DECISION SCIENCES
A JOURNAL OF THE DECISION SCIENCES INSTITUTE

Decision Sciences
Volume 44 Number 6
December 2013

Analyzing the Integration of WiMAX and Wi-Fi Services: Bandwidth Sharing and Channel Collaboration*

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ABSTRACT

Several emerging studies have focused on the pricing issue of bandwidth sharing between Wi-Fi and WiMAX networks; however, most either concentrate on the design of collaborated protocols or figure out the issue without the overall consideration of consumer preferences and contract design. In this study, we explore a wireless service market in which there are two wireless service providers operating Wi-Fi and WiMAX. One of the research dimensions given in this study is whether wireless service providers implement bandwidth sharing, while the other is whether they make decisions individually or jointly. By involving consumer preferences and a wholesale price contract in the present model, we find that bandwidth sharing would benefit a WiMAX service provider, yet a Wi-Fi service provider would make no significant savings under a wholesale price contract. In addition, the profit of a WiMAX service provider may increase with Wi-Fi coverage when bandwidth sharing has been implemented but decrease with Wi-Fi coverage when both wireless services operate without bandwidth sharing. Furthermore, the WiMAX service provider allocates more capacity when the average usage rate increases, but lowers the expenditure of capacity when the average usage rate is too high. [Submitted: October 23, 2011. Revised: May 30, 2012; November 26, 2012; May 3, 2013. Accepted: May 8, 2013.]

Subject Areas: Bandwidth Sharing, Noncooperative Games, Pricing, Telecommunications, Wi-Fi, and and WiMAX.

INTRODUCTION

Since Android and iTunes launched application stores in 2008, more than 55% of American adults have connected to the Internet wirelessly through either a

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*National Science Council of Taiwan supported this research (grant NSC 101–2410-H-266–002).

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Wi-Fi or a WiMAX connection via their laptops or smartphones. There were 302.9 million wireless connections as of December 2010; in addition, it is expected that consumers will download more than 44 billion applications by 2016 (Rainie, 2010; CTIA, 2011). All business reports identify the importance of the wireless industry because it offers unlimited revenue potential.

While wireless service providers are competitors for their market shares, they also collaboratively provide Internet connectivity. Wi-Fi and WiMAX are the two most promising technologies that have been implemented by wireless service providers. Wi-Fi technology is mostly used in laptops today and it is commonly available in coffee shops and other public places around the world. Wi-Fi operators aggregate the wireless networks provided by microcarriers, such as Starbucks coffee shops and Borders bookstores in the United States, in order to provide single access to the end user (Yaiparoj, Harmantzis, & Gunasekaran, 2008). The main disadvantage of using Wi-Fi networks is insufficient service coverage because Wi-Fi hotspots in a network operating in public bands must avoid interference with each other. As a result, the wireless signals they broadcast are fairly weak.

By contrast, WiMAX, an emerging technology, promises to offer faster data speeds than current wireless networks and over much longer distances than comparably fast Wi-Fi technology; hence, WiMAX can be considered to be a solution to fill the holes in the insufficient coverage provided by Wi-Fi hotspots and to enable wireless connectivity on public transportation (Ballon, 2007). Before long-term evolution (LTE) technology is available, Sprint, a wireless company utilizing WiMAX to offer access services, has stated that true download speeds are between 2 megabits per second (Mbps) and 4 Mbps, comparable with many digital subscriber line (DSL) and cable modem services (Reardon, 2007; Lawson, 2008; Myslewski, 2009). Currently, Sprint has joined Verizon and AT&T, who have already rolled out LTE, to become the third official carrier in the United States for the Apple iPhone. Therefore, Sprint plans to introduce its LTE network to accommodate Apple's upcoming iPhone with LTE. Despite the impact of LTE, Sprint will provide WiMAX services to its customers until 2015 according to the agreement announced by Sprint and Clearwire (Haselton, 2011; Bora, 2012).

The pricing and features of transmission media affect the allocation of wireless network resources in terms of Wi-Fi and WiMAX. Because of the distance limitation for Wi-Fi hotspots, the only solution for a Wi-Fi service provider to reduce users' inconvenience in finding the nearest hotspot is to increase the number of hotspots. Since each Wi-Fi hotspot needs a wired backhaul to offer Internet connectivity, the capacity cost of Wi-Fi services spent on wired backhauls can be saved if Internet connectivity is offered by a wireless backhaul, such as a WiMAX base station. The focus in recent years has been on real-time data services in wireless environments. Wireless service providers can charge users a service fee to compel their consumption decisions toward more efficient network usage. In general, wireless users are inherently time-sensitive and the aspects of coverage are highly subjective and depend heavily on consumer experiences. Thus, wireless service providers can rely on consumer preferences in regard to quality of service (QoS) and the characteristics of their own media to implement a traffic management and versioning strategy by pricing their services accordingly.

Table 1: Research architecture scenarios of the integration of Wi-Fi and WiMAX.

		Bandwidth Sharing	
		No	Yes
Channel collaboration	No	(I)	(II)
	Yes	(III)	(IV)

Problems and Motivation

WiMAX base stations can serve as wireless backhauls where the bandwidth of WiMAX networks is shared by Wi-Fi hotspots to provide Internet connectivity to mobile Wi-Fi users (Fantacci & Tarchi, 2006; Lin, Lin, Chang, & Cheng, 2009; Huang, Hu, Chen, Chen, & Chen, 2010). From the aspect of practicability, the integration of Wi-Fi and WiMAX can benefit Wi-Fi service providers because the cost of wired infrastructure can be avoided. However, the impact of bandwidth sharing on wireless service providers' service strategies is not clear. Since Internet access is almost homogeneous (Shin, Weiss, & Tucci, 2007), consumer preferences regarding wireless access service are largely affected by price, available bandwidth, and service coverage. Accordingly, this study emphasizes the service strategies universally adopted by wireless service providers, such as how to allocate service capacity and how to determine service coverage.

This research can be split into four scenarios, as shown in Table 1. The first dimension is whether Wi-Fi and WiMAX services operate with bandwidth sharing or not. The other dimension is channel collaboration in which both wireless service providers can make their decisions individually or jointly. Scenario I is an extension of typical Bertrand competition in which both service providers set prices simultaneously. In Scenario II, the WiMAX service provider offers a wholesale price contract for bandwidth sharing, and then the Wi-Fi service provider makes a take-it-or-leave-it decision. In Scenario III, the wireless service providers do not share bandwidth but make decisions jointly. Scenario IV is a fully integrated case in which both wireless service providers not only share bandwidth but also make decisions jointly. Therefore, for the two given dimensions, we aim to study the following questions:

- For a WiMAX service provider guaranteeing QoS access and a Wi-Fi service provider serving best effort traffic, how do system factors, such as average usage rate, bandwidth uncertainty, and capacity cost affect their decisions regarding capacity and coverage?
- Once both service providers decide to implement bandwidth sharing, what are the features of the contract and their profits? Further, what is the impact of channel collaboration on capacity investment and bandwidth sharing?

Findings and Contribution

We find that bandwidth sharing may benefit a WiMAX service provider, yet a Wi-Fi service provider would have no significant savings under a wholesale price contract. In addition, the profit of a WiMAX service may increase with Wi-Fi coverage when bandwidth sharing is implemented but decrease with Wi-Fi coverage when both wireless services operate without bandwidth sharing. The WiMAX service provider allocates more capacity when the average usage rate increases, but decreases the expenditure of capacity when the average usage rate is too high. Finally, we also identify the difference in service strategies between channel competition and channel collaboration.

LTE, a wireless broadband technology similar to WiMAX, has recently been developed by the Third Generation Partnership Project, and it will thus now compete with WiMAX for broadband wireless consumers. Since WiMAX and LTE aim to provide last mile access to Internet users, they deliver traffic to the Internet running under transmission control protocol/internet protocol (TCP/IP) despite the differences in their infrastructures. However, the network coverage problem will continue to exist until the infrastructure supporting the latest standard in the mobile network technology tree has been completed (Tofel, 2011). Conceptually, a wireless firm adopting LTE can also cooperate with Wi-Fi firms to extend its coverage by sharing bandwidth. Consequently, our findings from bandwidth sharing between Wi-Fi and WiMAX can also be considered to be a promising solution to Wi-Fi and LTE technologies.

LITERATURE REVIEW

It is widely accepted that the Internet industry in the United States is a vertical structure that is composed of Internet backbone providers (IBPs) and Internet service providers (ISPs) (Shin et al., 2007). The relationship between IBPs and ISPs can be seen as a wholesaler–retailer relationship, and IBPs can accordingly charge ISPs transit fees for Internet access. A prior study has supported the contention that mobile Internet services are mainly used for low-bandwidth applications (Ooteghem et al., 2009). Recently, several emerging studies have focused on the revenue and benefit in the integration of Wi-Fi and WiMAX. Gunasekaran and Harmantzis (2006) proposed a service model that utilizes Wi-Fi and WiMAX to deliver cost-effective broadband services in which Wi-Fi service providers adopt WiMAX backhaul systems to reduce infrastructure cost. Niyato and Hossain (2007) considered a single WiMAX base station serving multiple connections from WiMAX users and Wi-Fi hotspots. In their setting, the WiMAX network serves real-time traffic, while the Wi-Fi network serves best effort traffic. The authors show that the WiMAX service provider needs to increase the wholesale price paid by the Wi-Fi service provider for each Wi-Fi hotspot when the traffic arrival rate increases. The reason for this pricing strategy is to compensate for the loss in revenue resulting from degraded QoS performance for WiMAX users.

Niyato and Hossain (2008) utilized two oligopolistic models for price competition among service providers in a heterogeneous wireless environment composed of WiMAX and Wi-Fi access networks. Their research findings indicated that a WiMAX service provider can increase its offered price to gain a higher profit when transmission quality is enhanced. Maillé and Tuffin (2010) studied a pricing game between two wireless access providers, one operating Wi-Fi access and the other operating WiMAX access. Surprisingly, the authors found that the overall utility of the system is maximized at equilibrium. Ibrahim, Khawam, Samhat, and

Tohme (2009) provided tractable formulae of the end-user mean capacity and coverage probability in order to properly value the integration of Wi-Fi and WiMAX. Ognenoski, Rakovic, Bogatinovski, Atanasovski, and Gavrilovska (2009) considered a case in which a single service provider runs two wireless networks, including Wi-Fi and WiMAX, and observed how the mean utility, total system utility, and revenue change in a backup network offering voice and file download services.

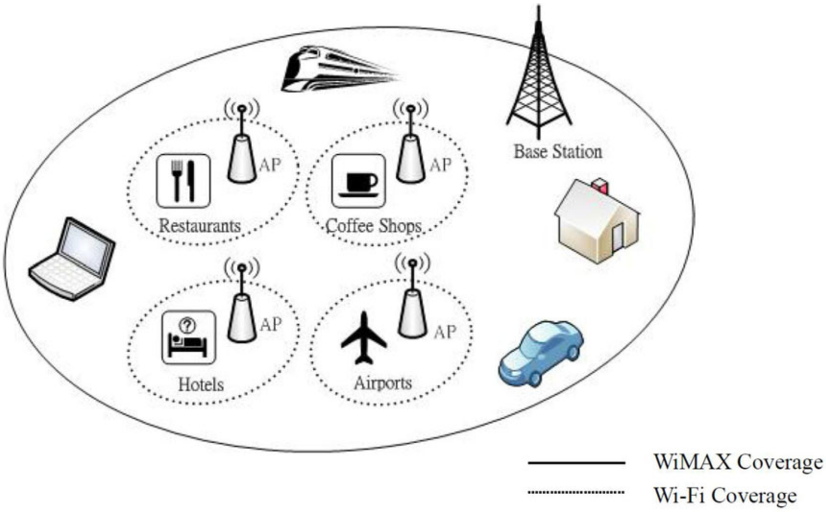
The present study is different from extant studies in that we concentrate on the following two dimensions. First, we rely on the aspect of bandwidth uncertainty and coverage uncertainty to model consumer preferences regarding available bandwidth and service coverage. Most prior studies have either adopted an inverse demand function to express the relation between price and the number of consumers or directly excluded human perception factors. Second, Wi-Fi and WiMAX service providers can make their pricing and service decisions individually or jointly. Most extant research has only considered bandwidth sharing under the assumption that both Wi-Fi and WiMAX service providers cooperate without conflict. Thus, we utilize a wholesale price contract to explore the issue of bandwidth sharing between Wi-Fi and WiMAX when both service providers make their service decisions individually.

Prior studies of channel collaboration emphasize the performance advantages of firms (Lee, Pak, & Lee, 2003; Tuominen, 2004; Chen & Chen, 2005, Min et al., 2005; Hyvönen & Tuominen, 2007). By contrast, bandwidth sharing is an innovative research issue, and most studies either concentrate on its technology feasibility rather than business models or treat the issue without the features of competition between wireless service providers (Gunasekaran & Harmantzis, 2006; Niyato & Hossain, 2007, 2008; Ibrahim et al., 2009; Islam, Rashid, & Tarique, 2011; Tang, 2012). A profitable innovative technology requires support from suitable business models. Therefore, we integrate the perspective of channel collaboration and bandwidth sharing to study the wireless industry and explore the economic feasibility of bandwidth sharing. In this study, our primary contribution to the literature is to demonstrate that competing wireless access retailers providing differentiated services (WiMAX and Wi-Fi) in the wireless market can reach a supplier–retailer relationship through bandwidth sharing. In addition, the performance of bandwidth sharing approaches that of channel collaboration when capacity cost does not bear heavily on suppliers, which can be considered to be an alternative to raising retailers' profits when channel collaboration is prohibited by law or other concerns such as security questions and asymmetric bargaining power.

MODEL

We consider a wireless service zone in which there are two wireless service providers operating access services under different wireless technologies, Wi-Fi and WiMAX. The number of consumers in the market who are interested in subscribing to wireless services is denoted as η_0. In this study, we are interested in a more general case in which there is demand for both Wi-Fi and WiMAX. For a Wi-Fi service provider, in order to make wireless signals so ubiquitous that consumers can receive them conveniently, it has to deploy a great number of

Figure 1: Comparison between a Wi-Fi network and a WiMAX network.



hotspots in the service zone. These hotspots deployed by the Wi-Fi service provider are kinds of wireless routers, which are used as access points that connect to fixed lines, such as T1 and T3. Similarly, WiMAX signals are broadcasted by a WiMAX base station, which works exactly like GSM network phone towers that reach high up in the air to broadcast radio signals. Figure 1 demonstrates the difference in service coverage between Wi-Fi and WiMAX, where the coverage of WiMAX is greater than that of Wi-Fi. Although there are other pricing mechanisms, such as usage-based pricing (Geng & Whinston, 2001), in this study, we only concentrate on the monthly subscription fees that both service providers charge consumers for offering access services. Thus, the price of a Wi-Fi service is denoted as p_F, whereas the price of a WiMAX service is denoted as p_M. All notations used in the present model can be found in Table 2.

The model we use for Scenarios I and II can be viewed as a two-stage game. When bandwidth sharing is not adopted, the Wi-Fi service provider chooses the number of hotspots in stage 1 and both the Wi-Fi and the WiMAX service providers engage in Bertrand pricing in stage 2. By contrast, when bandwidth sharing is adopted, the WiMAX service provider chooses a wholesale price in stage 1 and both the Wi-Fi and the WiMAX service providers still engage in Bertrand pricing in stage 2.

Features of Wireless Technology

The main factors that affect consumers' purchasing decisions regarding wireless services are service coverage, available bandwidth, and price (Yaiparoj et al., 2008). A well-known characteristic of the Internet industry is that Internet access service is almost homogeneous (Shin et al., 2007). Thus, without considering

Table 2: Definition of notations.

Notation	Description
$p_F(p_M)$	Price of Wi-Fi service (WiMAX service)
$\pi_F(\pi_M)$	Profit of Wi-Fi service (WiMAX service) without both bandwidth sharing and channel collaboration
$\pi_{FB}(\pi_{MB})$	Profit of Wi-Fi service (WiMAX service) with bandwidth sharing but without channel collaboration
$\pi_{(M+F)}(\pi_{(M+F)B})$	Total profit of channel collaboration without (with) bandwidth sharing
$c_d(\hat{c}_d)$	Disutility of bandwidth uncertainty without (with) bandwidth sharing
n	Number of hotspots
$c_a\,(n)$	Disutility of coverage uncertainty
$V_F(V_M)$	Consumers' willingness-to-pay for wireless access service under Wi-Fi (WiMAX) technology
ϕ	Difference between V_M and V_F
θ	Consumers' sensitivity to uncertainty in terms of coverage and bandwidth
η_0	Number of consumers who want to buy wireless services
$\eta_F(\eta_M)$	Demand for Wi-Fi service (WiMAX service)
$U_F(U_M)$	Consumers' utilities when using Wi-Fi service (WiMAX service)
$c_F(c_M)$	Service-related cost of Wi-Fi service (WiMAX service)
$\gamma_F(\gamma_M)$	Network-related cost of Wi-Fi service (WiMAX service)
μ	Service rate of a WiMAX base station
K_M	Marginal capacity cost of a WiMAX base station
K_F	Average capacity cost per hotspot
$\lambda(\beta)$	Consumers' arrival rates (usage rates)
d	Threshold of average delay in WiMAX networks
ω	Wholesale price per Wi-Fi consumer
EW	Expected time a request remains in the system from its arrival time until its processing has been completed
T1 (T3)	Cost for T1 (T3) line rental
$Rent_F(Rent_M)$	Cost for Wi-Fi (WiMAX) site rental
$Licence$	License fee to acquire the right to broadcast WiMAX signals
α	Revenue-sharing ratio

service coverage and available bandwidth, we assume that each consumer has the same maximum willingness-to-pay for both wireless services. Here, we regard the quality of the WiMAX service as a benchmark for the quality of wireless services. In fact, many businesses have considered WiMAX technology to be a standard-based technology that enables the delivery of "last mile" wireless broadband access, which can be used in place of traditional cable and DSL (Gunasekaran & Harmantzis, 2008).

One of the most substantial differences between Wi-Fi and WiMAX is that the maximal transmission distances provided by a WiMAX base station and a Wi-Fi hotspot are 9.6 km and 90 m long, respectively. Accordingly, in a certain geographical zone served by a WiMAX base station, if the number of hotspots deployed by a Wi-Fi service provider is not enough, consumers bear the inconvenience of having to find the nearest Wi-Fi hotspot. We refer to this as the disutility of coverage uncertainty because Wi-Fi coverage is restricted by a certain distance so that the signals broadcasted by Wi-Fi hotspots cannot be found everywhere.

In many Web sites operated by wireless service providers, users can find the number of hotspots in a specific area through search engines. For example, if searching Boingo Wireless Hotspots in Seattle, a user will receive information in the form of a map that shows the number and location of nearby hotspots. As the number of hotspots increases, the number of holes without wireless signals will decrease. Consequently, wireless users who randomly walk in a specific area will have a higher probability to connect their devices to the Internet. Therefore, compared to WiMAX (which guarantees users a signal over a larger service area of up to 9.6 km), we define the disutility of coverage uncertainty as a function with the number of hotspots as its argument. Thus, the disutility of coverage uncertainty resulting from the distribution of Wi-Fi hotspots is denoted as $c_a\,(n)$, where n is the number of hotspots, satisfying $\partial c_a(n)/\partial n < 0$ and $\partial^2 c_a(n)/\partial n^2 > 0$.

As for bandwidth, although Wi-Fi technology typically provides local network access over a radius of around a few hundred feet with speeds of up to 54 Mbps, the real available bandwidth is limited to fixed backhaul. Because of the cost of fixed backhaul, most Wi-Fi hotspots, in practice, are connected to T1 lines (or other media with lower transmission rates), which have a transmission speed rate of 1.5 Mbps. Furthermore, Wi-Fi and WiMAX adopt different channel access methods for shared medium networks. Because WiMAX technology operates in time division multiple access (TDMA), it can provide better QoS than Wi-Fi technology operating in carrier sense multiple access with collision avoidance (CSMA/CA). With TDMA, WiMAX can serve real-time traffic for users and guarantee its service level (i.e., the available bandwidth), while most Wi-Fi hotspots around the world, in general, only serve best effort traffic.

Owing to the characteristics of best effort traffic, the bandwidth that consumers enjoy from Wi-Fi technology is affected by several external factors. First, Wi-Fi uses a CSMA/CA mechanism that is inadequate for high-bandwidth applications such as video (Weiss, 2011). Second, spectrum limitation makes Wi-Fi systems vulnerable to congestion as public Wi-Fi becomes more prevalent (Taylor, 2012). Consequently, once Wi-Fi hotspots become congested due to VoIP applications and bandwidth demanding content, each user connecting to these hotspots will perceive that the Wi-Fi network is hopelessly overcrowded (Sauter, 2007), which leads to uncertainty in bandwidth availability.

Compared with Wi-Fi, because a WiMAX service provider can deploy a single WiMAX base station to serve consumers at a wider range than Wi-Fi, the cost of fixed backhaul can be reduced. Consequently, the WiMAX service provider can offer real-time traffic by connecting its base station to a high-speed fixed backhaul, such as a T3 line, which has a transmission rate of 44.736 Mbps. Therefore, in the present model, we assume that the Wi-Fi service network serves best effort traffic and the WiMAX service network serves real-time traffic. Consumers may expect to experience uncertain bandwidth when using Wi-Fi services. We refer to this as the disutility of bandwidth uncertainty, which is denoted as c_d. This consideration is reasonable because few Wi-Fi service providers, such as T-Mobile and Boingo, ever mention the available bandwidth on their webpages. Moreover, in order to make terminologies concise, we use "bandwidth uncertainty" and "uncertainty in bandwidth availability" interchangeably throughout the study.

Consumer Preference

Then, consumers choose their preferred wireless access services based on their individual utilities, which are affected by price and service quality. Because consumers have heterogeneous sensitivity to the disutility of uncertainty in terms of bandwidth and coverage, we follow prior studies (Chun & Kim, 2005; Fan, Kumar, & Whinston, 2007–2008, 2009) and denote the sensitivity of disutility as θ, which is randomly drawn from a uniform distribution with support [0, 1]. In addition, we consider these wireless access services to be perceived as homogeneous if the disutility of uncertainty disappears. However, whether wireless access services are identical or not does not matter. The key question is whether consumers in the market appreciate any differences in wireless access services. If not, then consumers regard wireless access services as substitutes even though they are not identical. In this study, in order to further investigate the impact of service differentiation between Wi-Fi and WiMAX services, we denote V_M and V_F as consumers' willingness-to-pay for wireless access services under Wi-Fi and WiMAX technologies, respectively. Consequently, consumers' utilities are $U_M(\theta) = V_M - p_M$ and $U_F(\theta) = V_F - p_F - \theta(c_a(n) + c_d)$ for using the WiMAX service and Wi-Fi service, respectively.

Wireless service is a mobile way to connect to the Internet. Just like broadband services and cell phone services, consumers have different preferences with respect to network speed and service coverage. Both network speed and service coverage are significantly affected by the nature of wireless technology. Compared with WiMAX, Wi-Fi covers a relatively short transmission distance and occupies a relatively narrow spectrum; as a result, a Wi-Fi user cannot be certain whether there will be a wireless signal or how much bandwidth will be available when he or she wants to surf the Internet—these are the components of the Wi-Fi user's disutility.

A consumer is indifferent about the two access services if $U_M = U_F$ holds. Given $\phi \equiv V_M - V_F$, solving the equation leads to $\theta^* = (p_M - p_F - \phi)/(c_a(n) + c_d)$, which is the point of indifference for consumers. Therefore, the demands for the Wi-Fi service and WiMAX service are $\eta_F = \theta^* \eta_0$ and $\eta_M = (1 - \theta^*)\eta_0$, respectively.

SERVICE COMPETITOIN

In this section, we consider the first scenario, in which a Wi-Fi service provider competes with a WiMAX service provider in a wireless access market without any hardware and channel integration. According to the comprehensive study conducted by Ooteghem et al. (2009), operational expenditures for a wireless access network can be split between network-related and service-related costs. Service-related costs are composed of service provisioning and customer relationship management, consisting of pricing and billing, helpdesks, and marketing. We denote c_F as the service-related costs of Wi-Fi services and c_M as the service-related costs of WiMAX services. Although their service-related costs are alike, both access services have many differences in network-related costs, including operations, administration, and maintenance.

First, the network-related costs of the Wi-Fi service increase with the number of hotspots, while those of the WiMAX service increase with the number of subscribing consumers because the WiMAX network serves real-time traffic. Second, because the WiMAX service uses a licensed spectrum to deliver a point-to-point connection to the Internet, the WiMAX service provider has to pay a license fee to acquire the right to broadcast WiMAX signals, which is denoted as $Licence$. Third, according to the chosen fixed lines, such as T1 and T3, the costs of fixed backhaul are also different. For convenience, we denote the cost of renting T1 and T3 lines as T1 and T3, respectively. Finally, both access service providers bear the cost for site rental, which is the multiplication of the number of sites with the cost per site to rent (Ooteghem et al., 2009). The costs per site to rent for Wi-Fi and WiMAX services are denoted as $Rent_F$ and $Rent_M$, respectively. Since the Wi-Fi network serves best effort traffic and each Wi-Fi hotspot connects to a fixed line, the expected capacity cost is estimated by $n \cdot K_F$, where K_F is the average capacity cost per hotspot. Although Wi-Fi coverage can be expanded by other approaches, such as Wi-Fi roaming, we simplify these considerations to make our model tractable. Therefore, the Wi-Fi service provider's profit function π_F can be written as

$$\pi_F = (p_F - c_F)\eta_F - n \cdot \gamma_F, \tag{1}$$

where

$$\gamma_F = \mathrm{T1} + Rent_F + K_F.$$

Dynamic Aspects of the Model

On the contrary, because of offering real-time traffic, the WiMAX service provider needs to consider the queuing delay and processing rate for its access service. When wireless subscribers access the wireless network, they presumably take account of service prices and benefits from usage, but ignore the congestion delay that they impose on other subscribers, which is known as congestion externality (MacKie-Mason & Varian, 1995). One of the ways to alleviate congestion is to establish rationing and quota systems under pricing mechanisms (Mendelson, 1985; Masuda & Whang, 2006). The goal of congestion prices is not only to decrease usage when congestion is presented, but also to generate revenue for capacity expansion. The importance of queuing effects in studying the performance of computer systems is well known and this helps top managers quantify the associated trade-off between making sound capacity decisions and improving service quality (Mendelson, 1985). In order to process customer requests, we denote the processing rate as μ, which is also used to represent IT capacity (Tan & Mookerjee, 2005). The marginal cost for the processing rate is denoted as K_M; thus, the total cost for capacity μ is $K_M \cdot \mu$.

Following prior studies (Mendelson, 1985; Masuda & Whang, 2006; Fan et al., 2009), it is natural to replace the quantity of wireless accessing by the arrival rate of transactions to the transport medium, enabling us to model a subscriber's arrival on the WiMAX network as a Poisson process with a mean arrival rate λ. In an M/M/1 queue, the average delay for a customer can be represented as $EW = 1/(\mu - \lambda)$, which represents the expected time a request remains in the system from its arrival time until its processing has been completed. Although the total number of consumers subscribing to the WiMAX service is η_M, only a proportion of all consumers use the access service at any one time. By letting β be the average usage rate of all consumers, we have $\lambda = \beta\eta_M$. In addition, in order to serve real-time traffic on the WiMAX network, the average delay for a subscriber cannot be higher than a specific threshold d, which is the average delay guarantee of the WiMAX service provider. Therefore, we have $1/(\mu - \lambda)d$, which further influences the WiMAX provider's pricing decision because its capacity cost $K_M \cdot \mu$ increases with WiMAX demand to satisfy the average delay guarantee. In this way, we can bridge the gap between the dynamics occurring over short timescales and longer term parameters such as WiMAX prices. Therefore, the WiMAX service provider's profit function π_M can be written as

$$\begin{aligned} &\pi_M = (p_M - c_M)\,\eta_M - (\gamma_M + K_M\mu)\,, \\ &s.t.\ \frac{1}{\mu - \lambda} \le d, \end{aligned} \tag{2}$$

where

$$\gamma_M = \mathrm{T3} + Rent_M + Licence.$$

In Equation (2), the WiMAX service provider needs to expand its IT capacity (that is, service rate μ) in order to fit the requirement $EW \le d$. Thus, its minimal capacity cost $K_M\mu$ is given by solving $EW = d$. This approach is consistent with previous studies (Wang & Gerchak, 2003; Bernstein & DeCroix, 2004; Fan et al., 2009), establishing the relation between EW and μ. As a result, given the arbitrary p_M, the WiMAX service provider can optimize its profit by choosing μ as follows:

$$\mu = \beta\eta_M + \frac{1}{d}. \tag{3}$$

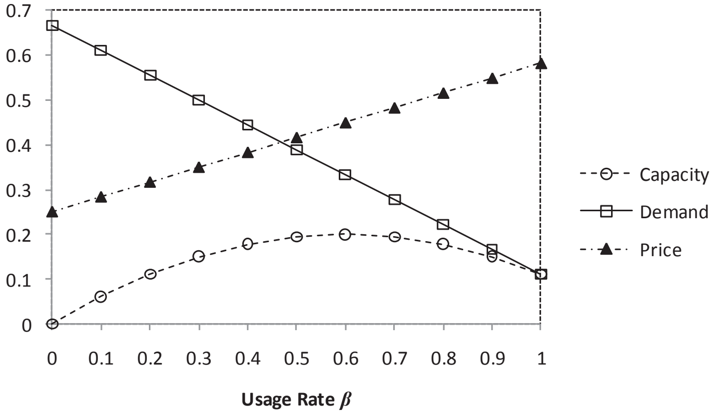
Proposition 1 (Scenario I):

The capacity investment made by the WiMAX service provider may increase with usage rate; however, when the proportion of all consumers using the WiMAX service at any time is too high, the WiMAX service provider will reduce capacity expenditure. Formally, $\partial\mu/\partial\beta > 0$ when $\beta < \beta^*$ and $\partial\mu/\partial\beta < 0$ when $\beta > \beta^*$, where β^* is given by

$$\beta^* = \left(2\,(c_a\,(n) + c_d) - c_M + c_F + \phi\right)\big/(2K_M). \tag{4}$$

In Proposition 1, we discuss the relation between service rate μ and average usage rate β. Service rate μ, in fact, can be viewed as IT capacity (Tan & Mookerjee, 2005). In addition, owing to the service requirements (that is, subscriber's average delay is less than *d*), the capacity cost increases with average usage rate. Therefore,

Figure 2: Relationship between usage rate and processing capacity (WiMAX price and demand).



we can observe the relation between capacity cost and usage rate by observing the relation between μ and β. This can be done by observing $\partial\mu/\partial\beta$. We find that $\partial\mu/\partial\beta = 0$ when $\beta = \beta^*$ holds. This means that the WiMAX service provider will have a different capacity policy when $\beta > \beta^*$ and $\beta < \beta^*$. If $\beta < \beta^*$ holds, the WiMAX service provider enhances its service capability, whereas if $\beta > \beta^*$ holds, it reduces its service capability but charges a higher price to reduce user demand.

When usage rate grows, it is intuitive that the WiMAX service provider will allocate more capacity to attain the delay guarantee. Nevertheless, when the average usage rate is too high, the capacity cost to guarantee service delay is so expensive that the WiMAX service provider has an incentive to reduce capacity cost and charge consumers a higher service fee to decrease the demand for the WiMAX service. In practice, WiMAX equipment is rather expensive and the total investment is even higher for a single WiMAX base station than it is for a group of Wi-Fi hotspots (Ooteghem et al., 2009). Thus, the government should take this into account and compensate WiMAX service providers in order to optimize the social welfare of wireless access services when the average usage rate grows. The relationship between the average usage rate and processing capacity (WiMAX price and demand) is shown in Figure 2. For the sake of convenience, all results are scaled and shown in the same graph, but the qualitative results remain unchanged as long as they satisfy the required conditions given in Proposition 1.

Optimal Wi-Fi Coverage

Subsequently, we consider the question of how a Wi-Fi service provider decides on the number of hotspots in a long-term competition. The scenario has two stages: (i) the Wi-Fi service provider decides on the number of hotspots and (ii) both service providers simultaneously decide on their prices. Because the equilibrium prices charged by the wireless service providers were derived from Proposition 1, we can only solve the first stage of the game. Thus, the Wi-Fi service provider's problem is

$$\begin{aligned}
&\operatorname*{Max}_{n}\ \pi_F = (p_F - c_F)\,\eta_F - n \cdot \gamma_F,\\
&\text{s.t.}\\
&p_M^*(n) = \text{Min}\left\{\frac{2c_a(n) + 2c_d + 2c_M + 2\beta K_M + c_F + \phi}{3}, V_M\right\},\\
&p_F^*\left(p_M^*(n)\right) = \frac{p_M^*(n) + c_F - \phi}{2}.
\end{aligned} \tag{5}$$

Proposition 2: (a) If the Wi-Fi service provider decides to expand its coverage, the WiMAX service provider must bear a high capacity cost but Wi-Fi consumers suffer from a lower disutility of bandwidth uncertainty. Formally, $c_M + \beta K_M - \phi - c_F > c_a(n^*) + c_d$ if $\partial \pi_F / \partial n = 0$ and $\partial \pi_F^2 / \partial n^2 < 0$.

(b) If WiMAX capacity does not cost the WiMAX service provider too much and Wi-Fi consumers suffer from a higher disutility of bandwidth uncertainty, the Wi-Fi service provider will lower its investment in the number of hotspots. Formally, $\partial \pi_F / \partial n < 0$ for all n if $c_M + \beta K_M - \phi - c_F < c_d$ holds.

Before the WiMAX service provider enters the market, it is sensible that the Wi-Fi service provider should enhance service coverage to gain an edge in the wireless service market. However, our results indicate that it is necessary that the Wi-Fi service provider understands the features of its service and the services of the other providers before deciding on its service coverage, because we find a corner solution and an interior solution when seeking the optimal number of hotspots.

In other words, the Wi-Fi service provider may largely invest in service coverage or keep the basic investment, depending on its service capability and its competitor's capacity costs. If capacity costs bear heavily on the WiMAX service provider and bandwidth uncertainty under the Wi-Fi service is enhanced, the Wi-Fi service provider can largely invest in service coverage to compete with the WiMAX service provider. In this case, owing to high capacity costs, the WiMAX service provider would not launch a price war. However, if WiMAX's capacity costs are very low, the Wi-Fi service provider's investment decision will incur its competitor's revenge with a lower price. In the latter case, since the WiMAX service provider does not dread investment in WiMAX capacity, the strategy of expanding Wi-Fi service coverage may lead to a cutthroat price competition; consequently, the Wi-Fi service provider should not expand its service coverage.

In fact, our results also indirectly address the driving force of information technology in Wi-Fi coverage. Although Wi-Fi naturally operates in CSMA/CA, many studies have proposed approaches to improve its QoS (Iera, Molinaro, Ruggeri, & Tripodi, 2005; Bruno, Conti, & Gregori, 2007). As a result, Wi-Fi service providers may consider expanding service coverage when these emerging technologies become mature in the wireless market. However, for Wi-Fi service providers, if WiMAX service providers can reduce capacity costs by more efficiently managing bandwidth (Panken & Hoekstra, 2007; Ntagkounakis et al., 2007), the investment decision of whether to expand service coverage should be evaluated more carefully.

Corollary 1: (a) If the Wi-Fi service provider has an incentive to expand its service coverage, its investment in service coverage increases with consumers' maximal willingness-to-pay for the WiMAX service, but decreases with consumers' maximal willingness-to-pay for the Wi-Fi service. Formally, the optimal number of hotspots is derived from the equation

$$\left\{1-\left(\frac{c_F-c_M-\beta K_M+V_M-V_F}{c_a\left(n^*\right)+c_d}\right)^2\right\}\frac{\partial c_a\left(n^*\right)}{\partial n}=\frac{9\gamma_F}{\eta_0}.$$

(b) If the Wi-Fi service provider has no incentive to expand its service coverage, its investment in service coverage decreases with consumers' maximal willingness-to-pay for wireless access services. Formally, the optimal number of hotspots is derived from the equation $c_a\left(n^*\right)=(2V_M+V_F-c_F)/2-(c_d+c_M+\beta K_M)$.

The interior solution and corner solution show two different perspectives. In the interior solution, the Wi-Fi service provider will invest in service coverage to compete with the WiMAX service provider actively. However, in the corner solution, the WiMAX service provider will adjust its investment to accommodate the WiMAX service provider's pricing strategy.

Therefore, considering the interior solution (Corollary 1a), we can observe that the Wi-Fi service provider will expand its service coverage when a customer's willingness-to-pay for the WiMAX service increases (that is, $\partial n^*/\partial V_M>0$), but will invest less in service coverage when a customer's willingness-to-pay for the Wi-Fi service increases (that is, $\partial n^*/\partial V_F<0$). In the former case, the Wi-Fi service provider needs to lower its price and offer better service quality to compete with the WiMAX service provider. In the latter case, the Wi-Fi service provider can raise its price but reduce its investment in service coverage to save the cost of maintaining hotspots.

By contrast, in the corner solution (Corollary 1b), the Wi-Fi service provider's best investment strategy is to entice the WiMAX service provider to charge the highest price. This can be done by investing less in service coverage because the WiMAX price increases when the coverage of the Wi-Fi service gets smaller. Therefore, when a customer's willingness-to-pay for the WiMAX service increases, the Wi-Fi service provider can invest less in service coverage because there is space for a rise in the WiMAX service price (that is, $\partial n^*/\partial V_M<0$). Likewise, the WiMAX service provider will reduce its price when a customer's willingness-to-pay for the Wi-Fi service increases so that the Wi-Fi service provider can also invest less to achieve the goal (that is, $\partial n^*/\partial V_F<0$).

Note that the WiMAX service provider may unilaterally deviate from the current equilibrium by blocking entry, which is known as market foreclosure. For brevity, the discussion of market foreclosure is relegated to Appendix A.

BANDWIDTH SHARING

Currently, there are many technology studies focusing on the development of the integration of Wi-Fi and WiMAX (Yang & Wu, 2007; Chen, Studer, & Perrig, 2008; Kim et al., 2010). In this section, we consider a business model in which the WiMAX service provider sells extra bandwidth to the Wi-Fi service provider by offering wireless backhaul support; consequently, the Wi-Fi service provider can avoid the cost of a wired infrastructure. The WiMAX service provider may charge the Wi-Fi service provider a wholesale price ω per Wi-Fi demand. Notice that we use $\hat{K}_F$ instead of K_F to express the influence of replacing T1 lines with the WiMAX backhaul on the capacity cost of Wi-Fi service. Thus, the Wi-Fi service provider's problem can be rewritten as

$$\operatorname*{Max}_{p_F} \ \pi_{FB} = (p_F - c_F - \omega)\,\eta_F - n \cdot \gamma_{FB}, \tag{6}$$

where

$$\gamma_{FB} = Rent_F + \hat{K}_F.$$

By contrast, the WiMAX service provider can receive the revenue of bandwidth sharing from the Wi-Fi service provider. In this case, in addition to all consumers who subscribe to the WiMAX service, the WiMAX base station also serves real-time traffic for each Wi-Fi hotspot to provide wireless backhaul support instead of wired backhaul support such as T1. The WiMAX service provider's problem can be rewritten as

$$\begin{aligned} &\operatorname*{Max}_{p_M} \ \pi_{MB} = (p_M - c_M)\,\eta_M - (\gamma_M + K_M \mu_B) + \omega \cdot \eta_F \\ &\text{s.t.} \ \tfrac{1}{\mu_B - \lambda_B} \le d. \end{aligned} \tag{7}$$

In addition to the usage rate derived from consumers who subscribe to the WiMAX service, the consumer arrival rate of a WiMAX base station has to involve the expected number of hotspots requesting an Internet connection, which is given by

$$H(n, \beta\eta_F) = n\left\{1 - \left(1 - \frac{1}{n}\right)^{\beta\cdot\eta_F}\right\}. \tag{8}$$

Equation (8), a result coming from a binomial distribution, represents the expected number of hotspots requesting an Internet connection, which is given by $n \cdot p$, where p can be derived from $(1 - (1 - \psi)^m)$. The notation ψ represents the probability that a hotspot is visited by a customer, which is given by $1/n$. Moreover, because there are $\beta\eta_F$ Wi-Fi customers requesting an Internet connection, we can plug $\beta\eta_F$ into m.

After bandwidth sharing, the customer arrival rate is different. Thus, we denote λ_B as the average customer arrival rate with bandwidth sharing. In addition, the consumer utility function for the Wi-Fi service after bandwidth sharing remains the same if the WiMAX service provider can guarantee that its service level agreement, which may include a congestion delay, jitter, and routing policy, is the same as that of a T1 line. To enhance the practicability of this study, we define the disutility of bandwidth uncertainty as $\hat{c}_d$ when the Wi-Fi service provider adopts bandwidth sharing instead of renting a T1 line.

While users may connect or disconnect their devices with wireless access points randomly, a Wi-Fi hotspot stays "active" at almost all times if enough users share the same hotspot. Therefore, the WiMAX service provider can consider the hotspot always active when the number of Wi-Fi consumers in the market is sufficiently large. That is, we can directly approximate $H(n, \beta\eta_F)$ as n. Thus, given an arbitrary p_M, the WiMAX service provider can maximize its profit by choosing capacity as

$$\mu_B = \lambda_B + \frac{1}{d}, \text{ where } \lambda_B = \beta\eta_M + H(n, \beta\eta_F). \tag{9}$$

Using the same approach mentioned in Proposition 1, we have $p_M^*(\omega)$ and $p_F^*(\omega)$. As a result, the problem of choosing the optimal wholesale price is given by

$$\begin{aligned} &\operatorname*{Max}_{\omega}\ \pi_{MB} \\ &\text{s.t.} \\ &\pi_{FB}^*(\omega) \geq \pi_F^*, \\ &p_M^*(\omega) = \text{Min}\left\{\frac{2(c_a(n) + \hat{c}_d + c_M + \beta K_M) + 3\omega + c_F + \phi}{3},\ V_M\right\}, \\ &p_F^*(\omega) = \frac{p_M^*(\omega) + c_F + \omega - \phi}{2}. \end{aligned} \tag{10}$$

Proposition 3 (Scenario II): When the number of consumers subscribing to the Wi-Fi service in the market is sufficiently large, the optimal wholesale price charged by the WiMAX service provider may decrease with the usage rate. Formally, $\partial\omega^*/\partial\beta < 0$.

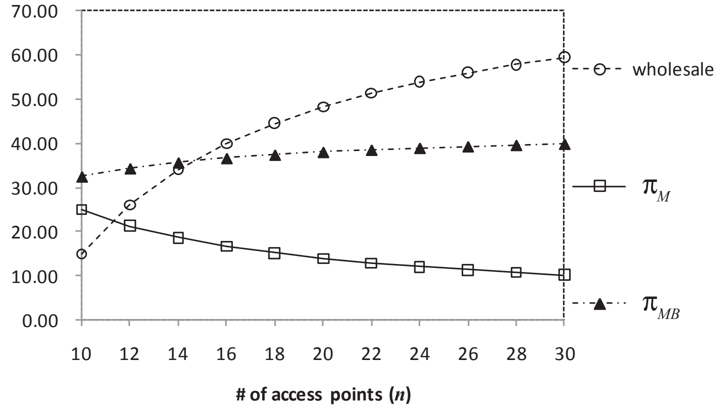
When one service provider raises its price, the other would also adopt the same pricing strategy until consumers cannot afford to pay such a high price. As a result, when the wholesale price is not too large, the Wi-Fi service provider can always raise its price to offset the loss resulting from the wholesale price charged by the WiMAX service provider. Accordingly, the result of bandwidth sharing does not benefit wireless consumers because it leads to higher prices for both access services. Examining the profit of the Wi-Fi service provider, we find the business model of bandwidth sharing, in general, cannot benefit the Wi-Fi service provider. Therefore, we examine the relation between Wi-Fi service coverage and the wholesale price when $\pi_{FB}^* = \pi_F^*$ holds.

Corollary 2: (a) The Wi-Fi service provider gains a higher profit after bandwidth sharing only when the cost of T1 is high enough.

(b) If the cost of T1 is not too high and bandwidth sharing does not make much difference to the uncertainty of bandwidth, the WiMAX service provider will raise the wholesale price when the number of Wi-Fi hotspots increases. Formally, $\partial\omega^*/\partial n > 0$ when $\pi_{FB}^* = \pi_F^*$ and $c_d \approx \hat{c}_d$.

A scaled numerical example is shown in Figure 3, where WiMAX profits without and with bandwidth sharing are denoted as π_M and π_{MB}, respectively.

Figure 3: Relationship between number of access points and wholesale price (WiMAX profit).



Without bandwidth sharing, we find that the WiMAX service provider's profit may decrease with Wi-Fi coverage; however, its profit may increase with Wi-Fi coverage when the WiMAX base station serves as the wireless backhaul for the hotspots in the Wi-Fi network. If the Wi-Fi service provider intends to deploy more hotspots, the WiMAX service provider can charge them a higher wholesale price to offset the impact of wider Wi-Fi service coverage. As a result, the WiMAX service provider can take advantage of the wholesale price contract to prevent the pitfalls of Bertrand competition in which the Wi-Fi service provider overinvests in service coverage. In addition, comparing π^*_{FB} with π^*_F, we find that the WiMAX service provider is willing to implement bandwidth sharing in a large wireless market; however, the high number of Wi-Fi hotspots may become an obstacle to bandwidth sharing because of real-time traffic.

CHANNEL COLLABORATION

Next, we consider the scenario in which both service providers can reach an agreement to collaborate in the wireless service market and split the collaborating profit. The channel collaboration problem without bandwidth sharing can be formulated as follows:

$$\begin{aligned}&\pi_{(M+F)} = (p_M - c_M)\eta_M + (p_F - c_F)\eta_F - (\gamma_M + K_M\mu) - n \cdot \gamma_F \\ &\text{s.t. } \frac{1}{\mu - \lambda} \leq d,\end{aligned} \tag{11}$$

where μ is given by Equation (3) and $\lambda = \beta\eta_M$. The channel collaboration problem with bandwidth sharing can be derived from Equation (11) by replacing γ_F with γ_{FB} and considering μ_B and λ_B from Equation (9).

Proposition 4 (Scenarios III and IV): (a) Without bandwidth sharing, the WiMAX capacity after adopting channel collaboration is more than that after adopting channel competition under the same Wi-Fi service coverage.

(b) When both service providers adopt channel collaboration, if bandwidth uncertainty can (cannot) be improved after bandwidth sharing and the cost of fixed lines is sufficiently higher (lower) than the marginal capacity cost of the WiMAX service, they can (cannot) gain a higher joint profit by adopting bandwidth sharing.

After adopting channel collaboration, the WiMAX service provider has to raise the amount of capacity to fit the requirement of real-time traffic because demand for the WiMAX service with channel collaboration is higher than that with channel competition. In addition, with channel collaboration, whether the WiMAX base station should share bandwidth with these Wi-Fi hotspots in the wireless service zone depends on the saved cost of T1 and the marginal cost of WiMAX capacity. When the saved cost of T1 is greater than the marginal cost of WiMAX capacity, bandwidth sharing can increase the profit from collaborating. In this case, deploying more Wi-Fi hotspots in the wireless service zone can further raise the profit from collaborating because the service providers can coordinate their prices accordingly. Otherwise, the wireless service providers should not integrate their access services to save the cost of WiMAX capacity.

EXTENSIONS

In this section, we aim to compare all profits in the four scenarios by viewing them as a part of the overall game. For splitting revenue generated from channel collaboration, there are several candidate mechanisms including joint ownership, agreements, quantity discounts, and profit sharing (Li & Huang, 1995). In terms of profit sharing, we need a sharing ratio to reflect the negotiation power between firms, and its value is usually determined in a negotiation process. While there are many bargaining models for dividing the increased profit gain between them (Nash, 1950; Eliashberg, 1986; Gupta & Livne, 1988), the negotiation process of the sharing ratio is not the focus of this article. The stages in the overall game are as follows.

Stage 1

Based on all known parameters such as the current number of hotspots, consumers' willingness-to-pay, and costs, both Wi-Fi and WiMAX service providers determine whether to integrate or not. If channel collaboration is adopted, both make decisions on bandwidth sharing and pricing jointly. The sharing ratio is determined by bargaining power, where the WiMAX and Wi-Fi service providers receive $\alpha \cdot \pi^e_{(M+F)}$ and $(1-\alpha) \cdot \pi^e_{(M+F)}$, respectively. If the WiMAX and Wi-Fi service providers do not agree to integrate, they move onto the next stage.

Stage 2

The WiMAX service provider decides on bandwidth sharing under the condition where the Wi-Fi service provider's profit with bandwidth sharing cannot be less than it would be without bandwidth sharing. If the WiMAX service provider decides to adopt bandwidth sharing, it proposes a wholesale price ω^*.

Stage 3

Without bandwidth sharing, the Wi-Fi service provider makes a decision on service coverage in terms of the number of hotspots.

Stage 4

The WiMAX and Wi-Fi service providers make pricing decisions independently and receive π_M^e and π_F^e, respectively.

The notations of joint profits in channel collaboration with and without bandwidth sharing are given in the proof of Proposition 4. (All proofs are located in Appendix A.) In other words, we have $\pi_{(M+F)}^e = \text{Max}\left\{\pi_{(M+F)}^*, \pi_{(M+F)B}^*\right\}$. Moreover, π_M^e and π_F^e are given in Lemma 1 as follows.

Lemma 1: Without channel collaboration, given $\omega^* = \text{Min}\,(\omega_1, \omega_2)$, the profits of the WiMAX and Wi-Fi service providers in stage 2 are given by

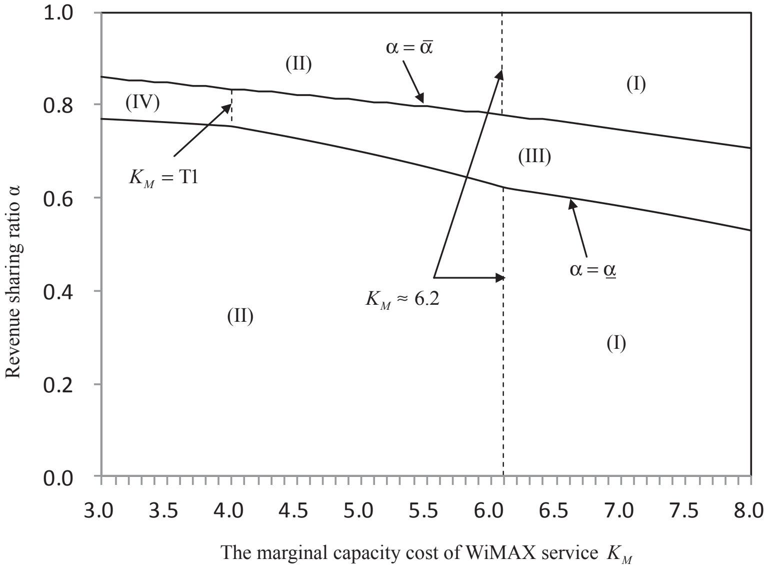
$$\pi_M^e = \begin{cases} \left\{(V_M - c_M - \beta K_M) - \dfrac{(V_M - \omega^* - c_M - \beta K_M)(V_M - \omega^* - c_F - \phi)}{2\,(c_a\,(n) + \hat{c}_d)}\right\}\eta_0 \\ -\left(\gamma_M + \left(n + \frac{1}{d}\right)K_M\right), \;\; \pi_{MB}^*\,(\omega_1) \geq \pi_M^* \; and \; \omega_3 \leq Min\,(\omega_1, \omega_2) \\ \dfrac{(2c_a\,(n^*) + 2c_d - c_M + c_F - \beta K_M + \phi)^2}{9\,(c_a\,(n^*) + c_d)}\eta_0 - \left(\gamma_M + \dfrac{K_M}{d}\right), \\ \qquad otherwise \end{cases}$$

$$\pi_F^e = \begin{cases} \dfrac{(V_M - c_F - \omega^* - \phi)^2}{4\,(c_a\,(n) + \hat{c}_d)}\eta_0 - n \cdot \gamma_{FB}, \\ \qquad \pi_{MB}^*\,(\omega_1) \geq \pi_M^* \text{ and } \omega_3 \leq \text{Min}\,(\omega_1, \omega_2) \\ \dfrac{(c_a\,(n^*) + c_d + c_M - c_F + \beta K_M - \phi)^2}{9\,(c_a\,(n^*) + c_d)}\eta_0 - n^* \cdot \gamma_F, \\ \qquad \text{otherwise.} \end{cases}$$

In Lemma 1, the notations ω_1 and ω_2 are given in the proof of Proposition 3, while ω_3 is defined in the proof of Lemma 1. With these auxiliary results, the equilibrium of the overall game is given as follows. The decision to adopt channel collaboration is made when $\alpha \cdot \pi_{(M+F)}^e > \pi_M^e$ and $(1-\alpha) \cdot \pi_{(M+F)}^e > \pi_F^e$ because channel collaboration arises when both service providers can gain a higher profit from the joint profit $\pi_{(M+F)}^e$. Otherwise, their equilibrium profits in the overall game are given by π_M^e and π_F^e.

Proposition 5: (a) If profit sharing ratio α can satisfy $\alpha \cdot \pi_{(M+F)}^e > \pi_M^e$ and $(1-\alpha) \cdot \pi_{(M+F)}^e > \pi_F^e$, both wireless service providers agree on channel

Figure 4: The equilibrium of the overall game.
Note: $n = 70, \eta_0 = 1,000, c_F = 0.1, c_M = 0.1, \gamma_M = 8, \gamma_F = 5, \beta = 0.2, d = 5, V_M = V_F = 3, \mathrm{T1} = 4, c_d = \hat{c}_d = 1, c_a(n) = 150/n, K_F = \hat{K}_F$



collaboration, implementing bandwidth sharing when $\Gamma < 0$ holds but operating their bandwidth independently when $\Gamma > 0$, where

$$\Gamma \equiv \left\{ \frac{(\hat{c}_d - c_d)(-c_M - \beta K_M + c_F + \phi)^2}{4\,(c_a(n) + c_d)(c_a(n) + \hat{c}_d)} \right\} \eta_0 - n \cdot \left(\mathrm{T1} + K_F - \hat{K}_F - K_M\right).$$

(b) If profit sharing ratio α cannot satisfy $\alpha \cdot \pi^e_{(M+F)} > \pi^e_M$ or $(1-\alpha) \cdot \pi^e_{(M+F)} > \pi^e_F$, both firms price their service independently, implementing bandwidth sharing when $\pi^*_{MB}(\omega_1) \geq \pi^*_M$ and $\omega_3 \leq \mathrm{Min}(\omega_1, \omega_2)$ but operating their bandwidth independently when the opposite holds.

Because there is no tractable form, we attempt to demonstrate the equilibrium results in a numerical way, as shown in Figure 4. For convenience, we consider $V_M = V_F$ and $c_d = \hat{c}_d$. The lower solid line is a plot of $\underline{\alpha} = \pi^e_M / \pi^e_{(M+F)}$, while the upper solid line is a plot of $\bar{\alpha} = 1 - \pi^e_F / \pi^e_{(M+F)}$.

The areas of channel collaboration, (III) and (IV), are surrounded by two solid lines (the plots of $\alpha = \bar{\alpha}$ and $\alpha = \underline{\alpha}$) and separated by a cutoff point where $K_M = \mathrm{T1}$. By excluding areas (III) and (IV), we have the areas of channel competition, (I) and (II), which are separated by two dashed lines. Examining Figure 4, we may conclude two points. First, bandwidth sharing reduces the space for channel collaboration when $K_M < \mathrm{T1}$, which shows that bandwidth sharing is a worthy alternative when both service providers fail to broker a revenue-sharing ratio that both sides can accept. Second, when $K_M > \mathrm{T1}$ holds, the space for channel collaboration grows. This happens because the benefit of bandwidth sharing decreases with the capacity costs of the WiMAX service provider and channel competition leads to a significant decline in the profits of both service providers.

To further explore the equilibrium results shown in a numerical way, Figure 4 helps justify that all scenarios (that is, the four subgames) in our study can arise under certain conditions; this enables us to concentrate on each scenario and discover interesting and useful insights from them.

LIMITATIONS AND MANAGERIAL IMPLICATIONS

Several limitations exist in the present study. First, we only study a wireless service market in which both wireless service providers offer homogeneous access services other than service coverage and available bandwidth because of their underlying network media. In fact, these wireless service providers can bundle their access services with differentiated value-added services to turn their infrastructure services into profitable businesses (Geng & Whinston, 2001). However, discussion of value-added services may dilute our efforts to analyze bandwidth sharing and channel collaboration; in addition, our model would be intractable if consumers were heterogeneous to value-added services. In other words, the bundle of access services and value-added services should be considered to be an isolated research question.

Second, we are only interested in the general case where there is demand for both Wi-Fi and WiMAX services; that is, some boundary cases are not explored in this study for the sake of convenience. For example, we did not consider that the Wi-Fi or WiMAX service provider can choose a cutthroat price to repel its competitor from the wireless market. Practically, the findings derived from the boundary cases seem mathematically sensible but contribute less to the wireless industry. In addition, a variety of business models can be further discussed but these are not treated in this research. For example, firms selling fixed lines are absent from our model, and we regard the price of fixed lines as exogenous. Cases can arise in which two or more firms sell fixed lines in a network market where there is less variance in the price of fixed lines because of homogeneous competition.

Third, we differentiate between Wi-Fi and WiMAX from the consumer's perspective about the disutility of uncertainty bandwidth and service coverage because both technologies show significant differences in the maximal transmission distance and available bandwidth. However, service providers may have an incentive to offer additional services such as online disk and email space to attract consumers. Thus, we may relax the assumption that each consumer has an identical willingness-to-pay for wireless access services and further examine the influence of service differentiation. But relaxing this assumption would create more mathematical work because the maximal price the WiMAX service provider can charge would no longer be fixed. Finally, some may raise the point that the WiMAX card is not a standard device in all laptops, however, this issue is not the focus of the study, and the impact of the lack of hardware support to WiMAX services may decrease with time. Subsequently, we summarize the implications for academic researchers and wireless service providers as follows.

Implications for academic researchers

Our primary contribution to academic researchers is that we demonstrate that competing wireless access retailers providing differentiated services (WiMAX and Wi-Fi) in the wireless market can reach a supplier–retailer relationship through hardware integration. If channel collaboration is prohibited by law or these wireless firms cannot reach an agreement to implement information sharing, such hardware integration, instead of channel collaboration, can enhance their profits. Thus, academic researchers can consider whether the concept of hardware integration can be applied into other services because it can slacken the effect of Bertrand competition. In addition, our model can also be extended to other scenarios, such as a company selling fixed lines and offering Wi-Fi services at the same time. It is clear that the positions of these wireless firms will significantly affect the results of bandwidth sharing and channel collaboration.

Implications for wireless service providers

The goal of this study was not to compare profits between the four scenarios, since each scenario represents a different degree of integration. In the case of channel collaboration, the total generated profits are the highest because both wireless service providers reach full integration just as in an alliance, sharing the same information and negotiating deals without conflicts of interest. If the two wireless service providers cannot attain channel collaboration, bandwidth sharing is the best alternative because it allows them to decide whether to integrate the infrastructures supporting wireless traffic. Therefore, the comparison in profits among these scenarios is omitted because total profit increases with the degree of integration.

For Wi-Fi service providers, our findings remind them of the pitfalls of investment in service coverage. Before a WiMAX service provider enters the wireless market, a Wi-Fi service provider must invest in expanding service coverage and improving QoS in order to increase profit. However, once a WiMAX service matures, expanding service coverage will lead to a price war because WiMAX takes advantage of service coverage and available bandwidth. Consequently, even if the Wi-Fi service provider can increase the number of hotspots, this strategy may cause a negative effect on its profit because the features of WiMAX outperform those of Wi-Fi from the consumer's point of view. If a Wi-Fi service provider intends to invest in service coverage, it has to confirm the following two critical points. First, the provider should be able to overcome the question of uncertain bandwidth resulting from the nature of Wi-Fi. Second, the cost of WiMAX capacity must be sufficiently high that the WiMAX service provider would have trouble in a price war.

WiMAX service providers should enhance consumers' impressions of available bandwidth because there is room for improvement in Wi-Fi service coverage. However, it is difficult to solve the question of bandwidth uncertainty in Wi-Fi networks because of the limitation of CSMA/CA. In addition, our results indicate that bandwidth sharing is a promising strategy for WiMAX service providers because it not only slackens price competition but also raises the WiMAX service provider's bargaining power with the firm selling fixed lines. A large wireless market is beneficial to the performance of bandwidth sharing because a WiMAX service provider's profit increases with the number of Wi-Fi consumers. Although the strategy of bandwidth sharing seems to have potential for a WiMAX service provider, this strategy still has the problem of applicability under certain conditions. For example, a WiMAX service provider has to carefully measure the impact of Wi-Fi coverage on capacity requirement because it has to guarantee available bandwidth for WiMAX consumers.

CONCLUSION

In this study, we considered a wireless service zone in which there are two wireless service providers operating different technologies: Wi-Fi and WiMAX. The aim of the study was to examine the impact of the integration of Wi-Fi and WiMAX on wireless operators' service strategies. Several emerging studies have focused on the pricing issue of bandwidth sharing between Wi-Fi and WiMAX networks; however, most have either concentrated on the design of collaborated protocols or have chosen not to consider consumers' preferences regarding uncertainty in terms of available bandwidth and service coverage. In contrast to prior studies, the present research applies a wholesale price contract to solve the issue of bandwidth sharing between Wi-Fi and WiMAX service providers; thus, our analytical results based on the aspects of enterprise operations and consumer preferences can serve as an important reference for current wireless service providers.

This study has conveyed important insights for wireless service providers, consumers, and academic researchers. First, our analysis reminds Wi-Fi service providers of the pitfalls of overinvestment in service coverage. Aggressively expanding Wi-Fi service coverage may reduce a service provider's profit because WiMAX takes advantage of service coverage and available bandwidth. If a Wi-Fi service provider intends to invest in service coverage, it has to evaluate whether it can overcome the question of uncertain bandwidth resulting from the nature of Wi-Fi and confirm whether the cost of WiMAX capacity creates substantial barriers to bandwidth investment for a WiMAX service provider. Second, bandwidth sharing is an important means to enhance the profit of a WiMAX service provider when its capacity expense is not too high. The strategy not only slackens price competition but also raises the WiMAX service provider's bargaining power vis-à-vis fixed line suppliers. In addition, the investment in Wi-Fi service coverage is also beneficial to the WiMAX service provider. However, bandwidth sharing does not benefit wireless subscribers because it leads to higher prices for both access services. Finally, academic researchers can consider how to apply the concept of hardware integration to other services since we have shown that it can slacken the effect of Bertrand competition and can create a profit margin as respectable as that of channel collaboration.

In future research, we can study service differentiation in the wireless industry. These wireless service providers can offer different service rates or additional service value to consumers. In addition, we plan to further examine how the bargaining power held by the wireless operator affects bandwidth sharing and service strategies, since the WiMAX service provider in the current study holds the bargaining power such that the Wi-Fi service provider cannot gain substantial benefit from bandwidth sharing. Moreover, it is an interesting research issue that a Wi-Fi service provider may only partially utilize bandwidth sharing, such that only a subset of hotspots is connected to WiMAX (with a certain price of wireless backhaul), while the other hotspots still connect to T1 lines.

Finally, two popular strategies have recently been adopted by wireless ISPs to extend their service regions. One strategy is that the ISPs increase the number of hotspots by themselves, whereas the other approach is to delegate this task to partners or peers. The latter case can be demonstrated by FON (http://fon.com), one of the largest Wi-Fi communities in the world, which offers two ways of encouraging individuals to install hotspots and share bandwidth. First, users who install their own hotspots and share bandwidth are allowed free access to the community's FON spots worldwide. Second, FON users who are interested in making money may change their profile to "bill" and enter their PayPal account information so that the users not sharing bandwidth can choose to pay to use their hotspots. Therefore, we can explore the difference between these two strategies in future research.

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APPENDIX A: PROOFS

Proof of Proposition 1: Solving $\partial\pi_M/\partial p_M = 0$ and $\partial\pi_F/\partial p_F = 0$ yields $p_M^*(p_F)$ and $p_F^*(p_M)$. Equating $p_M^*(p_F)$ and $p_F^*(p_M)$ leads to p_M^* and p_F^* as follows:

$$p_F^* = \frac{c_a(n) + c_d + c_M + \beta K_M + 2c_F - \phi}{3},$$

$$p_M^* = \frac{2c_a(n) + 2c_d + 2c_M + 2\beta K_M + c_F + \phi}{3}.$$

Plugging p_M^* and p_F^* into θ^*, we have

$$\mu^* = \beta\left\{\frac{2c_a(n) + 2c_d - \beta K_M - c_M + c_F + \phi}{3(c_a(n) + c_d)}\right\}\eta_0 + \frac{1}{d}. \tag{A1}$$

We complete the proof by examining $\partial\mu^*/\partial\beta$. □

Proof of Proposition 2: With $\eta_F^* = \frac{c_a(n)+c_d+c_M+\beta K_M-c_F-\phi}{3(c_a(n)+c_d)}\eta_0$ and $\pi_F^* = \left(p_F^* - c_F\right)\eta_F^* - n\cdot\gamma_F$, we have

$$\begin{aligned}\frac{\partial\pi_F^*}{\partial n} &= \frac{\partial p_F^*}{\partial n}\eta_F^* + \left(p_F^* - c_F\right)\frac{\partial\eta_F^*}{\partial n} - \gamma_F \\ &= \left(1 - \frac{c_M - c_F + \beta K_M - \phi}{c_a(n) + c_d}\right)\frac{\eta_F^*}{3}\frac{\partial c_a(n)}{\partial n} - \gamma_F.\end{aligned} \tag{A2}$$

Obviously, owing to $\partial c_a(n)/\partial n < 0$, $\partial\pi_F^*/\partial n < 0$ for all n if $c_M + \beta K_M - \phi - c_F < c_d$ holds. Moreover, if the optimal n^* satisfies $\partial\pi_F/\partial n = 0$ and $\partial\pi_F^2/\partial n^2 < 0$, its necessary condition is $c_M + \beta K_M - \phi - c_F > c_a(n^*) + c_d$. □

Proof of Corollary 1: Moreover, if $\partial\pi_F/\partial n < 0$ for all n, the Wi-Fi service provider will choose the minimal number of hotspots n^* up to $U_M = 0$. As a result, we have $c_a(n^*) = (2V_M + V_F - c_F)/2 - (c_d + c_M + \beta K_M)$ by solving $p_F^*(p_M) = p_M^*(p_F)$ and $p_M^*(p_F) = V_M$ simultaneously. Accordingly, we can verify $\partial n^*/\partial V_M < 0$ and $\partial n^*/\partial V_F < 0$. By contrast, if there exists n^* so that $\partial\pi_F^2/\partial n^2 < 0$ and $\partial\pi_F/\partial n = 0$, we can verify $\partial n^*/\partial V_M > 0$ and $\partial n^*/\partial V_F < 0$ because of Equation (A2). □

Proof of Proposition 3: Because $\partial\pi_{MB}/\partial\omega > 0$ when $p_M(\omega) < V_M$, we can only consider $p_M^*(\omega) = V_M$ due to $\partial p_M^*(\omega)/\partial\omega > 0$. Accordingly, the profit of the

WiMAX service provider can be expressed as

$$\begin{aligned}\pi_{MB} &= (p_M - c_M)\,\eta_M - (\gamma_M + K_M\mu_B) + \omega\cdot\eta_F\\ &= (V_M - c_M - \beta K_M)\,\eta_0\left(1 - \frac{V_M - c_F - \omega - \phi}{2\,(c_a\,(n) + \hat{c}_d)}\right)\\ &\quad + \omega\cdot\eta_0\left(\frac{V_M - c_F - \omega - \phi}{2\,(c_a\,(n) + \hat{c}_d)}\right) - \left(\gamma_M + \left(n + \frac{1}{d}\right)K_M\right).\end{aligned}$$

Consequently, we can derive $\omega_1 = \frac{2V_M - c_M - c_F - \beta K_M - \phi}{2}$ from $\partial\pi_{MB}/\partial\omega = 0$. As long as $\pi^*_{FB}\,(\omega_1) \geq \pi^*_F$ holds, the value of ω^*_1 is valid. If $\pi^*_{FB}\,(\omega_1) < \pi^*_F$, the Wi-Fi service provider will reject the contract proposed by the WiMAX service provider. Thus, the WiMAX service provider has to let ω^* satisfy $\pi^*_{FB}\,(\omega^*) = \pi^*_F$. Solving $\pi^*_{FB}\,(\omega^*) = \pi^*_F$ is equivalent to solving the following equation:

$$\begin{aligned}&\left(\frac{c_a\,(n) + c_d + c_M + \beta K_M - c_F - \phi}{3\sqrt{c_a\,(n) + c_d}}\right)^2 - \left(\frac{V_M - c_F - \omega^* - \phi}{2\sqrt{c_a\,(n) + \hat{c}_d}}\right)^2\\ &\quad = \frac{n\cdot\left(\mathrm{T1} + K_F - \hat{K}_F\right)}{\eta_0}.\end{aligned} \tag{A3}$$

Therefore, we can derive ω_2 from the above equation when $\pi^*_{FB}\,(\omega_1) < \pi^*_F$. Consequently, we complete the proof by examining $\partial\omega_1/\partial\beta$ and $\partial\omega_2/\partial\beta$. ☐

Proof of Corollary 2: If the Wi-Fi service provider can gain a higher profit after bandwidth sharing, the WiMAX service provider can charge the highest wholesale price $\omega_1 = \frac{2V_M - c_M - c_F - \beta K_M - \phi}{2}$. Given $\omega^* = \omega_1$, the profit of the Wi-Fi service provider can be expressed as $\pi^*_{FB} = (\frac{c_M - c_F + \beta K_M - \phi}{4})^2\frac{\eta_0}{c_a(n) + \hat{c}_d} - n\cdot\gamma_{FB}$. By comparing this with $\pi^*_F = \frac{(c_a(n) + c_d + c_M - c_F + \beta K_M - \phi)^2}{9(c_a(n) + c_d)}\eta_0 - n\cdot\gamma_F$, we find $\pi^*_{FB} > \pi^*_F$ only when the cost of T1 is high enough because the first term of π^*_{FB} is less than that of π^*_F. Moreover, when $c_d \approx \hat{c}_d$ and $\pi^*_{FB} = \pi^*_F$, we can examine $\partial\omega^*/\partial n$ in Equation (A3) to complete the proof. ☐

Proof of Proposition 4: The capacity investment without bandwidth sharing after adopting channel competition is given by Equation (A1). The capacity investment without bandwidth sharing after adopting channel collaboration is given by

$$\mu^* = \beta\eta_0\left(\frac{2\,(c_a\,(n) + c_d) - \beta K_M - c_M + c_F + \phi}{2\,(c_a\,(n) + c_d)}\right) + \frac{1}{d}. \tag{A4}$$

Obviously, Equation (A4) yields a greater capacity investment than Equation (A1). Moreover, the optimal profit of the channel collaboration without bandwidth sharing is given by

$\pi^*_{(M+F)} = \left\{(V_M - c_M - \beta K_M) + \frac{1}{c_a(n) + c_d}\left(\frac{-c_M - \beta K_M + c_F + \phi}{2}\right)^2\right\}\eta_0 -$ $\left(\gamma_M + \frac{K_M}{d}\right) - n\cdot\gamma_F$. The optimal profit of the channel collaboration with bandwidth sharing is given by

$\pi^*_{(M+F)B} = \left\{ (V_M - c_M - \beta K_M) + \frac{1}{c_a(n)+\hat{c}_d}\left(\frac{-c_M-\beta K_M+c_F+\phi}{2}\right)^2 \right\}\eta_0 - \left(\gamma_M + \frac{K_M}{d}\right) - n\cdot(K_M+\gamma_{FB})$. Obviously, if $c_d > \hat{c}_d$ holds, $\pi^*_{(M+F)B} > \pi^*_{(M+F)}$ when $K_M < \mathrm{T1} + K_F - \hat{K}_F$ holds. Likewise, if $c_d < \hat{c}_d$ holds, $\pi^*_{(M+F)B} < \pi^*_{(M+F)}$ when $K_M > \mathrm{T1} + K_F - \hat{K}_F$ holds. □

Proof of Lemma 1: To begin with, we need to make sure that the WiMAX service provider can gain a higher profit under bandwidth sharing. From Proposition 3, we know that the optimal wholesale price for the WiMAX service provider is ω_1. In other words, from the perspective of the WiMAX service provider, bandwidth sharing is feasible only when $\pi^*_{MB}(\omega_1) \geq \pi^*_M$ holds. If the constraint holds, our next goal is to find the minimal wholesale price to satisfy $\pi^*_{MB} = \pi^*_M$. However, owing to the boundary condition $p^*_M(\omega) = V_M$, we need to calculate it by considering the following two cases. To begin with, we find $\underline{\omega}$ so that $p^*_M\left(\underline{\omega}\right) = V_M$, where $p^*_M(\omega)$ and $p^*_F(\omega)$ are given in Equation (10). Therefore, in the first case where $\omega \leq \underline{\omega}$, the WiMAX profit is given by

$$\pi^*_{MB}(\omega) = \frac{(2c_a(n) + 2c_d - \beta K_M + c_F - c_M + \phi)^2}{9(c_a(n)+c_d)}\eta_0 - \left(\gamma_M + \frac{K_M}{d}\right) - nK_M + \omega\eta_0. \quad \text{(A5)}$$

In the second case where $\omega \geq \underline{\omega}$, we have $p^*_M = V_M$ and $p^*_F(p_M) = \frac{V_M + c_F + \omega - \phi}{2}$ so that the WiMAX service provider's profit is given by

$$\pi^*_{MB}(\omega) = \left\{ (V_M - c_M - \beta K_M) - \frac{(V_M - \omega - c_M - \beta K_M)(V_M - \omega - c_F - \phi)}{2(c_a(n)+c_d)} \right\}\eta_0 - \left(\gamma_M + \left(n + \frac{1}{d}\right)K_M\right). \quad \text{(A6)}$$

By comparing $\pi^*_{MB}(\omega)$ in Equations (A5) and (A6) with π^*_M, we find

$$\omega_3 = \begin{cases} \omega', & \pi^*_{MB}(\underline{\omega}) \geq \pi^*_M \\ \hat{\omega}, & otherwise, \end{cases}$$

so that $\pi^*_{MB}(\omega_3) = \pi^*_M$, where ω' and $\hat{\omega}$ are derived from $\pi^*_{MB} = \pi^*_M$ in Equations (A5) and (A6), respectively. Note that there are two roots in $\hat{\omega}$ when $\pi^*_{MB}(\omega_1) \geq \pi^*_M$, which are denoted as $\hat{\omega}_1$ and $\hat{\omega}_2$ where $\hat{\omega}_1 < \hat{\omega}_2$; thus, $\hat{\omega} = \max(\hat{\omega}_1, 0)$. As a result, if the WiMAX service provider has an incentive to adopt bandwidth sharing, its minimal acceptable wholesale price must be greater than ω_3. However, according to Proposition 3, if $\omega_2 < \omega_1$ holds where $\pi^*_{FB}(\omega_2) = \pi^*_F$, the maximal wholesale price the WiMAX service provider can charge is ω_2. Consequently, both service providers are willing to adopt bandwidth sharing when $\pi^*_{MB}(\omega_1) \geq \pi^*_M$ and $\omega_3 \leq \mathrm{Min}(\omega_1, \omega_2)$, while the optimal wholesale price is given by $\omega^* = \mathrm{Min}(\omega_1, \omega_2)$. □

Proof of Proposition 5: Based on the proof of Proposition 4, we have the following result:

$$\pi^*_{(M+F)} < \pi^*_{(M+F)B} \Leftrightarrow \left\{ \frac{(\hat{c}_d - c_d)(-c_M - \beta K_M + c_F + \phi)^2}{4\,(c_a\,(n) + c_d)\,(c_a\,(n) + \hat{c}_d)} \right\} \eta_0$$
$$- n \cdot \left(\mathrm{T1} + K_F - \hat{K}_F - K_M\right) < 0.$$

Combining this with the result derived from Lemma 1, we complete the proof. ☐

APPENDIX B: ANNEX TO MARKET FORECLOSURE

The purpose of this annex is to examine if foreclosing the market for WiMAX is not profitable under certain conditions. Note that we cannot show the conditions in terms of the exogenous parameters of the model if we do not have the closed form of the optimal service coverage (that is, n^*). Therefore, we make an attempt at establishing the conditions in a special case in which such a deviation (i.e., market foreclosure) does not exist. Note that $p^*_F(p_M) = (p_M + c_F - \phi)/2$. Substituting it in θ^* yields $\frac{p_M - c_F - \phi}{2(c_a(n)+c_d)}$. Therefore, we have $\pi_F(p^*_F, p_M) = (\frac{p_M - c_F - \phi}{2\sqrt{c_a(n)+c_d}})^2 \eta_0 - n \cdot \gamma_F$. For $\pi_F(p^*_F, \hat{p}_M) = 0$, we have $\hat{p}_M = \phi + c_F + 2\sqrt{(c_a(n) + c_d)\frac{n \cdot \gamma_F}{\eta_0}}$. When $p_M = \hat{p}_M$, the Wi-Fi service provider quits the market so that the WiMAX service provider obtains its monopolistic profit. That is, $\hat{\pi}_M = (\hat{p}_M - c_M - \beta K_M)\eta_0 - (\gamma_M + \frac{K_M}{d})$. Note that the profit of the WiMAX service provider is given by $\pi_M = \frac{(2c_a(n)+2c_d - c_M + c_F - \beta K_M + \phi)^2}{9(c_a(n)+c_d)}\eta_0 - (\gamma_M + \frac{K_M}{d})$ under the equilibrium of pure competition. As a result, solving $\pi_M - \hat{\pi}_M \geq 0$ yields $\frac{(4\chi_1 - \chi_2)(\chi_1 - \chi_2)}{18\chi_1} \geq \sqrt{\frac{n \cdot \chi_1 \cdot \gamma_F}{\eta_0}}$, where $\chi_1 \equiv c_a(n) + c_d$ and $\chi_2 \equiv \phi + c_F - c_M - \beta K_M$. Next, we examine whether n^* can make the equation hold. Note that $\chi_1 - \chi_2 \geq 0$ because $\eta^*_F = \frac{\chi_1 - \chi_2}{3(c_a(n)+c_d)}\eta_0 \geq 0$. Thus, we examine the relation between n^* and η_0 to ensure $\pi_M - \hat{\pi}_M \geq 0$. If a corner solution exists, we solve $c_a(n^*) = (2V_M + V_F - c_F)/2 - (c_d + c_M + \beta K_M)$. Note that n^* is irrelevant to η_0. Accordingly, we can find a large η_0 to ensure $\pi_M - \hat{\pi}_M \geq 0$. If an interior solution exists, we solve $\{1 - \frac{c_M - c_F + \beta K_M - \phi}{c_a(n)+c_d}\}\frac{\eta^*_F}{3}\frac{\partial c_a(n^*)}{\partial n} - \gamma_F = 0$. Since n^* is associated with η_0, we need a closed form of n^* to examine the equation $\pi_M - \hat{\pi}_M$. Therefore, we consider a special case in which $c_a(n) = 1/n$ and $c_d \approx 0$. Consequently, we have $n^* \approx \sqrt{\frac{\eta_0}{\eta_0 \chi_2^2 - 9\gamma_F}}$. Likewise, we can find a large η_0 to ensure $\pi_M - \hat{\pi}_M \geq 0$. ☐

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