Flux flow of Abrikosov vortices in type-II superconductors

J. L. Chen
Institute of Electro-optical Engineering, National Chiao-Tung University, Hsinchu, Taiwan

T. J. Yang
Department of Electro-Physics, National Chiao-Tung University, Hsinchu, Taiwan, Republic of China
(Received 21 December 1993)

The theory of flux flow developed by Bardeen and Stephen (BS) is modified and extended to the high-field case. The Clem model and Wigner-Seitz circle-cell approximation for vortices are used in our approach. The distinct boundary of the normal core of a vortex in BS theory is removed and treated naturally. Several interesting results come out as a consequence. The Lorentz force is determined by the normal current rather than the supercurrent. But the supercurrent can sustain the magnetic-field distribution of flux quanta. From energy dissipation considerations, the Lorentz force is equal to viscosity force automatically without assumption as made in BS theory. An expression for the viscosity is also obtained.

I. INTRODUCTION

In the mixed state of ideal type-II superconductors, Abrikosov vortices[1] penetrate the materials in the form of flux lines, each carrying a single flux quantum ϕ_0. The Lorentz force $\mathbf{F}_L$ due to a transport current $\mathbf{J}_T$ would drive vortices to move, and it can lead to electrical resistance. This phenomenon is called flux flow. The early flux-flow theory was developed by Bardeen and Stephen (BS).[2] We will review the BS theory in the following.

As illustrated in Fig. 1, a single vortex is driven to move in the direction $\mathbf{V}_L$ by a uniform transport flow indicated by $\mathbf{V}_T$. The circulation of a stationary vortex outside of a normal core is indicated by V_0. The total current is a superposition of the superconducting flow pattern of a stationary vortex, $J_0(r)$, and the transport current

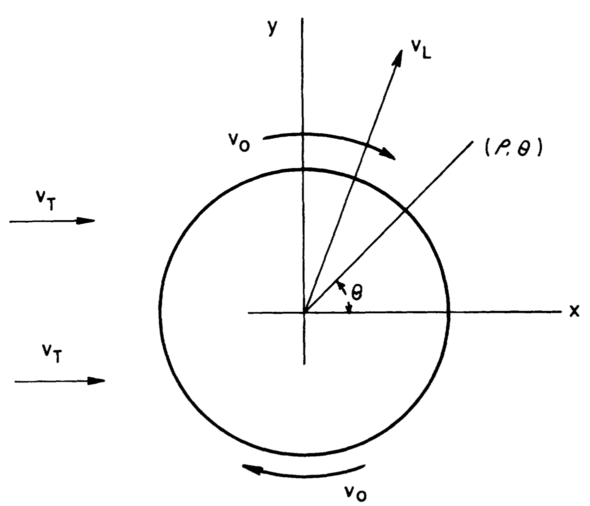

FIG. 1. A single vortex moving with a velocity $\mathbf{V}_L$ is driven by a uniform transport flow $\mathbf{V}_T$. The circulation of a stationary vortex outside of a normal core is V_0. The polar coordinate (ρ,θ) is used.

$$\mathbf{J}=\mathbf{J}_T+\mathbf{J}_0(|\mathbf{r}-\mathbf{V}_L t|)=\mathbf{J}_s+\mathbf{J}_n \;, \tag{1}$$

where $\mathbf{J}_s(\mathbf{J}_n)$ is the super (normal) current.

In the BS theory, the current velocity in the vortex core $\mathbf{V}_C=\mathbf{V}_T-\mathbf{V}_B$, where $\mathbf{V}_B$ is backflow current velocity due to the pinning center. In the no-pinning case, $\mathbf{V}_B=0$, i.e., $\mathbf{V}_C=\mathbf{V}_T$. Recently, Wang and Ting[3] proposed a theory of flux motion with backflow current in high-κ superconductors. Otherwise, BS thought that part of $\mathbf{J}_T$ came from $\mathbf{J}_s$ and part from $\mathbf{J}_n$. They assumed (disregarding the Hall effect) the equation of flux flow to be

$$\mathbf{F}_L=\eta\mathbf{V}_L \;, \tag{2}$$

where $\mathbf{F}_L=(1/c)\mathbf{J}_T\times\boldsymbol{\phi}_0$ and η is the flux-flow viscosity, which yields[4]

$$\eta=\phi_0 H_{c2}/\rho_n c^2 \;, \tag{3}$$

and flux-flow resistivity

$$\rho_f=\rho_n B/H_{c2} \;, \tag{4}$$

where ρ_n is the normal resistivity and B is the flux density.

But, after thinking it over, we have two questions about the BS theory: (1) BS assumed that there is a clear boundary between the normal state and the superconducting state in a vortex. The inside of the vortex core with radius $r\approx\xi$ is the region of normal conductivity. But, in fact, the clear boundary between the normal state and the superconducting state cannot be determined rigorously. (2) If the clear boundary between the normal state and the superconducting state of a vortex does not exist, what kind of result different from BS theory would we obtain? On the other hand, the BS theory is valid for an isolated vortex, i.e., applied magnetic field $H_a\approx H_{c1}$.

Therefore, we believe that the BS theory must be modified and extended to higher field situation. In this paper, we try to answer the two questions above. Our approach is valid for any applied magnetic field above first critical field H_{c1}. The current due to the electric field is assigned as $\mathbf{J}_n$, then we find the role of $\mathbf{J}_s$ alone circulates around the vortex core; moreover the Lorentz force $\mathbf{F}_L$ is due to $\mathbf{J}_n$, $\mathbf{J}_s$ has no contribution to $\mathbf{F}_L$. Finally, we derive the Lorentz force to be just equal to viscosity force automatically without regarding it as an assumption in the BS theory.

II. THEORY

In the mixed state of ideal type-II superconductors, vortices are arranged in the form of a triangular flux-line lattice [as in Fig. 2(a)].[5] In the following paragraph we will discuss two cases (i) stationary vortices and (ii) vortices moving along the x-axis with a constant velocity $\mathbf{V}_L$.

A. Stationary vortices

The Ginzburg-Landau (GL) equations[6] near H_{c2} have been solved by Abrikosov.[1] But, up to this time, there are not any exact solutions for $H_{c1} \ll H_a \ll H_{c2}$. Therefore we use the Clem model[7] to solve GL equations qualitatively. In Clem's original paper, he only discussed the single vortex case; but in this paper, we consider the overlap between vortices. For convenience, we adopt the Wigner-Seitz circle-cell approximation.[8] The hexagonal unit cell of the triangular flux-line lattice [as Fig. 2(a)] is replaced by a circular unit cell with equal area [as Fig. 2(b)]. The radius of the circular unit cell is ρ_0. We utilize cylindrical coordinates $\rho=\sqrt{x^2+y^2}$, $\theta=\tan^{-1}(y/x)$, and z, and approximate the GL order parameter[7] as $\Psi(\rho)=f(\rho)\exp(i\theta)$, where $f(\rho)=\rho/R=\rho/\sqrt{\rho^2+\xi_v^2}$, any ξ_v is a variational core value parameter. $\xi_v=\sqrt{2}\xi$ for $\kappa \gg 1$. Substituting $f(\rho)$ into the second GL equation, and with the help of Maxwell equations, we obtain the vector potential

$$\mathbf{A}=\frac{\phi_0}{2\pi\rho}[1-aRK_1(R/\lambda)+bRI_1(R/\lambda)]\hat{e}_\theta , \tag{5}$$

magnetic field

$$\mathbf{H}=\frac{\phi_0}{2\pi\lambda}[aK_0(R/\lambda)+bI_0(R/\lambda)]\hat{e}_z , \tag{6}$$

and supercurrent density

$$\mathbf{J}_s=\frac{\phi_0 c}{8\pi^2\lambda^2}\frac{\rho}{R}[aK_1(R/\lambda)-bI_1(R/\lambda)]\hat{e}_\theta , \tag{7}$$

where $K_n(x)$ and $I_n(x)$ are modified Bessel functions, λ is the penetration depth. a,b can be determined by two boundary conditions: $J_s(\rho=\rho_0)=0$ and $\int_{\text{unit cell}}\mathbf{H}\cdot d\mathbf{s}=\phi_0$. We have

$$a=\frac{I_1(R_0/\lambda)}{\xi_v[K_1(\xi_v/\lambda)I_1(R_0/\lambda)-K_1(R_0/\lambda)I_1(\xi_v/\lambda)]} , \tag{8a}$$

and

$$b=\frac{K_1(R_0/\lambda)}{\xi_v[K_1(\xi_v/\lambda)I_1(R_0/\lambda)-K_1(R_0/\lambda)I_1(\xi_v/\lambda)]} , \tag{8b}$$

where $R_0=\sqrt{\rho_0^2+\xi_v^2}$. Equations (5)–(8) reduce to the results obtained by Clem[7] in the low-field case, i.e., $\rho_0 \gg \lambda$.

B. Vortices moving along x axis with a constant velocity $\mathbf{V}_L$

In this case, we assume that vortices still sustain the form of a triangular flux-line lattice but move along the x axis with a constant velocity $\mathbf{V}_L$. Similarly, we still adopt the approximate methods in the case (i). So this case can be simplified to a single vortex as Fig. 2(b), but it moves with a constant velocity $\mathbf{V}_L$.

In principle, we must solve the time-dependent GL equations[9] in the moving vortices case. But the velocity $\mathbf{V}_L$ is very small, so the extra correct terms due to time variation are regarded as perturbations. We may neglect them. We just only replace x by $x-V_L t$ in the solutions which have been solved in case (i). But in case (ii), the cylindrical symmetry is broken because of vortices moving. We must choose the gauge very carefully. According to

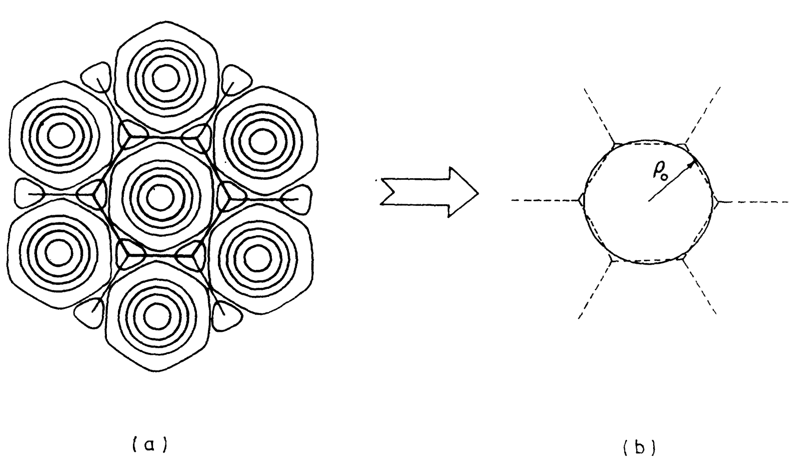

FIG. 2. The hexagonal unit cell of the triangular flux-line lattice [shown in (a)] is replaced by a circular unit cell with an equal area [shown in (b)].

the Josephson relation,[10] we know that the electric field **E** induced by vortices moving must include the uniform background term $\phi_0 V_L/\pi\rho_0^2 \hat{e}_y$. Therefore we choose a gauge in which the vector potential **A** includes the term $\phi_0(x - V_L t)/\pi\rho_0^2 \hat{e}_y$. We can get the vector potential

$$\mathbf{A} = \frac{\phi_0(x - V_L t)}{\pi\rho_0^2}\hat{e}_y + \frac{\phi_0}{2\pi}\left[\frac{1}{\rho} - \frac{\rho}{\rho_0^2} - \frac{aR}{\rho}K_1(R/\lambda) + \frac{bR}{\rho}I_1(R/\lambda)\right]\hat{e}_\theta \;, \tag{9}$$

magnetic field

$$\hat{\mathbf{H}} = \frac{\phi_0}{2\pi\lambda}[aK_0(R/\lambda) + bI_0(R/\lambda)]\hat{e}_z \;, \tag{10}$$

and supercurrent density

$$\mathbf{J}_s = \frac{\phi_0 c}{8\pi^2\lambda^2}\frac{\rho}{R}[aK_1(R/\lambda) - bI_1(R/\lambda)]\hat{e}_\theta \;. \tag{11}$$

Note: where $\rho = \sqrt{(x - V_L t)^2 + y^2}$, $\theta = \tan^{-1}[y/(x - V_L t)]$. We assume the electric potential is constant in this paper. Electric field

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = \frac{\phi_0 V_L}{2\pi c\rho_0^2}\hat{e}_y + \frac{\phi_0 V_L}{2\pi c}\left\{\left[\frac{1}{\rho^2} - a\left[\frac{RK_1(R/\lambda)}{\rho^2} + \frac{K_0(R/\lambda)}{2\lambda}\right] + b\left[\frac{RI_1(R/\lambda)}{\rho^2} - \frac{I_0(R/\lambda)}{2\lambda}\right]\right] \times (\sin 2\theta\hat{e}_x - \cos 2\theta\hat{e}_y) + \frac{aK_0(R/\lambda) + bI_0(R/\lambda)}{2\lambda}\hat{e}_y\right\} . \tag{12}$$

The average electric field is

$$\langle \mathbf{E} \rangle = \frac{1}{\pi\rho_0^2}\int_0^{2\pi} d\theta \int_0^{\rho_0} \mathbf{E}\rho\, d\rho = \frac{BV_L}{c}\hat{e}_y = \frac{\mathbf{B}\times\mathbf{V}_L}{c} \;, \tag{13}$$

where $B = \phi_0/\pi\rho_0^2$ is the flux density. Equation (13) is the Josephson relation.[10] The average normal current density due to $\langle \mathbf{E} \rangle$ is assumed to follow Ohm's law

$$\langle \mathbf{J}_n \rangle = \sigma\langle \mathbf{E} \rangle \;, \tag{14}$$

where σ is the conductivity. The moving vortices can be regarded as the ac magnetic field, and the ac magnetic field can excite the normal current. This is the reason why the current due to $\langle \mathbf{E} \rangle$ is assigned as the average normal current. The dissipation of energy per unit length of single vortex is

$$W = \sigma\int_0^{2\pi} d\theta \int_0^{\rho_0} E^2\rho\, d\rho = \frac{\sigma B\phi_0 V_L^2}{2c^2} + \frac{\sigma\phi_0^2 V_L^2}{8\pi c^2\lambda^2}\{1 + R_0^2[aK_0(R_0/\lambda) + bI_0(R_0/\lambda)]^2 - \xi_v^2[aK_0(\xi_v/\lambda) + bI_0(\xi_v/\lambda)]^2\} = \eta V_L^2 \;, \tag{15}$$

where η is flux-flow viscosity.

The Lorentz force per unit length on a vortex is

$$\mathbf{F}_L = \frac{1}{c}\int_0^{2\pi} d\theta \int_0^{\rho_0} (\mathbf{J}_n + \mathbf{J}_s)\times\mathbf{H}\rho\, d\rho = \frac{\sigma}{c}\int_0^{2\pi} d\theta \int_0^{\rho_0} \mathbf{E}\times\mathbf{H}\rho\, d\rho = \frac{\sigma B\phi_0 V_L}{2c^2}\hat{e}_x + \frac{\sigma\phi_0^2 V_L}{8\pi c^2\lambda^2}\{1 + R_0^2[aK_0(R_0/\lambda) + bI_0(R_0/\lambda)]^2 - \xi_v^2[aK_0(\xi_v/\lambda) + bI_0(\xi_v/\lambda)]^2\}\hat{e}_x \;. \tag{16}$$

In Eq. (16), we use Ohm's law $\langle \mathbf{J}_n \rangle = \sigma\langle \mathbf{E} \rangle$. The second term $\mathbf{J}_s \times \mathbf{H}$ will be canceled out by integrating all orientation. Thus, Lorentz force density is equal to $(1/c)\mathbf{J}_n \times \mathbf{H}$. That is, Lorentz force is determined by the normal current rather than the supercurrent.

From Eqs. (15) and (16), we get automatically

$$\mathbf{F}_L = \eta\mathbf{V}_L \;. \tag{17}$$

It is worthwhile to mention that Eq. (17) is an assumption in the BS theory, but here it is a derived result.

From Eq. (15), and letting $\kappa \gg 1$, we have

$$\eta = \begin{cases} \dfrac{\sigma\phi_0}{2c^2}\left[B + \dfrac{\phi_0}{4\pi\lambda^2}\right] & \text{for } \rho_0 \gg \lambda \\ \dfrac{\sigma\phi_0 B}{c^2} & \text{for } \rho_0 \ll \lambda \;. \end{cases} \tag{18}$$

Close to T_c, $\sigma \approx \sigma_n$, σ_n is normal conductivity. Because the dissipation energy of a vortex motion is proportional to flux density B, we obtain the viscosity η to be proportional to B. For $\rho_0 \ll \lambda$, the viscosity η given by Eq. (18) is the same as the result of BS theory. For $H_a \gg H_{c1}$, (i.e., $\rho_0 \ll \lambda$), the average dissipation power of flux flow per unit volume is

$$P = \langle \mathbf{J}_n \rangle \cdot \langle \mathbf{E} \rangle = \sigma \langle E \rangle^2 = \frac{\sigma B^2 V_L^2}{c^2} = \frac{\eta V_L^2}{\pi \rho_0^2} . \quad (19)$$

This is a reasonable result for physical requirements.

From the above demonstration our theory advances the BS theory to realize the mechanism of flux flow. When the applied transport current pushes the vortices to move, that will induce the electric field and generate the normal current. Then Lorentz force due to normal current will push the vortices to move continuously and be balanced by the viscosity force of flux flow. The supercurrent density $\mathbf{J}_s$ is just to sustain the magnetic-field distribution of flux quanta. It makes no contribution to $\mathbf{F}_L$.

III. CONCLUSIONS

Based on the Clem model and Wigner-Seitz circle-cell approximation, we have studied the mechanism of flux flow. In this paper, we obtain very important results which are valid in any applied magnetic field above H_{c1}. Otherwise, we realize Lorentz force is due to the normal current rather than the supercurrent. The supercurrent makes no contribution to the Lorentz force, but it can sustain the magnetic-field distribution of flux quanta. Finally, from the energy dissipation of flux flow, we naturally obtain that the Lorentz force is equal to the viscosity force without regarding it as an assumption, as in the BS theory. This is the reason why the vortices can move with a constant velocity in uniform applied transport current. Therefore, our theory can explain the flux-flow phenomenon clearly.

ACKNOWLEDGMENT

We are grateful to the financial support from the National Science Council of the Republic of China.

[1]A. A. Abrikosov, Zh. Eksp. Teor. Fiz. **32**, 1442 (1957) [Sov. Phys. JETP **5**, 1174 (1957)].

[2]John Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965).

[3]Z. D. Wang and C. S. Ting, Phys. Rev. B **46**, 284 (1992).

[4]M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).

[5]M. W. Kleiner, L. M. Roth, and S. H. Autler, Phys. Rev. **133**, A1226 (1964).

[6]V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950).

[7]John R. Clem, J. Low Temp. Phys. **18**, 4227 (1975).

[8]Neil W. Ashcroft and N. David Mermin, *Solid State Physics* (Saunders, Philadelphia, 1976), Chap. 11.

[9]L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].

[10]B. D. Josephson, Phys. Lett. **16**, 242 (1965).