



## Real options game over the business cycle

Hsing-Hua Huang\*, Wei-Liang Chuang

Department of Information Management and Finance, National Chiao Tung University, Taiwan



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### ABSTRACT

This paper studies the impact of business cycles on firms' strategic investment decisions by developing and solving a continuous time regime-dependent real options game in an asymmetric duopoly. The value functions, roles and optimal investment timing decisions of the two firms in the expansion and recession states are jointly determined. We show that the preemptive investment equilibrium, where the leader invests earlier than its own first-best investment timing, is pro-cyclical. Moreover, the simultaneous investment equilibrium, where the firms simultaneously invest late and enjoy waiting flexibility as a tacit collusion, is counter-cyclical. In addition, we specifically demonstrate that the values of the leader and follower in the expansion state are smaller than those in the recession state when the preemptive equilibrium prevails in the expansion state and the simultaneous equilibrium prevails in the recession state.

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### 1. Introduction

Since two seminal papers of McDonald and Siegel (1986) and Majd and Pindyck (1987) and one pronounced book by Dixit and Pindyck (1994), the real option approach has become a standard tool for analyzing firms' investment decisions under uncertainty. Recently, literature pays more attention to respectively explore the impacts of macroeconomic conditions and product market competition on firms' investment decisions due to the stylized facts that a firm's investment policy is usually dependent on business cycle and is frequently affected by its competitors' investment decisions as in an oligopoly. The two effects must be analyzed in a unified dynamic model, but surprisingly real options literature has not yet investigated how the interactions between a firm's and its rivals' investment decisions vary with macroeconomic conditions. This paper intends to fill this gap by developing and solving a continuous-time regime-dependent real options game model which integrates the setup of business cycle from Guo et al. (2005) into an asymmetric duopoly real options game framework of Pawlina and Kort (2006). In particular, we can investigate the effects of business cycle on the equilibriums of an investment timing game and on the firms' optimal investment strategies as well as values.

Using Markov chain to model regime shifts, Guo et al. (2005) analyze a firm's optimal investment policy, taking account of the possibility of future macroeconomic condition shift. This methodology is recently employed to investigate various issues, such as capital structure (Bhamra et al., 2010; Hackbarth et al., 2006), credit risk (Chen, 2010),

and agency problem (Chen and Manso, 2010). However, they are all based on a single-firm assumption, and therefore ignore the interdependent effect of the firm's and its rivals' investment policies. In a more related paper, Du and Mackay (2011) analyze investment and disinvestment timing decisions in both monopoly and competitive markets when firms are subject to macroeconomic conditions. They particularly show that monopoly and competitive firms still adopt identical policies under some realistic environment. Yet, they do not investigate a firm's investment decision in an oligopoly where the firm competes with its rivals in investing the same investment project.

Assuming firms are symmetric in Cournot–Nash oligopoly equilibrium, Grenadier (2002) analyzes a firm's delay option on an incremental investment project, while Jou and Lee (2008) focus on that option on a lumpy investment project. In addition, Aguerrevere (2009), with the same assumption, specifically demonstrates the relationship between the degree of competition and the assets' expected rates of return varies with product market demand. As mentioned by Back and Paulsen (2009), the symmetric Nash equilibriums in the models do not satisfy the requirement of subgame perfection and hence are open-loop equilibriums.

Firms, however, are seldom identical. The extensive literature on real options games suggests that, when a relative small number of firms compete, there often exists a first-mover advantage (FMA). For example, winning patent races can be characterized by a persistent FMA, that is, the first to invent gains an exclusive right over the technology. The simple asymmetric duopoly equilibrium is often employed to analyze a firm's irreversible investment decision while the two firms have different investment costs. Pioneered by Fudenberg and Tirole (1985) that capture the threat of preemptive investment, Pawlina and Kort (2006) and Mason and Weeds (2010) examine the irreversible

\* Corresponding author at: No. 1001 University Rd., Hsinchu City 300, Taiwan. Tel.: +886 3 5712121x57056; fax: +886 3 5729915.

E-mail address: [hhuang@mail.nctu.edu.tw](mailto:hhuang@mail.nctu.edu.tw) (H.-H. Huang).

investment behavior when there is a competitor who can potentially preempt the investment project. They show that a greater FMA will lead a firm to adopt a preemptive investment threshold which is significantly lower than its optimal investment trigger. Recently, Carlson et al. (2011) focus on the effects of a firm's expansion and contraction options on risk dynamics of the required returns when there exists a rival firm owning the same flexibilities. In sum, they generally find that competition will erode the values of wait-and-see options and their Nash equilibriums meet the requirement of Markov perfect closed-loop equilibriums which satisfy continuous-time dynamic subgame perfection. Nevertheless, none of existing real options game literature takes macroeconomic conditions into consideration.

Some empirical studies show supportive evidence that competition precipitates investment. For example, Driver et al. (2008) show that a FMA of investment created by R&D and advertising expenditures offsets the irreversibility effect of investment. In particular, Akdoğan and MacKay (2008) indicate that the value of investing strategically can outweigh the value of waiting in an oligopolistic industry. On the other hand, some studies propose that both macroeconomic conditions and industry-specific competition play important roles in determining a firm's optimal investment decision. For example, Martynova and Rennegog (2008) provide further evidence that waves of corporate takeovers tend to occur following economic recovery from previous recessions.<sup>1</sup>

As pointed out by Ghemawat (2009), "At the bottom of the business cycle, firms seem to overemphasize the financial risk of investing at the expense of the competitive risk of not investing. Once-in-a-cycle errors of this sort can create a lasting competitive disadvantage." This calls for a new dynamic model to analyze a firm's investment decision that encompasses both macroeconomic conditions and industry competition. By integrating the business cycle framework of Guo et al. (2005) into the two-player real options game model of Pawlina and Kort (2006), we investigate the interdependent effects between macroeconomic conditions (expansion and recession) and industry-specific strategic interaction on firms' optimal investment timing decisions and firm's value functions in an asymmetric duopoly.

Theoretically, we develop and solve a continuous time real options game, where the regime-dependent value functions, roles and optimal investment timing decisions of the two firms are jointly determined. We specifically demonstrate that the preemptive investment equilibrium, where the leader invests earlier than its own first-best investment timing, is pro-cyclical, i.e., the leader tends to adopt a more aggressive strategy to preempt in the expansion state. In addition, the simultaneous investment equilibrium, where the firms simultaneously invest late and enjoy waiting flexibility as a tacit collusion, is counter-cyclical, i.e., the tacit collusion to invest late is more significant in the recession state. We particularly show that the values of the leader and follower in the expansion state are smaller than those in the recession state as the preemptive equilibrium prevails in the expansion state and the simultaneous equilibrium prevails in the recession state.

The paper is organized as follows. In section 2 we present the model setup and two special cases. Section 3 demonstrates value functions and solution concept and section 4 explains three types of game equilibriums and provides numerical examples. Finally, section 5 concludes.

## 2. The model

This section details the basic setup of our model. We employ the basic framework of Pawlina and Kort (2006) with the essential difference that we consider the two-state regime shifts rather than only one state to reveal the characteristic of the business cycle. The two

<sup>1</sup> Some mergers and acquisitions can result from a strategic motive to compete with industry rivals.

risk neutral firms, Firm 1 and Firm 2, compete in the product market and have a single investment opportunity to raise their instantaneous profits. The common uncertainty of the two firms' profits,  $x(t)$ , is governed by

$$dx_t = \mu_{\varepsilon_t} x_t dt + \sigma_{\varepsilon_t} x_t dW_t, \text{ given } x(0, \varepsilon_0) = x \geq 0, \quad (1)$$

where  $\mu_{\varepsilon_t}$  and  $\sigma_{\varepsilon_t}$  are the drift and diffusion terms, and  $W_t$  is a Wiener process.  $\varepsilon_t$  is a continuous time Markov chain with two states  $R$  (Recession) and  $E$  (Expansion). The intensity  $\lambda_R$  ( $\lambda_E$ ) shows the leaving rate of state  $R$  ( $E$ ) to state  $E$  ( $R$ ). Consequently,  $\mu_{\varepsilon_t}$  and  $\sigma_{\varepsilon_t}$  can be respectively explained as the industry growth rate and volatility which vary over business cycle. The riskless interest rate is  $r$ , and we assume that  $r - \mu_{\varepsilon_t} > 0$  for ensuring finite valuation. Following Guo et al. (2005), we assume  $\mu_E > \mu_R$  and  $\sigma_E < \sigma_R$ , and the relationships between the optimal investment triggers of the leader and follower in the expansion and recession states are given by:  $x_i^{L,E} < x_i^{L,R}$ ,  $x_i^{F,E} < x_i^{F,R}$ ,  $x_i^{L,E} < x_i^{F,E}$  and  $x_i^{L,R} < x_i^{F,R}$ ,  $i = 1, 2$ , showing that the leader and follower both invest earlier in expansion state, and the leader invest earlier than the follower in both states. For simplicity, we further assume that  $x_i^{L,E} < x_j^{L,R} < x_i^{F,E} < x_j^{F,R}$ ,  $i \neq j$ , where  $ij = 1, 2$ .

The instantaneous profits of firms are given by  $\pi_{mn} = xD_{mn}$ ,  $m, n = 0, 1$ , in an asymmetric duopoly, where  $D_{mn}$  stand for the deterministic part of profit function. The profits of the leader and follower are  $xD_{00}$  when the two firms have not invested and are  $xD_{11}$  when the two firms have already invested.  $xD_{10}$  and  $xD_{01}$  are respectively the profits of the leader and follower when the leader has invested and the follower has not. We assume that  $D_{10} > D_{00}$ ,  $D_{10} > D_{11}$ ,  $D_{11} > D_{01}$  and  $D_{00} > D_{01}$  to assure that there is a first-mover advantage and a second-mover disadvantage.

The two firms both face a perpetual, irreversible investment (growth) opportunity. Without loss of generality, we suppose the investment cost of Firm  $i$  is  $I_i$ ,  $i = 1, 2$ , where  $I_1 = I$  and  $I_2 = \kappa I$ ,  $\kappa > 1$ . Firm 1 is therefore the low-cost firm and Firm 2 is the high-cost firm. We also assume the initial realizations of the process underlying both firms' profits are low enough in both macroeconomic states so that immediate investment decisions are not optimal for both firms.

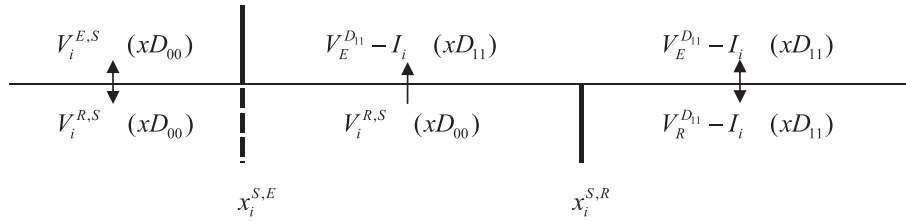
## 3. Value functions and investment thresholds

There are three possible investment timings for the two firms in both recession and expansion states. First, Firm  $i$  can invest first as the leader, and alternatively, Firm  $j$  can invest earlier and Firm  $i$  is hence the follower. Finally, the two firms can invest simultaneously. In this section, we will establish the two firms' value functions and investment thresholds associated with the three possibilities in two economic states. At the beginning, we analyze the case of simultaneous investment, which can be helpful to explain how possible regime shifts affect the firms' value functions and investment thresholds. Following the standard approach to solve a dynamic game backward in time, we subsequently introduce the leader's and follower's value functions and follower's optimal investment threshold when the leader has invested but the follower has not. Finally, we analyze the leader's and follower's value functions and leader's optimal investment threshold when both of the firms have not invested yet.

### 3.1. When the two firms invest simultaneously

Let  $V_i^{S,E}$  and  $V_i^{S,R}$  and  $x_i^{S,E}$  and  $x_i^{S,R}$ ,  $i = 1, 2$ , respectively denote the two firms' value functions and investment thresholds in states  $E$  and  $R$  when Firm 1 and Firm 2 invest simultaneously. Fig. 1 illustrates the relationships between the value functions and investment thresholds. When  $x \geq x_i^{S,R}$ , the two firms have already invested in both states and receive perpetual profit flows  $xD_{11}$ . The value functions  $V_E^{D_{11}}$  and  $V_R^{D_{11}}$  can switch between the two states even after the two firms have both invested.

In general,  $V_E^{D_{mn}}(x)$  and  $V_R^{D_{mn}}(x)$ ,  $m, n = 0, 1$ , denote the value functions when the two firm's instantaneous profit flow is given by  $xD_{mn}$



**Fig. 1.** The value functions and investment thresholds when the two firms invest simultaneously in the expansion and recession states. The vertical bold lines not only show that value-matching and smooth-pasting conditions hold, but also denote the optimal investment thresholds, while the vertical bold dotted line only shows that both value-matching and smooth-pasting conditions hold. The contents in parentheses are the cash flows for the corresponding value functions. The up-and-down arrows show that the value functions between the two states can two-way switch while the up arrow shows that the value function can only change one way.

in states  $E$  and  $R$ , respectively, which satisfy the following ordinary differential equations (ODE) system:

$$\begin{cases} rV_E^{D_{mn}} = xD_{ij} + \mu_E x \frac{\partial V_E^{D_{mn}}}{\partial x} + \frac{1}{2} \sigma_E^2 x^2 \frac{\partial^2 V_E^{D_{mn}}}{\partial x^2} + \lambda_E (V_R^{D_{mn}} - V_E^{D_{mn}}) \\ rV_R^{D_{mn}} = xD_{ij} + \mu_R x \frac{\partial V_R^{D_{mn}}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 V_R^{D_{mn}}}{\partial x^2} + \lambda_R (V_E^{D_{mn}} - V_R^{D_{mn}}) \end{cases} \quad (2)$$

Using four free boundary conditions in states  $E$  and  $R$ , the solutions of  $V_E^{D_{mn}}(x)$  and  $V_R^{D_{mn}}(x)$  are:

$$V_\varepsilon^{D_{mn}}(x) = xD_{mn}K_\varepsilon, \quad \varepsilon = E, R, \quad (3)$$

where  $K_E = \frac{(r+\lambda_R+\lambda_E-\mu_R)}{(r+\lambda_E-\mu_E)(r+\lambda_R-\mu_R)-\lambda_E\lambda_R}$  and  $K_R = \frac{(r+\lambda_R+\lambda_E-\mu_E)}{(r+\lambda_E-\mu_E)(r+\lambda_R-\mu_R)-\lambda_E\lambda_R}$ .

$V_\varepsilon^{D_{mn}}(x)$ , in fact, denotes the expected present value of perpetual profit flow  $xD_{mn}$  where the uncertainty of the profit flow can switch between expansion and recession states in the future. In other words,  $V_\varepsilon^{D_{mn}}(x) = E \left[ \int_0^\infty e^{-rt} x(t) D_{mn} dt \mid x(0) = x; \varepsilon(0) = \varepsilon \right], \varepsilon = E, R$ .

In view of Fig. 1, the two value functions,  $V_i^{S,E}$  and  $V_i^{S,R}$ , and the two investment thresholds,  $x_i^{S,E}$  and  $x_i^{S,R}$ , are interdependent and should be solved together.  $V_i^{S,E}$  and  $V_i^{S,R}$  must satisfy the following ODE system.

For  $x < x_i^{S,E}$ ,

$$\begin{cases} rV_i^{S,E} = xD_{00} + \mu_E x \frac{\partial V_i^{S,E}}{\partial x} + \frac{1}{2} \sigma_E^2 x^2 \frac{\partial^2 V_i^{S,E}}{\partial x^2} + \lambda_E (V_i^{S,R} - V_i^{S,E}) \\ rV_i^{S,R} = xD_{00} + \mu_R x \frac{\partial V_i^{S,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 V_i^{S,R}}{\partial x^2} + \lambda_R (V_i^{S,E} - V_i^{S,R}) \end{cases}, \text{ and} \quad (4)$$

for  $x_i^{S,E} \leq x < x_i^{S,R}$ ,

$$rV_i^{S,R} = xD_{00} + \mu_R x \frac{\partial V_i^{S,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 V_i^{S,R}}{\partial x^2} + \lambda_R (V_E^{D_{11}}(x) - I_i - V_i^{S,R}). \quad (5)$$

The general solutions of  $V_i^{S,E}$  and  $V_i^{S,R}$  are given by

$$V_i^{S,E}(x) = \begin{cases} V_E^{D_{00}}(x) + \sum_{z=1}^4 a_{iz}^E x^{\theta_z}, & x < x_i^{S,E} \\ V_E^{D_{11}}(x) - I_i, & x_i^{S,E} \leq x \end{cases}, \text{ and} \quad (6)$$

$$V_i^{S,R}(x) = \begin{cases} V_R^{D_{00}}(x) + \sum_{z=1}^4 a_{iz}^R x^{\theta_z}, & x < x_i^{S,R} \\ \frac{x D_{00}}{r + \lambda_R - \mu_R} + \frac{\lambda_R V_E^{D_{11}}(x)}{r + \lambda_R - \mu_R} - \frac{\lambda_R I_i}{r + \lambda_R} + b_{11} x^{\beta_1^R} + b_{12} x^{\beta_2^R}, & x_i^{S,E} \leq x < x_i^{S,R} \\ V_R^{D_{11}}(x) - I_i, & x_i^{S,R} \leq x \end{cases} \quad (7)$$

where  $\theta_1 > \theta_2 > 1 > 0 > \theta_3 > \theta_4$  are the four real roots of the following characteristic function.  $g_E(\theta)g_R(\theta) - \lambda_E\lambda_R = 0$ , and  $\beta_1^E > 1 > 0 > \beta_2^E$  are the two real roots of  $g_E(\beta) = 0$  in which  $g_\varepsilon(\beta) = (r + \lambda_\varepsilon) - \mu_\varepsilon\beta - \frac{1}{2}$

$\sigma_\varepsilon^2\phi(\phi-1)$ ,  $\varepsilon = E, R$ . All the unknown parameters can be solved by the following boundary conditions:

- (i)  $\lim_{x \downarrow 0} V_i^{S,E}(x) < \infty$  and  $\lim_{x \downarrow 0} V_i^{S,R}(x) < \infty$  (free boundary conditions);
- (ii)  $\lim_{x \uparrow x_i^{S,E}} V_i^{S,R}(x) = \lim_{x \uparrow x_i^{S,E}} V_i^{S,R}(x)$  and  $\lim_{x \uparrow x_i^{S,E}} \frac{\partial V_i^{S,R}(x)}{\partial x} = \lim_{x \uparrow x_i^{S,E}} \frac{\partial V_i^{S,R}(x)}{\partial x}$  (value-matching and smooth-pasting conditions at  $x_i^{S,E}$ );
- (iii)  $\lim_{x \uparrow x_i^{S,E}} V_i^{S,E}(x) = V_E^{D_{11}}(x_i^{S,E}) - I_i$  and  $\lim_{x \uparrow x_i^{S,R}} V_i^{S,R}(x) = V_R^{D_{11}}(x_i^{S,R}) - I_i$  (value-matching conditions at  $x_i^{S,E}$  and  $x_i^{S,R}$ ); and
- (iv)  $a_{i1}^R = \frac{\lambda_R}{g_R(\theta_1)} a_{i1}^E$  and  $a_{i2}^R = \frac{\lambda_R}{g_R(\theta_2)} a_{i2}^E$  (auxiliary conditions).

The optimal investment decisions of the two firms in states  $E$  and  $R$ ,  $x_i^{S,E}$  and  $x_i^{S,R}$ , are then jointly determined by the following two smooth-pasting conditions:  $\lim_{x \uparrow x_i^{S,E}} \partial V_i^{S,E}(x) / \partial x = D_{11}K_E$  and  $\lim_{x \uparrow x_i^{S,R}} \partial V_i^{S,R}(x) / \partial x = D_{11}K_R$ .

### 3.2. When the leader has invested but the follower has not

In this subsection, to solve the game backward, we first introduce the value functions of the leader and follower when the leader has invested but the follower has not. To make the following value functions of the leader and follower clearer, we assume that Firm  $i$  is the follower and Firm  $j$  is the leader without loss of generality.

Let  $\bar{V}_i^{F,E}$  and  $\bar{V}_i^{F,R}$ , and  $x_i^{F,E}$  and  $x_i^{F,R}$  respectively denote the follower's value functions and investment thresholds in states  $E$  and  $R$  when the leader has invested but the follower has not. The cash flow of the follower is  $xD_{01}$ . The value functions,  $\bar{V}_i^{F,E}$  and  $\bar{V}_i^{F,R}$ , can switch according to transition probability  $\lambda_E$  and  $\lambda_R$  when the follower has not invested in both states. Since the follower's optimal investment trigger in the expansion state is lower than that in the recession state, the follower may invest immediately in the expansion state, but have not yet invested in the recession state. In the recession state, the follower will invest either when  $x$  goes up and touches his optimal investment trigger or when the economy switch from the recession to expansion state. As a consequence,  $\bar{V}_i^{F,E}$  and  $\bar{V}_i^{F,R}$  satisfy the following ODE system.

For  $x < x_i^{F,E}$ ,

$$\begin{cases} r\bar{V}_i^{F,E} = xD_{01} + \mu_E x \frac{\partial \bar{V}_i^{F,E}}{\partial x} + \frac{1}{2} \sigma_E^2 x^2 \frac{\partial^2 \bar{V}_i^{F,E}}{\partial x^2} + \lambda_E (\bar{V}_i^{F,R} - \bar{V}_i^{F,E}) \\ r\bar{V}_i^{F,R} = xD_{01} + \mu_R x \frac{\partial \bar{V}_i^{F,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 \bar{V}_i^{F,R}}{\partial x^2} + \lambda_R (\bar{V}_i^{F,E} - \bar{V}_i^{F,R}) \end{cases}, \text{ and} \quad (8)$$

for  $x_i^{F,E} \leq x < x_i^{F,R}$ ,

$$r\bar{V}_i^{F,R} = xD_{01} + \mu_R x \frac{\partial \bar{V}_i^{F,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 \bar{V}_i^{F,R}}{\partial x^2} + \lambda_R (V_E^{D_{11}} - I_i - \bar{V}_i^{F,R}). \quad (9)$$

The general solutions of  $\bar{V}_i^{F,E}$  and  $\bar{V}_i^{F,R}$  are given by

$$\bar{V}_i^{F,E}(x) = \begin{cases} V_E^{D_{01}}(x) + \sum_{z=1}^4 c_{iz}^E x^{\theta_z}, & x < x_i^{F,E} \\ V_E^{D_{11}}(x) - I_i, & x_i^{F,E} \leq x \end{cases}, \text{ and} \quad (10)$$

$$\bar{V}_i^{F,R}(x) = \begin{cases} V_R^{D_{01}}(x) + \sum_{z=1}^4 c_{iz}^R x^{\theta_z}, & x < x_i^{F,E} \\ \frac{x D_{01}}{r + \lambda_R - \mu_R} + \frac{\lambda_R V_E^{D_{11}}(x)}{r + \lambda_R - \mu_R} - \frac{\lambda_R I_i}{r + \lambda_R} + d_{i1} x^{\beta_1^R} + d_{i2} x^{\beta_2^R}, & x_i^{F,E} < x < x_i^{F,R} \\ V_R^{D_{11}}(x) - I_i, & x_i^{F,R} \leq x. \end{cases} \quad (11)$$

All the unknown parameters can be solved by the following boundary conditions:

- (i)  $\lim_{x \rightarrow 0} \bar{V}_i^{F,E}(x) < \infty$  and  $\lim_{x \rightarrow 0} \bar{V}_i^{F,R}(x) < \infty$  (free boundary conditions);
- (ii)  $\lim_{x \uparrow x_i^{F,E}} \bar{V}_i^{F,R}(x) = \lim_{x \downarrow x_i^{F,E}} \bar{V}_i^{F,R}(x)$  and  $\lim_{x \uparrow x_i^{F,E}} \frac{\partial \bar{V}_i^{F,R}(x)}{\partial x} = \lim_{x \downarrow x_i^{F,E}} \frac{\partial \bar{V}_i^{F,R}(x)}{\partial x}$  (value-matching and smooth-pasting conditions at  $x_i^{F,E}$ );
- (iii)  $\lim_{x \uparrow x_i^{F,E}} \bar{V}_i^{F,E}(x) = V_E^{D_{11}}(x_i^{F,E}) - I_i$  and  $\lim_{x \uparrow x_i^{F,R}} \bar{V}_i^{F,R}(x) = V_R^{D_{11}}(x_i^{F,R}) - I_i$  (value-matching conditions at  $x_i^{F,E}$  and  $x_i^{F,R}$ ); and
- (iv)  $c_{i1}^R = \frac{\lambda_R}{g_R(\theta_1)} c_{i1}^E$  and  $c_{i2}^R = \frac{\lambda_R}{g_R(\theta_2)} c_{i2}^E$  (auxiliary conditions).

The optimal investment decisions of the follower,  $x_i^{F,E}$  and  $x_i^{F,R}$ , are jointly determined by the following smooth-pasting conditions:  $\lim_{x \uparrow x_i^{F,E}} \partial \bar{V}_i^{F,E}(x) / \partial x = D_{11} K_E$  and  $\lim_{x \uparrow x_i^{F,R}} \partial \bar{V}_i^{F,R}(x) / \partial x = D_{11} K_R$ .

Let  $\bar{V}_j^{L,E}$  and  $\bar{V}_j^{L,R}$  respectively denote the leader's value functions in states E and R when the leader has invested but the follower has not, and satisfy the following ODE system.

For  $x < x_j^{L,E}$ ,

$$\begin{cases} r\bar{V}_j^{L,E} = xD_{10} + \mu_E x \frac{\partial \bar{V}_j^{L,E}}{\partial x} + \frac{1}{2} \sigma_E^2 x^2 \frac{\partial^2 \bar{V}_j^{L,E}}{\partial x^2} + \lambda_E (\bar{V}_j^{L,R} - \bar{V}_j^{L,E}) \\ r\bar{V}_j^{L,R} = xD_{10} + \mu_R x \frac{\partial \bar{V}_j^{L,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 \bar{V}_j^{L,R}}{\partial x^2} + \lambda_R (\bar{V}_j^{L,E} - \bar{V}_j^{L,R}) \end{cases}, \text{ and} \quad (12)$$

for  $x_i^{F,E} \leq x < x_j^{L,R}$ ,

$$r\bar{V}_j^{L,R} = xD_{10} + \mu_R x \frac{\partial \bar{V}_j^{L,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 \bar{V}_j^{L,R}}{\partial x^2} + \lambda_R (V_E^{D_{11}}(x) - \bar{V}_j^{L,R}). \quad (13)$$

The general solutions of  $\bar{V}_j^{L,E}$  and  $\bar{V}_j^{L,R}$  are given by

$$\bar{V}_j^{L,E}(x) = \begin{cases} V_E^{D_{10}}(x) + \sum_{z=1}^4 e_{jz}^E x^{\theta_z}, & x < x_j^{L,E} \\ V_E^{D_{11}}(x), & x_i^{F,E} \leq x \end{cases}, \text{ and} \quad (14)$$

$$\bar{V}_j^{L,R}(x) = \begin{cases} V_R^{D_{10}}(x) + \sum_{z=1}^4 e_{jz}^R x^{\theta_z}, & x < x_j^{L,E} \\ \frac{x D_{10}}{r + \lambda_R - \mu_R} + \frac{\lambda_R V_E^{D_{11}}(x)}{r + \lambda_R - \mu_R} + f_{j1} x^{\beta_1^R} + f_{j2} x^{\beta_2^R}, & x_i^{F,E} < x < x_j^{L,R} \\ V_R^{D_{11}}(x), & x_i^{F,R} \leq x \end{cases}, \quad (15)$$

All the unknown parameters can be solved by the boundary conditions:

- (i)  $\lim_{x \rightarrow 0} \bar{V}_j^{L,E}(x) < \infty$  and  $\lim_{x \rightarrow 0} \bar{V}_j^{L,R}(x) < \infty$  (free boundary conditions);
- (ii)  $\lim_{x \uparrow x_j^{L,E}} \bar{V}_j^{L,R}(x) = \lim_{x \downarrow x_j^{L,E}} \bar{V}_j^{L,R}(x)$  and  $\lim_{x \uparrow x_j^{L,E}} \frac{\partial \bar{V}_j^{L,R}(x)}{\partial x} = \lim_{x \downarrow x_j^{L,E}} \frac{\partial \bar{V}_j^{L,R}(x)}{\partial x}$  (value-matching and smooth-pasting conditions at  $x_j^{L,E}$ );
- (iii)  $\lim_{x \uparrow x_j^{L,E}} \bar{V}_j^{L,E}(x) = V_E^{D_{11}}(x_j^{L,E})$  and  $\lim_{x \uparrow x_j^{L,R}} \bar{V}_j^{L,R}(x) = V_R^{D_{11}}(x_j^{L,R})$  (value-matching conditions at  $x_j^{L,E}$  and  $x_j^{L,R}$ ); and
- (iv)  $e_{j1}^R = \frac{\lambda_R}{g_R(\theta_1)} e_{j1}^E$  and  $e_{j2}^R = \frac{\lambda_R}{g_R(\theta_2)} e_{j2}^E$  (auxiliary conditions).

### 3.3. When the leader and follower have not invested

Let  $V_j^{L,E}$  and  $V_j^{L,R}$ , and  $x_i^{L,E}$  and  $x_i^{L,R}$  respectively denote the leader's value functions and investment thresholds in states E and R when the leader has not invested, satisfying the following ODE system.

For  $x < x_j^{L,E}$ ,

$$\begin{cases} rV_j^{L,E} = xD_{00} + \mu_E x \frac{\partial V_j^{L,E}}{\partial x} + \frac{1}{2} \sigma_E^2 x^2 \frac{\partial^2 V_j^{L,E}}{\partial x^2} + \lambda_E (V_j^{L,R} - V_j^{L,E}) \\ rV_j^{L,R} = xD_{00} + \mu_R x \frac{\partial V_j^{L,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 V_j^{L,R}}{\partial x^2} + \lambda_R (V_j^{L,E} - V_j^{L,R}) \end{cases}, \text{ and} \quad (16)$$

for  $x_j^{L,E} \leq x < x_j^{L,R}$ ,

$$rV_j^{L,R} = xD_{00} + \mu_R x \frac{\partial V_j^{L,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 V_j^{L,R}}{\partial x^2} + \lambda_R \left( \left( V_E^{D_{10}}(x) + \sum_{z=1}^4 e_{jz}^E x^{\theta_z} - I_j \right) - V_j^{L,R} \right). \quad (17)$$

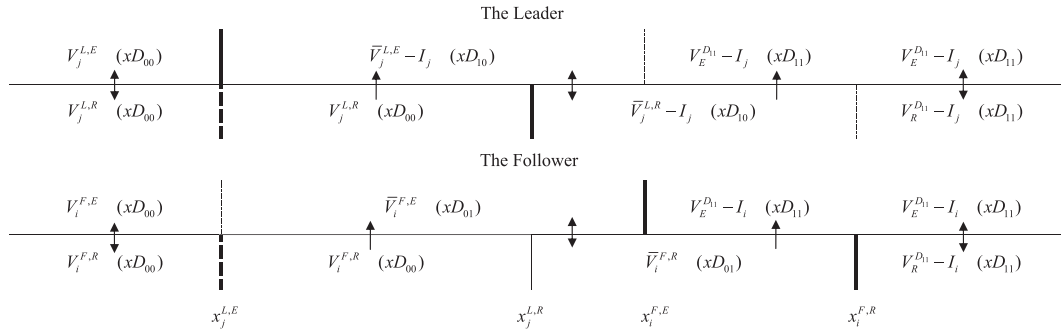
The general solutions of  $V_j^{L,E}$  and  $V_j^{L,R}$  are given by

$$V_j^{L,E}(x) = \begin{cases} V_E^{D_{00}}(x) + \sum_{z=1}^4 h_{jz}^E x^{\theta_z}, & x < x_j^{L,E} \\ \bar{V}_j^{L,E}(x) - I_j, & x_j^{L,E} \leq x < x_i^{F,E} \end{cases}, \text{ and} \quad (18)$$

$$V_j^{L,R}(x) = \begin{cases} V_R^{D_{00}}(x) + \sum_{z=1}^4 h_{jz}^R x^{\theta_z}, & x < x_j^{L,E} \\ \frac{x D_{00}}{r + \lambda_R - \mu_R} + k_{j1} x^{\beta_1^R} + k_{j2} x^{\beta_2^R} \\ + \lambda_R \left( \frac{V_E^{D_{10}}(x)}{r + \lambda_R - \mu_R} - \frac{I_j}{r + \lambda_R} + \sum_{z=1}^4 \frac{e_{jz}^E x^{\theta_z}}{g_R(\theta_z)} \right), & x_i^{F,E} < x < x_j^{L,R} \\ x_j^{L,E} \leq x < x_j^{L,R} \bar{V}_j^{L,R}(x) - I_j, & x_j^{L,R} \leq x < x_i^{F,R}. \end{cases}, \quad (19)$$

All the unknown parameters can be solved by the boundary conditions:

- (i)  $\lim_{x \rightarrow 0} V_j^{L,E}(x) < \infty$  and  $\lim_{x \rightarrow 0} V_j^{L,R}(x) < \infty$  (free boundary conditions);
- (ii)  $\lim_{x \uparrow x_j^{L,E}} V_j^{L,R}(x) = \lim_{x \downarrow x_j^{L,E}} V_j^{L,R}(x)$  and  $\lim_{x \uparrow x_j^{L,E}} \frac{\partial V_j^{L,R}(x)}{\partial x} = \lim_{x \downarrow x_j^{L,E}} \frac{\partial V_j^{L,R}(x)}{\partial x}$  (value-matching and smooth-pasting conditions at  $x_j^{L,E}$ );
- (iii)  $\lim_{x \uparrow x_j^{L,E}} V_j^{L,E}(x) = \bar{V}_j^{L,E}(x_j^{L,E}) - I_j$  and  $\lim_{x \uparrow x_j^{L,R}} V_j^{L,R}(x) = \bar{V}_j^{L,R}(x_j^{L,R}) - I_j$  (value-matching conditions at  $x_j^{L,E}$  and  $x_j^{L,R}$ ); and
- (iv)  $h_{j1}^R = \frac{\lambda_R}{g_R(\theta_1)} h_{j1}^E$  and  $g_{j2}^R = \frac{\lambda_R}{g_R(\theta_2)} g_{j2}^E$  (auxiliary conditions).



**Fig. 2.** The relationships of the value functions and investment thresholds between the leader and follower in the expansion and recession states. The vertical bold solid lines not only show that value-matching and smooth-pasting conditions hold, but also denote the optimal investment thresholds, while the vertical bold dotted lines only show that both value-matching and smooth-pasting conditions hold. The vertical dotted lines just denote the value-matching condition. The contents in parentheses are the cash flows for the corresponding value functions.

The optimal investment decisions of the leader,  $x_j^{L,E}$  and  $x_j^{L,R}$  in states  $E$  and  $R$ , are jointly determined by the following two smooth-pasting conditions:  $\lim_{x \uparrow x_j^{L,E}} \partial V_j^{L,E}(x) / \partial x = \lim_{x \downarrow x_j^{L,E}} \partial \bar{V}_j^{L,E}(x) / \partial x$  and  $\lim_{x \uparrow x_j^{L,R}} \partial V_j^{L,R}(x) / \partial x = \lim_{x \downarrow x_j^{L,R}} \partial \bar{V}_j^{L,R}(x) / \partial x$ .

Let  $V_i^{F,E}$  and  $V_i^{F,R}$  respectively denote the follower's value functions in states  $E$  and  $R$  when the leader and follower both have not invested, which satisfy the following ODE system.

For  $x < x_j^{L,E}$ ,

$$\begin{cases} rV_i^{F,E} = xD_{00} + \mu_E x \frac{\partial V_i^{F,E}}{\partial x} + \frac{1}{2} \sigma_E^2 x^2 \frac{\partial^2 V_i^{F,E}}{\partial x^2} + \lambda_E (V_i^{F,R} - V_i^{F,E}), \\ rV_i^{F,R} = xD_{00} + \mu_R x \frac{\partial V_i^{F,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 V_i^{F,R}}{\partial x^2} + \lambda_R (V_i^{F,E} - V_i^{F,R}), \end{cases} \text{ and} \quad (20)$$

for  $x_j^{L,E} \leq x < x_j^{L,R}$ ,

$$rV_i^{F,R} = xD_{00} + \mu_R x \frac{\partial V_i^{F,R}}{\partial x} + \frac{1}{2} \sigma_R^2 x^2 \frac{\partial^2 V_i^{F,R}}{\partial x^2} + \lambda_R \left( \left( V_E^{D_{01}}(x) + \sum_{z=1}^4 c_{iz}^E x^{\theta_z} \right) - V_i^{F,R} \right). \quad (21)$$

The general solutions of  $V_i^{F,E}$  and  $V_i^{F,R}$  are given by

$$V_i^{F,E}(x) = \begin{cases} V_E^{D_{00}}(x) + \sum_{z=1}^4 I_{iz}^E x^{\theta_z}, & x < x_j^{L,E} \\ \bar{V}_i^{F,E}(x), & x_j^{L,E} \leq x < x_i^{F,E} \end{cases} \text{ and} \quad (22)$$

$$V_i^{F,R}(x) = \begin{cases} V_R^{D_{00}} + \sum_{z=1}^4 I_{iz}^R x^{\theta_z}, & x < x_j^{L,E} \\ \frac{x D_{00}}{r + \lambda_R - \mu_R} + o_{i1} x^{\beta_1^R} + o_{i2} x^{\beta_2^R} \\ + \lambda_R \left( \frac{V_E^{D_{01}}(x)}{r + \lambda_R - \mu_R} + \sum_{z=1}^4 \frac{c_{iz}^E x^{\theta_z}}{g_R(\theta_z)} \right), & x_j^{L,E} \leq x < x_j^{L,R} \\ \bar{V}_i^{F,R}(x), & x_j^{L,R} \leq x < x_i^{F,R} \end{cases} \quad (23)$$

All the unknown parameters can be solved by the boundary conditions:

- (i)  $\lim_{x \rightarrow 0} V_i^{F,E}(x) < \infty$  and  $\lim_{x \rightarrow 0} V_i^{F,R}(x) < \infty$  (free boundary conditions);
- (ii)  $\lim_{x \uparrow x_j^{L,E}} V_i^{F,R}(x) = \lim_{x \downarrow x_j^{L,E}} V_i^{F,R}(x)$  and  $\lim_{x \uparrow x_j^{L,E}} \frac{\partial V_i^{F,R}(x)}{\partial x} = \lim_{x \downarrow x_j^{L,E}} \frac{\partial V_i^{F,R}(x)}{\partial x}$  (value-matching and smooth-pasting conditions at  $x_j^{L,E}$ );

- (iii)  $\lim_{x \uparrow x_j^{L,E}} V_i^{F,E}(x) = \bar{V}_i^{F,E}(x_j^{L,E})$  and  $\lim_{x \uparrow x_j^{L,R}} V_i^{F,R}(x) = \bar{V}_i^{F,R}(x_j^{L,R})$  (value-matching conditions at  $x_j^{L,E}$  and  $x_j^{L,R}$ ); and
- (iv)  $I_{j1}^R = \frac{\lambda_R}{g_R(\theta_1)} I_{j1}^E$  and  $I_{j2}^R = \frac{\lambda_R}{g_R(\theta_2)} I_{j2}^E$  (auxiliary conditions).

In sum, we employ Fig. 2 to illustrate the relationships among the leader's and follower's value functions and investment thresholds in states  $E$  and  $R$ . To solve the game backward, we first jointly solve the follower's value functions,  $V_i^{F,E}$  and  $V_i^{F,R}$ , and investment thresholds,  $x_i^{F,E}$  and  $x_i^{F,R}$ , in states  $E$  and  $R$  when the leader has invested but the follower has not. Second, given the follower's optimal investment thresholds, we then jointly derive the leader's value functions,  $V_j^{L,E}$  and  $V_j^{L,R}$ , in both states when the leader has invested but the follower has not. Third, we jointly solve the leader's value functions,  $V_j^{L,E}$  and  $V_j^{L,R}$ , and investment thresholds,  $x_j^{L,E}$  and  $x_j^{L,R}$ , in states  $E$  and  $R$  when the leader has not yet invested. Finally, given the leader's optimal investment thresholds, we jointly derive the follower's value functions,  $V_i^{F,E}$  and  $V_i^{F,R}$ , in both states when the leader has not yet invested.

The solution procedure mentioned above clearly demonstrates that the two firms' value functions and investment thresholds are not only relevant to the rival's decisions but also dependent on the macroeconomic conditions (the expansion and recession states) or business cycle. Until now, the two firms' roles (leader or follower) of the investment timing game are pre-specified, and will be endogenously determined later. We analyze the game equilibria in the following section.

#### 4. Equilibria and numerical examples

There are three types of the equilibria in states  $E$  and  $R$  due to the interaction between the two firms' investment timing decisions, namely preemptive, sequential and simultaneous equilibria.<sup>2</sup>

The first type of equilibrium, the preemptive equilibrium, occurs when the cost asymmetry between the two firms is relatively small. In other words, both of the firms have an incentive to be the leader. The low-cost Firm 1, therefore, must consider the possibility that Firm 2 can preempt the investment project. To analyze the preemptive equilibrium, we first define  $\xi_i^\varepsilon(x) \equiv (\bar{V}_i^{L,\varepsilon}(x) - I_i) - V_i^{F,\varepsilon}(x)$ ,  $i = 1, 2$  and  $\varepsilon = E, R$ . Next, we define  $x_2^{\varepsilon,E} = \inf\{x > 0 : \xi_2^\varepsilon(x) = 0\}$ ,  $\varepsilon = E, R$ , which is the lowest realization of  $x$  for a given regime  $\varepsilon$  so that Firm 2 is indifferent between the leader and follower. It is also the lowest investment trigger of  $x$  for a given regime  $\varepsilon$  so that Firm 2 still has an incentive to be the leader. If  $x_2^{\varepsilon,E}$  exists in the regime  $\varepsilon$ , then the prevailing preemptive equilibrium leads the low-cost Firm 1 to be the leader who chooses the optimal investment trigger as  $\min(x_1^{L,\varepsilon}, x_2^{\varepsilon,E})$  and Firm 2 to be the follower who chooses  $x_2^{\varepsilon,E}$ ,  $\varepsilon = E, R$ .

<sup>2</sup> The definitions of the three types of equilibria are mentioned in Pawlina and Kort (2006).

Secondly, the sequential equilibrium occurs when Firm 2 has no incentive to preempt to be the leader, i.e., when  $\xi_2^\varepsilon(x) < 0$  for all  $x < x_2^{\varepsilon,E}$ . In this case, the low-cost Firm 1 is still the leader whose optimal investment trigger is  $x_1^{\varepsilon,E}$ , while the follower, Firm 2, chooses  $x_2^{\varepsilon,E}$ ,  $\varepsilon = E, R$ .

The final type of equilibrium is the simultaneous equilibrium in which the two firms invest at the same time. If  $\zeta_1^\varepsilon(x) \leq 0$  where  $\zeta_1^\varepsilon(x) = V_1^{\varepsilon,E}(x) - V_2^{\varepsilon,E}(x)$ ,  $\forall x \leq x_1^{\varepsilon,E}$  and  $x_2^{\varepsilon,E} < x_1^{\varepsilon,E}$ , then the two firms invest simultaneously at  $x_1^{\varepsilon,E}$ ,  $\varepsilon = E, R$ . The above-mentioned condition shows that the two firms both prefer investing simultaneously to being the leader. Although this type of equilibrium is non-cooperative, it is often referred to as a tacit collusion since the two firms choose joint investment, implying that some implicit coordination to increase rents over their preemption level.

To illustrate the conditions for the three equilibria, we first define  $x_\varepsilon^*$  and  $\kappa_\varepsilon^* > 1$  as the solutions of  $\xi_2^\varepsilon(x; \kappa) = 0$  and  $\partial \xi_2^\varepsilon(x; \kappa) / \partial x = 0$ ,  $\varepsilon = E, R$ , and  $x_\varepsilon^{**}$  and  $\kappa_\varepsilon^{**} > 1$  as the solutions of  $\zeta_1^\varepsilon(x; \kappa) = 0$  and  $\partial \zeta_1^\varepsilon(x; \kappa) / \partial x = 0$ ,  $\varepsilon = E, R$ , and the conditions of three types of equilibria in state  $\varepsilon \in (E, R)$  are defined as: (i) the preemptive equilibrium is characterized by  $\kappa < \kappa^*(\varepsilon)$  and  $\kappa \geq \kappa^{**}(\varepsilon)$ ; (ii) the sequential equilibrium is characterized by  $\kappa \geq \kappa^*(\varepsilon)$  and  $\kappa \geq \kappa^{**}(\varepsilon)$ ; and (3) the simultaneous equilibrium is characterized by  $\kappa < \kappa^{**}(\varepsilon)$ .<sup>3</sup> It is worthwhile emphasizing that our results are not only related to macroeconomic conditions but also interdependent. Therefore, the above solutions and conditions of the two regimes need to be solved simultaneously.

Fig. 3 illustrates the regions of the three equilibria for different first-mover advantages and cost asymmetry, with the following basic parameters:  $r = 0.05$ ,  $\mu_E = 0.015$ ,  $\mu_R = 0.01$ ,  $\sigma_E = 0.1$ ,  $\sigma_R = 0.15$ ,  $\lambda_E = 0.1$ ,  $\lambda_R = 0.15$ ,  $I = 50$ ,  $D_{01} = 0.25$ ,  $D_{00} = 0.5$ , and  $D_{11} = 0.9$ . The level of first-mover advantage is defined as  $\gamma = D_{10}/D_{11}$  and the level of cost asymmetry is denoted as  $\kappa$ . For both expansion and recession regimes, there are three general observations. First, when the cost asymmetry is relatively small and there is no significant first-mover advantage, the two firms invest simultaneously (the southwest region). Second, when the first-mover advantage becomes significant and the cost asymmetry is insignificant, the low-cost Firm 1 prefers being the leader to investing simultaneously, resulting in the preemptive equilibrium (the southeast region). Third, when there is a significant cost asymmetry between the two firms, Firm 1 is the leader and the follower, Firm 2, does not have any incentive to preempt, leading to the sequential equilibrium.

In Fig. 3, we first demonstrate that the region of the simultaneous equilibrium in the recession state is larger than that in the expansion state. As mentioned in Boyer et al. (2005), the simultaneous equilibrium can be viewed as a tacit collusion. Both of the two firms tend to defer their investment and enjoy waiting flexibility, and the optimal investment timing is even later than the optimal investment timing when the high-cost Firm 2 is the follower. This shows that the tacit collusion to invest late is more significant in the recession state than in the expansion state, i.e., the simultaneous equilibrium (tacit collusion) is counter-cyclical. In addition, the region of the preemptive equilibrium becomes larger as in the expansion state. The leader in the preemptive equilibrium will invests earlier than its own single-firm first-best optimal investment timing, i.e., the preemptive equilibrium is pro-cyclical. It therefore implies that firms' strategic investments (e.g., patent races, mergers and acquisitions, new product introductions and new technology adoptions) are often pro-cyclical, which is generally in line with the real world observation that some larger firms are apt to merge or acquire firms before or just at the early beginning of economic recovery.

Fig. 4 displays the two firms' value functions of the three types of equilibria for different investment cost asymmetries. First, the relationships between the leader's and follower's value functions in the expansion and recession states are generally consistent with Pawlina and Kort (2006), and demonstrate that increasing high-cost Firm 2's

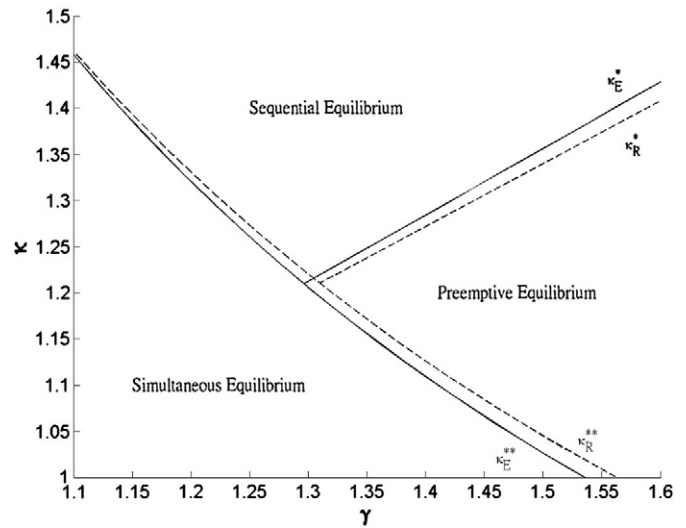


Fig. 3. The regions of the sequential, preemptive and simultaneous equilibria for various first-mover advantages and cost asymmetries. The first-mover advantage is defined as  $\gamma = D_{10}/D_{11}$  and  $\kappa$  denotes the cost asymmetry. The parameters are given by  $r = 0.05$ ,  $\mu_E = 0.015$ ,  $\mu_R = 0.01$ ,  $\sigma_E = 0.1$ ,  $\sigma_R = 0.15$ ,  $\lambda_E = 0.1$ ,  $\lambda_R = 0.15$ ,  $I_1 = 50$ ,  $I_2 = \kappa I_1$ ,  $D_{01} = 0.25$ ,  $D_{00} = 0.5$ , and  $D_{11} = 0.9$ .

investment costs ( $\kappa$  increases) can particularly raise its own firm value when the preemptive equilibrium prevails. Next, as mentioned above, the region of the preemptive equilibrium is enlarged in the expansion state, while the area of the simultaneous equilibrium becomes greater in the recession state. Third, in both states, the value functions of the leader are always higher than those of the follower, which is consistent with the case of single-state economy. Finally, the values of the leader and follower in the expansion state, as expected, are generally larger than those in the recession state. However, in the region that the preemptive equilibrium prevails in the expansion state while the simultaneous equilibrium prevails in recession state, the values in the expansion state are smaller than those in the recession state. The economic intuition is given as below. When the two firms tend to collude to invest late in the recession state and tend to compete in investing early in the expansion state, the values of the two firms which enjoy the waiting flexibility in

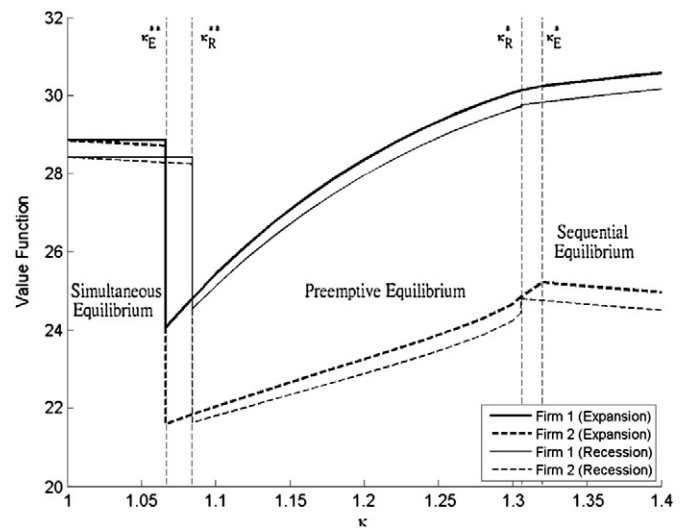


Fig. 4. The value functions of the two firms for various cost asymmetries.  $\kappa$  denotes the cost asymmetry, and the parameters are given by  $r = 0.05$ ,  $\mu_E = 0.015$ ,  $\mu_R = 0.01$ ,  $\sigma_E = 0.1$ ,  $\sigma_R = 0.15$ ,  $\lambda_E = 0.1$ ,  $\lambda_R = 0.15$ ,  $I_1 = 50$ ,  $I_2 = \kappa I_1$ ,  $D_{01} = 0.25$ ,  $D_{00} = 0.5$ ,  $D_{11} = 0.9$ ,  $\gamma = D_{10}/D_{11} = 1.45$  and  $x = 2$ .

<sup>3</sup> The proof of proposition is similar to that of Pawlina and Kort (2006) and thus omitted.

the recession state can be greater than those in the expansion state where the flexibility value is eroded by the rival's preemption.

## 5. Concluding remarks

This paper studies the impacts of business cycle on firms' strategic investment decisions in a duopolistic preemption game by integrating the business cycle framework of Guo et al. (2005) into the two-player real options game model of Pawlina and Kort (2006). We theoretically develop and solve a continuous time real options game, where the regime-dependent value functions, roles and optimal investment timing decisions of the two firms are endogenously determined. On one hand, we demonstrate that the preemptive investment equilibrium, where the leader invests earlier than its own first-best investment timing, is pro-cyclical. This may particularly provide an explanation why the strategically preemptive investments (e.g., patents, mergers and acquisitions, new product introductions and new technology adoptions) are usually considered as pro-cyclical indicators. On the other hand, the simultaneous investment equilibrium, where the firms simultaneously invest late and enjoy waiting flexibility together as a tacit collusion, is counter-cyclical. Furthermore, we show that the values of the leader and follower in the expansion state are smaller than those in the recession state when the preemptive equilibrium prevails in the expansion state and the simultaneous equilibrium prevails in the recession state.

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