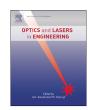
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# Heterodyne moiré interferometry for measuring corneal surface profile



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## ABSTRACT

This study proposes an accurate method for reconstructing the corneal surface profile. By applying a constant velocity to the projection grating along the grating plane, a series of sampling points of the sinusoidal wave, which behaves in the manner of heterodyne interferometric signals, can be recorded using a CMOS camera. The phase distribution of the moiré fringes can then be obtained using the IEEE 1241 least-square sine fitting algorithm and two-dimensional (2D) phase unwrapping. Finally, the corneal surface profile can be reconstructed by substituting the phase distribution into a specially derived equation. To validate the proposed method, the corneal surface of a pig eyeball was measured. The measurement resolution was approximately 3.5 µm. Because of the introduction of the Talbot effect, the projection moiré method, and heterodyne interferometry, this approach provides the advantages of a simple optical setup, ease of operation, high stability, and high resolution.

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## 1. Introduction

The refractive power of the cornea is more than two-thirds of the total refractive power of the human eye; therefore, a slight variation of the corneal surface can drastically affect normal human vision. Accurate reconstruction of the corneal surface profile can thus provide essential information for vision diagnosis. Various optical methods have been proposed for this purpose. Placido-disc topography [1–3], which involves analyzing reflected images and using the spherical approximation algorithm, is the most common technique used for reconstructing the corneal surface profile. However, this approach can incur height estimation errors at the same curvature, and the test edge yields a lowerquality measurement result. Other methods, such as the slit scanning method [4,5] and the Scheimpflug imaging method [6,7], have also been proposed. These methods can be used to acquire information on the corneal thickness and the profiles of the anterior and posterior corneal surfaces. However, during the measurement process, fringe projection scanning or rotating camera scanning is required, which renders the process timeconsuming and may generate fixed-position errors. Although the described methods demonstrate the benefits of structural simplicity and ease of operation, the accuracy of measurement and resolution tend to be limited by fluctuations in the light intensity of the test. Therefore, this study proposes a simple approach for reconstructing the corneal surface profile based on the Talbot effect, the projection moiré method, and heterodyne interferometry. A linear grating is obliquely illuminated by an expanding collimated light, and a self-image of this grating can be generated and projected onto the corneal surface. The deformed grating fringes are imaged onto the second grating to form the moiré fringes. If the first grating is moved at a constant velocity along the grating plane, then a series of sampling points of the sinusoidal wave, which behaves in the manner of heterodyne interferometric signals, can be recorded using a CMOS camera. The phase distribution of the corneal surface can then be obtained using the IEEE 1241 least-square sine fitting algorithm and twodimensional (2D) phase unwrapping. Finally, the corneal surface profile can be reconstructed by substituting the phase distribution into a special derived equation. This method demonstrates the benefits of the projection moiré method, the Talbot effect, and heterodyne interferometry, including simple optical setup, ease of operation, high stability, and high resolution.

## 2. Principle

Fig. 1 shows the optical configuration of the proposed method. For convenience, the *z*-axis is assigned as the observation axis of the CMOS camera, and the *y*-axis is set perpendicular to the paper

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plane. A laser light at wavelength  $\lambda$  passes through a beam expander to form an expanded and collimated light, and then impinges on a linear grating  $G_1$  at an incident angle of  $\alpha$  to the normal of the grating plane. The self-images of grating  $G_1$  can be generated at the Talbot distances  $Z_T$  and can be expressed as [8]

$$Z_T = \frac{mp^2}{\lambda} \cos^3(\alpha), \quad m = 1, 2, 3...$$
 (1)

where p is the pitch of the grating  $G_1$ . After arranging the test sample on the first Talbot distance (m=1) of grating  $G_1$ , the grating fringes are self-imaged on the sample and deformed by the height distribution. The deformed grating fringes can be expressed as

$$I_1(x,y) = \frac{1}{2} \left\{ 1 + \cos \left[ \frac{2\pi}{p} x + \theta(x,y) + \phi_1 \right] \right\},$$
 (2)

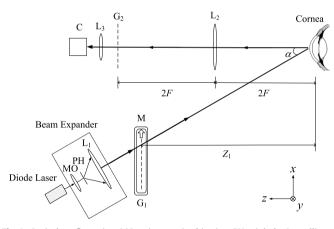
where  $\phi_1$  is the initial phase of grating  $G_1$ , and  $\theta(x, y)$  denotes the phase of the height distribution on the sample, expressed as

$$\theta(x,y) = \frac{2\pi}{p} H(x,y) \tan \alpha. \tag{3}$$

In Eq. (3), H(x, y) is the height distribution on the sample. Eq. (3) can be rewritten as

$$H(x,y) = \frac{p}{2\pi \tan \alpha} \theta(x,y). \tag{4}$$

According to Eq. (4),  $\theta(x, y)$  is a function of H(x, y); therefore, the height distribution of the sample H(x, y) can be estimated by accurately measuring the phase  $\theta(x, y)$ .



**Fig. 1.** Optical configuration. MO: microscopic objective; PH: pinhole;  $L_1$ : collimating lens;  $L_2$ : imaging lens;  $L_3$ : camera lens;  $G_1$  and  $G_2$ : linear grating; M: motorized translation stage;  $Z_1$ : first Talbot distance;  $\alpha$ : projection angle; F: focal length of imaging lens; C: CMOS camera.



Fig. 2. The tested sample of the pig eyeball.

In the subsequent step, the deformed fringes are imaged at  $1 \times 1$  magnification on the grating  $G_2$  of a pitch p to form the moiré fringes and are captured using a camera lens  $L_3$  on the CMOS camera C. The captured fringes can be expressed as

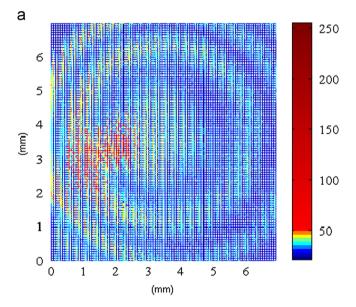
$$I_{2}(x,y) = I_{1}(x,y) \times T$$

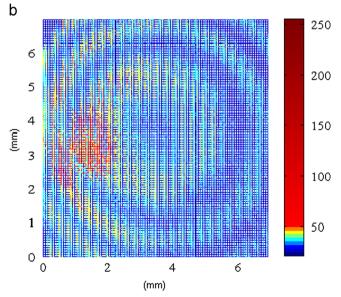
$$= \frac{1}{2} \left\{ 1 + \cos \left[ \frac{2\pi}{p} x + \theta(x,y) + \phi_{1} \right] \right\} \times \frac{1}{2} \left[ 1 + \cos \left( \frac{2\pi}{p} x + \phi_{2} \right) \right],$$
(5)

where *T* is the transmission of the grating  $G_2$ , and  $\phi_2$  is the initial phase of grating  $G_2$ . Eq. (5) can then be expanded as

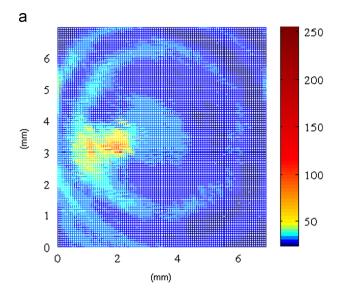
$$I_{2}(x,y) = \frac{1}{4} \left\{ 1 + \cos \left[ \frac{2\pi}{p} x + \theta(x,y) + \phi_{1} \right] + \cos \left( \frac{2\pi}{p} x + \phi_{2} \right) + \frac{1}{2} \cos \left[ \frac{4\pi}{p} x + \theta(x,y) + \phi_{1} + \phi_{2} \right] + \frac{1}{2} \cos \left[ \theta(x,y) + \phi_{1} - \phi_{2} \right] \right\}.$$
(6)

In Eq. (6), the second, third, and fourth terms are the high-order harmonics, and the final term is a moiré fringe. The moiré fringes





**Fig. 3.** Moiré fringes recorded by CMOS camera. (a and b) Moiré fringes at  $0 \, \text{s}$  and  $7/15 \, \text{s}$ .



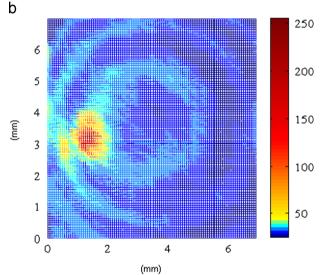


Fig. 4. (a and b) The results from applying the 2D median filter to filter Fig. 3(a) and (b).

can be extracted by filtering the high-order harmonics, and can be expressed as

$$I(x, y) = I_0(x, y) + \gamma(x, y) \cos[\theta(x, y) + \eta],$$
 (7)

where  $\gamma(x, y)$  is the visibility of the signal, and  $\eta = \phi_1 - \phi_2$  is the relative phase induced by the relative displacement between two gratings. When a motorized translation stage M moves the grating  $G_1$  along the grating plane at a constant velocity v, the time-varying phase can be induced and expressed as

$$\eta = \frac{2\pi vt}{p} = 2\pi f_h t,\tag{8}$$

where  $f_h$  is the heterodyne frequency resulting from the timevarying phase. Each pixel of the CMOS camera records a series of sampling points of the sinusoidal waves. These sinusoidal waves behave in the manner of heterodyne interferometric signals. Therefore, the light intensity at the CMOS camera C can be expressed as

$$I(x,y) = I_0(x,y) + \gamma(x,y)\cos[2\pi f_h t + \theta(x,y)], \tag{9}$$

To obtain  $\theta(x, y)$ , Eq. (9) can be rewritten as

$$I(x, y) = A\cos(2\pi f_h t) + B\sin(2\pi f_h t) + C,$$
(10)

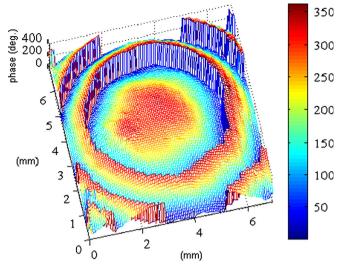


Fig. 5. Wrapped phase distribution of the corneal surface of the pig cornea.

where *A*, *B*, and *C* are real numbers, and the relationship between *A*, *B*, and  $\theta(x, y)$  can be shown as

$$\theta(x,y) = \tan^{-1}\left(\frac{-B}{A}\right). \tag{11}$$

The terms A and B can be estimated using the least-squares sine wave fitting algorithm. Substituting A and B into Eq. (11) yields the corresponding phase  $\theta(x, y)$  of the pixel. The phase distribution of the sample can be obtained by applying this procedure to each pixel and using 2D phase unwrapping. The sample surface profile can then be reconstructed using Eq. (4).

## 3. Experimental results and discussions

The proposed method was validated by using it to measure the corneal surface of a pig eyeball, as shown in Fig. 2. The experiment setup included a diode laser at a wavelength of 473 nm, two linear gratings at a pitch of 0,2822 mm, an imaging lens with a focal length of 200 mm, a motorized translation stage (Sigma Koki/SGSP (MS)26-100) with 0.05 µm resolution to produce a heterodyne frequency  $f_h$  = 2 Hz (v = 0.5644 mm/s), and a CMOS camera (Basler/ A504k) with an 8-bit gray level and  $1280 \times 1024$  image resolution. For convenience, the projection angle was 30°, the frame rate of the camera  $f_s$ =30 fps, the exposure time  $\Delta t$ =0.033 s, and the total recording time  $\Delta T = 1$  s to record the interferometric signals at different time points. To improve the visibility of the projected grating fringes, fluorescein was used to stain the corneal surface. The experimental results are displayed in Figs. 3-6. Fig. 3(a) and (b) shows the moiré fringes recorded by the camera at 0 s and 7/15 s, respectively. Fig. 4(a) and (b) shows the results obtained after applying the 2D median filter to filter Fig. 3 (a) and (b), respectively. Fig. 5 displays the phase distribution estimated using the least-squares sine fitting algorithm. Using 2D phase unwrapping and Eq. (4), the corneal surface profile can be reconstructed, as shown in Fig. 6. To prove the feasibility of proposed method, the identical vertical principle meridian of the corneal surface was extracted from the experimental results shown in Fig. 6 and the side view of the cornea shown in Fig. 7. The side view of the corneal sample in x–z plane was captured using the CMOS camera with the same magnification of the camera lens L<sub>3</sub>. The vertical principle meridian of the corneal surface could be drawn along the boundary where the light intensity decreases substantially from the black part to the gray

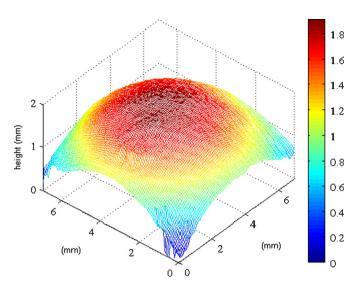


Fig. 6. Reconstructed surface profile of the pig cornea.

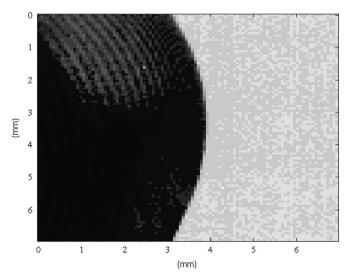


Fig. 7. Side view of cornea.

part, and was used to be the reference data. Fig. 8 shows the vertical principle meridians extracted from the experimental data and the reference data. The experimental data exhibit the same height trend as that of the reference data and, consequently, can prove the feasibility of proposed method, despite the reference data providing an unsmooth curve because of considerably lower resolution.

According to Eq. (4), the measurement height error  $\Delta H$  can be expressed as

$$\Delta H = \left| \frac{\partial H}{\partial p} \Delta p \right| + \left| \frac{\partial H}{\partial \alpha} \Delta \alpha \right| + \left| \frac{\partial H}{\partial \theta} \Delta \theta \right|. \tag{12}$$

where  $\Delta p$  is the grating pitch error,  $\Delta \alpha$  is the projection angle error, and  $\Delta \theta$  is the phase error. The grating pitch error  $\Delta p$  is caused by grating fabrication. The gratings are fabricated by applying chromium plating onto a glass substrate by using a mask laser writer. The grating pitch error  $\Delta p$  is approximately 0.1  $\mu$ m. The projection angle error  $\Delta \alpha$  is introduced according to the axis alignment error and the resolution of the rotary stage for

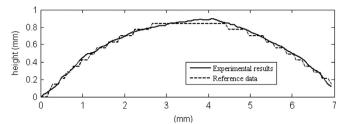


Fig. 8. Vertical principle meridians of cornea surface from experimental results and reference data

alignment. Considering that the axis alignment error in the experiment is  $0.05^{\circ}$ , and the resolution of the rotary stage is 10', the projection angle error  $\Delta \alpha$  can be estimated at approximately  $0.22^{\circ}$ . The phase error  $\Delta \theta$  is introduced according to the sampling error [9]. Considering that the visibility of the moiré fringes is 0.3, the stability of light intensity resulted from the stability of the light source and the vibration of the grating is 1%, and according to heterodyne interferometry, the phase error  $\Delta \theta$  can be estimated at approximately  $0.9^{\circ}$ . By substituting the experimental conditions,  $\Delta p$ ,  $\Delta \alpha$ , and  $\Delta \theta$  into Eq. (12), the measurement height error  $\Delta H$  is approximately  $3.5~\mu m$ . This error analysis indicates that the proposed technique yields high accuracy and high resolution.

## 4. Conclusion

This study proposes a simple method for reconstructing the corneal surface profile. The proposed method was validated by using it to measure the corneal surface of a pig eyeball. The measurement resolution was approximately 3.5  $\mu m$ . The proposed approach demonstrates the benefits of the projection moiré method, the Talbot effect, and heterodyne interferometry, including simple optical setup, ease of operation, high stability, and high resolution.

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#### References

- [1] Corbett MC, Rosen ES, O'Brart DP. Corneal topography: principles and applications. London: BMJ Books; chapter 1–3.
- [2] Jongsma FHM, Brabander Jd, Hendrikse F. Review and classification of corneal topographers. Lasers Med Sci 1999;14:2–19.
- [3] Massig JH, Lingelbach E, Lingelbach B. Videokeratoscope for accurate and detailed measurement of the cornea surface. Appl Opt 2005;44(12):2281–7.
- [4] Liu Z, Huang AJ, Pflugfelder SC. Evaluation of corneal thickness and topography in normal eyes using the Orbscan corneal topography system. Br J Ophthalmol 1999:83:774–8.
- [5] Cairns G, Collins A, McGhee CN. A corneal model for slit-scanning elevation topography. Ophthalmic Physiol Opt 2003;3(3):193–204.
- [6] Swartz T, Marten L, Wang M. Measuring the cornea: the latest developments in corneal topography. Curr Opin Ophthalmol 2007;18(4):325–33.
- [7] Oliveira CM, Ribeiro C, Franco S. Corneal imaging with slit-scanning and Scheimpflug imaging techniques. Clin Exp Optom 2011;94(1):33–42.
- Scheimpflug imaging techniques. Clin Exp Optom 2011;94(1):33–42.
  [8] Testorfa M, Jashnsa J, Khilob NA, Goncharenkob AM. Talbot effect for oblique angle of light propagation. Opt Commun 1996;129(3–4):167–72.
- [9] Jian ZC, Chen YL, Hsieh HC, Hsieh PJ, Su DC. Optimal condition for full-field heterodyne interferometry. Opt Eng 2007;46(11):115604-1–8.