

# A Blind Fine Synchronization Scheme for SC-FDE Systems

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**Abstract**—This work presents a blind fine synchronization scheme, which estimates and compensates residual carrier-frequency offset (RCFO) and symbol timing offset (STO), for single-carrier frequency-domain equalization (SC-FDE) systems. Existing fine synchronization schemes for SC-FDE systems rely on time-domain unique words (UW) sequences as reference signals to assure the estimation accuracy, at the cost of decreased system throughput. The proposed technique, named simplified weighted least-square method for single-carrier systems (SWLS-SC), combines the decision feedback structure and SWLS estimator for OFDM systems. Together with specifically derived weighting factors, it has much better estimation accuracy than the well-known linear least-square (LLS) method for SC-FDE systems, and its BER performance can approach that of the ideal synchronization condition. The proposed technique is more effective than existing techniques, in terms of both performance and throughput. Theoretical estimation bounds are also derived to verify the effectiveness of the proposed method.

**Index Terms**—Synchronization operations, symbol timing offset, residual carrier-frequency offset, simple weighted least square, SC-FDE systems.

## I. INTRODUCTION

SINCE recent years, SC-FDE techniques have been popularly applied to many modern wireless communication systems, such as IEEE 802.11.ad [1] and IEEE 802.16 Wimax [2]. The data format of a SC-FDE system is generally composed of data blocks, each preceded with a cyclic prefix (CP) such that low-complexity and accurate frequency-domain channel equalization can be effectively applied. Similar to SC-FDE systems, distributed and localized single-carrier frequency-division multiple access (SC-FDMA) schemes adopted in LTE standard [17] also facilitate low-complexity frequency-domain equalization in combating intersymbol interferences [3]–[5]. Major advantages of SC-FDE over OFDM are its lower peak-to-average power ratio (PAPR) and less sensitivity to carrier-frequency offset (CFO).

However, compared to OFDM systems, problems and techniques of fine synchronization for SC-FDE systems are much less discussed in the literature. The issue of fine synchronization is mainly to estimate and compensate STO and RCFO, which can be estimated jointly by extracting the phases of

certain frequency-domain signals. Since accumulated STO and RCFO effects can significantly degrade system performances, in order to resolve this problem, various data-aided methods [6]–[13] have been proposed to perform fine synchronization in frequency domain, for either OFDM or SC-FDE systems. Techniques of linear least-square (LLS) [11] and simple weighted least square (SWLS) [8] were proposed to estimate STO and RCFO based on known reference signals, such as pilot subcarriers in OFDM systems. The SWLS method has been proven to be more robust to multipath channels than LLS method, because it weights the received signal with estimated power of channel frequency response such that the estimation error due to deep-fading frequency response can be minimized. Further, [12] proposes an ML estimation method on RCFO and STO for OFDM systems. However, these frequency-domain estimation methods are developed specifically only for OFDM systems by exploiting pre-defined pilot subcarriers but cannot be applied to SC-FDE systems. In order to apply SWLS concept to SC-FDE systems, the method in [9] performs joint RCFO and STO estimations by taking advantage of a special SC-FDE block format defined in [2]. Nevertheless, it only works for such a special block format, which requires multiple unique word (UW) sequences arranged sequentially in a row, and is not a general solution for the fine-synchronization problem of SC-FDE systems. More importantly, the insertion of multiple UWs causes degradation in system throughput.

Based on our preliminary work [18], this work generalizes the concept to SC-FDE systems and presents a blind technique, named SWLS-SC, for joint RCFO and STO estimations without needing additional UW sequences. In addition, a receiver structure based on the proposed SWLS-SC estimator is also illustrated in this work. The proposed technique mainly combines techniques of decision feedback [7] and SWLS estimator to perform fine synchronization. In addition, to further deal with random frequency responses of SC signals and multipath effects, a new method for refining weighting coefficients are also derived. Numerical performance analysis shows that SWLS-SC method is robust to severe multipath channels. Moreover, BER simulations demonstrate that the proposed blind synchronization scheme can produce results very close to those under ideal synchronization condition for light-of-sight (LOS) channels [14]. Theoretical estimation error of the proposed scheme is also derived and verified with simulations in order to demonstrate its effectiveness.

This paper is organized as follows. Section II describes the signal model of SC-FDE systems under effects of STO and RCFO. In Section III, an existing work [9] on applying SWLS method to SC-FDE systems is briefly discussed. The proposed

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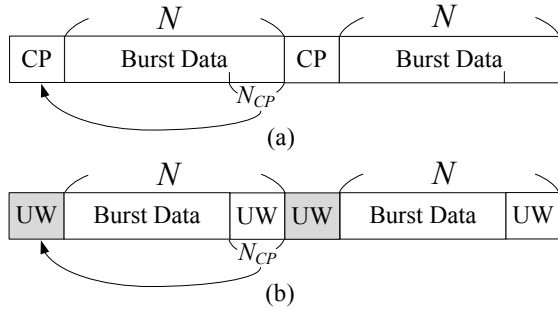


Fig. 1. Block structure of the (a) data block without UW, (b) data block with UW.

algorithm along with its theoretical performance analysis is presented and discussed in Section IV. Simulation results and conclusions are given in sections V and VI, respectively.

## II. SYSTEM MODEL FOR SC-FDE SYSTEMS WITH STO AND RCFO

Fig. 1 shows two different data block formats for a SC-FDE system. Fig. 1(a) illustrates a general format of a SC-FDE block where cyclic-prefix (CP) is a copy of the last  $N_{CP}$  samples in the block of  $N$  data samples. With the help of CP, frequency-domain equalization can be applied to such systems as in OFDM systems to lessen the multipath effect with low computational cost. However, unlike OFDM, SC-FDE systems do not have pilot subcarriers to help fine synchronizations in frequency domain. Therefore, as shown in Fig. 1(b), the predefined UW sequence is inserted and served as a reference sequence to ensure the estimation accuracy based on existing fine synchronization algorithms. However, for higher throughput consideration, this work only considers the data block format without UW sequences in Fig. 1(a), and only its system model is discussed in the remaining part of this section.

In Fig. 2, a basic block diagram of a SC-FDE system is shown. One of its major differences with conventional SC systems is the insertion of CP at the transmitter side as well as the CP removal at the receiver side. If denoting  $z(t)$  and  $T_s$  as the received signal subject to multipath fading effect  $h(t)$  and the duration of one symbol, respectively,  $z(t)$  can be written as

$$z(t) = \int_0^{(M-1)/T_s} h(\tau)x(t-\tau)d\tau + w(t), \quad (1)$$

where  $x(t)$ ,  $w(t)$  and  $M$  are the transmitted signal, the AWGN and the number of channel taps, respectively. By defining normalized RCFO and STO factors as  $\epsilon$  and  $\delta$ , the  $n$ -th received sample in the  $i$ -th block in the presence of fine synchronization error, RCFO  $\epsilon/(NT_s)$  and STO  $\delta T_s$ , is

$$\begin{aligned} z_i[n] &= e^{j2\pi[t\epsilon/(NT_s)]} z(t)|_{t=T_s(1+\delta)(i(N+N_{CP})+N_{CP})n} \\ &= (e^{j2\pi(i(N+N_{CP})+N_{CP})(1+\delta)n\epsilon/N} \\ &\quad \times \frac{1}{N} \sum_{m=-N/2+1}^{N/2} e^{j2\pi nm/N} e^{j2\pi n\delta m/N} \\ &\quad H_i[m]X_i[m]) + w_i[n], \text{ for } 0 \leq n \leq N-1, \end{aligned} \quad (2)$$

where  $H_i[m]$  and  $X_i[m]$  are the channel frequency response and transmitted signal at the  $m$ -th frequency point, respec-

tively. In addition,  $w_i[n] \sim \mathcal{CN}(0, N_0)$  denotes the time-domain noise. Furthermore, by taking discrete Fourier transform (DFT) of (2), the received sample in the  $k$ -th frequency point of the  $i$ -th block is

$$\begin{aligned} Z_i[k] &= (e^{j2\pi(i(N+N_{CP})+N_{CP})\phi_{kk}/N} e^{j2\pi(N-1)\phi_{kk}/N} \times \\ &\quad \lambda_N(\phi_{kk})H_i[k]X_i[k]) + I_i[k] + W_i[k], \end{aligned} \quad (3)$$

where  $W_i[k] \sim \mathcal{CN}(0, NN_0)$  and  $I_i[k]$  is the inter-carrier interference (ICI) given by

$$\begin{aligned} I_i[k] &= \sum_{m=-N/2+1, m \neq k}^{N/2} e^{j2\pi(i(N+N_{CP})+N_{CP})\phi_{km}/N} \\ &\quad e^{j2\pi(N-1)\phi_{km}/N} \times \lambda_N(\phi_{km})H_i[m]X_i[m]. \end{aligned} \quad (4)$$

In (3) and (4),  $\phi_{km}$  is defined as  $\phi_{km} \equiv (1+\delta)(\epsilon+m) - k$  and  $\lambda(\phi_{km})$  is:

$$\lambda_N(\phi_{km}) = \frac{\sin\{\pi[(1+\delta)(\epsilon+m) - k]\}}{N \sin\{\frac{\pi[(1+\delta)(\epsilon+m) - k]}{N}\}}. \quad (5)$$

Note that, in this work, RCFO is assumed to be relatively small as compared with the frequency sample spacing and, hence,  $I_i[k]$  is ignored. Besides, if  $\delta$  is also assumed to be small, then  $\phi_{kk}$  can be reduced to  $\epsilon + \delta k$ . As a result, (3) can be approximated as

$$\begin{aligned} Z_i[k] &\approx \frac{1}{N} e^{j2\pi(i(N+N_{CP})+N_{CP})\phi_{kk}/N} e^{j2\pi(N-1)(\epsilon+\delta k)/N} \\ &\quad H_i[k]X_i[k] + W_i[k], \end{aligned} \quad (6)$$

In (6), the effect of STO contributes to a slope change in the phase shift which is proportional to the frequency index  $k$ , while the phase shift caused by RCFO is independent of  $k$ .

## III. THE SWLS JOINT ESTIMATIONS OF STO AND RCFO FOR SC-FDE SYSTEMS

In this section, an SWLS fine synchronization estimator [9] for SC-FDE systems is briefly discussed. In Fig. 1(b), it is noted that there are two consecutive UW sequences, where the first one is the last  $N_{CP}$  samples of the first data block and the second one is CP of the second data block. Moreover, since the UW sequence is known in advance at the receiver, the preceding UW sequence can be served as the CP of its following UW sequence. SWLS estimator [9] takes  $N_{CP}$ -point DFT operation of the latter UW sequence for frequency-domain SWLS estimations. Similar to (6), one can obtain the following frequency-domain signal  $\tilde{Z}_i[k]$  by taking DFT operation of the  $i$ -th received UW sequence, and treating it as the CP of each data block:

$$\begin{aligned} \tilde{Z}_i[k] &= \lambda_{N_{CP}}(\phi_{kk}) e^{j2\pi i(N+N_{CP})\phi_{kk}/N_{CP}} \\ &\quad + \tilde{H}_i[k]\tilde{X}_i[k] + \tilde{W}_i[k], \end{aligned} \quad (7)$$

where  $\tilde{H}_i[k]$ ,  $\tilde{X}_i[k]$  and  $\tilde{W}_i[k]$  denote the channel frequency response, the UW sequence and combined effects of AWGN and ICI, respectively. Without loss of generality,  $\tilde{H}_i[k]$  is assumed to be relatively static between successively received blocks and  $\tilde{X}_i[k]$  (with its average power  $\tilde{E}_i[k]$ ) is also known at the receiver. Then, assuming negligible ICI effect, the information of  $\tilde{H}_i[k]$  can be extracted by taking the conjugate product of  $\tilde{X}_i[k]$  between two successively received

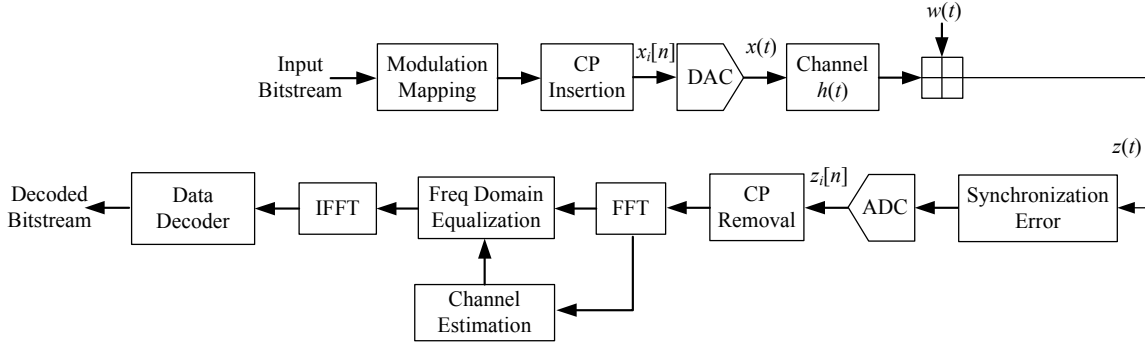


Fig. 2. A block diagram of conventional SC-FDE systems

data blocks as

$$\begin{aligned} \tilde{M}_i[k] &= \tilde{Z}_i[k] \tilde{Z}_{i-1}^*[k] \\ &\approx e^{j2\pi(N+N_{CP})\phi_{kk}/N_{CP}} |\tilde{H}_i[k]|^2 \tilde{E}_i[k] + \text{noise}, \end{aligned} \quad (8)$$

where the noise term is the sum of all other interfering effects. Subsequently, the total phase shift due to STO and RCFO can be obtained as

$$\tilde{\theta}_i[k] = \frac{N_{CP}}{2\pi(N+N_{CP})} \arg(\tilde{M}_i[k]) + \tilde{e}_i[k], \quad (9)$$

where  $\tilde{e}_i[k]$  denotes the overall phase error at the  $k$ -th frequency point of the  $i$ -th received block. To avoid effects of the direct-current offset, the technique in [9] excludes frequency samples at the DC with indices  $k \in \mathcal{K}$ , where  $\mathcal{K} = \{k : k = \pm 1, \pm 2, \dots, \pm Q\}$ . Parameter  $Q$  is chosen empirically as  $0.4N_{CP}$  as suggested in [9] based on the tradeoff between the computational complexity and estimation performance. As a result, the RCFO and STO estimates due to SWLS estimator are given as

$$\tilde{\epsilon}_i = \frac{N_{CP}}{2\pi(N+N_{CP})} \frac{\sum_{k \in \mathcal{K}} \tilde{C}_i[k] \tilde{\theta}_i[k]}{\sum_{k \in \mathcal{K}} \tilde{C}_i[k]}, \quad (10)$$

and

$$\tilde{\delta}_i = \frac{N_{CP}}{2\pi(N+N_{CP})} \frac{\sum_{k \in \mathcal{K}} k \tilde{C}_i[k] \tilde{\theta}_i[k]}{\sum_{k \in \mathcal{K}} k^2 \tilde{C}_i[k]}. \quad (11)$$

Coefficient  $\tilde{C}_i[k]$  is derived to be roughly equal to  $|\tilde{H}_i[k]|$ . The purpose of this weighting coefficient is to reduce the effect for those heavily-faded frequency samples owing to the frequency-selective channel.

However, when applying SWLS estimator to the frame structure as in Fig. 1(a), the assumption of constant signal power in (8) fails to hold, because transmitted time-domain symbols are randomly generated and independent between successive data blocks. Hence, SWLS estimator is not a general solution for all frame formats of SC-FDE systems.

#### IV. THE PROPOSED SWLS-SC ESTIMATOR FOR SC-FDE SYSTEMS

In this section, the proposed SWLS-SC estimator along with its corresponding receiver structure for the data block format of Fig. 1(a) will be introduced, followed by its theoretical performance analysis and discussion.

##### A. The SWLS-SC Estimator

Fig. 3 shows a receiver structure based on SWLS-SC estimator. Prior to minimum-mean-square-error frequency-domain equalizer (MMSE-FDE) [4][5], three additional modules, including an interpolator, a frequency compensator and DFT window adjustment, are required for fine synchronization, compared with Fig. 2. As shown, the frequency compensator module is to eliminate the RCFO effect in the received signal by utilizing the estimated RCFO. Besides, DFT window adjustment literally means to adjust the DFT sample window and remove CP from the received signal, by using the estimated STO. The interpolator serves to oversample the received signal in order to achieve better decoding performance for either SC or SC-FDE systems.

In Fig. 3,  $\hat{X}_i[k]$  is defined as the frequency-domain signal of  $\hat{x}_i[n]$ , and is equivalent to  $X_i[k] + P_i[k]$ , where  $P_i[k]$  denotes the overall decoding error. Different from SWLS estimator, the proposed estimator extracts RCFO and STO by taking conjugate product  $M_i[k]$  of  $\hat{X}_i[k]$  and  $Y_i[k]$  as shown below

$$\begin{aligned} M_i[k] &= \hat{X}_i^*[k] Y_i[k] \\ &\approx \frac{1}{N} e^{j2\pi(i(N+N_{CP})+N_{CP})\phi_{kk}} e^{j2\pi(i(N-1)(\epsilon+\delta k)/N)} \\ &\quad \times \frac{|H_i[k]|^2 \hat{X}_i^*[k] X_i[k]}{|H_i[k]|^2 + 1/\eta_i} + \frac{1}{N} \frac{H_i^*[k] \hat{X}_i^*[k] W_i[k]}{|H_i[k]|^2 + 1/\eta_i}. \end{aligned} \quad (12)$$

Next, assuming that the RCFO and STO of previously received block are well estimated and compensated, then (12) can be approximated as

$$\begin{aligned} M_i[k] &\approx \frac{1}{N} e^{j2\pi(i(N-1)(\epsilon+\delta k)/N)} \frac{|H_i[k]|^2 |\hat{X}_i^*[k]|^2}{|H_i[k]|^2 + 1/\eta_i} \\ &\quad + \frac{1}{N} \frac{H_i^*[k] \hat{P}_i^*[k] X_i[k]}{|H_i[k]|^2 + 1/\eta_i} \\ &\quad + \frac{1}{N} \frac{H_i^*[k] \hat{X}_i^*[k] W_i[k]}{|H_i[k]|^2 + 1/\eta_i} \\ &\quad + \frac{1}{N} \frac{H_i^*[k] \hat{P}_i^*[k] W_i[k]}{|H_i[k]|^2 + 1/\eta_i}. \end{aligned} \quad (13)$$

In high SNR conditions, is equal or very close to zero. Therefore, the extracted phase of (13) can be further simplified as

$$\begin{aligned} \hat{\theta}_i[k] &= \arg(M_i[k]) \\ &\approx \frac{2\pi(i(N-1)(\epsilon+\delta k)}{N} + \hat{e}_i[k], \end{aligned} \quad (14)$$

where  $\hat{e}_i[k]$  is the overall phase error due to AWGN and decoding error. Similar to (10) and (11), estimated RCFO and

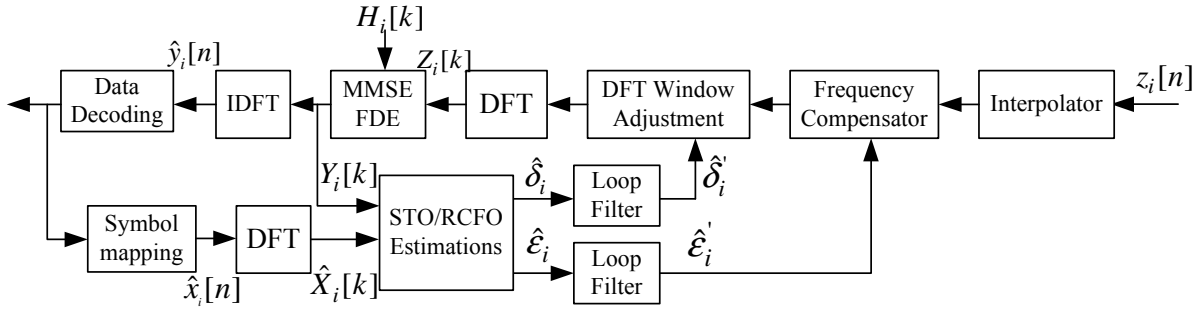


Fig. 3. The receiver structure based on the SWLS-SC estimator for SC-FDE systems.

STO due to SWLS-SC estimator can be derived as

$$\hat{\epsilon}_i = \frac{N}{2\pi(N-1)} \frac{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} C_i[k] \hat{\theta}_i[k]}{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} C_i[k]}, \quad (15)$$

and

$$\hat{\delta}_i = \frac{N}{2\pi(N-1)} \frac{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} k C_i[k] \hat{\theta}_i[k]}{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} k^2 C_i[k]}, \quad (16)$$

where

$$C_i[k] \approx |H_i[k]|^2 \frac{|\hat{X}_i[k]|^2}{|\tilde{W}_i[k]|^2} = |H_i[k]|^2 \hat{\eta}_i[k], \quad (17)$$

and  $\hat{\eta}_i[k]$  denotes the estimated SNR at the  $k$ -th frequency point. As given in the Appendix, (17) shows the quasi-optimal weighting coefficient for SWLS-SC estimator. In addition to reducing estimation error in frequency-selective channels, (17) also contains instantaneous SNR information, which can enhance estimation accuracy particularly at the locations of deep frequency notches resulted from the transmitted SC signal itself. Besides, estimations of (15) and (16) degenerates to an LLS estimation only when is equal to 1 for all  $k$ .

In order to further increase the estimation accuracy, the technique of closed-loop tracking is incorporated and the resultant estimated RCFO and STO are

$$\hat{\epsilon}'_i = \hat{\epsilon}_i + \alpha_\epsilon \hat{\epsilon}_i, \quad (18)$$

and

$$\hat{\delta}'_i = \hat{\delta}_i + \alpha_\delta \hat{\delta}_i, \quad (19)$$

Finally, the estimated RCFO and STO are then passed to modules of frequency compensator and DFT window adjustment for fine synchronization processing.

### B. Theoretical Performance Analysis

If denoting the average amplitude of  $x_i[n]$  in Fig. (2) as  $\sqrt{E_s}$  for all  $n$ , then the average power of  $X_i[k]$  in (2) can be shown to be  $NE_s$ . In (13), the STO and RCFO of previously received block are assumed to be well compensated and negligible in the performance analysis. Under AWGN channel ( $H_i[k]=1$ ) and error-free reference feedback signal  $M_i[k]$  becomes

$$M_i[k] \approx \frac{e^{j2\pi(N-1)(\epsilon+\delta k)/N}}{N} \left( \frac{\eta_i}{1+\eta_i} \right) |X_i[k]|^2 + \tilde{W}_i[k], \quad (20)$$

where

$$\tilde{W}_i[k] = \frac{1}{N} \left( \frac{1}{1+1/\eta_i[k]} \right) X_i^*[k] W_i[k]. \quad (21)$$

In order to analyze the mean-square-error (MSE) of RCFO estimation, one can firstly multiply (20) with a normalization factor  $\alpha$ , which is defined as

$$\alpha \equiv N e^{-j(2\pi(N-1)\epsilon)/N} \frac{(1+\eta_i)/\eta_i}{|X_i[k]|^2}. \quad (22)$$

Then, (20) becomes

$$\alpha M_i[k] \approx e^{j(2\pi(N-1)\delta k)/N} + \alpha \tilde{W}_i[k], \quad (23)$$

where  $\alpha \tilde{W}_i[k] \sim \mathcal{CN}(0, N_0/E_s)$ . Note that roughly only STO factor and scaled noise remain in (23), which constitute the error in estimating the RCFO. Next, if  $\delta$  is assumed to be small enough, the RCFO estimation error  $e_{\epsilon,i}$  for the  $i$ -th received block can be expressed as

$$e_{\epsilon,i} \equiv \hat{\epsilon}_i - \epsilon = \frac{N}{2\pi(N-1)} \frac{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} C_i[k] \angle(\alpha M_i[k])}{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} C_i[k]}. \quad (24)$$

If further assuming that weighting coefficient  $C_i[k]$  can be averagely approximated as  $E_s/N_0$ , then (24) can be rewritten as

$$e_{\epsilon,i} \approx \frac{N}{2\pi(N-1)} \frac{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} \frac{E_s}{N_0} \angle(\alpha M_i[k])}{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} \frac{E_s}{N_0}} \approx \frac{N}{2\pi(N-1)^2} \sum_{k=-N/2+1, k \neq 0}^{N/2-1} \Im(\alpha M_i[k]) \approx \frac{N}{2\pi(N-1)^2} \sum_{k=-N/2+1, k \neq 0}^{N/2-1} \Im(\alpha \tilde{W}_i[k]) \quad (25)$$

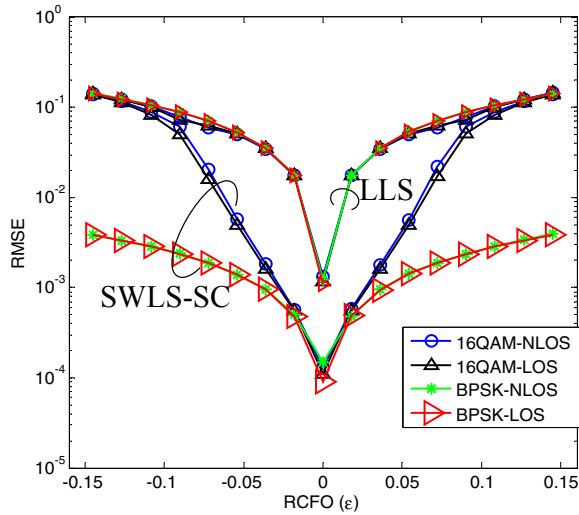
where  $\Im(\bullet)$  denotes the imaginary part of a complex variable. The derivation reduction to the last line of (25) is by using the approximation  $\sin(\theta) \approx \theta$  due to small STO. Then, knowing that  $\Im(\alpha \tilde{W}_i[m]) \sim \mathcal{N}(0, N_0/2E_s)$ , one can show that  $e_{\epsilon,i}$  is unbiased. As a result, the MSE for one-shot RCFO estimation error is

$$\text{MSE}(\hat{\epsilon}_i) \equiv E[e_{\epsilon,i}^2] = \frac{N^2}{8\pi^2(N-1)^3(E_s/N_0)} \quad (26)$$

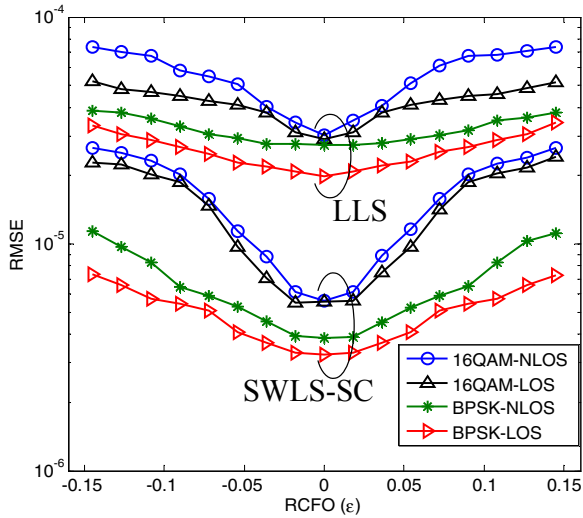
Similarly, one can also follow procedures from (23)-(26) to obtain the MSE for one-shot STO estimation as

$$\text{MSE}(\hat{\delta}_i) \equiv E[e_{\delta,i}^2] = \frac{3N}{2\pi^2(N-1)^3(N-2)(E_s/N_0)} \quad (27)$$

Following (26) and (27), theoretical MSEs of the incorporated closed-loop tracking scheme can also be derived by referring



(a)



(b)

Fig. 4. RMSEs of (a) RCFO and (b) STO estimations versus RCFO, in stationary multipath fading channels, SNR is 24dB and  $\delta$  is 20ppm.

to [16]. Therefore, in the case of small tracking error, steady-state MSEs for  $\hat{\epsilon}'_i$  and  $\hat{\delta}'_i$  are

$$\text{MSE}(\hat{\epsilon}'_i) = \frac{\alpha_\epsilon^2}{2 - \alpha_\epsilon} \text{MSE}(\hat{\epsilon}_i) \quad (28)$$

and

$$\text{MSE}(\hat{\delta}'_i) = \frac{\alpha_\delta^2}{2 - \alpha_\delta} \text{MSE}(\hat{\delta}_i) \quad (29)$$

## V. SIMULATION RESULTS AND ANALYSIS

The simulation profile is listed in Table I based on the system parameters of the IEEE 802.11.ad SC mode. In the simulation, normalized minimum square error (NMSE) as defined by (30) of the reference feedback signal is firstly analyzed.

$$\text{NMSE} = \frac{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} |\hat{X}_i[k] - X_i[k]|^2}{\sum_{k=-N/2+1, k \neq 0}^{N/2-1} |X_i[k]|^2}. \quad (30)$$

TABLE I  
SIMULATION PROFILE

Channel Models	CM1.1(LOS) and CM2.1(NLOS) [12]
Symbol Rate	1.7GHz for SC mode
Equalizer	MMSE-FDE
DFT Length	512
Cyclic Prefix	128
$\alpha_\epsilon$ and $\alpha_\delta$	0.1

Subsequently, in order to evaluate estimation performance, root-mean-square error (RMSE) is adopted as the comparison measure. However, since SWLS estimator for SC systems cannot be performed without inserting known sequences, this work only compares LLS estimator as well as the perfect synchronization scenario for evaluating the effectiveness of the proposed scheme. Next, LOS and non-LOS (NLOS) channel models defined in [14] are adopted to simulate the robustness of SWLS-SC under severe channel conditions.

Fig. 4 depicts RMSEs of joint RCFO and STO estimations under different levels of RCFOs where STO and SNR are fixed at 20ppm and 24dB, respectively. In Fig. 4(a), simulation results due to LLS estimator almost overlap with each other regardless of channel conditions and modulation schemes. The reason is that, the frequency response of the reference signal is random and deeply faded at certain frequency points where their SNRs are not high enough for accurate estimation and the performance is thus severely degraded. Note that, by employing SWLS-SC estimator, RMSE performance is improved by nearly an order of magnitude, compared with LLS method when RCFO is small, because weighting coefficients effectively reduce the estimation error caused by either the multipath fading effect or the reference signal itself. However, for the case of 16QAM with SWLS-SC, the RMSE of the estimated STO increases with the CFO value, because the constellation of 16QAM is more sensitive to ICI effect, and hence, the quasi-optimality of the weighting coefficients in (17) is decreased. Fig. 4(b) compares estimated STOs in different channel conditions and modulation schemes. The results also clearly show that the proposed scheme is much less sensitive to ICI effect than LLS estimator regardless of channel conditions.

In addition, similar simulations were also conducted to compare STO and RCFO RMSE performances versus STO in Fig. 5, in which a constant RCFO,  $\epsilon = 0.02$ , is injected and the SNR is fixed at 24dB. Results show that the proposed SWLS-SC estimator outperforms LLS estimator for both estimations in low ICI conditions. Particularly, one can observe that STO and RCFO estimates are less sensitive to STO even when STO is 40ppm. The main reason is that the ICI effect due to STO is only about several tens ppm of the sub-frequency spacing, comparing with the amount caused by the given RCFO effect,  $\epsilon = 0.02$  which is also equal to 20000ppm of the sub-frequency spacing.

Next, joint estimation performances versus SNR as well as their theoretical bounds are provided in Fig. 6. Due to multipath channels, both LLS and SWLS-SC estimators deviate from the theoretical bounds. In Fig. 6(a), SWLS-SC

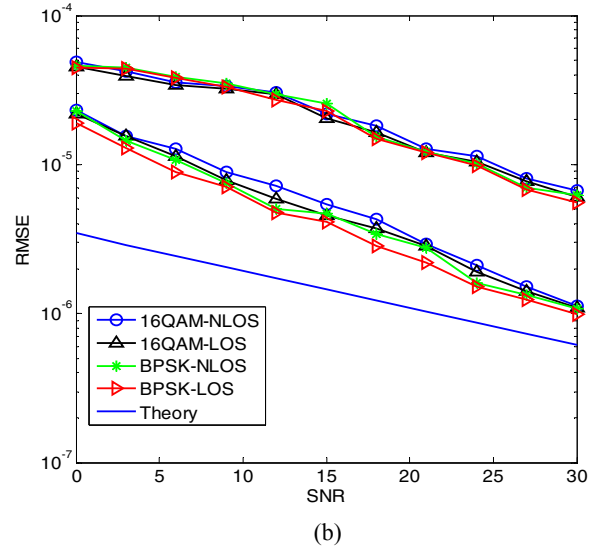
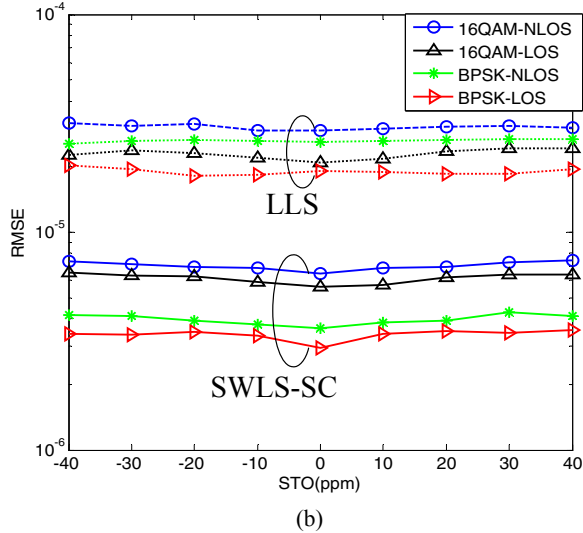
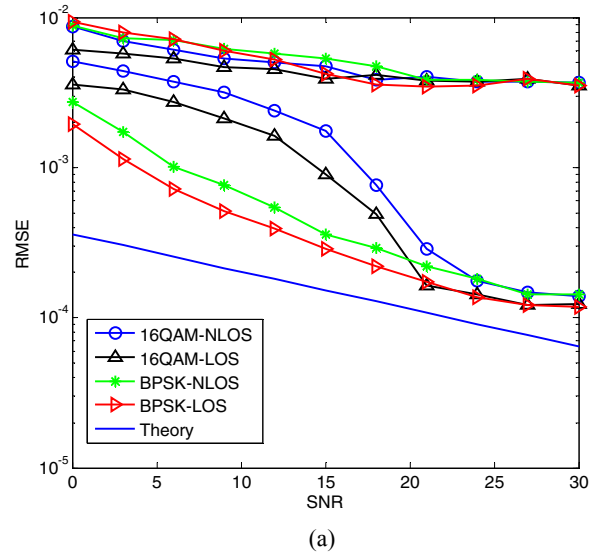
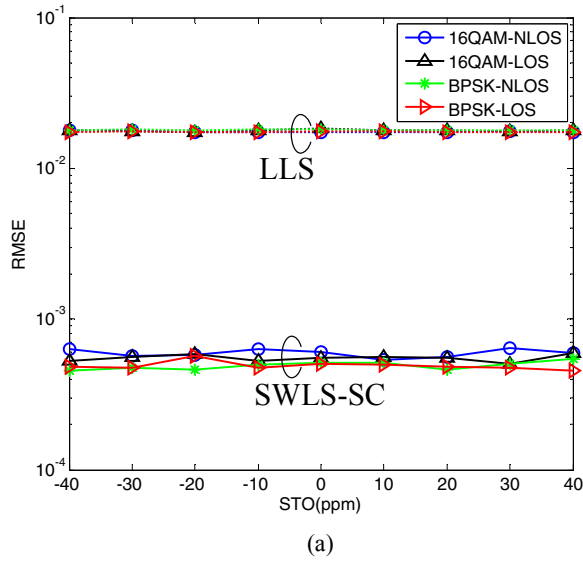


Fig. 5. RMSEs of (a) RCFO and (b) STO estimations versus STO, in stationary multipath fading channels, SNR is 24dB and  $\epsilon$  is 0.02.

Fig. 6. RMSEs of (a) RCFO and (b) STO estimations versus SNR, in stationary multipath fading channels,  $\epsilon = 0.02$  and  $\delta = 20$ ppm.

estimator is shown to perform much closer to the theoretical bounds especially in high SNR regions. However, for the 16QAM case, its RCFO estimation performance degrades to LLS one in the low SNR region, because the reference signal is much less accurate due to high BER. Hence, once BER is small enough, estimation performances for 16QAM and BPSK quickly converges the derived bounds. On the other hand, Fig. 6(b) shows simulation results of RMSEs versus SNR for STO estimations. It can be easily observed that SWLS-SC estimator also outperforms LLS estimator.

Fig. 7(a) and Fig. 7(b) show NMSE comparison results for different modulation schemes and different DFT points of the reference signal, respectively. Note that, each incorrectly decoded BPSK, QPSK and 8PSK symbols contribute minimal phase errors of  $\pi$ ,  $\pi/2$  and  $\pi/8$  to their respective time-domain symbols. Therefore, under the condition of the same symbol error rate (SER), the reference signal utilizing lower modulation orders would have higher NMSEs than those with higher modulation orders. As for QAM modulations, since the

Euclidean distance between adjacent constellation points are closer than PSK schemes, the overall NMSE is much less than PSK schemes. In Fig. 7(a), one can observe that NMSEs of the reference signal  $\hat{X}_i[k]$  are respectively as low as -34dB, -37dB, -42 dB and -44dB for BPSK, QPSK, 8PSK and 16QAM modulation schemes when SER is at  $10^{-4}$ . Furthermore, NMSEs for various DFT lengths are also analyzed in Fig. 7(b) for BPSK example. Simulation results show that NMSEs of different DFT points are almost the same. Therefore, for applications with different DFT lengths, the reference signals remain accurate enough for SWLS-SC estimator. It is because that the time-domain decoding error after DFT operation is distributed throughout the entire bandwidth and its resultant effect to each individual frequency point is much less severe than the fine synchronization algorithm for OFDM systems in [7] which adopts decision-feedback structure.

Fig. 8 shows performance comparisons of fine synchronization schemes based on decision-feedback structure for SC-FDE and OFDM systems. Simulation parameters for the

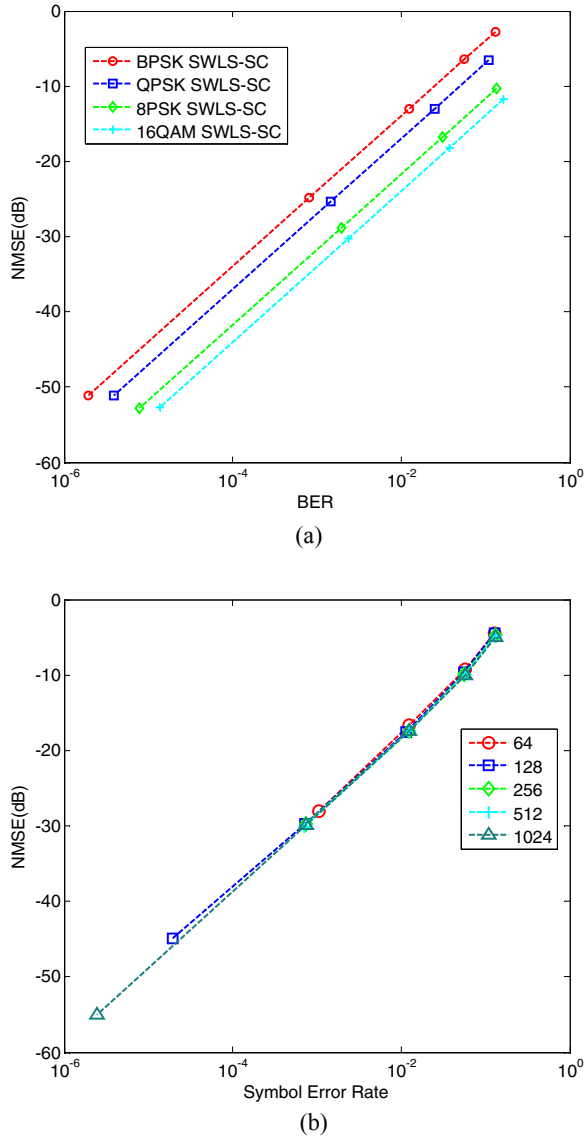


Fig. 7. NMSE comparisons for (a) different modulation schemes with 512-point DFT, and (b) different DFT points for BPSK case.

OFDM case are the same as the SC-FDE one for fair comparison consideration. The SWLS-SC estimator provides better estimation accuracies than OFDM systems under various modulation schemes and channel conditions. The performance gap between these two systems is especially larger for higher modulation schemes because of more severe error-propagation effect in OFDM system. Note that differences of signal powers between subcarriers in OFDM systems vary much less than that of SC-FDE systems. Therefore, fine-synchronization performance of OFDM systems outperforms the LLS estimator of SC-FDE system as shown in Fig. 6.

Comparisons of BER performance are illustrated in Fig. 9. Fig. 9 (a) gives the result due to 16QAM scheme. It shows that, the fine synchronization with SWLS-SC estimator only degrades from the ideal synchronization by 0.2dB and 0.5dB for LOS and NLOS cases, respectively. In addition, Fig. 9(b) presents the BER performance for BPSK case. Performance degradations of SWLS-SC estimations from the ideal estimations are much less than the LLS one. One can

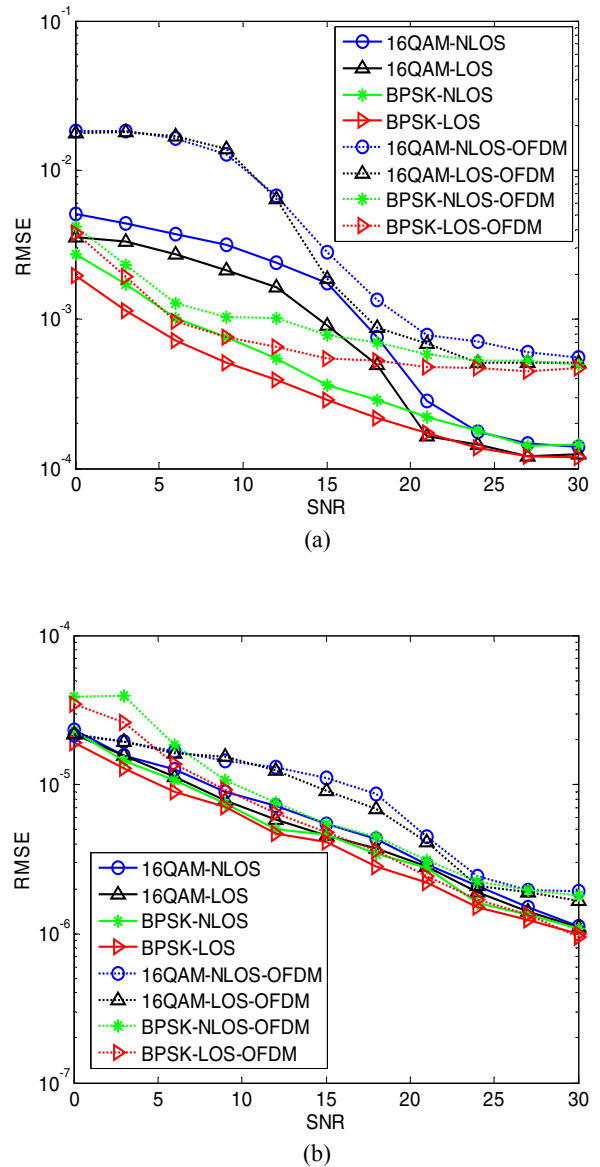
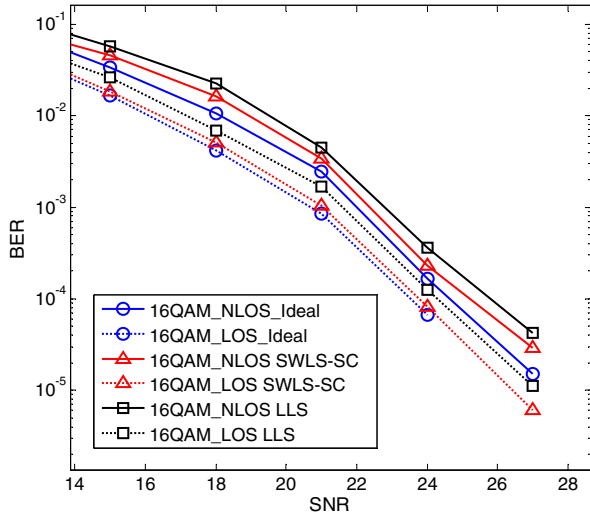


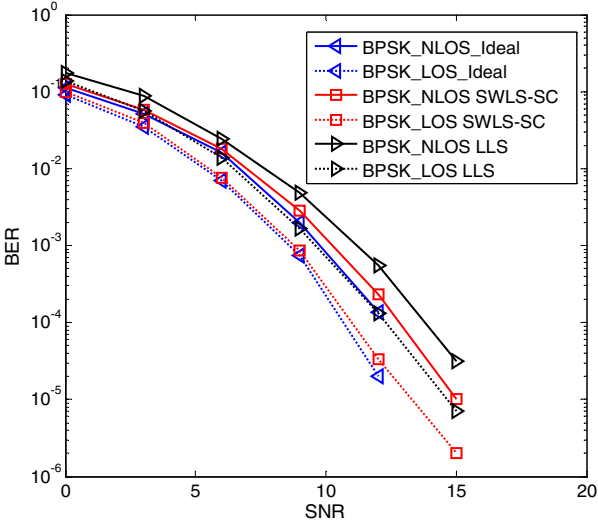
Fig. 8. Comparative simulation results for (a) RCFO and (b) STO estimations for  $\epsilon=0.02$  and  $\delta=20$ ppm.

notice that the performance improvement of BPSK case is greater than that of 16QAM case. The main reason is that every single error bit of the BPSK modulation induces a 180 degree phase error to its time-domain symbol while one error bit for 16QAM case only induces much less phase error due to the Gray-coding scheme.

Performance comparisons of SWLS-SC and LLS estimators subject to channel estimation error are illustrated in Fig. 10, where "Ch Err" denotes the power ratio of estimation error over original channel response. Results show that channel estimation error is a crucial factor for fine synchronizations, especially for residual carrier-frequency-offset (RCFO) estimations. In summary, the proposed estimator maintain better robustness than the conventional LLS one. Nevertheless, in IEEE 802.11.ad system, channel estimation error [19][20] is very small due to well-designed preamble sequences and generally slow-fading environments.



(a)

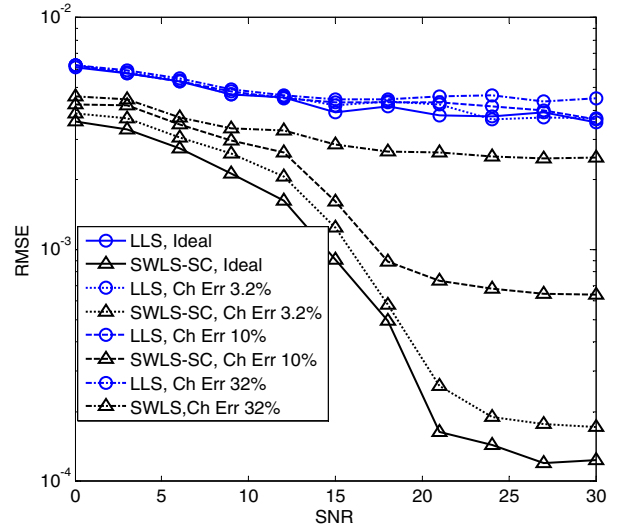


(b)

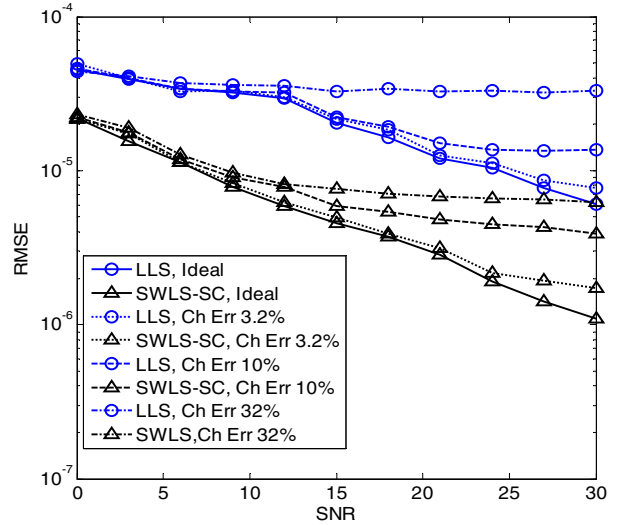
Fig. 9. Error rate performance for (a) 16QAM and (b) BPSK modulation schemes,  $\epsilon=0.02$  and  $\delta=20$ ppm.

## VI. CONCLUSION

In this work, an effective SWLS-SC joint estimation method for STO and RCFO is proposed. With the proposed estimator, one can perform fine-synchronizations without using UW for SC-FDE systems. It is developed based on the SWLS estimator and assisted with a decision-feedback structure to obtain the reference signal. Moreover, the derivation of quasi-optimal weights specifically for SC-FDE system is also given along with its theoretical bound for estimation error. Simulation results show that the BER performance of the proposed estimation is close to those with ideal synchronizations. Additionally, the feedback reference signal is also demonstrated to be accurate enough for various modulation schemes and different lengths of DFT operations without the need of inserting known sequences. Therefore, by using the proposed estimation method, both the system throughput and performance can be simultaneously increased.



(a)



(b)

Fig. 10. Performance comparisons subject to estimation error on (a) RCFO and (b) STO estimations for the case of 16QAM scheme and LOS channel, given that  $\epsilon=0.02$  and  $\delta=20$ ppm.

## APPENDIX

The quasi-optimal weighting coefficients in (17) are derived based on (13) and (14) as

$$\begin{aligned}\hat{\theta}_i[k] &= \arg(M_i[k]) \\ &= \arg\left(N\hat{Z}_i[k] \frac{|H_i[k]|^2 + 1/\eta_i}{|H_i[k]|^2 |X_i[k]|^2}\right) \\ &= \arg\left(e^{j\pi(N-1)(\epsilon+\delta k)/N} \left[1 + \frac{H_i[k]\hat{X}_i[k]\hat{W}_i^*[k]}{|H_i[k]|^2 |X_i[k]|^2}\right]\right)\end{aligned}\quad (31)$$

Assuming high SNR, the approximation,  $\sin\theta \approx \theta$ , can be applied to simplify the derivation. As a result, with error-free reference signal,  $\hat{e}_i[k]$  in (14), for small RCFO and STO, is approximately equivalent to

$$\begin{aligned}\hat{e}_i[k] &\approx \arg\left(e^{j\pi(N-1)(\epsilon+\delta k)/N} \left[\frac{H_i[k]\hat{X}_i[k]\hat{W}_i^*[k]}{|H_i[k]|^2 |X_i[k]|^2}\right]\right) \\ &\approx \frac{H_i[k]\hat{X}_i[k]\hat{W}_i^*[k]}{|H_i[k]|^2 |X_i[k]|^2}.\end{aligned}\quad (32)$$



In [15], it shows that the weighted least-square (WLS) estimation has minimum variance in estimation error if the coefficient,  $D_i[k]$ , in (16) and (17) is adopted as

$$\begin{aligned} D_i[k] &= E[\hat{e}_i[k]\hat{e}_i^*[k]]^{-1} \\ &\approx |H_i[k]|^2 \frac{|\hat{X}_i[k]|^2}{|\hat{W}_i[k]|^2} \\ &= |H_i[k]|^2 \hat{\eta}_i[k]. \end{aligned} \quad (33)$$

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