Exploring steering effects using Bell tests

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(Received 22 February 2013; published 18 February 2014)

We study the interconnection among steering, uncertainty, and quantum correlation. The practical steering criteria for bipartite quantum entanglement are proposed using the multiple applications of the Maassen-Uffink uncertainty relation. The results show that distant steering causes less uncertainty than that in any local-hidden-variable theories. The reduction of uncertainty in the quantum steering of entangled states leads to quantum correlation. The steering visibility for the mixed states is also discussed. The experimental results on the proposed steering criteria are reported.

DOI: 10.1103/PhysRevA.89.022115 PACS number(s): 03.65.Ud, 03.67.Mn

I. INTRODUCTION

When two entangled qubits are spatially separated, the state of one qubit can be "steered" or piloted (as it was termed by Schrödinger) by the prior measurement of the other one. Historically, this effect has led to disputes on the completeness of quantum theory. To depict the Einstein, Podolsky, and Rosen (EPR) paradox in a nutshell, here we consider a composite system composed of two qubits, which are entangled and spatially separated. According to quantum theory, once the first qubit (system) is measured using one of two noncommuting observables, the second qubit (system) will be left in different wave functions. To preserve the locality, Einstein, Podolsky, and Rosen (EPR) [1] asserted "No real change can take place in the second system in consequence of anything that may be done to the first system." Furthermore, EPR claimed that the same reality may be assigned to two different wave functions required by the consequences of measuring the remote systems. The above argument regarding locality and reality led EPR to the conclusion that quantum theory is incomplete, and implied the existence of local hidden variables (LHV).

Thanks to Schrödinger, local hidden variables as an explanation of steering was implicitly rejected then [2]. In addition, Cavalcanti *et al.* have verified that the notions of EPR paradox and steering are an equivalent depiction of nonlocality [3]. These previous works motivated us to tackle the steering effect using Bell tests on quantum nonlocality. In the following, we consider the following scenario. Spatially separated observers Alice and Bob each hold half of the entangled systems. To perform local measurements, each party randomly chooses one of two or more noncommuting operators corresponding to physical quantities. To ensure that Alice is to steer Bob's qubit, they initially agree that, in each round, Alice performs her local measurement prior to Bob's. Without classical communication, the no-signal theorem states that Bob has no information about Alice's local operation, which, nevertheless, lets Bob's qubit behave as a steerable state resulting in violation of some specific bipartite Bell-type inequality.

On the other hand, according to EPR's criteria for reality, Bob may indeed predict the value of "a" physical quantity with certainty [4]. However, Bob never predicts the values of two noncommuting physical quantities with certainty. It is well known that uncertainty relations among noncommuting observables play an essential role in quantum theory. A useful steering criterion as an extended version of EPR's reality was proposed by Reid [5], which can be stated as follows. Without disturbing a system in any way, one can predict the value of a physical quantity with some specific uncertainty. Cavalcanti *et al.* further developed Reid's proposal as quantitative criteria. In detail, due to Alice's measurement and hence the steering effect on Bob's observables, she can estimate or infer Bob's observables using her knowledge of local measurement outcomes. Such interference or estimation can indeed reduce the uncertainty, which has been experimentally validated in the continuous variable case [6]. As shown by Cavalcanti *et al.*, the product uncertainty relations including the inference variances of noncommuting observables can be served as the steering criteria [3,7,8]. The other kind of steering criteria are based on additive convexity of uncertainty relations, which are sums of convex functions [3,7,8].

In this paper, we attempt to explore the steering effect in the Bell tests. The basic idea is that the steering effect in the Bell tests can be revealed with the knowledge of the time order of local measurements. Here we propose the uncertainty relations as the steering criteria, which can be regarded as alternative expressions of Clauser-Horne-Shimony-Holt (CHSH) inequality [9,10] and chained Bell inequalities [11] in terms of conditional marginal probabilities. On this basis, the experimental setup of the proposed criteria is almost the same as the corresponding Bell tests, except that the time ordering of the local measurements must be recorded or previously settled. Our results show that the steering effect can reduce less uncertainty than LHV theory in the Bell tests, which is the reason why quantum theory can violate Bell inequalities.

Before proceeding further, here we compare the proposed criteria with Cavalcanti *et al.*'s steering criteria and Bell-type inequalities. First, researches on quantum entanglement are unfolded in two different perspectives, the nonlocality and steering effect. The former focuses on correlation relations, whereas the latter focuses on interference ability. The joint probabilities revealing correlations are usually exploited in Bell-type inequalities, whereas, based on Alice's estimation, the steering criteria usually include conditional probabilities or inference variances of local observables [3]. Second, the conditional probabilities are multiplied in the proposed criteria, whereas the joint probabilities are summed in Bell-type inequalities. Cavalcanti *et al.* constructed the steering criteria using either the product or the sums of inference invariance [3,7,8].

The paper is organized as follows. Section II introduces Maassen and Uffink uncertainty relation as the building blocks of the proposed criteria. Then we manipulate them to construct the proposed criteria in Sec. III. The two-setting and infinite-setting cases are studied respectively. Experimental verification of two-setting case is shown in Sec. IV. In Sec. V, we make discussions on local hidden states and visibility of the steering effect, followed by the conclusions.

II. MAASSEN AND UFFINK UNCERTAINTY RELATION

In 1961, Pollack *et al.* [12,13] proposed the uncertainty relation for continuous time-limiting and band-limiting signal processing. In 1988, Maassen and Uffink (MU) [14] revised the relation as quantum uncertainty relation as follows. Given a qubit $|\psi\rangle$, one performs the measurement using the orthonormal basis $|B_y^b\rangle$, where y is the index of the basis, and $b = \{0,1\}$ is the measurement outcome. The maximum probability of a certain outcome given the basis is $p_y = \max\{p(b=0|\psi,y),\ p(b=1|\psi,y)\}$. The MU uncertainty relation can be formulated as

$$p_0 p_1 \leqslant \left(\frac{1+c}{2}\right)^2, \tag{1}$$

where $c = \max_{b,b'} |\langle B_0^b|B_1^{b'}\rangle|$. Here, $|\psi\rangle$ is the maximally probable state, if the equality of Eq. (1) holds.

In this paper, we exploit Eq. (1) as the building block of the proposed uncertainty relations, which can serve as the steering criteria. We demonstrate that the steering effect in quantum theory can result in less uncertainty than that in theories with no steering (e.g., local hidden variables). Wehner and Oppenheimer [15] first considered the connection between uncertainty relations and Bell-type inequalities. The relations to be proposed in the present work are much simpler than theirs from the operational viewpoint. In the following, $|\psi\rangle\langle\psi| = \frac{1}{2}[I + (-1)^b \widehat{\psi} \cdot \overrightarrow{\sigma}]$, $|B_y^b\rangle\langle B_y^b| = \frac{1}{2}[I + (-1)^b \widehat{n}_y \cdot \overrightarrow{\sigma}]$, and $|A_x^a\rangle\langle A_x^a| = \frac{1}{2}[I + (-1)^a \widehat{\chi}_x \cdot \overrightarrow{\sigma}]$, where the vector $\overrightarrow{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$ and therein σ_x, σ_y, and σ_z are the Pauli matrices. The vectors $\widehat{\chi}_x$ and $\widehat{n}_y$ are depicted in Fig. 1.

Before proceeding further, we assign two operational meanings to Eq. (1). In the one-qubit case, we consider a single photon impinging on a series of three polarizers. The first and third polarizer axes are fixed at $\widehat{n}_0$ and $\widehat{n}_1$, respectively, and $\widehat{n}_0 \cdot \widehat{n}_1 = c$. Without loss of generality, let the measurement outcome $b = 0$ denote that the state of polarization is $|B_0^0\rangle$ ($|B_1^0\rangle$) when the photon passes the first (third) polarizer. Then designate the second polarizer axis as the direction vector $\widehat{\psi}$. After a photon passes through this polarizer, the state of polarization then becomes $|\psi\rangle$. Once the photon passes through the first polarizer, the probability passing through the second and third polarizer is $P = p(B_1^0|\psi)p(\psi|B_0^0)$. On the

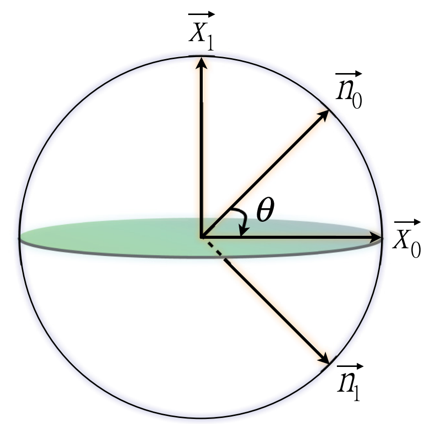

FIG. 1. (Color online) The four coplanar state vectors are on the Bloch sphere. The angle θ is $\frac{\pi}{4}$ and 0 for the best-case and worst-case scenarios, respectively.

other hand, in quantum theory, the probability

$$p(m|n) = |\langle m|n\rangle|^2 = p(n|m)$$

representing the state $|n\rangle$ ($|m\rangle$) is measured using the orthonormal basis $\{|m\rangle,|m^\perp\rangle\}$ ($\{|n\rangle,|n^\perp\rangle\}$), and the posts-elected state is $|m\rangle$ ($|n\rangle$). Here, m and n can represent B_y^0 and ψ or vice versa. Therefore, $p(\psi|B_0^0) = p(B_0^0|\psi)$ and $p(B_1^0|\psi)p(\psi|B_0^0) = p(B_1^0|\psi)p(B_0^0|\psi)$, which was the probability of photon ψ passing both polarizers, $\widehat{n}_0$ and $\widehat{n}_1$. As a result, let $p_y = p(B_y^0|\psi)$; then the maximal probability $P_{\max}$ is the MU uncertainty relation. Finally, the equality holds when $\widehat{n}_y \cdot \widehat{\psi} = \sqrt{\frac{1+c}{2}}$. From the perspective of local hidden variables, the measurement result can always be set to $b = 0$, and the photon always passes through the second polarizer. In the LHV case, $p_0 = p_1 = 1$, there is no uncertainty.

In the two-qubit case, Alice prepares a two-qubit system and then sends Bob one of these two qubits. Even though Reid showed that there is unavoidable uncertainty inherent to the steering effect [5], Alice and Bob still can collaborate to reduce the uncertainty to the minimum from the perspective of the MU relation. As for Alice's operation, she attempts to pilot the state of Bob's qubit into a certain maximally probable state. On the other hand, Bob also adjusts his measurement observables to maximize the probability product of Eq. (1). Notably, even though the equality in Eq. (1) holds, it is not enough to guarantee that Alice can steer Bob's state. One of the examples is the case when the qubit at Bob's hand is initially in the maximally probable state, which is unentangled with Alice's qubit.

Here we sketch the proposed quantum steering criteria as follows. Alice can convince Bob of her steering ability with piloting his qubit into a set of nonorthogonal states. Mathematically speaking, we exploit the MU relation repeatedly with different conditional states. In this case, if the uncertainty is too large, such that it can be explained using local hidden variables, Bob can reject her claim of steerability. In contrast, if the uncertainty were too small (as in the finite multisetting case), it cannot be explained using quantum theory.

III. STEERING CRITERIA

Wiseman *et al.* [16] revisited Schrödinger's concept of steerability in terms of tasks. From this foundation, we propose the following steering-convincing task. In the general scheme of this task, Alice first prepares two bipartite steerable boxes, and then sends one of the boxes to remote Bob. Then, in the steering phase, Alice inputs the random bit x into her accessible box, which outputs a bit α. Finally, in the checking phase, Bob also inputs the random bit y into his accessible box, which outputs a bit β. If the nonlocal input-output relation were physically realized as

$$\alpha + \beta = xy + c_a x + c_b y \bmod 2,$$

where x, y, α, β, c_a, and $c_b \in \{0,1\}$, given Alice's steering information (x,α), Bob can predict with certainty the output $\beta = \alpha + xy + c_a x + c_b y \bmod 2$. According to Reid's criteria, Bob can predict with some specific uncertainty. Here we evaluate and quantify the uncertainty due to the steering effect as follows. We define the value

$$M_{\alpha\beta} = \prod_{x,y=0}^{1} p(\beta = \alpha + yx + c_a x + c_b y|\alpha,x,y). \quad (2)$$

In the proposed protocol, Bob and Alice are to minimize the uncertainty. That is, they aim to maximize both $M_{\alpha\beta}$ and $M_{\overline{\alpha}\overline{\beta}}$. If Bob were to predict with certainty, $M_{\alpha\beta} = M_{\overline{\alpha}\overline{\beta}} = 1$.

In the quantum version, the boxes are replaced by two bipartite qubits. The two-setting scenario is described in detail as follows. In the preparation phase, Alice initially prepares the two-qubit Bell state

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) = \frac{1}{\sqrt{2}}\left(|\chi_x^0\rangle|\chi_x^0\rangle + |\chi_x^1\rangle|\chi_x^1\rangle\right),$$

where $|\chi_x^0\rangle = \cos\varphi|0\rangle + \sin\varphi|1\rangle$, and $\langle\chi_x^0|\chi_x^1\rangle = 0$. In the steering phase, Alice randomly chooses the orthonormal basis $\{|A_x^0\rangle,|A_x^1\rangle\}$, $x \in \{0,1\}$, as the measurement basis. As a result, the states of both qubits are simultaneously collapsed into the same state $|\chi_x^j\rangle = |A_x^j\rangle$, $j \in \{0,1\}$. In the checking phase, Bob randomly chooses $\{|B_y^0\rangle,|B_y^1\rangle\}$, $y \in \{0,1\}$, as the measurement basis. Now we consider the ideal case: they agree beforehand that these vectors, ($\widehat{\chi}_0$, $\widehat{\chi}_1$, $\widehat{n}_0$, and $\widehat{n}_1$), are always coplanar in the Bloch sphere. Without loss of generality, we set $|A_0^0\rangle = |0\rangle$, $|A_0^1\rangle = |1\rangle$, $|A_1^a\rangle = \frac{1}{\sqrt{2}}(|0\rangle + (-1)^a|1\rangle)$, and $\widehat{n}_y = \frac{1}{\sqrt{2}}[\widehat{z} + (-1)^y\widehat{x}]$ so that the value of c in the MU relation is $\frac{1}{\sqrt{2}}$. Consequently, $|\chi_0^0\rangle$, $|\chi_0^1\rangle$, $|\chi_1^0\rangle$, and $|\chi_1^1\rangle$ are all maximally probable states. Hereafter, the probability with which they obtain the results a and b from the x and y measurement directions, respectively, is $p(B_y^b|A_x^a) = p(b|a,x,y)$. In the following, we state the steering criteria as the main result. And then, Bob's qubit is then steerable so long as either

$$\frac{1}{4} < M_{00}, \quad M_{11} \leqslant \left[\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)\right]^4, \quad (3)$$

or

$$\frac{1}{4} < M_{01}, \quad M_{10} \leqslant \left[\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)\right]^4 \quad (4)$$

holds. Without loss of generality, hereafter we set $c_a = c_b = 0$, and hence we have $p(\beta = yx|0,x,y) = p(\beta = 1 + yx|1,x,y) = \frac{1}{2}(1 + \frac{1}{\sqrt{2}})$, and therefore $M_{00} = M_{11} = [\frac{1}{2}(1 + \frac{1}{\sqrt{2}})]^4$. In this way, Alice can maximally steer Bob's state. In contrast, we consider two worst-case scenarios that result in $M_{00} = M_{11} = \frac{1}{4}$. First, the measurement basis is set as $\widehat{\chi}_0 = \widehat{n}_0$ and $\widehat{\chi}_1 = -\widehat{n}_1$, and the measurement probabilities are $p(z|z,0,0) = p(\overline{z}|z,1,1) = 1$ and $p(\overline{z}|z,x \neq y) = \frac{1}{2}$ for $\forall z \in \{0,1\}$. In the perspective of local hidden variables, take a predetermined output that can be represented as binary vector $(a|_{x=0}, a|_{x=1}, b|_{y=0}, b|_{y=1})$; M_{00} and M_{11} can then be obtained with the equal mixing vectors (0, 0, 0, 0), (0, 0, 0, 1), (1, 1, 1, 0), and (1, 1, 1, 1). Second, let $\widehat{\chi}_0 = \widehat{n}_0 = \widehat{n}_1$ and $\widehat{\chi}_1 \cdot \widehat{\chi}_0 = \frac{1}{\sqrt{2}}$, the probabilities are then $p(z|z,0,0) = p(z|z,0,1) = 1$ and $p(0|z,1,0) = p(0|z,1,1) = \frac{1}{2}$. Alice cannot convince Bob that his qubit is steerable in such cases.

As the end of this section, we interpret M_{00} and M_{11} as a polarized single photon $\widehat{n}_1$ that passes through a series of five polarizers. The polarizations of the first through fourth polarizers are set to be $\widehat{\chi}_0$, $\widehat{n}_0$, $\widehat{\chi}_1$, and $\widehat{n}_1^{\perp}$, respectively. Here, M_{00} is the probability of a photon passing through the fifth polarizer once it has passed the first one. If we simultaneously rotate these polarizers by 90°, the corresponding probability becomes M_{11}. However, in the single-photon case for local hidden variables, this detection probability is zero, as the photon's original polarization and the final polarizers are orthogonal to one another. Thus, from the perspective of local hidden variables, the single-qubit case is not completely equivalent to the two-qubit one.

IV. EXPERIMENTAL SETUP AND RESULTS

We consider the polarization-entangled photon pair to demonstrate the steering process, and the experimental setup is shown in Fig. 2. Here, we use second harmonic (390 nm) of a Ti:sapphire pulse laser (780 nm). The second harmonic pumps a type-II BBO (β-barium borate) crystal to generate the polarization-entangled photon pair. Half wave plates (HWPs) and BBO crystals are used to compensate for group delay and walkoff in the photon pair. The local measurement was performed using a quarter wave plate and an HWP. The filters were used to suppress noise, passing only photons with a wavelength of 780 ± 3 nm. The polarizers were set at 0° (90°) for the horizontal (vertical) polarization measurement. Each of the entangled photons was coupled to a single-mode fiber connected to a single-photon-counting module (SPCM). The photon-counting signal of Alice (SPCM A) was used as the stop trigger, introducing electronic delay. Meanwhile, the nondelayed signal of Bob (SPCM B) was used as the start trigger in the coincidence measurement. Note that Alice always measured the photon earlier than Bob. It is an obvious advantage that it is possible to use the same experimental setup to test the CHSH inequality ($S = 2.76 \pm 0.06$). The only difference is that we must ensure the measurement order of the two local measurements. Experimentally, the results satisfied our steering criteria of Eq. (3), as $M_{00}^{\text{exp}} = 0.45 \pm 0.02$, and $M_{11}^{\text{exp}} = 0.51 \pm 0.03$. Consequently, Alice can exactly steer Bob's qubit into the maximally probable states in the entangled photon pair.

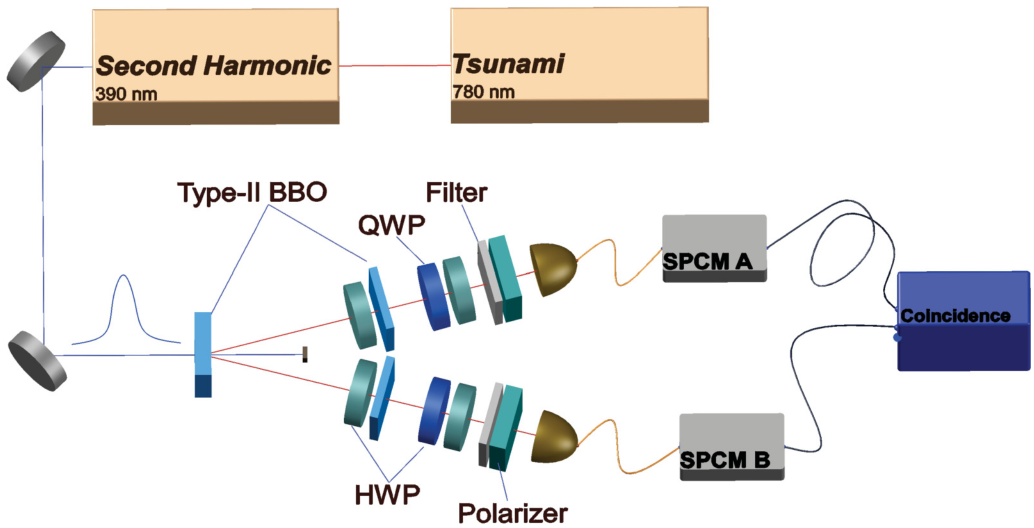

FIG. 2. (Color online) The experimental setup [9]. Tsunami: the laser system; QWP: quarter wave plate; HWP: half wave plate.

V. DISCUSSIONS AND CONCLUSIONS

A. Local hidden states

Wiseman *et al.* pointed out the fundamental differences among quantum nonseparability, Bell nonlocality, and steering in between [16]. In brief, quantum states and local hidden variables are involved and considered in quantum nonseparability and Bell nonlocality, respectively. To test the steering criteria, the local hidden states were proposed [16]. Once the other remote quantum system is measured, the steerable states in the quantum system cannot be described as hidden local states [16], which can be stated as follows. Given the preparation procedure c, the joint probability for a local hidden state can be written as

$$p(a,b|x,y,c) = \sum\nolimits_{\lambda} p(\lambda|c)p(a|x,c,\lambda)p_Q(b|y,c,\lambda),$$

where $p_Q(b|y,c,\lambda)$ represent probability distributions, which are compatible with a quantum state. However, the measurement outcomes of the one-qubit system can be simulated using local hidden variables, which was pointed out by Bell [17]. In this sense, the local hidden states can be completely simulated by local hidden variables, and hence cannot be exploited for the violation of either Eq. (3) or (4).

B. Multisetting case

Now we consider the bipartite multisetting case. The following proposed uncertainty relation can be regarded as an alternative description of chained Bell inequalities. In this process, Alice and Bob each randomly input A_x and B_y, where x, $y \in \{1,2,\ldots,N\}$, and the outputs are a and $b \in \{0,1\}$, respectively. We define $M_{\alpha\beta}^N$ as

$$M_{\alpha\beta}^N = \prod_{y=1}^{N} p(b=\beta|a=\alpha,x=y+1) \times p(b=\beta+\delta_{y,N}|a=\alpha,x=y), \quad (5)$$

where $N+1 \equiv 1$, and the parameter $\delta_{y,N}$ is unity when $y = N$ and zero otherwise. Using a similar approach for $N=2$, to reach the upper bound in local hidden variables, we set $a(x)=\alpha$ and $b(y)=\beta$, and furthermore, $b(N)$ can be set deterministically to 0 or 1 with equal probability. As a result, the maximal $M_{\alpha\beta}^N$ is $\frac{1}{4}$ for local hidden variables.

For the quantum version, we can exploit the same setup for chained Bell inequalities. Without loss of generality, Alice performs her measurement $\widehat{A}_x = \widehat{\chi}_x \cdot \overrightarrow{\sigma}$ with the measurement outcome $\alpha=0$. This measurement steers Bob's qubit state into $|\chi_x\rangle\langle\chi_x| = \frac{1}{2}(I+\widehat{A}_x)$. As for Bob, he performs the measurement $\widehat{B}_y$ with the basis $\{|n_y\rangle,|n_y^{\perp}\rangle\}$. The outcome b is denoted by 0 or 1 if the postselected state is $|n_y\rangle$ or $|n_y^{\perp}\rangle$, respectively. Let the angle between $\widehat{\chi}_y$ and $\widehat{n}_y$ be $\frac{\pi}{2N}$. As a result, $c = |\langle\chi_i|n_i\rangle| = |\langle\chi_{i+1}|n_i\rangle| = \cos\frac{\pi}{2N} > |\langle\chi_i|n_i^{\perp}\rangle| = |\langle\chi_{i+1}|n_i^{\perp}\rangle|, \forall i \in \{1,2,\ldots,N\}$, with $\chi_{N+1}=\chi_1$. According to the MU relation, max $p(n_{y-1}|\chi_y)p(n_y|\chi_y) = (\frac{1+c}{2})^2$, $M_{00}^N \leqslant (\frac{1+c}{2})^{2N}$, and $M_{11}^N \leqslant (\frac{1+c}{2})^{2N}$, replacing χ_i and n_i with $\chi_i^{\perp}$ and $n_i^{\perp}$, respectively. Then the steering criteria can be stated as follows. The box systems are steerable if, for some α, $\beta \in \{0,1\}$,

$$\frac{1}{4} \leqslant M_{\alpha\beta}^N, \quad M_{\overline{\alpha}\overline{\beta}}^N \leqslant \left(\frac{1+c}{2}\right)^{2N}. \quad (6)$$

Here, $M_{\alpha\beta}^{N=2} = M_{\alpha\beta}$ in Eq. (2). In particular, note that $\lim_{N\to\infty}(\frac{1+c}{2})^{2N} = 1$, and therefore, $M_{\alpha\beta}^{\infty} = 1$.

Again, one should consider the operational interpretation of $M_{\alpha\beta}^N$. Notably, M_{00}^N can be modified to

$$M_{00}^N = \prod_{y=1}^{N} |\langle n_y|\chi_y\rangle|^2 |\langle\chi_{y+1}|n_y\rangle|^2 = \prod_{y=1}^{N} p(\chi_{y+1}|n_y)p(n_y|\chi_y),$$

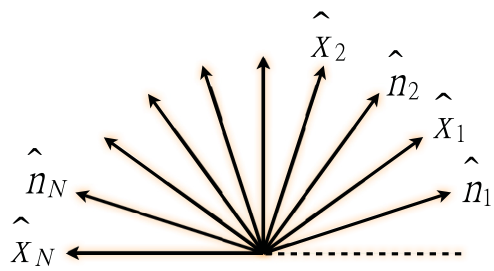

FIG. 3. (Color online) The $2N$ coplanar state vectors for the chained Bell inequalities are on the Bloch sphere.

where we set $\chi_{y+1} = \chi_1^\perp$. In the proposed scenario, a single photon impinges on $2N$ polarizers, as shown in Fig. 3, which represents the quantum Zeno effect [18]. Let the polarization of the incoming photon be $\widehat{\chi}_1$. The kth polarizer is oriented along the direction of vector $\widehat{\chi}_{k/2+1}$ ($\widehat{n}_{(k+1)/2}$) if k is even (odd) with respect to Alice and Bob. In this case, M_{00}^N can be interpreted as the probability of the photon passing through all $2N$ polarizers. Once the polarization of the photon and the orientation of the polarizers are rotated by 90°, the transmission probability becomes M_{11}^N.

C. Nonideal cases

First, suppose that a nonmaximal entanglement is prepared, e.g., $|\phi\rangle = s|0\rangle|0\rangle + t|1\rangle|1\rangle(\{s,t\} \in R$ and $s^2 + t^2 = 1)$. Alice can steer Bob's state into either $|0\rangle$ or $|1\rangle$ with certainty. However, from the unitary operation, $|\phi\rangle$ can be rewritten as $\frac{1}{\sqrt{2}}(|+\rangle|\varphi_+\rangle + |-\rangle|\varphi_-\rangle)$, where $|\varphi_\pm\rangle = s|0\rangle \pm t|1\rangle$ is a nonorthogonal basis $\langle\varphi_+|\varphi_-\rangle \neq 0$. Here, Alice can exploit the unambiguous quantum measurement of nonorthogonal states by a selective filter $\mathcal{F} = \frac{1}{\sqrt{1+|s^2-t^2|}}(|+\rangle\langle\varphi_-^\perp| + |-\rangle\langle\varphi_+^\perp|)$. Probabilistically, Alice's qubit can pass through the filter, ensuring that Bob's state will be steered into the same state as hers. Second, in the biased two-base local measurements, the upper and lower bounds of $M_{\alpha\beta}$ in Eqs. (3) and (4) are unchanged. Third, consider the Werner isotropic state [19],

$$W = (1-\eta)\tfrac{1}{4} I \otimes I + \eta|\Psi^+\rangle\langle\Psi^+|,$$

where $0 \leqslant \eta \leqslant 1$, I is the identity. Such states can be regarded as depolarizing disturbances from the environment. Isotropic states are inseparable if $\eta \geqslant \frac{1}{3}$ [20] and steerable if $\eta \geqslant \frac{1}{2}$ [16] in the two-level case.

As for the proposed N-setting scheme, we consider the critical value of η_N such that $M_{00}^N = M_{11}^N = \frac{1}{4}$. That is,

$$\frac{1}{4} = \left[\frac{1}{2}(1-\eta_N) + \eta_N\left(\frac{1+\cos\frac{\pi}{2N}}{2}\right)\right]^{2N},$$

and hence

$$\eta_N = (2^{1-1/N} - 1)\sec\frac{\pi}{2N}.$$

That is, the steering effect becomes "visible" if $\eta > \eta_N$ as shown in Fig. 4. Obviously, $\eta_2 = 2 - \sqrt{2} > \frac{1}{2}$ and

$$\lim_{N\to\infty} \eta_N = 1.$$

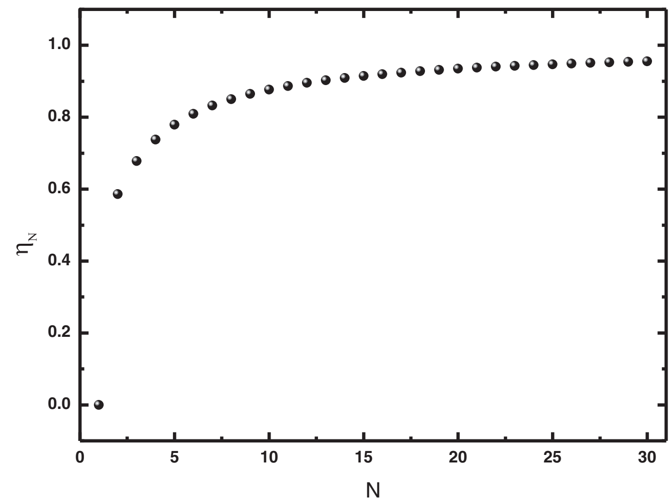

FIG. 4. The visibility of the steering effect.

The numerical result is shown in Fig. 4. As a result, the two-setting setup is optimal for witnessing quantum steering.

D. Min-entropy representation

Finally, we express Eq. (2) in terms of the min-entropy $H_\infty(X|Y) = -\log_2 \max P(X|Y)$, with a logarithm of base 2; for a random variable $X \in \{x_1,\ldots,x_N\}$ and $Y = y$, let the arbitrary conditional probability distribution be $P(X|Y) = \{p(x_1|y),\ldots,p(x_N|y)\}$. According to Eq. (2), we have $\sum_{x,y=0}^{1} H_\infty(b|a,x,y) \leqslant -\log_2 M$, and equivalently, $4[2 - \log_2(2+\sqrt{2})] \leqslant -\log_2 M \leqslant 2$.

In conclusion, a practical criterion for quantum steering is proposed that allows us to relate the steering, the uncertainty relation, and Bell-type inequalities. Indeed, quantum steering can reduce less uncertainty than LHV. From the operational perspective, the proposed criteria can be physically realized using the same set of Bell tests. In particular, the two-setting case can be performed using the experimental setup of CHSH inequality. Characteristically, our proposed criteria exploit the MU relation to revise the joint probability terms of Bell-type inequalities as the product form of conditional probabilities. It can be easily verified that, once the measurement outcomes result in the maximal values of both M_{00} and M_{11} ($[\frac{1}{2}(1+\frac{1}{\sqrt{2}})]^4$), CHSH inequality is maximally violated. In addition, the two-setting case is the most robust against depolarized noise. Finally, we believe that the Bell tests and the steering criteria can be physically performed simultaneously.

ACKNOWLEDGMENTS

We acknowledge the financial support from the National Science Council of the Republic of China under Contracts No. NSC.102-2112-M-033-006-MY3, and No. NSC.101-2112-M-009-002-MY2.

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