-leg intersection			15.6%
Multiple-leg intersection			7.2%
Roundabout			0.3%
Roadway type			
Major arterial			2.2%
Minor street			96.7%
Alley			1.1%
Time			
18:00–24:00			31.9%
24:00–06:00			11.4%
06:00–18:00			56.7%
Lighting conditions			
Daylight			62.8%
Night with illumination			20.1%
Night without illumination			17.1%
Total crashes			2661 (100.0%)

Table 2
Cross-tabulation by the severity levels of two parties.

First party	Second party				
	Property damage only	Possible injury	Evident injury	Disabling injury and fatality	Total
Property damage only	527(37.7%)	723(51.8%)	85(6.1%)	62(4.4%)	1397(100.0%)
Possible injury	249(45.0%)	253(45.8%)	28(5.1%)	23(4.2%)	553(100.0%)
Evident injury	24(49.0%)	17(34.7%)	5(10.2%)	3(6.1%)	49(100.0%)
Disabling injury and fatality	31(60.8%)	13(25.5%)	3(5.9%)	4(7.8%)	51(100.0%)
Total	831	1006	121	92	2050

are binary coded to represent certain types of drivers, vehicles, violations, collisions, intersections, roadways, times and lighting conditions. However, the age variable is examined separately in model estimation in its continuous and discretized form.

Three levels of severity in raw accident data are typically used—property damage only, injury, and fatality. However, under this classification system, sample distribution is generally uneven with too few fatal crash cases (<0.1%). Therefore, this study reclassifies cases using four severity levels—property damage only, possible injury, evident injury, and disabling injury and fatality. Table 2 gives a cross-tabulation of severity levels of the two parties.

As shown in Table 2, injury severity levels of the first and second parties are strongly correlated in severity levels of property damage only and possible injury. Notably, the total number of cases without injury to the first party (property damage only) is much larger than that of the second party (1397 vs. 831), while the total number of serious-injury cases for the first party is less than that of the second party (evident injury, 1006 vs. 553; disabling injury and fatality, 121 vs. 49), suggesting that the second party is more vulnerable to serious injury than the first party, and explaining the importance of simultaneously examining factors contributing to severity levels for different parties.

4. Results

To identify the factors contributing to crash severity for both parties and to demonstrate the importance of incorporating generalized thresholds, both the BOP and BGOP models are estimated and compared. Policy implications are also proposed based on estimation results.

4.1. Model estimation

Tables 3 and 4 present the estimation results for the BOP and BGOP models, respectively.

The goodness-of-fit indices of the BOP and BGOP models are pretty well (Tables 3 and 4). Correlation parameters (ρ) in both models are significant, demonstrating the need to model injury severity levels for both parties simultaneously. For comparisons, two separate univariate models for the first party and the second

Table 3
Estimation result of the BOP model.

Types	Variables		Latent propensity			
			First party		Second party	
			Estimate	*t*-Stat.	Estimate	*t*-Stat.
	Constant		−0.563	−1.30	−1.088	−10.27
	u_2		1.901	28.81	2.001	38.78
	u_3		2.250	28.51	2.458	40.09
Driver	Male	1st	−0.313	−3.57		
		2nd			−0.134	−1.82
	Age > 65	1st	0.275	1.61		
	ln(Age)	2nd	−0.218	−1.92		
Violation	Alcoholic use	1st	0.966	7.52	0.679	6.20
		2nd			0.586	2.72
	Liability	2nd			−0.226	−3.73
Vehicle	Bus	1st			0.504	3.20
		2nd	1.142	7.11		
	Car	1st			0.255	4.12
	Motorcycle	1st	2.569	29.21		
		2nd	−0.494	−5.79	2.115	31.86
Intersection	Three-leg				0.173	2.27
	Multiple-leg				0.281	2.87
Roadway	Major arterial				−1.963	−3.35
Collision	Rear-end		0.574	2.58		
Lighting conditions	Timing (24:00–06:00)		0.236	2.33		
	Lighting (night without illumination)				0.264	3.19
Number of observations (number of parameters)		2050 (27)				
LL(*C*)		−3261.17				
LL(β)		−2375.98				
ρ (*t*-Stat.)		0.29 (8.87)				
adj-ρ^2		0.27				
AIC		4805.95				
BIC		4957.85				

Note: Only the variables with significantly tested parameters at α = 0.10 are reported.

Table 4
Estimation results of the BGOP model.

Types	Variables	Party	Threshold function								Latent propensity			
			$\tilde{u}_{n=1,k=2}$		$\tilde{u}_{n=1,k=3}$		$\tilde{u}_{n=2,l=2}$		$\widetilde{u}_{n=2,l=3}$		1st		2nd	
			Estimate	*t*-Stat.	Estimate	*t*-Stat.	Estimate	*t*-Stat.	Estimate	*t*-Stat.	Estimate	*t*-Stat.	Estimate	*t*-Stat.
	Constant		−0.568	−3.01	0.874	26.86	−0.417	−2.51	0.945	25.70	−0.636	−1.42	−1.217	−11.14
Driver	Male	1st									−0.326	−3.62		
		2nd							−0.068	−1.98			−0.165	−2.19
	Age > 65	1st									0.287	1.65		
	ln(Age)	2nd									−0.217	−1.84		
Violation	Alcoholic use	1st					0.132	2.62			0.932	6.19	0.699	5.84
		2nd											0.587	2.64
	Liability	2nd											−0.229	−3.62
Vehicle	Bus	1st											0.530	3.40
		2nd									1.031	5.66		
	Car	1st											0.260	4.10
	Motorcycle	1st	1.320	6.98							2.737	28.05		
		2nd					1.116	6.77			−0.498	−5.81	2.285	31.41
Intersection	Three-leg												0.172	2.18
	Multiple-leg												0.287	2.74
Roadway	Major arterial												−1.975	−3.35
Collision	Rear-end											0.612	2.77	
Lighting conditions	Time (24:00~06:00)											0.256	2.29	
	Lighting (night without illumination)						0.296	4.95	0.218	3.86			0.640	4.77

Number of observations (number of parameters)	2050 (33)
$LL(C)$	−3261.17
$LL(\beta)$	−2265.29
ρ (*t*-Stat.)	0.247 (6.26)
$adj\text{-}\rho^2$	0.31
AIC	4596.57
BIC	4782.22
Likelihood ratio test with BOP model	$-2[LL(\beta_{BGOP}) - LL(\beta_{BOP})] = 221.38$

Note: Only the variables with significantly tested parameters at $\alpha = 0.10$ are reported.

party have also been estimated, respectively. The results show that the estimated standard deviations of bivariate models (i.e., BOP and BGOP) are lower than those of the univariate models and the estimated coefficients of the bivariate and the univariate models are different. Moreover, the bivariate models perform better than the univariate models in terms of MSE and MAPE, suggesting the high correlation between the severity levels of two parties and the needs of simultaneously modeling. However, for brevity, the estimation results of univariate models are omitted. Since the goodness-of-fit indices (such as *adj*-ρ^2, AIC, and BIC) of the BGOP model perform better than that of the BOP model, demonstrating the importance of using generalized thresholds. Moreover, the likelihood ratio test result shows that the chi-squared value is 221.38, which exceeds the 5% significance level ($\chi^2_{(5,0.95)} = 11.07$), suggesting the BGOP model performs significantly better than the BOP model. Accordingly, implications, the discussion and conclusions are interpreted based on BGOP model results.

To classify latent propensity into four severity levels, three thresholds are required for each party—$\tilde{u}_{n(n=1,2),l(l=1,2,3)}$. By setting two thresholds of $\tilde{u}_{n(n=1,2),1}$ as the reference, four threshold functions of $\tilde{u}_{n(n=1,2),l(l=2,3)}$ are estimated. Several explanatory factors, including vehicle type (motorcycle), driver type (male), and violation type (alcohol use), have significant effects on the shift of thresholds, and changing severity level classification results (Table 4). Notably, the value of correlation coefficient ρ decreases slightly when compared with that of the BOP model (Table 3), because these explanatory variables are incorporated into threshold functions.

The estimated parameters with a positive sign in threshold functions indicate that when the associated type of a condition is present, the threshold shifts to the right and then increases the interval of the defined severity level and increases probability, resulting in a lower severity level compared with that of the other party (Eluru et al., 2008). A negative estimated parameter has the opposite effects. The estimated parameters in threshold functions are interpreted by the elasticity effect in the following section.

Based on the estimated parameters in latent propensity functions, a parameter with positive sign indicates that when the type of an associated explanatory variable is present in an accident, the severity level for this party increases. These estimated parameters for the first and second parties are markedly different and some factors related to one party have significant effects on the severity level of the other party, explaining the interaction between two parties involved in an accident (Table 4).

For the estimated parameters for driver type (gender and age), when the driver is male, the latent propensity to injury himself can be curtailed by 0.326 and 0.165 for the first and the second parties, respectively. This estimation result is in agreement with those in previous studies (Kockelman and Kweon, 2002; Abdel-Aty, 2003; Yamamoto and Shankar, 2004; Holdridge et al., 2005; Eluru and Bhat, 2007; de Lapparent, 2008; Eluru et al., 2008; Gkritza and Mannering, 2008; Yamamoto et al., 2008) and the gendered stereotype that posits that male drivers are in better physical condition for resisting potential dangers and are faster in responding to risk, resulting in male drivers being less injured in accidents than female drivers. However, the magnitude of estimated parameter of corresponding variable for the second party is much lower than that of the first party, implying that although the second party is male, he is still more vulnerable to a severe injury than the male first party. Additionally, the parameter of the male second party in the threshold function $\tilde{u}_{n=2,l=3}$ is negative (−0.068), suggesting that a male second party has a higher risk for severe injury than a female second party; this finding runs counter to our expectation. This may be because the driver overacts and this increases injury severity.

The age variable (continuous form in the logarithmic term and discretized form for drivers aged > 65) only affects the latent propensity of the first party, suggesting that when the first party is aged > 65, his/her injury severity in an accident increases. This finding is also consistent with that in other studies (Kockelman and Kweon, 2002; Zajac and Ivan, 2003; Abdel-Aty, 2003; Ulfarsson and Mannering, 2004; Yamamoto and Shankar, 2004; Eluru and Bhat, 2007; de Lapparent, 2008; Eluru et al., 2008; Yamamoto et al., 2008). Conversely, the *ln*(Age) parameter of the second party has a negative sign, implying that older drivers may have more experience than young drivers and respond to potential crashes better, thereby reducing the possible injury to the other party.

The estimated parameters for violation type indicate that when the first driver is drunk, injury severity of the first and the second parties increases by 0.932 and 0.699, respectively. This analytical result is also in agreement with that in many previous studies (O'Donnell and Connor, 1996; Kockelman and Kweon, 2002; Zajac and Ivan, 2003; Abdel-Aty, 2003; Ulfarsson and Mannering, 2004; Yamamoto and Shankar, 2004; Holdridge et al., 2005; Eluru and Bhat, 2007; Eluru et al., 2008; Wang and Abdel-Aty, 2008; Yamamoto et al., 2008), because alcohol adversely affects driver responses to risk by prolonging reaction time. However, if a drunk driver is the second party, only his/her injury severity increases, and has no significant effect on that of the first party. Surprisingly, negative effects of other behaviors that break traffic laws on injury severity of the second party are significant, implying that the second party breaking traffic laws may reduce his/her injury severity. This unexpected finding resembles that obtained by Abdel-Aty (2003).

The effects of vehicle type on injury severity of the first or second parties are also significant. Unlike previous studies of univariate models (Kockelman and Kweon, 2002; Zajac and Ivan, 2003; Yamamoto and Shankar, 2004; Holdridge et al., 2005; Eluru and Bhat, 2007; Eluru et al., 2008; Gkritza and Mannering, 2008; Yamamoto et al., 2008) which cannot further identify the effects of vehicle type to both parties, the BGOP model can identify the interrelation between the vehicle types used by two parties. According to the estimation results, several findings are identified. First, when the vehicle of the first party is a bus, the injury severity of the second party increases, since damage typically increases as vehicle size of another party increases and vice versa. Additionally, if the vehicle of the first party is a car, then the injury severity of the second party also increases, but not vice versa. Notably, motorcycles have the largest effect on injury severity of the riders themselves, since the estimated parameters have the largest value among all explanatory variables. Those riding motorcycles typically suffer serious injury regardless of whether they are the first or second party in an accident. This finding underscores the dangers associated with this transportation mode, as motorcycles lack external protection, and coincides with the fact that motorcyclist fatalities in Taiwan account for 56.69% of all traffic deaths (Wen et al., 2012). However, when the vehicle type of the second party is a motorcycle, injury severity of the first party declines slightly, due to reduced impact from a motorcycle. This finding is similar to the previous works of Evans (2004) on two-vehicle crashes from an aggregate perspective.

In this analysis, intersection types affect injury severity of the second party, not that of the first party. When an intersection is three-legged or multiple-legged, the injury severity of the second party increases. Because multiple-legged intersections usually have more conflicting traffic flows with an insufficient sight distance. Although at three-legged intersections, one vehicle always has to make a turn (at least on the third leg - the one without on-coming traffic), which reduces the speed of the first or second vehicle, this may lead to a reduction of crash severity. However, in Taiwan, motorcycles are prevailing at urban streets. To improve traffic safety at most of four-legged intersections, motorcycles are not allowed to turn left during the green phase. Instead, they

Table 5
Elasticity effects for the first party.

Types	Variables		BOP				BGOP			
			Property damage only	Possible injury	Evident injury	Disabling injury and fatality	Property damage only	Possible injury	Evident injury	Disabling injury and fatality
Driver	Male	1st	0.17	−0.56	−1.41	−1.76	−1.36	−0.56	−0.99	−2.06
		2nd	0.00	0.00	0.00	0.00	−2.63	0.00	0.00	0.00
	Age > 65	1st	−0.11	0.36	0.86	1.06	3.50	0.49	0.88	1.83
	ln(Age)	2nd	0.13	−0.32	−0.65	−0.76	0.12	−0.30	−0.48	−0.77
Violation	Alcoholic use	1st	−0.47	3.04	17.18	27.40	1.27	2.50	5.93	29.91
		2nd	0.00	0.00	0.00	0.00	3.95	0.00	0.00	0.00
	Liability	2nd	0.00	0.00	0.00	0.00	−0.77	0.00	0.00	0.00
Vehicle	Bus	1st	0.00	0.00	0.00	0.00	3.93	0.00	0.00	0.00
		2nd	−0.52	4.07	29.21	50.32	1.75	2.92	7.35	43.42
	Car	1st	0.00	0.00	0.00	0.00	3.38	0.00	0.00	0.00
	Motorcycle	1st	−2.59	14.60	655.46	2348.98	−4.53	23.14	3.11	5231.09
		2nd	0.29	−0.94	−2.73	−3.57	2.16	−0.88	−1.61	−4.04
Intersection	Three-leg		0.00	−0.01	−0.03	−0.03	2.75	−0.01	−0.01	−0.02
	Multiple-leg		0.01	−0.02	−0.05	−0.06	3.52	−0.01	−0.02	−0.04
	Major arterial		−0.01	0.06	0.13	0.16	3.58	0.03	0.05	0.09
Collision	Rear-end		−0.28	1.33	4.68	6.44	3.40	1.31	2.66	8.53
Lighting conditions	Time (24:00–06:00)		−0.15	0.51	1.29	1.61	1.00	0.50	0.89	1.84
	Lighting (night without illumination)		0.01	−0.02	−0.04	−0.05	2.39	0.00	0.00	0.00

are required to stop in the motorcycle left-turn waiting area and wait for the green phase of this approach to reduce potential conflicts between through traffic and left-turning motorcycles. Such a design does not present at many three-legged intersections due to space limitation. Consequently, motorcycles can directly make a left-turning without the need to stop and wait for the green phase of the approach, which become a potential risk to higher crash severity. Therefore, to re-design three-legged intersections for accommodating the waiting areas for left-turning motorcycles is strongly recommended. Furthermore, when one roadway at an intersection is a major arterial, injury severity for the second party is decreased. This is because major arterials have a high geometrical design standard. Drivers on such roads have better sight distance and longer time to respond than drivers on other minor roads.

Rear-end collision is the only collision type that affects injury severity. Similar findings were obtained by Chang and Mannering (1999), Khattak (2001), Kockelman and Kweon (2002), Eluru and Bhat (2007), de Lapparent (2008), and Gkritza and Mannering (2008). Injury severity of the first party increases in a rear-end collisions (Table 4). The reason is likely that in rear-end accidents, drivers of following vehicles are usually the first party and fail to maintain a safe distance from the front vehicles. For a rear-end collision, drivers of front vehicles and drivers of following vehicles tend to have more-severe injuries than the other drivers.

Last, lighting condition factors also affect injury severity of the first or second parties. The injury severity level of the first party is higher when a crash occurs at night (24:00–6:00), because vehicles during that period are often moving at relatively high speeds and drivers can become distracted (Eluru and Bhat, 2007; Eluru et al., 2008). Additionally, injury severity of the second party is higher when an accident occurs under a poor lighting condition (Abdel-Aty, 2003; Zajac and Ivan, 2003; Holdridge et al., 2005; Eluru and Bhat, 2007). Under this condition, drivers have poor visualization and lack sufficient reaction time to avoid serious accidents.

4.2. Elasticity effect

The estimated parameters of explanatory variables (Table 4) do not directly show the magnitude of effects on the probability of each injury severity level. Some explanatory variables have different effects on injury severity level of one party in threshold functions and latent propensity functions. To elucidate the impact of contributing factors, this study computes the aggregate level elasticity effects of variables (Tables 5 and 6) for the first and second parties using the BOP and BGOP models. Thus, one can calculate aggregate level elasticity of any dummy variable by changing the variable value to 1 for the subsample of observations for which the variable has a value of 0, and to 0 for a subsample of observations for which the variable has a value of 1. Following computation, this study sums shifts in expected aggregate shares in the two subsamples after reversing the signs of shifts in the second subsample, and computes an effective percentage change in expected aggregate shares in the entire sample after changing the dummy variable value from 0 to 1 (Eluru and Bhat, 2007; Eluru et al., 2008). The elasticity effect can be interpreted as percentage change in probability of an injury severity category due to changing the variable to 1 from 0, except for the continuous variable, *ln*(Age).

Without considering flexible thresholds, all elasticity effects of the BOP model increase monotonically or decrease across severity levels (Tables 5 and 6). However, for the BGOP model, elasticity effects may exhibit a bi-modal pattern (i.e., have a larger effect on the two extreme severity levels), which can better describe the effect of a factor. Most variables considered in this study are categorical. The heterogeneity of variables may be rather large. The estimation results of the BOP and BGOP models differ, not only for signs of elasticity, but also for magnitude.

To avoid redundant implications and discussions based on estimation results for latent propensity, the following only focuses on factors that have relatively large elasticity effects. Vehicle type of motorcycle has the largest positive elasticity effect on disabling injury and fatality (5231.09), underlining the danger to the first party when they are riding a motorcycle. Notably, the elasticity effect of this variable does not increase monotonically with injury severity, but has a large effect on severity levels of possible injury, and disabling injury and fatality. Additionally, this variable only reduces the probability of property damage, again emphasizing the risk of riding a motorcycle. This estimations result is in agreement with that obtained by Wen et al. (2012). The second-largest positive elasticity effect is colliding with a bus (43.42), and this variable (vehicle type = bus driven by the second party) exhibits a monotonic pattern from the least-serious severity level to the most-serious severity level. The third-largest positive elasticity effect is

Table 6
Elasticity effects for the second party.

Types	Variables		BOP				BGOP			
			Property damage only	Possible injury	Evident injury	Disabling injury and fatality	Property damage only	Possible injury	Evident injury	Disabling injury and fatality
Driver	Gender (Male)	1st	0.00	0.00	0.00	0.00	−4.02	0.00	0.00	0.00
		2nd	0.14	−0.09	−0.30	−0.40	−1.95	−0.10	−0.53	0.02
	Age > 65	1st	0.00	0.00	0.00	0.00	6.06	0.00	0.00	0.00
Violation	Alcoholic use	1st	−0.56	0.56	3.27	5.32	4.83	0.69	0.65	6.25
		2nd	−0.48	0.46	2.48	3.92	6.06	0.41	1.46	4.24
	Liability	2nd	0.24	−0.14	−0.51	−0.70	−0.59	−0.14	−0.41	−0.74
Vehicle	Bus	1st	−0.44	0.38	1.88	2.89	6.01	0.37	1.27	3.41
	Car	1st	−0.28	0.16	0.57	0.78	−0.27	0.15	0.46	0.83
		2nd	0.00	0.00	0.00	0.00	6.68	0.00	0.00	0.00
	Motorcycle	1st	0.00	0.00	0.00	0.00	3.32	0.00	0.00	0.00
		2nd	−3.62	5.81	211.52	624.11	−7.98	10.86	1.35	1514.77
Intersection	Three-leg		−0.18	0.11	0.40	0.55	3.32	0.10	0.31	0.57
	Multiple-leg		−0.28	0.19	0.76	1.08	4.68	0.18	0.57	1.16
Roadway	Major arterial		3.20	−3.68	−100.97	−262.45	7.50	−4.00	−14.19	−361.55
Collision	Rear-end		0.00	0.00	0.00	0.00	6.45	0.00	0.00	0.00
Lighting conditions	Time (24:00–06:00)		0.00	0.00	0.00	0.00	4.85	0.00	0.00	0.00
	Lighting (night without illumination)		−0.27	0.18	0.69	0.96	0.10	0.81	0.38	0.13

drunk driving. Drunk drivers tend to get injured with monotonically increasing probability. Conversely, the largest negative elasticity effect is to collide with a motorcycle (−4.04).

For the second party (Table 6), similar to the first party, the largest elasticity effect is riding a motorcycle (1514.77), underscoring the danger in using this transportation mode regardless of whether the motorcycle is driven by the first or second party. In Taipei City, which has a convenient public transportation system, motorcycle ridership accounts for approximately 30% of total trips; however, in other cities in Taiwan, ridership is as high as 70%, similar to that in many Asian cities. Thus, motorcycles should be equipped with enhanced riders protection and collision warning systems. Of course, the most effective countermeasure is to reduce motorcycle usage.

The second- and third-largest positive elasticity effects are drunk drivers as the first or second party. According to the magnitudes of these elasticity effects of the two parties, being hit by a drunk driver is more dangerous than a drunk driver hitting another vehicle. Notably, for major arterials, the injury severity level of the second party is markedly reduced (−361.55), due to high design standards and improved sight distance.

4.3. Safety implications

Several key risk factors of two-vehicle accident severity at signalized intersections are identified. One of the most dangerous factors is motorcycle use. Potentially effective strategies to improve the safety of motorcycles or reduce motorcycle usage are allocating lanes to motorcycles on urban streets so as to mitigate interactions among different vehicle types, to strictly enforce mandatory use of helmets for motorcycle riders, discouraging motorcycle usage by proving a convenient and affordable public transportation system,

Table 7
Validation result of the BOP and BGOP models.

Dataset	Actual		Predictions			
			BOP		BGOP	
Estimation sample						
Shares (%)	1st	2nd	1st	2nd	1st	2nd
Property damage only	68.15	40.54	69.33	41.60	68.88	41.21
Possible injury	26.98	49.07	24.99	47.30	25.76	47.70
Evident injury	2.39	5.90	2.29	5.62	2.30	5.99
Disabling injury and fatality	2.49	4.49	3.39	5.49	3.06	5.11
MAPE (RMSE)						
Property damage only			0.50(0.43)	0.80(0.55)	0.52(0.45)	0.84(0.57)
Possible injury			1.00(0.54)	0.50(0.38)	1.11(0.60)	0.57(0.43)
Evident injury			1.22(0.22)	0.68(0.19)	0.81(0.17)	0.50(0.15)
Disabling injury and fatality			1.64(0.46)	0.97(0.27)	1.51(0.42)	0.88(0.24)
Overall MAPE			4.36	2.95	3.95	2.79
Validation sample						
Shares (%)	1st	2nd	1st	2nd	1st	2nd
Property damage only	71.36	36.82	71.25	42.56	70.88	42.22
Possible injury	24.71	53.52	23.83	46.67	24.35	47.02
Evident injury	1.80	4.91	2.06	5.47	2.18	5.88
Disabling injury and fatality	2.13	4.75	2.86	5.30	2.59	4.88
MAPE (RMSE)						
Property damage only (%)			0.45(0.41)	0.86(0.58)	0.47(0.43)	0.91(0.61)
Possible injury (%)			1.06(0.55)	0.44(0.38)	1.17(0.61)	0.51(0.42)
Evident injury (%)			1.45(0.24)	0.84(0.21)	0.97(0.18)	0.58(0.17)
Disabling injury and fatality (%)			1.66(0.41)	0.90(0.25)	1.54(0.38)	0.80(0.22)
Overall MAPE			4.62	3.04	4.14	2.80

and increasing motorcycle parking fees. For further discussion of motorcycle safety measures, see Pai (2011).

The second key factor is alcohol use. Obviously, cracking down on drunk driving is an effective countermeasure, especially during the night (24:00–6:00). Additionally, drivers aged > 65 are at high risk for severe accidents. Programs that educate aged people not to ride motorcycles and drive cars instead should be considered. Further, a periodic review of the physical condition of aged drivers (riders) is also important. Moreover, the geometric design of intersections and roadways is also a factor key to injury severity. Well-designed roadway systems (e.g., major arterials) reduce injury severity, while poorly designed intersections (e.g., multi-leg intersections) increase injury severity. In summary, crash severity can be reduced through education, engineering, enforcement, and encouragement, the so-called 4Es.

4.4. Model validation

Another dataset consisting of 611 cases is used to validate the BOP and BGOP models in terms of two commonly used performance indices—MAPE and RMSE—for each crash level and overall performance (Table 7). For comparison, both indices are computed for the estimation dataset. Both models perform extreme well with an MAPE < 5% and hit ratio >80% (Table 5). Additionally, the BGOP model outperforms the BOP model with a lower overall MAPE. Notably, the BGOP model has better prediction accuracy than the BOP model for two most-severe injury levels of evident injury and disabling injury and fatality.

5. Conclusions

The injury severity of two parties involved in two-vehicle accidents differs markedly. Thus, factors contributing to injury severity warrant analysis. This study applies the BGOP model, which relaxes the assumption of fixed threshold values for injury severity levels. Estimation results show that the BGOP model performs better than the conventional BOP model in terms of goodness-of-fit and prediction accuracy, and provides a superior understanding of factors contributing to different injury severity levels.

According to estimated parameters in latent propensity functions and threshold functions and elasticity effects, several key risk factors are identified—driver type (aged > 65), vehicle type (motorcycle), violation type (alcohol use), intersection type (three-leg and multiple-leg intersections), collision type (rear-end), and lighting conditions (night and night without illumination). Corresponding countermeasures for such risk factors are also proposed. Although most of our findings are consistent with previous studies, two issues should be raised. First, in this study, the liability and classification of the involved parties mainly rely on police subjective judgment. To be more scientific, an artificial neural network model for accident appraisal proposed by Chiou (2006) can be used to systematically determine the liabilities of two parties at the data processing stage. Second, underreporting effect is indeed a problem which may lead to biased estimation results. Ordered probability models are particularly susceptible to underreporting of crash-injury data. To rectify this problem, to obtain the underreporting rates in the population for using the weighted maximum likelihood function to estimate the proposed model or to develop a new model, such as bivariate sequential logit and probit models, is worthy of a further study.

Additionally, several research directions for future studies are identified. First, based on our proposed BGOP model, more generalized multivariate models can be developed for modeling multi-vehicle crashes or analyzing injury-severities of drivers and passengers in an accident. Second, additional explanatory variables (e.g., road width, number of lanes, and signal control) with regard to two-vehicle accidents can be collected and analyzed to propose relatively more effective improvement strategies. Third, the injury severity levels can be re-designed according to injured body parts of involved parties; different body parts can be injured by different contributing factors such as collision type (e.g., head-on, rear-end, and sideswipe collisions), crash sites (e.g., intersection and segment), and driver traits (helmet use and traffic violations). Last but not least, the random coefficient specification and latent class approach can be utilized to derive the mixed BGOP model and latent class BGOP model to reveal the unobserved heterogeneity of latent injury propensity of the two parties.

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References

Abdel-Aty, M., 2003. Analysis of driver injury severity levels at multiple locations using ordered probit models. Journal of Safety Research 34 (5), 597–603.
Abdel-Aty, M., Keller, J., 2005. Exploring the overall and specific crash severity levels at signalized intersections. Accident Analysis and Prevention 37 (3), 417–425.
Al-Ghamdi, A., 2002. Using logistic regression to estimate the influence of accident factors on accident severity. Accident Analysis and Prevention 34 (6), 729–741.
Aptech Systems, 1995. Gauss Applications: Maximum Likelihood. Aptech Systems Inc., Maple Valley, WA.
Bijleveld, F.D., 2005. The covariance between the number of accidents and the number of victims in multivariate analysis of accident related outcomes. Accident Analysis and Prevention 37 (4), 591–600.
Carson, J., Mannering, F., 2001. The effect of ice warning signs on accident frequencies and severities. Accident Analysis and Prevention 33 (1), 99–109.
Chang, L.Y., Mannering, F., 1999. Analysis of injury severity and vehicle occupancy in truck- and non-truck-involved accident. Accident Analysis and Prevention 31 (5), 579–592.
Chiou, Y.C., 2006. An artificial neural network-based expert system for the accident appraisal of two-car crash accidents. Accidents Analysis and Prevention 38 (4), 777–785.
Duncan, C., Khattak, A., Council, F., 1998. Applying the ordered probit model to injury severity in truck-passenger car rear-end collisions. Transportation Research Record 1635, 63–71.
de Lapparent, M., 2008. Willingness to use safety belt and levels of injury in car accidents. Accident Analysis and Prevention 40 (3), 1023–1032.
Eluru, N., Bhat, C., Hensher, D., 2008. A mixed generalized ordered response model for examining pedestrian and bicyclist injury severity level in traffic crashes. Accident Analysis and Prevention 40 (3), 1033–1054.
Eluru, N., Bhat, C., 2007. A joint econometric analysis of seat belt use and crash – related injury severity. Accident Analysis and Prevention 39 (5), 1037–1049.
Evans, L., 2004. Traffic Safety, 2nd ed. Science Serving Society, Bloomfield Hills, USA.
Geedipally, S.R., Lord, D., 2010. Investigating the effect of modeling single-vehicle and multi-vehicle crashes separately on confidence intervals of Poisson-gamma models. Accident Analysis and Prevention 42 (4), 1273–1282.
Gray, R., Quddus, M., Evans, A., 2008. Injury severity analysis of accidents involving young male drivers in Great Britain. Journal of Safety Research 39 (5), 483–495.
Gkritza, K., Mannering, F., 2008. Mixed logit analysis of safety-belt use in single- and multi-occupant vehicles. Accident Analysis and Prevention 40 (2), 443–451.
Hutchinson, T.P., 1986. Statistical modelling of injury severity, with special reference to driver and front seat passenger in single-vehicle crashes. Accident Analysis and Prevention 18, 157–167.
Holdridge, J., Shankar, V., Ulfarsson, G., 2005. The crash severity impacts of fixed roadside objects. Journal of Safety Research 36 (2), 139–147.
Khattak, A., 2001. Injury severity in multivehicle rear-end crashes. Transportation Research Record 1746, 59–68.
Kockelman, K., Kweon, Y.-J., 2002. Driver injury severity: an application of ordered probit models. Accident Analysis and Prevention 34 (3), 313–321.
Lee, C., Abdel-Aty, M., 2005. Comprehensive analysis of vehicle–pedestrian crashes at intersections in Florida. Accident Analysis and Prevention 37 (4), 775–786.
Lee, C., Abdel-Aty, M., 2008. Presence of passengers: does it increase or reduce driver's crash potential? Accident Analysis and Prevention 40 (5), 1703–1712.
Lee, J., Mannering, F., 2002. Impact of roadside features on the frequency and severity of run-off-roadway accidents: an empirical analysis. Accident Analysis and Prevention 34 (2), 149–161.

Lewis, C.D., 1982. International and Business Forecasting Methods. Butterworths, London.

Maher, M.J., 1990. A bivariate negative binomial model to explain traffic accident migration. Accident Analysis and Prevention 22 (5), 487–498.

Milton, J., Shankar, V., Mannering, F., 2008. Highway accident severities and the mixed logit model: an exploratory empirical analysis. Accident Analysis and Prevention 40 (1), 260–266.

N'Guessan, A., Essai, A., N'Zi, M., 2006. An estimation method of the average effect and the different accident risks when modeling a road safety measure: a simulation study. Computational Statistics and Data Analysis 51, 1260–1277.

O'Donnell, C.J., Connor, D.H., 1996. Predicting the severity of motor vehicle accident injuries using models of ordered multiple choice. Accident Analysis and Prevention 28 (6), 739–753.

Pai, C., Saleh, W., 2007. An analysis of motorcyclist injury severity under various traffic control measures at three-legged junctions in the UK. Safety Science 45, 832–847.

Pai, C., Saleh, W., 2008. Exploring motorcyclist injury severity in approach-turn collisions at T-junctions: focusing on the effects of driver's failure to yield and junction control measures. Accident Analysis and Prevention 40 (2), 479–486.

Pai, C., Hwang, K., Saleh, W., 2009. A mixed-logit analysis of motorists' right-of-way violation in motorcycle accidents at priority T-junctions. Accident Analysis and Prevention 41, 565–573.

Pai, C., 2011. Motorcycle right-of-way accidents – a literature review. Accident Analysis and Prevention 43 (3), 971–982.

Rana, T.,2009. A joint analysis of drivers' injury severity in road accidents involving two vehicles. In: Proceedings of the ITE Annual Meeting and Exhibit 2009. University of South Florida, USA.

Renski, H., Khattak, A., Council, F., 1999. Effect of speed limit increases on crash injury severity: analysis of single-vehicle crashes on North Carolina interstate highways. Transportation Research Record 1665, 100–108.

Savolainen, P., Mannering, F., Lord, D., Quddus, M., 2011. The statistical analysis of highway crash-injury severities: a review and assessment of methodological alternatives. Accident Analysis and Prevention 43 (5), 1666–1676.

Savolainen, P., Mannering, F., 2007. Probabilistic models of motorcyclists' injury severities in single- and multi-vehicle crashes. Accident Analysis and Prevention 39 (5), 955–963.

Shankar, V., Mannering, F., 1996. An exploratory multinomial logit analysis of single-vehicle motorcycle accident severity. Journal of Safety Research 27 (3), 183–194.

Shankar, V., Mannering, F., Barfield, W., 1996. Statistical analysis of accident severity on rural freeways. Accident Analysis and Prevention 28 (3), 391–401.

Shibata, A., Fukuda, K., 1994. Risk factors of fatality in motor vehicle traffic accidents. Accident Analysis and Prevention 26 (3), 391–397.

Sze, N., Wong, S.C., 2007. Diagnostic analysis of the logistic model for pedestrian injury severity in traffic crashes. Accident Analysis and Prevention 39 (6), 1267–1278.

Tunaru, R., 2002. Hierarchical Bayesian models for multiple count data. Austrian Journal of Statistics 31, 221–229.

Ulfarsson, G., Mannering, F., 2004. Differences in male and female injury severities in sport-utility vehicle, minivan, pickup and passenger car accidents. Accident Analysis and Prevention 36 (2), 135–147.

Wang, X., Abdel-Aty, M., 2008. Analysis of left-turn crash injury severity by conflicting pattern using partial proportional odds models. Accident Analysis and Prevention 40 (5), 1674–1682.

Wen, C.H., Chiou, Y.C., Huang, W.L., 2012. A dynamic analysis of motorcycle ownership and usage: a panel data modeling approach. Accident Analysis and Prevention 49 (1), 193–202.

Williams, R., 2006. Generalized ordered logit/partial proportional odds models for ordinal dependent variables. The Statistic Journal 6, 58–82.

Winston, C., Maheshri, V., Mannering, F., 2006. An exploration of the offset hypothesis using disaggregate data: the case of airbags and antilock brakes. Journal of Risk and Uncertainty 32 (2), 83–99.

Yamamoto, T., Hashiji, J., Shankar, V., 2008. Underreporting in traffic accident data, bias in parameters and the structure of injury severity models. Accident Analysis and Prevention 40 (4), 1320–1329.

Yamamoto, T., Shankar, V., 2004. Bivariate ordered-response probit model of driver's and passenger's injury severities in collision with fixed objects. Accident Analysis and Prevention 36 (5), 869–876.

Zajac, S., Ivan, J., 2003. Factors influencing injury severity of motor vehicle–crossing pedestrian crashes in rural Connecticut. Accident Analysis and Prevention 35 (3), 369–379.