Anomalous Hall effect from vortex motion in high-T_c superconductors

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In this work, the unusual Seebeck effect is taken into consideration in explaining the possible origin of the anomalous Hall effect for high-T_c superconductors. Combining Maki's theory of transport entropy and Tinkham's theory of resistive transition, we explain why the anomalous Hall effect can be observed in high-T_c superconductors, but is absent in most conventional superconductors. The behavior of $\rho_{xy}(H,T)$ in our theory is qualitatively consistent with experiments. In addition, our theory not only predicts that ρ_{xy} will become positive from $\rho_{xy}<0$ when the temperature is decreased in constant magnetic field, but also predicts that $|\rho_{xy}|\propto\rho_{xx}^2$ in the region of $\rho_{xy}<0$ and that the negative ρ_{xy} will diminish with increasing defect concentration.

I. INTRODUCTION

The early theories of flux flow developed by Bardeen and Stephen (the BS model)[1] and Nozieres and Vinen (the NV model)[2] predict that the Hall resistivity ρ_{xy} in the mixed state of a superconductor should be of the same sign in the normal state. This conclusion is valid in most conventional superconductors except Nb and V.[3–5] In high-T_c superconductors, however, ρ_{xy} usually shows a change of sign under low magnetic field and at temperatures near T_c.[6–13] This strange phenomenon is called the anomalous Hall effect.

Various theories have been proposed to explain the anomalous Hall effect, including the fluctuation model,[14] the two-band model,[15] and the flux-flow model proposed by Hagen *et al.*[9] Although these models can indeed predict the existence of the anomalous Hall effect, it is difficult to explain the qualitative behavior of ρ_{xy} (H,T) completely using these models.

Wang and Ting[16] developed a theory of flux motion with backflow current due to pinning centers. They suggested that the backflow current induces a negative Hall electric field in the region of vortex cores, causing the negative Hall effect to occur. On the other hand, high-T_c superconductors have the characteristic of high-T_c, short coherence length and strong anisotropy. So the vortex-antivortex pairs can be excited very easily and Kosterlitz-Thouless (KT) transition[17] occurs near T_c. This has been confirmed in experiments.[18,19] Motivated by this idea, Li and Zhang[20] proposed a vortex-antivortex model to explain the anomalous Hall effect. The above two models suggest that the anomalous Hall effect is related to pinning centers. This suggestion contradicts the experimental results of Budani, Liou, and Cai,[21] however, in which the negative ρ_{xy} was found to diminish with increasing defect concentration.

A thermoelectric model was proposed by Freimuth, Hohn, and Galffy.[22] According to their theory, however, the values of the thermal conductivity κ obtained from fitting experimental data are of order of 10^2 W/K m for $Bi_2Sr_2Ca_2Cu_3O_x$ and of order 0.2 W/K m for $YBa_2Cu_3O_{7-\delta}$. This is one or two orders of magnitude smaller than expected from direct measurements.[23,24]

Because none of the above models is completely satisfactory, in this paper we propose a new theory to try to explain experimental results concerning the anomalous Hall effect. Our theory relates the anomalous Hall effect to the unusual Seebeck effect for high-T_c superconductors, which is about 1000 times larger than expected from the conventional flux-creep–flow model.[25] This unusual Seebeck effect has been observed in many experiments.[12,13,26,27] Because the sign of the Seebeck voltage is the opposite of that of the flux-flow voltage, we believe the Seebeck voltage overcomes the flux-flow voltage, causing the anomalous Hall effect to occur. Therefore, the unusual Seebeck effect plays an important role in causing the anomalous Hall effect in high-T_c superconductors. This may be the reason the anomalous Hall effect can be observed in high-T_c superconductors but is absent in most conventional superconductors.

Our theory is based on the thermoelectric model in Ref. 22, but we modify the model to take into consideration the unusual Seebeck effect. Our theory offers a reasonable explanation for the anomalous Hall effect and agrees with experimental results qualitatively. Moreover, our theory explains why ρ_{xy} becomes positive from $\rho_{xy}<0$[11–13] when the temperature is decreased in a constant magnetic field, and it predicts that $|\rho_{xy}|\propto\rho_{xx}^2$ in the region of $\rho_{xy}<0$. This prediction is consistent with the results of the experiment of Lou, Orlando, and Graybeal.[28] This scaling behavior is also predicted by other theoretical models.[29] In addition, our theory can also explain the experimental result of Budhani, Liou, and Cai,[21] the negative ρ_{xy} diminishing with increasing defect concentration.

II. THEORY

The temperature gradient ∇T due to flux flow has been measured in Y-Ba-Cu-O single crystal.[23] In addition, the experimental results have shown that the unusual Seebeck effect and anomalous Hall effect occur near T_c together,[12,13] and the negative ρ_{xy} diminishes with increasing defect concentration.[21] We think that the anomalous Hall effect occurs mainly in the flux-flow region and is closely related to the unusual Seebeck effect, but not closely related to the pinning centers. As in Ref. 22, we assume the applied current density j_x is in the x direction in the mixed state of type-II superconductors. The Lorentz force will push vortices to move in the y direction. Then the heat current due to flux flow is given by

$$q_{v,y}=nTs_{\phi}V_{L,y} \ , \tag{1}$$

where $n=B/\phi_0$ is the vortex density and ϕ_0 is the flux quantum. s_{ϕ} is the entropy per unit length of a vortex, and $\mathbf{V}_L$ is the velocity of vortex motion. According to the Josephson relation,[30] $V_{L,y}$ is given by

$$V_{L,y}=-E_x/B=-j_x\rho_{xx}/B \ , \tag{2}$$

where ρ_{xx} is the flux-flow resistivity.

The heat current due to flux flow $q_{v,y}$ will establish the temperature gradient in the y direction $(\nabla T)_y$. In a stationary state $q_{v,y}$ is compensated by normal heat current $q_{n,y}$, that is,

$$q_{v,y}=-q_{n,y}=\kappa(\nabla T)_y \ , \tag{3}$$

where κ is the thermal conductivity. $(\nabla T)_y$ will cause an electric field in the y direction.

$$E_y=S(\nabla T)_y \ , \tag{4}$$

where S is the Seebeck coefficient in the mixed state. Then the Hall resistivity due to the direct effect of flux flow and the Seebeck effect caused by the flux motion in the y direction is

$$\rho_{xy}=\rho_{xx}\tan\theta_H=\rho_{xx}\tan\theta_f+S(\nabla T)_y/j_x \ , \tag{5}$$

where $\tan\theta_f$ is the Hall angle due to direct flux flow.

Combining Eqs. (1)–(3), we obtain the following expression for the transport line energy of a vortex:[23]

$$U_{\phi}=Ts_{\phi}=-\kappa\phi_0(\nabla T)_y/E_x \ . \tag{6}$$

By Maki's theory,[31] U_{ϕ} is given by

$$U_{\phi}=\phi_0\langle M\rangle L(T) \ , \tag{7}$$

where $\langle M\rangle=[H_{c2}(t)-H]/[1.16(2\kappa_{\rm GL}^2-1)+1]$ is the equilibrium magnetization, $\kappa_{\rm GL}$ is the Ginzburg-Landau parameter, and $L(T)$ is a transport coefficient of the order of l/ξ in the pure limit, with l being the mean free path. In this paper, we take $L(t)$ in the pure limit because of the short coherence length in high-T_c superconductors.

From Eqs. (5)–(7), we have

$$\rho_{xy}=\rho_{xx}[\tan\theta_f-S\langle M\rangle L/\kappa] \ . \tag{8}$$

The first term on the right-hand side of Eq. (8) is due to flux flow. It is contributed by normal electrons. According to the experimental results of Hagen, Lobb, and Greene,[11] we can assume $\tan\theta_f\propto H$, just as is the case for the Hall angle of normal conductors. From Ref. 26, the Seebeck coefficient in the mixed state is given by

$$S(T)=\frac{\rho_{xx}(T)}{\rho_n(T)}S_n(T) \ , \tag{9}$$

where ρ_n is normal resistivity and S_n is the Seebeck coefficient in the normal state. The magnetic-field dependence of ρ_n and S_n can be neglected. Furthermore, the temperature dependence of these two quantities can be neglected over a relatively small temperature range.[26] The change in κ is only $\sim 10\%$ over a wide temperature range close to T_c for various H.[12,23,32,33] So we can regard κ as a constant in the temperature region of interest. $L(T)\propto T^2$ approximately in the pure limit.[31,34] Therefore we can rewrite Eq. (8) in the following form:

$$\rho_{xy}(H,t)\propto\bar{\rho}_{xx}(H,t)\{H-C\bar{\rho}_{xx}(H,t)t^2[H_{c2}(t)-H]\} \ , \tag{10}$$

where $\bar{\rho}_{xx}$ is equal to ρ_{xx} normalized by the normal resistivity $\rho_n(T_c)$ and $t=T/T_c$ is the reduced temperature. C is an adjustable parameter. $\rho_{xx}(H,t)$ is given by Tinkham[35] qualitatively as

$$\rho_{xx}/\rho_n=\{I_0[A(1-t)^{3/2}2H]\}^{-2} \ , \tag{11}$$

where I_0 is the modified Bessel function and A is a fitting parameter. Combining Eqs. (10) and (11), we can describe the behavior of $\rho_{xy}(H,t)$ completely.

We adopt $A=1.2\times10^7$ $G\|c$ axis fitted by Tinkham,[35] and $T_c=92.2$ K, $dH_{c2}^{\|}/dT=-1.9$ T/K obtained from the experiment of Welp *et al.*[36] Then $H_{c2}(T)\approx175(1-t)$ tesla near T_c. We choose $C=4.5$. Figure 1 depicts ρ_{xy} as a function of temperature at various fixed magnetic fields H. We see that ρ_{xy} is indeed negative in the region of lower magnetic field and temperature below T_c. The negative Hall effect disappears when the magnetic field H increases. It is worth mentioning that ρ_{xy} will become pos-

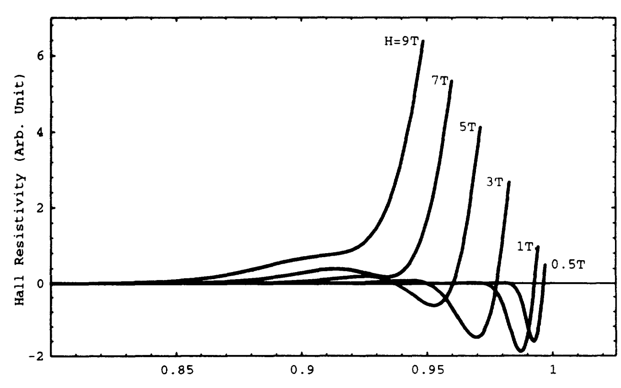

FIG. 1. Plot of Hall resistivity vs reduced temperature $t=T/T_c$ for various magnetic fields H. These curves end at $t=t_c(H)$.

itive from $\rho_{xy} < 0$ at some fixed magnetic field when T is decreased. This result has been observed in many experiments.[11–13]

Figure 2 shows the magnetic-field dependence of ρ_{xy} at constant temperature. When the temperature decreases from T_c, the range of the negative Hall effect ΔH becomes broader and broader until it is saturated and then becomes smaller and smaller until it disappears. The maximum of the negative Hall resistivity ρ_m also increases to saturation and then decreases until it disappears. The position of ρ_m shifts toward larger H. These results agree very well with experiments.[6–10,13]

In the region of $\rho_{xy} < 0$, the first term on the right-hand side of Eq. (8) is smaller than the second term. Thus we have $|\rho_{xy}| \propto \rho_{xx}^2$ when $\rho_{xy} < 0$. This has been observed in Ref. 28. Comparing the experimental data of Ref. 6 with Ref. 8, we find the maximum of the negative Hall angle $\theta_{H,\max} \approx 10^{-2}$ for high-quality Y-Ba-Cu-O single crystal and $\theta_{H,\max} \approx 10^{-3}$ for polycrystal Y-Ba-Cu-O bulk. This is consistent with $L(t) \sim l/\xi$ in Eq. (10). If l is shortened, i.e., the defect concentration is increased, $L(t)$ will become smaller and the negative ρ_{xy} will diminish. This prediction is consistent with the experimental results of Budhani, Liou, and Cai[21] Therefore, on the whole, our theory agrees with experiments qualitatively.

Hagen *et al.*[37] objected to the thermoelectric model by measuring Hall coefficient R_H and Seeback coefficient S for $Nd_{1.85}Ce_{0.15}CuO_{4-y}$.[38] As evidence against the thermoelectric model, they showed $R_H < 0$ and $S > 0$ for $x = 0.15$ (x is doping of Ce). But both R_H and S are positive for increasing Ce doping up to $x = 0.22$. Therefore we think their evidence against the thermoelectric model was insufficient. We suggest the anomalous Hall behavior for $Nd_{1.85}Ce_{0.15}CuO_{4-y}$ might be due to two types of carriers or anisotropic vortex flow.[39,40] But this is beyond the scope of our theory. In addition, Hagen *et al.*[37] referred to the importance of l/ξ for interpreting the anomalous Hall effect. Our theory also includes l/ξ in $L(t)$. But the reasons of anomalous Hall behaviors for Nb and V are not clear.[3–5] We suggest the anomalous Hall behaviors for Nb and V may be due to other mechanisms, because the behavior of ρ_{xy} for Nb and V is qualitatively unlike that of high-T_c superconductors.

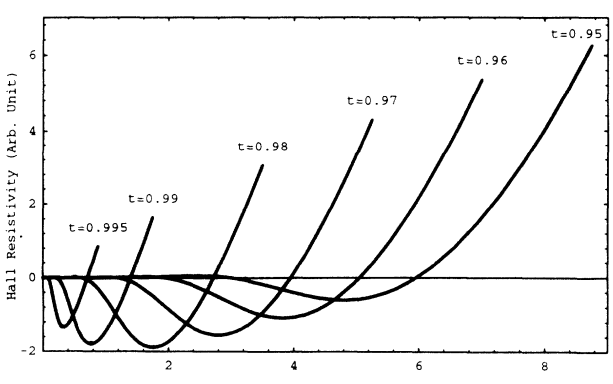

FIG. 2. Plot of Hall resistivity vs magnetic field H for various t. These curves end at $H = H_{c2}(t)$.

III. DISCUSSION

We will now discuss the difference between our theory and the theory of Freimuth, Hohn, and Galffy.[22] In the stationary state, the vortices move with a constant velocity V_L, which is determined by the balance of the driving force and the viscosity force. If we apply the Lorentz force as the driving force, the balance of forces is given by

$$-j_x\phi_0 = \eta V_L \ . \tag{12}$$

If the driving force is the thermal force, then

$$-s_\phi(\nabla T)_y = \eta V_L \ . \tag{13}$$

Substituting Eq. (12) into Eq. (13) and using $E_x = QB(\nabla T)_y$, Freimuth, Hohn, and Galffy[22] obtained

$$\frac{s_\phi}{\phi_0} = \frac{j_x}{(\nabla T)_y} = \frac{QB}{\rho_{xx}} \ , \tag{14}$$

where Q is the Nerst coefficient. In the original paper of Freimuth, Hohn, and Galffy[22] they used Eq. (14) in their derivation. The values of the thermal conductivity κ they obtained from fitting experimental data were one to two orders of magnitude smaller than those obtained from direct measurements. This is because Freimuth, Hohn, and Galffy neglected the importance of the unusual Seebeck effect in contributing to the anomalous Hall effect in high-T_c superconductors. The angle α between ∇T and $\mathbf{V}_L$ is very large in the mixed state of high-T_c superconductors. Near T_c, $\alpha \approx 60°-90°$ for $Bi_{1.76}Pb_{0.24}Sr_2Ca_2Cu_3O_\delta$, and $\alpha \approx 80°-90°$ for $Tl_2Ba_2Ca_2Cu_3O_\delta$.[13] Since the Seebeck effect is not observed in conventional superconductors, Eqs. (13) and (14) are valid for conventional superconductors. In contrast, they are not valid for high-T_c superconductors.

In our theory, the Seebeck coefficient is obtained directly from experimental results, and Maki's theory[31] is valid for conventional and high-T_c superconductors. So our theory avoids the discrepancy between experiments and theoretical fitting found in Ref. 22. From Eq. (8) we see that because the Seeback coefficient is very small in conventional superconductors, the anomalous Hall effect does not appear. In contrast, the Seebeck coefficient is large in high-T_c superconductors, so the anomalous Hall effect is observed.

IV. CONCLUSION

The anomalous Hall effect and unusual Seebeck effect are common phenomena in high-T_c superconductors. We have developed a theory that relates the unusual Seebeck effect to the anomalous Hall effect for high-T_c superconductors. Our theory explains why the anomalous Hall effect can be observed in high-T_c superconductors but is absent in most conventional superconductors. In this paper, we adopt some of the concept of the thermoelectric model proposed by Freimuth, Hohn, and Galffy[22] and combine Maki's theory of transport entropy[31] with Tinkham's theory of resistive transition.[35] Our theory is simplified by some estimates from experimental results.[11,23,26,32–34] If, on the other hand, $\rho_{xx}(H,T)$ and

$H_{c2}(T)$ can be obtained from direct measurements, our theory has only one parameter, C, to fit. Figures (1) and (2) show that our theory is qualitatively consistent with experiments. On the other hand, we predict that ρ_{xy} becomes positive from $\rho_{xy}<0$ when the temperature decreases and $|\rho_{xy}| \propto \rho_{xx}^2$ in the region of $\rho_{xy}<0$. This scaling behavior is also predicted by other theoretical models.[29] We also predict that the negative ρ_{xy} diminishes with increasing defect concentration, as observed by Budhani, Liou, and Cai.[21]

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