$$= -\mathbf{B}^{(2)}\mathbf{B}^{(2)^t}.$$

We must emphasize that there is no flaw in the theoretical development of [1].

Again, we are very thankful to Dr. Xiao for the careful examination of the results in [1].

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Efficient Time Domain Synthesis of Pipelined Recursive Filters

Chien-Piao Lan and Chein-Wei Jen

***Abstract*— An efficient technique is developed for the synthesis of pipelined recursive filters directly from the time domain specifications. Based on the modified least-squares approximation, the pipelinable denominator form that contains only the powers of z^{-R} is designed inherently by expressing the design criterion in terms of only the nonzero denominator coefficients and a final design is derived. Due to the lack of the conventional transformation overheads, the multiplications needed are fewer in the resulted filters. Moreover, we introduce a new kind of companion matrix that characterizes the stable properties of pipelined recursive filters. Using this result, we prove that the proposed direct synthesis method can always guarantee the stability.**

I. Introduction

The data dependence between successive outputs of recursive filters restricts the use of pipelining techniques to derive high throughput rate [1]. A new scattered look-ahead method to pipeline recursive filters has been proposed by Parhi [2]. It releases the dependence of $y(n)$ on $y(n-1)$ to $y(n)$ on $y(n-R)$. The new recursive loop can then be pipelined by R stages and achieve R times throughput rate. Conventionally, the design of these pipelined recursive filters is employing a pole-zero cancellation technique [2]. A simple nonpipelined filter $H_1(z)$

$$H_1(z) = \frac{\sum_{i=0}^{Q} b_i \cdot z^{-i}}{1 + \sum_{i=1}^{P} a_i \cdot z^{-i}}$$

is first designed to satisfy some specifications. For each pole of this filter, we add $(R-1)$ new canceling pole and zero pairs uniformly located with the same radius of that pole, the resulted transfer function $H_2(z)$ will only contain z^{-R} terms in the denominator as

$$H_2(z) = \frac{\left(\sum_{i=0}^{Q} b_i z^{-i}\right) \cdot L(z)}{\left(1 + \sum_{i=1}^{P} a_i z^{-i}\right) \cdot L(z)} = \frac{\sum_{i=0}^{(R-1)P+Q} b'_i \cdot z^{-i}}{1 + \sum_{i=1}^{P} a'_i \cdot z^{-Ri}}.$$

This releases the output dependence. However, $(R-1)P$ new coefficients are added in numerator. They are usually too large and unnecessary for specification demands.

A frequency-sampling technique was first explored by Soderstrand [3] to reduce the above overheads. It takes the high sampling form of the denominator of $H_1(z)$ only, and then designs the canceling FIR filter. The final numerator could have fewer coefficients. This is a two-step design style. However, more freedom of the numerator

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order will be gained if we synthesize a pipelined recursive filter

$$\hat{H}(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m}}{1 + a_1 z^{-R} + a_2 z^{-2R} + \cdots + a_n z^{-nR}} \quad (1)$$

directly. We can adopt either the frequency domain approach or the time domain approach. For the frequency domain approach, the methods of designing recursive decimators [4], [5] can all be employed as in Chung–Parhi's work [6]. Recently, Lan has also explored a constrained iterative technique [7] to synthesize pipelined recursive filters directly. For the design problems with nonstandard frequency responses [8]–[11] or the specifications are only known from the time domain such as the voice modeling filter in the speech synthesizer [12], we may prefer the time domain approach. However, little literature concerned with the synthesis of pipelined recursive filters from the time domain approach. In this paper, we want to formulate the direct synthesis problem of (1) based on the time domain approach. We expect to develop an efficient design method to save the pole-zero cancellation overheads.

II. Synthesis of Pipelined Recursive Filters

Let a desired infinite impulse response h_k and the corresponding transfer function be specified as

$$H(z) = h_0 + h_1 z^{-1} + \cdots + h_k z^{-k} + \cdots$$

$$h_k = 0 \quad \text{for } k < 0.$$

The problem is to find the coefficients $\{a_i \mid i = 1, 2, \cdots, n\}$ and $\{b_j \mid j = 1, 2, \cdots, m\}$ of the pipelined recursive filter (1) so that the error measure

$$\epsilon = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| H(e^{j\omega}) \cdot D(e^{j\omega}) - N(e^{j\omega}) \right|^2 d\omega \quad (2)$$

is minimized. This is equivalent to design a filter

$$\hat{H}(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m}}{1 + q_1 z^{-1} + q_2 z^{-2} + \cdots + q_{nR} z^{-nR}}$$

where

$$q_{iR} = a_i \quad i = 1, 2, \cdots, n$$
$$q_j = 0 \quad j \neq iR$$

to minimize the error (2). If we substitute the new denominator coefficients $\{q_j \mid j = 1, 2, \cdots, nR\}$ into (2) and apply the Parseval's theorem, we find

$$\epsilon = \sum_{k=0}^{m} b_k^2 - 2 \cdot \sum_{k=0}^{m} b_k \sum_{j=0}^{nR} q_j h_{k-j} + \sum_{i=0}^{nR} \sum_{j=0}^{nR} q_i q_j r_{|i-j|} \quad (3)$$

where $q_0 = a_0 = 1$, and $r_i = \sum_{-\infty}^{\infty} h_k h_{k+i}$ is the autocorrelation sequence about h_k. Minimizing this error with respect to b_k yields the coefficients b_k to be

$$b_k = \sum_{j=0}^{nR} q_j \cdot h_{k-j} \quad (4)$$

$$= \sum_{i=0}^{n} a_i \cdot h_{k-iR}. \quad (5)$$

Equation (3) with these b_k (4) becomes

$$\epsilon = \sum_{i=0}^{nR} \sum_{j=0}^{nR} q_i q_j r_{|i-j|} - \sum_{k=0}^{m} \left(\sum_{j=0}^{nR} q_j h_{k-j} \right)^2. \quad (6)$$

Some coefficients q_j can be constrained to be zero inherently if we rewrite (6) as

$$\epsilon = [a_n, a_{n-1}, \cdots, a_1, a_0] \cdot \boldsymbol{K} \cdot \begin{bmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_1 \\ a_0 \end{bmatrix} \quad (7)$$

where $\boldsymbol{K}$ is a symmetrical matrix whose elements are given by

$$k_{ij} = r_{|iR-jR|} - \sum_{k=-\infty}^{m-nR+\min(iR,jR)} h_k h_{k+|iR-jR|} \quad i, j = 0, 1, \cdots, n. \quad (8)$$

There are not restrictions about the coefficients a_i. The minimization of (7) does not need any constrained programming methods.

By the Lagrange multipliers method [13], the minimal value of (7), $\epsilon_{\min}$, and the corresponding coefficient $\boldsymbol{a} = [a_n \ a_{n-1} \ \cdots \ a_1 \ 1]^T$, are found to satisfy

$$\boldsymbol{K} \cdot \begin{bmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_1 \\ a_0 \end{bmatrix} = \epsilon_{\min} \cdot \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad a_0 = 1. \quad (9)$$

Coefficient $\boldsymbol{a}$ can be solved from (9) by computing the last column of $\boldsymbol{K}^{-1}$ and normalizing it so that the last element is unity. Coefficients b_k are then calculated from (5).

Although our design criterion ϵ in (2) is a modification of the standard least-squares error [10]

$$\left\| h - \hat{h} \right\|^2 = \sum_{k=0}^{\infty} \left(h_k - \hat{h}_k \right)^2$$
$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| H(e^{j\omega}) - \frac{N(e^{j\omega})}{D(e^{j\omega})} \right|^2 d\omega \quad (10)$$

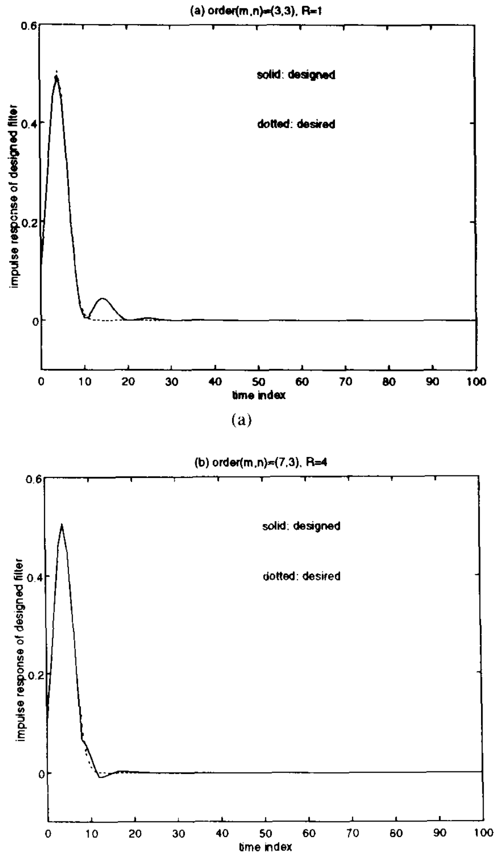
the designed result can be reasoned in the sense of least-squares. This is because the least-squares error (10) is bounded by the product of the error ϵ and the maximum value of $\left| D(e^{j\omega}) \right|^{-2}$. If the designed ϵ vanishes, so does the least-squares error for a stable filter. The designs by our approach, therefore, have the similar meaning as those by the least-squares criterion. Some differences between these two methods are discussed in [14] and not mentioned here. We mainly use the important fact that the modified least-squares provides a quadratic form of the designed variables. Thus the original constrained filter design can be easily simplified.

III. Stability Characterization

For a pipelined recursive filter with transfer function as that in (1), we define a power companion matrix$\hat{\boldsymbol{A}}$ in terms of the denominator coefficients as

$$\hat{\boldsymbol{A}} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}.$$

This matrix has a very useful property that the Rth power of each pole of the filter corresponds to one eigenvalue of$\hat{\boldsymbol{A}}$. Thus, if matrix$\hat{\boldsymbol{A}}$ has all eigenvalue magnitudes less than unit, the filter will be stable.



(b)

Fig. 1. The desired and designed impulse responses of example 1.

TABLE I
THE DESIGN RESULTS OF EXAMPLE 1

Pipeline stages	Filter order(m,n)	$\overline{\epsilon_2}^{(99)}$
1	(3,3)	8.7301
2	(5,3)	4.5717
3	(6,3)	5.7238
4	(7,3)	4.3886
5	(7,3)	6.8273

Let us refer to the upper left $n \times n$ portion of matrix $\boldsymbol{K}$, denoted as $\boldsymbol{K}_1$, and the matrix $\hat{\boldsymbol{A}}$. Using the solution (9), we find that

$$\hat{\boldsymbol{A}}\boldsymbol{K}_1\hat{\boldsymbol{A}}^T = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} - \epsilon_{\min} \cdot \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} [0\ 0\ \cdots\ 0\ 1] = \boldsymbol{K}_2 - \epsilon_{\min} \cdot \tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^T.$$

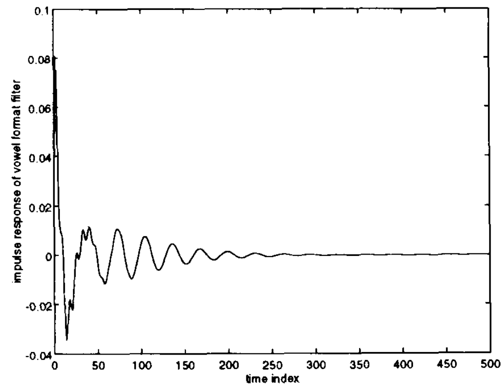


Fig. 2. The impulse response of the vowel formant filter of example 2.

The matrix $\boldsymbol{K}_2$ represents the lower right $n \times n$ portion of $\boldsymbol{K}$. From the matrix element definition (8), we obtain the matrices $\boldsymbol{K}_1$, $\boldsymbol{K}_2$ to be related as

$$\begin{aligned} \boldsymbol{K}_2 &= \boldsymbol{K}_1 - \left(\boldsymbol{h}_1 h_1^T + \boldsymbol{h}_2 h_2^T + \cdots + \boldsymbol{h}_R h_R^T\right) \\ &= \boldsymbol{K}_1 - \boldsymbol{L} \end{aligned}$$

where

$$\boldsymbol{h}_k = \left[h_{m-nR+k}\ h_{m-nR+R+k}\ \cdots\ h_{m-nR+(n-1)R+k}\right]^{\boldsymbol{T}}, \quad k = 1, 2, \cdots, R.$$

Finally, we obtain the result

$$\hat{\boldsymbol{A}}\boldsymbol{K}_1\hat{\boldsymbol{A}}^T = \boldsymbol{K}_1 - \boldsymbol{L} - \epsilon_{\min} \cdot \tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^T. \tag{11}$$

The measure defined in (2) is nonnegative and reaches to zero for the optimal design. But in real synthesis, we can only design the best $N(z)$ and $D(z)$ to approximate a truncated version of the impulse response

$$\bar{H}(z) = h_0 + h_1 z^{-1} + \cdots + h_k z^{-k} + \cdots + h_F z^{-F}.$$

rather than the entire infinite one. It follows that (7) can be merely minimized to a positive value. The matrix $\boldsymbol{K}$ is positive definite. As a result, all the matrices $\hat{\boldsymbol{A}}$, $\tilde{\boldsymbol{b}}$, $\boldsymbol{K}_1$, $\boldsymbol{L}$ and (11) satisfy the Lyapunov stability theorem [15]. All the eigenvalues of the power companion matrix $\hat{\boldsymbol{A}}$ therefore have the magnitudes less than unit. The filter designed by our method is then proved to be stable.

IV. ILLUSTRATIVE EXAMPLES

To show the effectiveness of our method, we design two different filters in this section. The quantity

$$\overline{\epsilon_2}^{(\xi)} = \frac{\left[\sum_{k=0}^{\xi}\left(h_k - \hat{h}_k\right)^2\right]^{\frac{1}{2}}}{\left[\sum_{k=0}^{\xi} h_k^2\right]^{\frac{1}{2}}} \times 100$$

defined in [16], [17] for time domain design comparison is used as the specification.

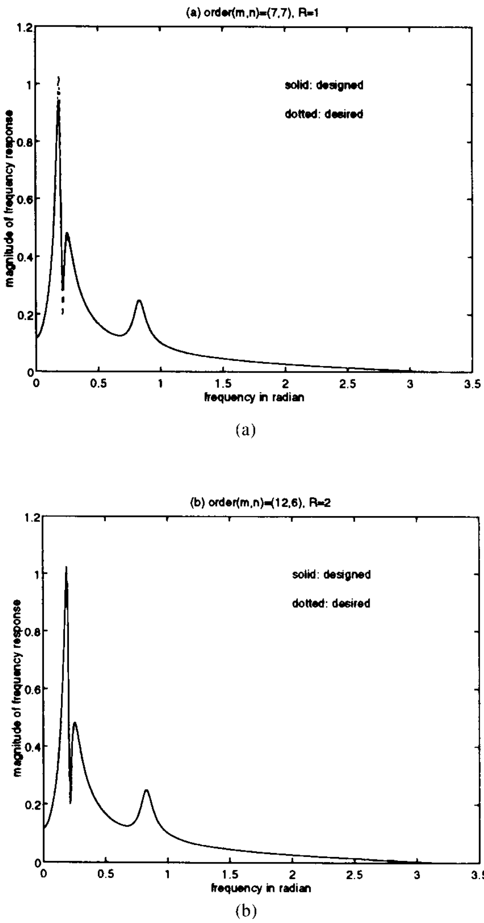


Fig. 3. The designed responses of example 2.

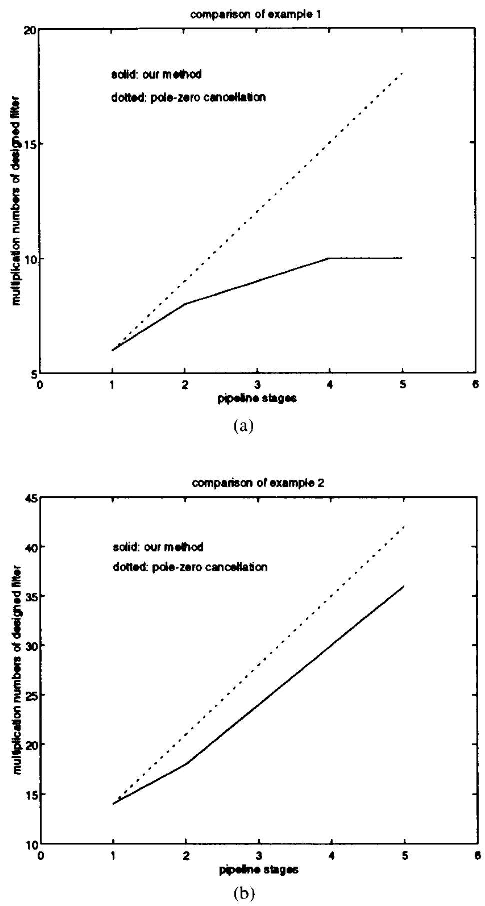


Fig. 4. The comparison between the multiplication numbers of filters designed by different methods.

TABLE II
THE DESIGN RESULTS OF EXAMPLE 2

Pipeline stages	Filter order(m,n)	$\overline{\epsilon_2}^{(499)}$
1	(7,7)	7.3884
2	(12,6)	0.0379
3	(18,6)	0.0002
4	(24,6)	0.0001
5	(30,6)	0.00005

Example 1—A Causal Gaussian Filter: In this example, the problem is to approximate the impulse response of a causal Gaussian filter given by

$$h_k = 0.256322 \cdot e^{-0.103203\cdot(k-4)^2} \quad k = 0, 1, \ldots$$

The finite truncation, F, is selected to be 99. The specification is $\overline{\epsilon_2}^{(99)} \leq 9$. The designed results for different pipelining stages are summarized in Table I. Some coefficients of the designed filter are given as

$$R = 1$$

$$N(z) = 0.0971 - 0.0024z^{-1} + 0.0791z^{-2} + 0.0412z^{-3}$$

$$D(z) = 1 - 2.0846z^{-1} + 1.6575z^{-2} - 0.4997z^{-3}$$

$$R = 4$$

$$N(z) = 0.0971 + 0.2000z^{-1} + 0.3351z^{-2} + 0.4566z^{-3} + 0.4926z^{-4} + 0.4285z^{-5} + 0.2879z^{-6} + 0.1358z^{-7}$$

$$D(z) = 1 - 0.1406z^{-4} + 0.0412z^{-8} - 0.0140z^{-12}$$

The desired and designed impulse responses are shown in Fig. 1.

Example 2—A Vowel Formant Filter: Fig. 2 shows the impulse response of a filter corresponding to the formant for the vowel "ɔ" (as in law) in a speech synthesis system [18], [19]. The finite truncation, F, is selected to be 499. The specification is $\overline{\epsilon_2}^{(499)} \leq 8$. The design results are summarized in Table II. Some coefficients are given as

$R = 1$

$$\begin{aligned} N(z) &= 0.0410 - 0.0930z^{-1} + 0.0158z^{-2} \\ &\quad + 0.1264z^{-3} - 0.1177z^{-4} - 0.0085z^{-5} \\ &\quad + 0.0609z^{-6} - 0.0247z^{-7} \\ D(z) &= 1 - 4.2290z^{-1} + 6.9478z^{-2} - 4.4936z^{-3} \\ &\quad - 1.4692z^{-4} + 4.4596z^{-5} - 2.8884z^{-6} \\ &\quad + 0.6749z^{-7} \end{aligned}$$

$R = 2$

$$\begin{aligned} N(z) &= 0.0411 + 0.0807z^{-1} - 0.0635z^{-2} \\ &\quad - 0.2121z^{-3} + 0.0035z^{-4} + 0.2476z^{-5} \\ &\quad + 0.0511z^{-6} - 0.1958z^{-7} - 0.0570z^{-8} \\ &\quad + 0.1259z^{-9} + 0.0516z^{-10} \\ &\quad - 0.0430z^{-11} - 0.0237z^{-12} \\ D(z) &= 1 - 3.2876z^{-2} + 5.0197z^{-4} - 5.1398z^{-6} \\ &\quad + 4.2001z^{-8} - 2.3845z^{-10} + 0.6459z^{-12} \end{aligned}$$

The designed responses for these two cases are shown in Fig. 3. They are plotted in the frequency domain to compare the result apparently.

Finally, we compare the proposed method with the pole-zero cancellation method in terms of the multiplication numbers of the designed filter. The numbers for the pole-zero cancellation are evaluated assuming R-stage scattered look-ahead is applied to the original nonpipelined filter ($R = 1$). This comparison is made for both example 1 and example 2. Fig. 4 indicates the results. Our method can generally reduces the multiplications number by about 60 to 80%.

V. Conclusions

An efficient technique has been developed in this paper for the synthesis of pipelined recursive filters directly from their time domain specifications. Based on the modified least-squares approximation, the error measure is expressed in the quadratic form of the denominator coefficients. Furthermore, the demand that the denominator polynomial contains only powers of z^{-R} is served simply by constructing a special matrix $\boldsymbol{K}$ rather than constraining some denominator coefficients to be zero. Design result is derived via a matrix inversion operation and pipelinability has been satisfied inherently. No any complex programming is need. Several examples have been illustrated to show the effective reduction of the pole-zero cancellation overheads. Besides, we have proved the designed filters can always be ensured to be stable. This offers more attractions to the proposed method.

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