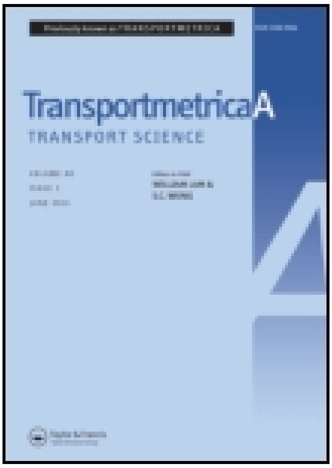

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Integrated fuzzy data envelopment analysis to assess transport performance

Lawrence W. Lan[a,b], Yu-Chiun Chiou[b] & Barbara T.H. Yen[b]
[a] Department of Tourism Management, Ta Hwa University of Science and Technology, No. 1 Tahwa Rd., Chiunglin, Hsinchu 30740, Taiwan, Republic of China
[b] Institute of Traffic and Transportation, National Chiao Tung University, 4F, 118 Sec.1, Chung-Hsiao W. Rd., Taipei 10012, Taiwan, Republic of China
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Lawrence W. Lan[a,b], Yu-Chiun Chiou[b]* and Barbara T.H. Yen[b]

[a] *Department of Tourism Management, Ta Hwa University of Science and Technology, No. 1 Tahwa Rd., Chiunglin, Hsinchu 30740, Taiwan, Republic of China;* [b] *Institute of Traffic and Transportation, National Chiao Tung University, 4F, 118 Sec.1, Chung-Hsiao W. Rd., Taipei 10012, Taiwan, Republic of China*

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Previous fuzzy data envelopment analysis (FDEA) models separately solving for the lower- and upper-bound efficiency frontiers under a specific α-cut level may lead to inconsistent efficiency rankings, unreasonable efficiency scores, and cumbersome slack computations. To rectify such shortcomings, this paper proposes two novel integrated fuzzy data envelopment analysis (IFDEA) models wherein both efficiency frontiers are incorporated into a single modelling formulation in ways that the slack values for lower- and upper-bound input/output variables are determined simultaneously. A numerical example shows that the proposed IFDEA models are more generalised and with greater simplicity than an existent FDEA model. A case study further demonstrates that the proposed IFDEA models can satisfactorily assess the relative efficiency for bus transport companies provided that a portion of the variables are measured qualitatively with vagueness (passenger satisfaction in this study).

Keywords: fuzzy data envelopment analysis; integrated fuzzy data envelopment analysis; bus transport

1. Introduction

A comprehensive performance assessment for transport services must consider both 'crisp' quantitative measures (e.g. labour, fleet, fuel consumption, service frequency, vehicle-kilometres, ton-kilometres, passenger-kilometres, revenues) and 'fuzzy' qualitative measures (e.g. crew member's attitude, vehicle's quality, customer's satisfaction). However, the qualitative measures have been ignored in most conventional data envelopment analysis (DEA) applications in transport systems (Lan and Lin 2005; Chiou and Chen 2006; Chiou, Lan, and Yen 2010; Lin, Lan, and Chiu 2010) because it is hard to precisely measure the qualitative variables, which are often in linguistic forms, e.g. 'old' vehicle, 'good' service, or 'comfortable' environment (Lertworasirikul et al. 2003). To be more comprehensive while assessing the transport services, it is believed that the qualitative measures are as important as the quantitative ones, at least from the users' perspectives. Here arises a challenging issue as to how to logically incorporate the qualitative measures into the quantitative measures while using the DEA-based modelling to carry out the performance evaluation of transport services.

In literature, DEA is a useful technique to measure the relative efficiency or effectiveness of decision-making units (DMUs) that produce similar (homogeneous) products or services. DEA has some good merits in benchmarking the efficient DMUs that can reveal the improvement directions/magnitudes for the inefficient units without the needs of pre-specifying the functional

*Corresponding author. Email: ycchiou@mail.nctu.edu.tw

Table 1. Input and output data provided by León et al. (2003).

DMU	X	Y
A	$\tilde{3} = (3, 2)$	$\tilde{3} = (3, 1)$
B	$\tilde{4} = (4, 0.5)$	$\tilde{2.5} = (2.5, 1)$
C	$\tilde{4.5} = (4.5, 1.5)$	$\tilde{6} = (6, 1)$
D	$\tilde{6.5} = (6.5, 0.5)$	$\tilde{4} = (4, 1.25)$
E	$\tilde{7} = (7, 2)$	$\tilde{5} = (5, 0.5)$
F	$\tilde{8} = (8, 0.5)$	$\tilde{3.5} = (3.5, 0.5)$
G	$\tilde{10} = (10, 1)$	$\tilde{6} = (6, 0.5)$
H	$\tilde{6} = (6, 0.5)$	$\tilde{2} = (2, 1.5)$

forms or assigning subjective weights to inputs and outputs; therefore, various DEA-based formulations have been proposed and applied in different industries (André, Herrero, and Riesgo 2010; Chiou, Lan, and Yen 2010; Siriopoulos and Tziogkidis 2010; Simon, Simon, and Arias 2011; Chiou, Lan, and Yen 2012). Conventional DEA models are in nature formulated with quantitative variables, which are measured in a 'crisp' manner – hereinafter termed as crisp data envelopment analysis (CDEA). If one also wishes to measure the qualitative variables expressed in linguistic terms, one may formulate the DEA models with partial variables measured in a 'fuzzy' manner – hereinafter termed as fuzzy data envelopment analysis (FDEA). In this sense, FDEA models can be regarded as a generalisation of CDEA models.

Recently, several FDEA models have been proposed, most of which adopted two CDEA models by separately determining the evaluation results corresponding to lower- and upper-bound under a specific α-cut level. Kao and Liu (2000, 2005), for instance, transformed a fuzzy DEA model into a group of CDEA models by applying the α-cut method and Zadeh's extension principle to determine the imprecise efficiency values. Based on the α-cut transformation, Liu (2008) and Liu and Chuang (2009) further introduced the concept of assurance region (AR) and proposed a fuzzy DEA/AR model to calculate the lower- and upper-bound efficiency scores. Azadeh and Alem (2010) converted fuzzy input and output data into interval numbers by using the α-cut method and determined the interval efficiency scores of DMUs. These studies used the α-cut method to separately determine the efficiency interval scores and then reformulated the fuzzy numbers accordingly. The main problems encountered by these FDEA models include inconsistent efficiency rankings and unreasonable efficiency scores because the reformulated fuzzy numbers can be distorted by lacking integration among various α-cut levels with their associated upper- and lower-bounds. For instance, the numerical data (Table 1) provided by León et al. (2003) have been used to generate the lower- and upper-bound frontiers with two separated CDEA models proposed by Kao and Liu (2000). Figure 1 shows the lower-bound efficiency scores greater than the upper-bound efficiency scores for DMUs 4, 5, 6 and 7 under $\alpha = 0$, which are apparently unreasonable. This example clearly depicts the inconsistent efficiency frontiers encountered by previous FDEA models. In light of this, the present study aims to rectify such problems by proposing two novel models – termed as integrated fuzzy data envelopment analysis (IFDEA) models.

Moreover, the scale and slack analyses are difficult to obtain from previous FDEA models because the computation process of fuzzy efficiency scores is repeatedly determined from the interval values (lower- and upper-bound) under various α-cut levels, which is rather cumbersome. The proposed IFDEA models also attempt to improve this drawback with the underlying logic to simultaneously optimise the lower- and upper-bound values under a specific α-level and then to derive a crisp efficient frontier without the needs of additional fuzzy rankings. Specifically, the proposed IFDEA models will incorporate both lower- and upper-bound values into

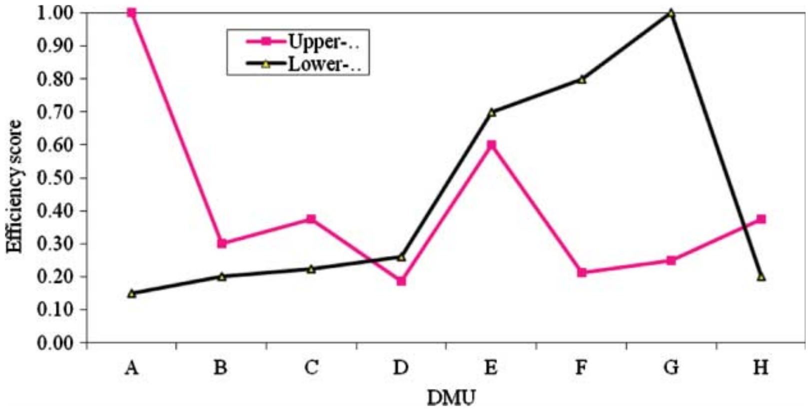

Figure 1. Efficiency frontiers determined by separate CDEA models under $\alpha = 0$.

objective functions and constraints, coupled with self-determined weights; as such, the relative efficiency of DMUs can be obtained without the needs of employing any fuzzy ranking methods.

The rest of this paper is organised as follows. Section 2 derives the mathematical formulation of the proposed IFDEA models under constant-returns-to-scale (CRS) and variable-returns-to-scale (VRS) technologies, respectively. Section 3 presents the efficiency (technical and scale) and slack analyses associated with the proposed IFDEA models. Section 4 demonstrates the superiority of the proposed IFDEA model over an existent FDEA model (León et al. 2003) using the same numerical dataset. Section 5 applies the proposed IFDEA models to evaluate the performance of 35 intercity bus companies in Taiwan. Section 6 further discusses the advantages of the proposed model by comparing with the FDEA models proposed by Kao and Liu (2000). Finally, conclusions with suggestions for future studies are addressed.

2. Models formulation

Following the conventional CCR model (Charnes, Cooper, and Rhodes 1978) for CRS technology and BCC model (Banker, Charnes, and Cooper 1984) for VRS technology, two basic IFDEA formulations are developed in this study – hereinafter termed as integrated fuzzy CCR (IFCCR) model and integrated fuzzy BCC (IFBCC) model.

2.1. *IFCCR model*

Consider n DMUs to be evaluated. Each DMU utilises m inputs to produce s outputs, and some of the inputs and/or outputs are measured qualitatively with fuzziness. To develop the IFCCR model, we first look into a fuzzy CCR (FCCR) model, which can be formulated as follows:

$$\text{(FCCR)} \quad \underset{u_r, v_i}{\text{Max}} \quad \tilde{h}_k = \sum_{r=1}^{s} u_r \tilde{y}_{rk} \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_i \tilde{x}_{ik} = \tilde{1}, \tag{2}$$

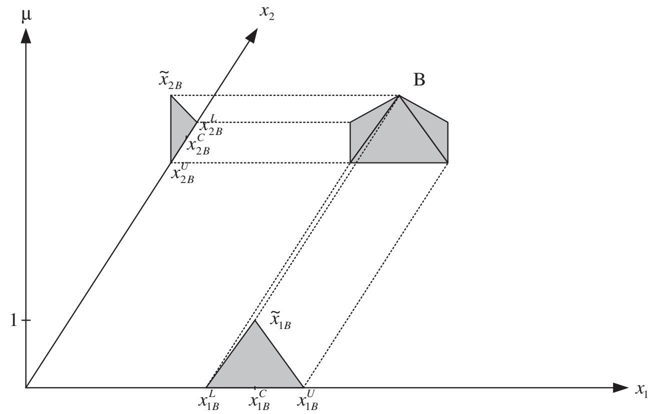

Figure 2. Projection of membership function of DMU B.

$$\sum_{r=1}^{s} u_r\tilde{y}_{rj} - \sum_{i=1}^{m} v_i\tilde{x}_{ij} \leq \tilde{0}, j = 1, \ldots, n, \tag{3}$$

$$u_r, v_i \geq \varepsilon > 0, r = 1, \ldots, s, i = 1, \ldots, m; \tag{4}$$

where $\tilde{h}_k$ is the fuzzy efficiency score of DMU k, $\tilde{x}_{ik}$ is the fuzzy input i of DMU k, $\tilde{y}_{rk}$ is the fuzzy output r of DMU k, u_r and v_i are the multipliers corresponding to output r and input i, respectively. To solve FCCR, the α-cut technique proposed by Dubois and Prade (1980) is employed to convert the associated fuzzy numbers into crisp formulation. The α-cut of $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are defined as follows:

$$\tilde{x}_{ij\alpha} = \{x_{ij} \in S(\tilde{x}_{ij}) | u_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\}, \quad \forall i, j, \tag{5}$$

$$\tilde{y}_{rj\alpha} = \{y_{rj} \in S(\tilde{y}_{rj}) | u_{\tilde{y}_{rj}}(y_{rj}) \geq \alpha\}, \quad \forall r, j; \tag{6}$$

where $u_{\tilde{x}_{ij}}$ and $u_{\tilde{y}_{rj}}$ are the membership functions of $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$, $S(\tilde{x}_{ij})$ and $S(\tilde{y}_{rj})$ are the support of $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$. The α-cut of a fuzzy number is an interval number defined by lower- and upper-bound. That is, $\tilde{x}_{ij\alpha} = [x^L_{ij\alpha}, x^U_{ij\alpha}]$ and $\tilde{y}_{rj\alpha} = [y^L_{rj\alpha}, y^U_{rj\alpha}]$ under α-cut level, where $x^L_{ij\alpha}, x^U_{ij\alpha}$ and $y^L_{rj\alpha}, y^U_{rj\alpha}$ respectively denote the lower- and upper-bound of $\tilde{x}_{ij\alpha}$ and $\tilde{y}_{rj\alpha}$.

Without loss of generality, the values of all inputs and outputs can be regarded as fuzzy numbers because any crisp value can be represented by a degenerated membership function having only one value in its domain. Hence, previous relevant works formulating the FCCR model in two separated 'crisp' CCR models can be associated with lower-bound and upper-bound, respectively. However, as demonstrated by the above example, this can lead to inconsistent evaluation results. The proposed IFDEA model, therefore, combines both lower- and upper-bounds into a single model. The concept can be depicted in Figures 2 and 3, wherein five DMUs (A, B, C, D and E) are considered. For simplicity, each DMU is assumed using two inputs to produce one output. Figure 2 demonstrates the projection of membership function for DMU B, provided that the efficiency frontier is formed by DMUs A, C, D and E. Under a specific α-cut level, the lower-, centre-, and upper-bound efficiency frontiers are respectively denoted as F^L_α, F^C_α, and F^U_α as shown in Figure 3. The range between F^L_α and F^U_α represents the bandwidth of the efficiency frontiers. In order to integrate the lower- and upper-bound efficiency frontiers, a preference weight is further introduced to generate a weighted efficiency frontier; the crisp efficiency can therefore be determined by the IFCCR model, explained as follows.

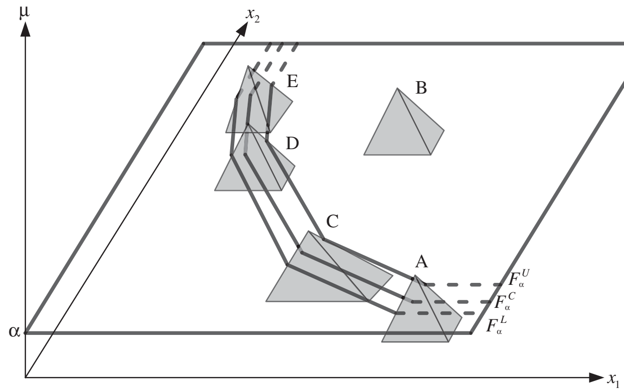

Figure 3. Efficiency frontier formed by DMUs A, C, D, E.

Equivalently, to maximise Equation (1) is to simultaneously maximise the summed lower-bound ($\sum_{r=1}^{s} u_r y_{rk\alpha}^L$) and the summed upper-bound ($\sum_{r=1}^{s} u_r y_{rk\alpha}^U$), depicted in Figure 4 and expressed below:

$$\operatorname*{Max}_{u_r,v_i}(\tilde{h}_k)_\alpha = \operatorname*{Max}_{u_r,v_i}\sum_{r=1}^{s} u_r[y_{rk\alpha}^L, y_{rk\alpha}^U] = \begin{cases} \operatorname*{Max}_{u_r,v_i}\sum_{r=1}^{s} u_r y_{rk\alpha}^L \\ \operatorname*{Max}_{u_r,v_i}\sum_{r=1}^{s} u_r y_{rk\alpha}^U \end{cases}. \tag{7}$$

In order to integrate these two objection functions, a preference weight β is introduced. The preference weights β is the weight of lower-bound under certain α-cut of $\tilde{y}_{rk}$. Let $\beta = 1$ denote a pessimistic opinion in maximising $\tilde{y}_{rk}$ because the worst situation (lower-bound) is considered; in contrast, $\beta = 0$ should be regarded as an optimistic opinion. Furthermore, to ensure the convex combination of lower- and upper-bound, a constraint $0 \leq \beta \leq 1$ should be added. Therefore,

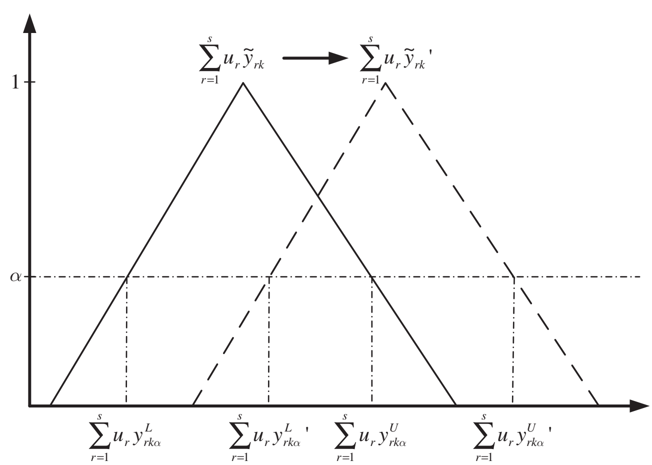

Figure 4. The summed fuzzy output of DMU k towards maximisation.

Equation (7) can be converted into a single objective function as shown in Equation (8):

$$\underset{u_r, v_i}{\text{Max}} \left\{ \sum_{r=1}^{s} u_r \beta y_{rk\alpha}^{L} + \sum_{r=1}^{s} u_r (1-\beta) y_{rk\alpha}^{U} \right\} = \underset{u_{r1}, u_{r2}, v_i}{\text{Max}} \left\{ \sum_{r=1}^{s} u_{r1} y_{rk\alpha}^{L} + \sum_{r=1}^{s} u_{r2} y_{rk\alpha}^{U} \right\}, \tag{8}$$

where $u_{r1} = u_r\beta$ and $u_{r2} = u_r(1-\beta)$. Since $0 \le \beta \le 1$ and $u_r \ge 0$, both u_{r1} and u_{r2} are non-negative.

Similarly, by substituting the α-cut interval number into Equation (2), we obtain an equivalent crisp constraint as:

$$\sum_{i=1}^{m} v_i \tilde{x}_{ik} = \sum_{i=1}^{m} v_i \left[x_{ik\alpha}^{L}, x_{ik\alpha}^{U} \right] = \tilde{1}, \tag{9}$$

where $\tilde{1}$ represents a fuzzy number distributed within proximity of 1. The constraint of $\sum_{i=1}^{m} v_i \tilde{x}_{ik}$ equal to $\tilde{1}$ indicates that the range between the summed lower-bound ($\sum_{i=1}^{m} v_i x_{ik\alpha}^{L}$) and the summed upper-bound ($\sum_{i=1}^{m} v_i x_{ik\alpha}^{U}$) should contain the value of 1. Hence, Equation (9) can be expressed by the following two inequalities:

$$\sum_{i=1}^{m} v_i x_{ik\alpha}^{L} \le 1, \tag{10}$$

$$\sum_{i=1}^{m} v_i x_{ik\alpha}^{U} \ge 1. \tag{11}$$

Following the same vein in converting the objective function, a preference weight variable γ $(0 \le \gamma \le 1)$ is introduced to integrate both Equations (10) and (11) into one equation as below:

$$\sum_{i=1}^{m} v_i \gamma x_{ik\alpha}^{L} + \sum_{i=1}^{m} v_i (1-\gamma) x_{ik\alpha}^{U} = 1. \tag{12}$$

Let $v_{i1} = v_i\gamma$ and $v_{i2} = v_i(1-\gamma)$, Equation (12) can be rewritten as:

$$\sum_{i=1}^{m} v_{i1} x_{ik\alpha}^{L} + \sum_{i=1}^{m} v_{i2} x_{ik\alpha}^{U} = 1. \tag{13}$$

Both v_{i1} and v_{i2} are non-negative because $v_i \ge 0$ and $0 \le \gamma \le 1$.

By substituting α-cut interval numbers of inputs and outputs into Equation (3), the constraint Equation (3) can be expressed as:

$$\sum_{r=1}^{s} u_r \left[y_{rj\alpha}^{L}, y_{rj\alpha}^{U} \right] - \sum_{i=1}^{m} v_i \left[x_{ij\alpha}^{L}, x_{ij\alpha}^{U} \right] \le \tilde{0}. \tag{14}$$

Using the addition operation of interval numbers, Equation (14) can further be expressed as:

$$\left[\sum_{r=1}^{s} u_r y_{rj\alpha}^{L}, \sum_{r=1}^{s} u_r y_{rj\alpha}^{U} \right] - \left[\sum_{i=1}^{m} v_i x_{ij\alpha}^{L}, \sum_{i=1}^{m} v_i x_{ij\alpha}^{U} \right] \le \tilde{0}. \tag{15}$$

Note that the left-hand side of Equation (15) is a minus of two interval numbers. To satisfy an interval number always smaller than the other, we let any arbitrary value in the former interval number be smaller than that in the latter one. That is:

$$\left(\beta\sum_{r=1}^{s}u_r y_{rj\alpha}^{L}+(1-\beta)\sum_{r=1}^{s}u_r y_{rj\alpha}^{U}\right)\le\left(\gamma\sum_{i=1}^{m}v_i x_{ij\alpha}^{L}+(1-\gamma)\sum_{i=1}^{m}v_i x_{ij\alpha}^{U}\right). \tag{16}$$

Equation (16) can therefore be expressed as:

$$\left(\sum_{r=1}^{s}u_{r1}y_{rj\alpha}^{L}+\sum_{r=1}^{s}u_{r2}y_{rj\alpha}^{U}\right)\le\left(\sum_{i=1}^{m}v_{i1}x_{ij\alpha}^{L}+\sum_{i=1}^{m}v_{i2}x_{ij\alpha}^{U}\right). \tag{17}$$

With Equations (8), (13), and (17), the above FCCR model can be easily transformed into our proposed IFCCR model as follows:

$$\text{(IFCCR)}\ \underset{u_{r1},u_{r2},v_{i1},v_{i2}}{\text{Max}}\quad h_{k\alpha}=\underset{u_{r1},u_{r2},v_{i1},v_{i2}}{\text{Max}}\left\{\sum_{r=1}^{s}u_{r1}y_{rk\alpha}^{L}+\sum_{r=1}^{s}u_{r2}y_{rk\alpha}^{U}\right\} \tag{18}$$

$$\text{s.t.}\quad \sum_{i=1}^{m}v_{i1}x_{ik\alpha}^{L}+\sum_{i=1}^{m}v_{i2}x_{ik\alpha}^{U}=1, \tag{19}$$

$$\left(\sum_{r=1}^{s}u_{r1}y_{rj\alpha}^{L}+\sum_{r=1}^{s}u_{r2}y_{rj\alpha}^{U}\right)\le\left(\sum_{i=1}^{m}v_{i1}x_{ij\alpha}^{L}+\sum_{i=1}^{m}v_{i2}x_{ij\alpha}^{U}\right), \tag{20}$$

$$u_{r1},u_{r2},v_{i1},v_{i2}\ge 0, j=1,\ldots,n;\quad i=1,\ldots,m;\quad r=1,\ldots,s; \tag{21}$$

where $h_{k\alpha}$ represents the crisp efficiency score of DMU k. If h_k equals 1, the DMU is regarded as relatively efficient; otherwise, it is inefficient. The variables $u_{r1}, u_{r2}, v_{i1}, v_{i2}$ are the corresponding virtual multipliers of the rth output and the ith input; n, m and s denote the number of DMUs, inputs and outputs, respectively.

The dual form of our proposed IFCCR model can be expressed as follows:

$$\text{(IFCCR-D)}\ \underset{\theta,\lambda}{\text{Min}}\quad \theta-\varepsilon\left(\sum_{i=1}^{m}s_{i_1}^{-}+\sum_{i=1}^{m}s_{i_2}^{-}+\sum_{r=1}^{s}s_{r_1}^{+}+\sum_{r=1}^{s}s_{r_2}^{+}\right) \tag{22}$$

$$\text{s.t.}\quad \theta x_{ik\alpha}^{L}-\sum_{j=1}^{n}\lambda_j x_{ij\alpha}^{L}-s_{i_1}^{-}=0, \tag{23}$$

$$\theta x_{ik\alpha}^{U}-\sum_{j=1}^{n}\lambda_j x_{ij\alpha}^{U}-s_{i_2}^{-}=0, \tag{24}$$

$$\sum_{j=1}^{n}\lambda_j y_{rj\alpha}^{L}-y_{rk\alpha}^{L}-s_{r_1}^{+}=0, \tag{25}$$

$$\sum_{j=1}^{n}\lambda_j y_{rj\alpha}^{U}-y_{rk\alpha}^{U}-s_{r_2}^{+}=0, \tag{26}$$

$$\lambda_j, s_{i_1}^{-}, s_{i_2}^{-}, s_{r_1}^{+}, s_{r_2}^{+}\ge 0,\ j=1,\ldots,n;\quad i=1,\ldots,m;\quad r=1,\ldots,s, \tag{27}$$

$$\theta\ \text{unrestricted in sign;} \tag{28}$$

where θ represents the efficiency score of DMU k. If θ equals 1, the DMU is regarded as relatively efficient; otherwise, it is inefficient. λ_j is the influence from DMU j; $(s_{i_1}^{-}, s_{i_2}^{-})$ are slack variables of the ith input and $(s_{r_1}^+, s_{r_2}^+)$ are slack variables of the rth output for lower-bound and upper-bound corresponding to a specific α-level, respectively.

2.2. *IFBCC model*

Following the above IFCCR procedures, the IFBCC model for VRS technology can be easily derived by simply adding a convexity constraint. The dual form of the proposed IFBCC model can be expressed as follows:

$$\text{(IFBCC-D)} \ \min_{\theta,\lambda} \quad \theta - \varepsilon\left(\sum_{i=1}^{m} s_{i_1}^- + \sum_{i=1}^{m} s_{i_2}^- + \sum_{r=1}^{s} s_{r_1}^+ + \sum_{r=1}^{s} s_{r_2}^+\right) \tag{29}$$

$$\text{s.t.} \quad \theta x_{ik\alpha}^L - \sum_{j=1}^{n} \lambda_j x_{ij\alpha}^L - s_{i_1}^- = 0, \tag{30}$$

$$\theta x_{ik\alpha}^U - \sum_{j=1}^{n} \lambda_j x_{ij\alpha}^U - s_{i_2}^- = 0, \tag{31}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj\alpha}^L - y_{rk\alpha}^L - s_{r_1}^+ = 0, \tag{32}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj\alpha}^U - y_{rk\alpha}^U - s_{r_2}^+ = 0, \tag{33}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{34}$$

$$\lambda_j, s_{i_1}^-, s_{i_2}^-, s_{r_1}^+, s_{r_2}^+ \geq 0, j = 1, \ldots, n; \quad i = 1, \ldots, m; \quad r = 1, \ldots, s, \tag{35}$$

$$\theta \text{ unrestricted in sign.} \tag{36}$$

3. Efficiency and slack analyses

Technical efficiency, scale efficiency, and slack analysis corresponding to the proposed IFCCR-D and IFBCC-D models are further derived.

3.1. *Technical efficiency*

The crisp efficiency score for each DMU can be determined by the proposed IFCCR-D and IFBCC-D models. Three types of efficiency scores are addressed:

(1) If $\theta_k^* < 1$, DMU k is defined as relatively inefficient. Equations (23) and (24) show that $\sum_{j=1}^{n} \lambda_j x_{ij\alpha}^L + s_{i_1}^- = \theta x_{ik\alpha}^L < x_{ik\alpha}^L$ and $\sum_{j=1}^{n} \lambda_j x_{ij\alpha}^U + s_{i_2}^- = \theta x_{ik\alpha}^U < x_{ik\alpha}^U$, suggesting that DMU k needs to reduce some amount of its inputs so as to achieve the efficiency frontier (e.g. DMU B in Figure 2).

(2) If $\theta_k^* = 1$ and $s_{i_1}^-, s_{i_2}^-, s_{r_1}^+, s_{r_2}^+$ are not all equal to zero, DMU k is defined as having radical efficiency. If $\theta_k^* = 1$ and $s_{i_1}^- \neq 0$, Equations (23) and (24) show that $\sum_{j=1}^{n} \lambda_j x_{ij\alpha}^L + s_{i_1}^- = x_{ik\alpha}^L$ and $\sum_{j=1}^{n} \lambda_j x_{ij\alpha}^U = x_{ik\alpha}^U$, suggesting that the lower-bound of input i of DMU k is larger than the weighted lower-bound of input i of DMUs on the efficiency frontier. If $\theta_k^* = 1$ and $s_{i_2}^- \neq 0$, Equations (23) and (24) show that $\sum_{j=1}^{n} \lambda_j x_{ij\alpha}^L = x_{ik\alpha}^L$ and $\sum_{j=1}^{n} \lambda_j x_{ij\alpha}^U + s_{i_2}^- = x_{ik\alpha}^U$,

suggesting that the upper-bound of input i of DMU k is larger than the weighted upper-bound of input $i$of DMUs on the efficiency frontier. If $\theta_k^* = 1$ and $s_{r_1}^+ \neq 0$, Equation (25) shows that $\sum_{j=1}^n \lambda_j y_{rj\alpha}^L > y_{rk\alpha}^L$, suggesting that the lower-bound of output r of DMU k is less than the weighted lower-bound of output r of DMUs on the efficiency frontier. If $\theta_k^* = 1$ and $s_{r_2}^+ \neq 0$, Equation (26) shows that $\sum_{j=1}^n \lambda_j y_{rj\alpha}^U > y_{rk\alpha}^U$, suggesting that the upper-bound of output r of DMU k is less than the weighted upper-bound of output r of DMUs on the efficiency frontier. These DMUs are defined as relatively inefficient (e.g. DMUs A and E in Figure 2).

(3) If $\theta_k^* = 1$ and $s_{i_1}^-, s_{i_2}^-, s_{r_1}^+, s_{r_2}^+$ are all equal to zero, DMU k is defined as relatively efficient. Equations (23)–(26) show that $\sum_{j=1}^n \lambda_j x_{ij\alpha}^L = x_{ik\alpha}^L$, $\sum_{j=1}^n \lambda_j x_{ij\alpha}^U = x_{ik\alpha}^U$, $\sum_{j=1}^n \lambda_j y_{rj\alpha}^L = y_{rk\alpha}^L$, and $\sum_{j=1}^n \lambda_j y_{rj\alpha}^U = y_{rk\alpha}^U$, suggesting that the lower- and upper-bound of fuzzy inputs and outputs of DMU k are equal to the weighted lower- and upper-bound of inputs and outputs of DMUs on the efficiency frontier. Under this circumstance, further improvement is not needed. Such DMUs are defined as relatively efficient (e.g. DMUs C and D in Figure 2).

3.2. *Scale efficiency*

To deal with both crisp and fuzzy data, the above IFBCC-D model can be further transformed into the following IFBCC-D* model, where Equations (38)–(41) are for fuzzy data, and Equations (43) and (44) are for crisp data.

$$\text{(IFBCC-D)* } \min_{\theta,\lambda} \quad \theta - \varepsilon\left(\sum_{i=1}^m s_{i_1}^- + \sum_{i=1}^m s_{i_2}^- + \sum_{r=1}^s s_{r_1}^+ + \sum_{r=1}^s s_{r_2}^+\right) \tag{37}$$

$$\text{s.t.} \quad \theta x_{ik\alpha}^L - \sum_{j=1}^n \lambda_j x_{ij\alpha}^L - s_{i_1}^- = 0, \tag{38}$$

$$\theta x_{ik\alpha}^U - \sum_{j=1}^n \lambda_j x_{ij\alpha}^U - s_{i_2}^- = 0, \tag{39}$$

$$\sum_{j=1}^n \lambda_j y_{rj\alpha}^L - y_{rk\alpha}^L - s_{r_1}^+ = 0, \tag{40}$$

$$\sum_{j=1}^n \lambda_j y_{rj\alpha}^U - y_{rk\alpha}^U - s_{r_2}^+ = 0, \tag{41}$$

$$\sum_{j=1}^n \lambda_j = 1, \tag{42}$$

$$\theta x_{ik} - \sum_{j=1}^n \lambda_j x_{ij} - s_i^- = 0, \tag{43}$$

$$\sum_{j=1}^n \lambda_j y_{rj} - y_{rk} - s_r^+ = 0, \tag{44}$$

$$\lambda_j, s_{i_1}^-, s_{i_2}^-, s_i^-, s_{r_1}^+, s_{r_2}^+, s_r^+ \geq 0; \quad r = 1, \ldots, s; \quad i = 1, \ldots, m;$$

$$j = 1, \ldots, n, \tag{45}$$

$$\theta \text{unrestricted in sign.} \tag{46}$$

If $\sum_{j=1}^{n} \lambda_j < 1$, the DMU is regarded as increasing returns to scale (IRS); if $\sum_{j=1}^{n} \lambda_j > 1$, the DMU is decreasing returns to scale (DRS); if $\sum_{j=1}^{n} \lambda_j = 1$, it is CRS.

3.3. *Slack analysis*

Slack values of each input variable provide useful information about initiating proper improvement strategies for the inefficient DMUs. For an inefficient DMU k, its fuzzy input and output under α-level can be expressed as $([x_{ij\alpha}^{L}, x_{ij\alpha}^{U}], [y_{rj\alpha}^{L}, y_{rj\alpha}^{U}])$. If the efficiency score and optimal multipliers of DMU k are $\theta^*, \lambda_j^*, s_{i_1}^{-*}, s_{i_2}^{-*}, s_{r_1}^{+*}, s_{r_2}^{+*}$, the shadows of DMU k on the efficiency frontier are:

$$x_{ij\alpha}^{L*} = \theta^* x_{ik\alpha}^{L} - s_{i_1}^{-*}, \tag{47}$$

$$x_{ij\alpha}^{U*} = \theta^* x_{ik\alpha}^{U} - s_{i_2}^{-*}, \tag{48}$$

$$y_{rj\alpha}^{L*} = y_{rk\alpha}^{L*} + s_{r_1}^{+*}, \tag{49}$$

$$y_{rj\alpha}^{U*} = y_{rk\alpha}^{U*} + s_{r_2}^{+*}. \tag{50}$$

The DMUs with $\lambda_j^* \neq 0$ determined by the IFBCC-D* model form a reference set – the efficiency frontier of DMU k. The coordinates of these benchmarked DMUs are denoted as:

$$\left(\left[\sum_{j=1}^{n} \lambda_j^* x_{ij\alpha}^{L}, \sum_{j=1}^{n} \lambda_j^* x_{ij\alpha}^{U} \right], \left[\sum_{j=1}^{n} \lambda_j^* y_{rj\alpha}^{L}, \sum_{j=1}^{n} \lambda_j^* y_{rj\alpha}^{U} \right] \right). \tag{51}$$

From Equations (47)–(50), the slack values of DMU k can be expressed as follows.

$$\Delta x_{ik\alpha}^{L} = x_{ik\alpha}^{L} - x_{ij\alpha}^{L*}, \tag{52}$$

$$\Delta x_{ik\alpha}^{U} = x_{ik\alpha}^{U} - x_{ij\alpha}^{U*}, \tag{53}$$

$$\Delta y_{rk\alpha}^{L} = y_{rk\alpha}^{L} + y_{rj\alpha}^{L*}, \tag{54}$$

$$\Delta y_{rk\alpha}^{U} = y_{rk\alpha}^{U} + y_{rj\alpha}^{U*}; \tag{55}$$

where $\Delta x_{ik\alpha}^{L}$ and $\Delta x_{ik\alpha}^{U}$ are the slack values of the lower- and upper-bounds of input i of DMU k, respectively; $\Delta y_{rk\alpha}^{L}$ and $\Delta y_{rk\alpha}^{U}$ are the slack values of the lower- and upper-bounds of input i of DMU k, respectively.

4. A numerical example

To demonstrate the superiority of the proposed IFDEA models, a numerical comparison with the existent FDEA model proposed by León et al. (2003) is conducted, both using the same dataset (Table 1) given by León et al. (2003).

First, the efficiency scores under CRS determined by the proposed IFCCR model under various α-levels are presented in Table 2. We note that only DMU C is benchmarked as efficient under all α-levels and that DMU A is evaluated as efficient for $\alpha \leq 0.5$. The FDEA model proposed by León et al. (2003) did not evaluate the CRS case.

Second, the efficiency scores under VRS determined by the proposed IFBCC model and León's model are presented in Tables 3 and 4, respectively. From Table 3 (León's model), two DMUs (A and C) are evaluated as efficient under all α-levels, and another two DMUs (G and B) become efficient as $\alpha \leq 0.9$ and $\alpha \leq 0.3$, respectively. The same results are also found in Table 4 (the

Table 2. Efficiency scores under various α-levels determined by the proposed IFCCR model.

	DMU							
α-Level	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
0.0	1.0000	0.6667	1.0000	0.6429	0.6000	0.4235	0.6000	0.4615
0.1	1.0000	0.6478	1.0000	0.6252	0.5931	0.4140	0.5841	0.4403
0.2	1.0000	0.6287	1.0000	0.6074	0.5863	0.4045	0.5684	0.4191
0.3	1.0000	0.6094	1.0000	0.5895	0.5797	0.3950	0.5529	0.3979
0.4	1.0000	0.5899	1.0000	0.5715	0.5732	0.3855	0.5377	0.3766
0.5	1.0000	0.5701	1.0000	0.5534	0.5668	0.3760	0.5227	0.3554
0.6	0.9418	0.5502	1.0000	0.5352	0.5604	0.3665	0.5079	0.3342
0.7	0.8839	0.5301	1.0000	0.5169	0.5542	0.3570	0.4932	0.3130
0.8	0.8337	0.5098	1.0000	0.4985	0.5480	0.3474	0.4787	0.2919
0.9	0.7895	0.4894	1.0000	0.4801	0.5418	0.3378	0.4643	0.2709
1.0	0.7500	0.4688	1.0000	0.4615	0.5357	0.3281	0.4500	0.2500

Table 3. Efficiency scores under various α-levels determined by the León's model.

	DMU							
α-Level	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
0.0	1.0000	1.0000	1.0000	0.7500	0.6429	0.6050	1.0000	0.6923
0.1	1.0000	1.0000	1.0000	0.7399	0.6398	0.5952	1.0000	0.6899
0.2	1.0000	1.0000	1.0000	0.7292	0.6369	0.5857	1.0000	0.6875
0.3	1.0000	1.0000	1.0000	0.7084	0.6310	0.5660	1.0000	0.6850
0.4	1.0000	0.9767	1.0000	0.6853	0.6244	0.5446	1.0000	0.6667
0.5	1.0000	0.9412	1.0000	0.6623	0.6172	0.5227	1.0000	0.6400
0.6	1.0000	0.9048	1.0000	0.6383	0.6094	0.5004	1.0000	0.6129
0.7	1.0000	0.8675	1.0000	0.6144	0.6010	0.4776	1.0000	0.5854
0.8	1.0000	0.8293	1.0000	0.5894	0.5919	0.4543	1.0000	0.5574
0.9	1.0000	0.7901	1.0000	0.5645	0.5821	0.4305	1.0000	0.5289
1.0	1.0000	0.7500	1.0000	0.5385	0.5714	0.4062	0.4500	0.5000

Table 4. Efficiency scores under various α-levels determined by the proposed IFBCC model.

	DMU							
α-Level	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
0.0	1.0000	1.0000	1.0000	0.7500	0.6429	0.6050	1.0000	0.6923
0.1	1.0000	1.0000	1.0000	0.7396	0.6398	0.5953	1.0000	0.6899
0.2	1.0000	1.0000	1.0000	0.7292	0.6369	0.5857	1.0000	0.6875
0.3	1.0000	1.0000	1.0000	0.7081	0.6311	0.5660	1.0000	0.6850
0.4	1.0000	0.9767	1.0000	0.6853	0.6244	0.5446	1.0000	0.6667
0.5	1.0000	0.9412	1.0000	0.6620	0.6172	0.5227	1.0000	0.6400
0.6	1.0000	0.9048	1.0000	0.6383	0.6094	0.5004	1.0000	0.6129
0.7	1.0000	0.8675	1.0000	0.6141	0.6010	0.4776	1.0000	0.5854
0.8	1.0000	0.8293	1.0000	0.5894	0.5919	0.4543	1.0000	0.5574
0.9	1.0000	0.7901	1.0000	0.5642	0.5821	0.4305	1.0000	0.5289
1.0	1.0000	0.7500	1.0000	0.5385	0.5714	0.4063	0.4500	0.5000

proposed IFBCC model). We also find that the efficiency scores of the IFBCC model (Table 4) are almost exactly the same as those of the León's model (Table 3).

Third, by using the proposed IFBCC model, the scale efficiency scores can be easily computed as shown in Table 5. We note that except for DMU *G* (characterised with DRS for $\alpha \leq 0.9$) and DMUs *A* and *C* (characterised with CRS for $\alpha \leq 0.3$ and for all α-levels, respectively), the

Table 5. Scale efficiency scores under various α-levels determined by the IFDEA model.

	DMU															
α-Level	*A*		*B*		*C*		*D*		*E*		*F*		*G*		*H*	
0.0	1.00	CRS	0.50	IRS	1.00	CRS	0.75	IRS	0.90	IRS	0.60	IRS	1.10	DRS	0.50	IRS
0.1	1.00	CRS	0.49	IRS	1.00	CRS	0.74	IRS	0.89	IRS	0.60	IRS	1.09	DRS	0.49	IRS
0.2	1.00	CRS	0.49	IRS	1.00	CRS	0.74	IRS	0.88	IRS	0.60	IRS	1.08	DRS	0.47	IRS
0.3	1.00	CRS	0.48	IRS	1.00	CRS	0.73	IRS	0.88	IRS	0.59	IRS	1.07	DRS	0.46	IRS
0.4	1.00	CRS	0.47	IRS	1.00	CRS	0.72	IRS	0.87	IRS	0.59	IRS	1.06	DRS	0.44	IRS
0.5	1.00	CRS	0.46	IRS	1.00	CRS	0.71	IRS	0.86	IRS	0.59	IRS	1.05	DRS	0.42	IRS
0.6	0.53	IRS	0.45	IRS	1.00	CRS	0.70	IRS	0.86	IRS	0.59	IRS	1.04	DRS	0.41	IRS
0.7	0.52	IRS	0.44	IRS	1.00	CRS	0.69	IRS	0.85	IRS	0.59	IRS	1.03	DRS	0.31	IRS
0.8	0.52	IRS	0.44	IRS	1.00	CRS	0.69	IRS	0.84	IRS	0.59	IRS	1.02	DRS	0.37	IRS
0.9	0.51	IRS	0.43	IRS	1.00	CRS	0.68	IRS	0.84	IRS	0.58	IRS	1.01	DRS	0.35	IRS
1.0	0.50	IRS	0.42	IRS	1.00	CRS	0.67	IRS	0.83	IRS	0.58	IRS	1.00	CRS	0.33	IRS

Table 6. Slack values for the lower-bound of input variable under various α-levels.

	DMU							
α-Level	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
0.0	0.0000	0.0000	0.0000	1.5000	1.7857	2.9622	0.0000	1.3846
0.1	0.0000	0.0000	0.0000	1.5756	1.8732	3.0557	0.0000	1.4109
0.2	0.0000	0.0000	0.0000	1.6519	1.9609	3.1486	0.0000	1.4375
0.3	0.0000	0.0000	0.0000	1.7952	2.0661	3.3203	0.0000	1.4646
0.4	0.0000	0.0860	0.0000	1.9512	2.1785	3.5067	0.0000	1.5667
0.5	0.0000	0.2206	0.0000	2.1123	2.2969	3.6989	0.0000	1.7100
0.6	0.0000	0.3619	0.0000	2.2787	2.4217	3.8968	0.0000	1.8581
0.7	0.0000	0.5102	0.0000	2.4505	2.5537	4.1008	0.0000	2.0110
0.8	0.0000	0.6659	0.0000	2.6279	2.6935	4.3109	0.0000	2.1689
0.9	0.0000	0.8290	0.0000	2.8110	2.8420	4.5272	0.0000	2.3318
1.0	0.0000	1.0000	0.0000	3.0000	3.0000	4.7500	5.5000	2.5000

remaining DMUs (*B*, *D*, *E*, *F*, and *H*) are all characterised with IRS (for all α-levels), suggesting that most of the DMUs need expanding their scales. In contrast, it would be difficult for León's model to obtain the scale efficiency scores.

Furthermore, using the proposed IFBCC model two slack values can be easily computed for lower- and upper-bounds under various α-levels, as shown in Tables 6 and 7, respectively. $\alpha = 1.0$ represents a crisp input data, thus the slack values for lower- and upper-bounds must be the same. From Tables 6 and 7, we note that except for the efficient DMUs (*A* and *C* for all α-levels; or *A*, *C* and *G* for $\alpha \leq 0.9$), all inefficient DMUs require reducing their input amounts to achieve efficiency. Taking DMU *D* as an example, one requires decreasing the input amounts by 1.50 to 3.00 for the lower-bound and by 1.75 to 3.00 for the upper-bound. With consideration of all required reductions in lower- and upper-bound under various α-levels, the fuzzy input for DMU *D* should decrease to a value of $\tilde{3} = (3, 0.375)$ to achieve efficiency, suggesting that both cortex and spread of the fuzzy input should simultaneously decrease. Once again, it is difficult for León's model to compute the slack values.

Compared with an existent FDEA model, the proposed IFDEA models can reach the same results in technical efficiency scores; more importantly, the proposed models can compute scale efficiency scores and slack values without difficulties. In sum, the proposed IFDEA models are more generalised and with greater simplicity than an existent FDEA model.

Table 7. Slack values for the upper-bound of input variable under various α-levels.

α-Level	DMU *A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
0.0	0.0000	0.0000	0.0000	1.7500	3.2143	3.3571	0.0000	1.6923
0.1	0.0000	0.0000	0.0000	1.8100	3.1700	3.4200	0.0000	1.6899
0.2	0.0000	0.0000	0.0000	1.8686	3.1229	3.4800	0.0000	1.6875
0.3	0.0000	0.0000	0.0000	1.9996	3.0992	3.6242	0.0000	1.6850
0.4	0.0000	0.1000	0.0000	2.1400	3.0800	3.7800	0.0000	1.7667
0.5	0.0000	0.2500	0.0000	2.2813	3.0625	3.9375	0.0000	1.8900
0.6	0.0000	0.4000	0.0000	2.4233	3.0467	4.0967	0.0000	2.0129
0.7	0.0000	0.5500	0.0000	2.5663	3.0325	4.2575	0.0000	2.1354
0.8	0.0000	0.7000	0.0000	2.7100	3.0200	4.4200	0.0000	2.2574
0.9	0.0000	0.8500	0.0000	2.8546	3.0092	4.5842	0.0000	2.3789
1.0	0.0000	1.0000	0.0000	3.0000	3.0000	4.7500	5.5000	2.5000

5. Case study

A case study on the intercity bus companies in Taiwan is conducted by using the proposed IFDEA models. The data and evaluation results are delineated below.

5.1. *Data*

Referring to previous relevant literature (Gillen and Lall 1997a, 1997b; Lan and Lin 2005; Chiou and Chen 2006; Bhadra 2009; Karlaftis 2010; Lin, Lan, and Hsu 2010), this study selects number of employees, length of operating network, capital cost and fuel cost as the input variables; total

Table 8. Correlation coefficients among crisp input and output variables.

Variable	Input: Bus	Labour	Operating network	Capital cost	Fuel cost	Output: Bus-km	Passenger-km	Revenue
Bus	1.00							
Labour	0.95	1.00						
Operating network	0.52	0.61	1.00					
Capital cost	0.53	0.51	0.25	1.00				
Fuel cost	0.90	0.96	0.54	0.52	1.00			
Bus-km	0.84	0.90	0.39	0.58	0.96	1.00		
Passenger-km	0.72	0.81	0.43	0.54	0.91	0.96	1.00	
Revenue	0.94	0.98	0.55	0.52	0.98	0.95	0.87	1.00

Table 9. Regression results for input and output variables.

Dependent variables	Independent variables: Bus	Labour	Operating network	Capital cost	Fuel cost
Bus-km	26,991.389 (8.351)	7304.43 (2.409)	3872.509 (3.500)	0.015 (2.801)	0.217 (7.700) $R^2 = 0.979$
Passenger-km	790,437.011 (2.385)	693,665.200 (1.395)	27,678.15 (3.885)	0.258 (2.730)	4.842 (6.014) $R^2 = 0.921$
Revenue	551,132.550 (3.245)	127,018.628 (2.421)	25,300.793 (4.042)	0.015 (3.132)	2.524 (4.041) $R^2 = 0.970$

Note: *t*-Values in parentheses.

Table 10. Passenger satisfaction for 35 intercity bus companies.

DMU	Passenger satisfaction	DMU	Passenger satisfaction	DMU	Passenger satisfaction
1	Fair service	13	Fair service	25	Fair service
2	Fair service	14	Fair service	26	Fair service
3	Fair service	15	Good service	27	Poor service
4	Poor service	16	Fair service	28	Good service
5	Poor service	17	Poor service	29	Fair service
6	Poor service	18	Poor service	30	Poor service
7	Fair service	19	Poor service	31	Fair service
8	Fair service	20	Poor service	32	Fair service
9	Fair service	21	Poor service	33	Fair service
10	Fair service	22	Fair service	34	Fair service
11	Poor service	23	Fair service	35	Poor service
12	Poor service	24	Poor service		

Table 11. Efficiency scores of 35 intercity bus companies under various α-levels.

	CRS				VRS			
DMU	$\alpha = 0.0$	$\alpha = 0.4$	$\alpha = 0.8$	$\alpha = 1.0$	$\alpha = 0.0$	$\alpha = 0.4$	$\alpha = 0.8$	$\alpha = 1.0$
1	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
2	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
3	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
4	0.4640	0.4640	0.4640	0.4640	0.4645	0.4645	0.4645	0.4645
5	0.5436	0.5436	0.5436	0.5436	0.6753	0.6753	0.6753	0.6753
6	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
7	0.8904	0.8904	0.8903	0.8902	0.9452	0.9452	0.9452	0.9452
8	0.6915	0.6915	0.6915	0.6915	0.8702	0.8702	0.8702	0.8702
9	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
10	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
11	0.5613	0.5613	0.5613	0.5613	0.8262	0.8262	0.8262	0.8262
12	0.9468	0.9468	0.9468	0.9468	0.9842	0.9842	0.9842	0.9842
13	0.6669	0.6668	0.6667	0.6666	0.7942	0.7942	0.7942	0.7942
14	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
15	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
16	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
17	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
18	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
19	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
20	0.5843	0.5842	0.5842	0.5842	0.7826	0.7826	0.7826	0.7826
21	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
22	0.6075	0.6075	0.6075	0.6075	0.7943	0.7943	0.7943	0.7943
23	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
24	0.9127	0.9127	0.9127	0.9127	1.0000*	1.0000*	1.0000*	1.0000*
25	0.7768	0.7768	0.7768	0.7768	1.0000*	1.0000*	1.0000*	1.0000*
26	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
27	0.5784	0.5783	0.5783	0.5782	0.5813	0.5813	0.5813	0.5813
28	0.4789	0.4789	0.4789	0.4789	1.0000*	1.0000*	1.0000*	1.0000*
29	0.5457	0.5457	0.5457	0.5457	0.5571	0.5571	0.5571	0.5571
30	0.8027	0.8027	0.8027	0.8027	0.9213	0.9213	0.9213	0.9213
31	0.8051	0.8051	0.8051	0.8051	1.0000*	1.0000*	1.0000*	1.0000*
32	0.9978	0.9978	0.9977	0.9977	1.0000*	1.0000*	1.0000*	1.0000*
33	0.8003	0.8003	0.8003	0.8003	1.0000*	1.0000*	1.0000*	1.0000*
34	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
35	0.4704	0.4703	0.4702	0.4701	0.4759	0.4759	0.4759	0.4759

Note: '*' represents the optimal efficient score.

Table 12. Scale efficiency scores of 35 intercity bus companies under various α-levels.

	α-level							
DMU	0.0		0.4		0.8		1.0	
1	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
2	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
3	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
4	0.9167	IRS	0.9148	IRS	0.9128	IRS	0.9118	IRS
5	1.5088	DRS	1.5088	DRS	1.5088	DRS	1.5088	DRS
6	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
7	1.0190	DRS	1.0185	DRS	1.0181	DRS	1.0179	DRS
8	2.0600	DRS	2.0600	DRS	2.0600	DRS	2.0600	DRS
9	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
10	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
11	1.1935	DRS	2.2882	DRS	2.2882	DRS	2.2882	DRS
12	1.7514	DRS	1.7514	DRS	1.7514	DRS	1.7514	DRS
13	1.1128	DRS	1.1102	DRS	1.1077	DRS	1.1065	DRS
14	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
15	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
16	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
17	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
18	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
19	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
20	1.1211	DRS	1.1181	DRS	1.1151	DRS	1.1137	DRS
21	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
22	1.0693	DRS	1.0676	DRS	1.0660	DRS	1.0652	DRS
23	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
24	4.4237	DRS	4.4237	DRS	4.4237	DRS	4.4237	DRS
25	3.5736	DRS	3.5736	DRS	3.5736	DRS	3.5736	DRS
26	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
27	0.9416	IRS	0.9403	IRS	0.9389	IRS	0.9382	IRS
28	1.3082	DRS	1.3082	DRS	1.3082	DRS	1.3082	DRS
29	1.3515	DRS	1.3515	DRS	1.3515	DRS	1.3515	DRS
30	1.6929	DRS	1.6929	DRS	1.6929	DRS	1.6929	DRS
31	1.2837	DRS	1.2837	DRS	1.2837	DRS	1.2837	DRS
32	1.0099	DRS	1.0097	DRS	1.0095	DRS	1.0094	DRS
33	2.7128	DRS	2.7128	DRS	2.7128	DRS	2.7128	DRS
34	1.0000	CRS	1.0000	CRS	1.0000	CRS	1.0000	CRS
35	0.9564	IRS	0.9554	IRS	0.9544	IRS	0.9539	IRS

passenger-km, total bus-km, total revenue, and passenger satisfaction as the output variables. It should be noted that passenger satisfaction is the only qualitative variable (conducted by a questionnaire survey) and the remaining quantitative variables are all crisp. All the data are available from the annual report published by the Institute of Transportation, Ministry of Transportation and Communications (Taiwan) in 2005.

Table 8 gives the correlation coefficients among the crisp variables. All correlation coefficients between input and output variables are significantly positive, confirming that the dataset satisfies the isotonicity property. To ensure that the selected input/output variables are important and relevant, regression analyses are further conducted as shown in Table 9. Note that all the explanatory variables show positive and significant effects on at least one of the associated dependent variables, suggesting the appropriateness of the above selected input variables.

The fuzzy variable, passenger satisfaction, is represented by three linguistic degrees: poor service (75, 5), fair service (85, 5) and good service (95, 5), with half-overlapped triangular membership functions. The original data of this fuzzy variable are summarised in Table 10.

5.2. *Efficiency analyses*

Table 11 presents the efficiency scores of the bus companies under CRS and VRS technologies, respectively. From Table 11, we note that 16 and 22 companies have been benchmarked as efficient with IFCCR and IFBCC models, respectively. Interestingly, the efficiency scores do not vary much with different α-levels. Similar to the numerical example presented in Section 4, the efficiency scores of inefficient companies increase as the α-level goes higher.

Table 12 further gives the scale efficiency scores of these bus companies. We note that most of the bus companies are characterised with DRS, implying the necessity of downsizing their scales. Only three bus companies (4, 27 and 35) are characterised with IRS, suggesting that they have the advantages to scale up.

5.3. *Slack analysis*

The slack values for the input variables of inefficient companies are computed by the IFBCC model. Table 13 gives the slack values for the input variables under $\alpha = 0.8$, from which one notice that the percentages of reduction in input amounts for the inefficient companies can range

Table 13. Slack values of input variables for 35 intercity bus companies ($\alpha = 0.8$).

DMU	Bus	Labour	Operating network	Capital cost	Fuel cost
1	0.00%	0.00%	0.00%	0.00%	0.00%
2	0.00%	0.00%	0.00%	0.00%	0.00%
3	0.00%	0.00%	0.00%	0.00%	0.00%
4	60.75%	63.10%	63.89%	71.11%	66.15%
5	46.12%	44.16%	36.83%	58.23%	45.18%
6	0.00%	0.00%	0.00%	0.00%	0.00%
7	37.00%	34.70%	4.62%	14.42%	63.19%
8	16.61%	15.40%	43.03%	67.43%	22.97%
9	0.00%	0.00%	0.00%	0.00%	0.00%
10	0.00%	0.00%	0.00%	0.00%	0.00%
11	40.06%	43.03%	55.90%	91.45%	47.18%
12	29.68%	23.37%	39.25%	81.48%	12.67%
13	27.83%	4.73%	5.48%	40.32%	16.15%
14	0.00%	0.00%	0.00%	0.00%	0.00%
15	0.00%	0.00%	0.00%	0.00%	0.00%
16	0.00%	0.00%	0.00%	0.00%	0.00%
17	0.00%	0.00%	0.00%	0.00%	0.00%
18	0.00%	0.00%	0.00%	0.00%	0.00%
19	0.00%	0.00%	0.00%	0.00%	0.00%
20	41.04%	39.79%	27.09%	9.09%	40.15%
21	0.00%	0.00%	0.00%	0.00%	0.00%
22	47.57%	39.01%	28.18%	52.64%	41.57%
23	0.00%	0.00%	0.00%	0.00%	0.00%
24	0.00%	0.00%	0.00%	0.00%	0.00%
25	0.00%	0.00%	0.00%	0.00%	0.00%
26	0.00%	0.00%	0.00%	0.00%	0.00%
27	50.43%	68.37%	68.60%	79.64%	73.01%
28	0.00%	0.00%	0.00%	0.00%	0.00%
29	68.51%	64.02%	89.35%	94.88%	51.98%
30	42.72%	42.26%	54.25%	48.53%	30.97%
31	0.00%	0.00%	0.00%	0.00%	0.00%
32	0.00%	0.00%	0.00%	0.00%	0.00%
33	0.00%	0.00%	0.00%	0.00%	0.00%
34	0.00%	0.00%	0.00%	0.00%	0.00%
35	66.77%	58.66%	82.03%	71.32%	70.72%

from 4.73% to 94.88%. Taking Company 11 as an example, reducing the fleet size by 40.06%, the labour force by 43.03%, the operating network by 55.90%, the capital by 91.45%, and the fuel by 47.18% will move the company towards efficiency.

6. Discussion

The major merit of the proposed IFDEA models is to integrate the lower- and upper-bound efficiency frontiers to generate a crisp efficiency value. With the determined crisp efficiency frontier, the scale efficiency scores and the slack values for DMUs can be easily computed. As such, and the improvement directions for the inefficient DMUs can be clearly identified.

To further highlight the advantages of the proposed IFCCR model, a comparison with the FDEA models proposed by Kao and Liu (2000) is conducted. Table 14 presents the slack values of input variables for DMU 13 determined by the IFCCR model. Figure 5 further compares the efficiency scores for DMU 13 by the FDEA model (Kao and Liu 2000) and by the proposed IFCCR model. We note that the efficiency value for DMU 13 decreases as α gets larger, showing that the proposed IFCCR model computes lower efficiency value with higher α value (i.e. more pessimistic than FDEA). The proposed IFCCR model becomes a crisp model and shows DMU 13 being inefficient as $\alpha = 1$. From Figure 5, it is apparent that the results of IFCCR model lie between lower- and upper-efficiency frontiers, which are in effect derived from two CDEA models (Kao and Liu 2000). In contrast, the proposed IFCCR model has reasonably integrated the lower- and upper-efficiency frontiers.

Figure 6 further displays the slack values of input variables for the inefficient DMU 13 under different α values by the IFCCR model. When α value becomes larger, DMU 13 requires curtailing

Table 14. Slack values of input variables for DMU 13 by the IFCCR model.

α Value	Bus	Labour	Operating network	Capital cost	Fuel cost
0.0	31.57%	10.09%	5.66%	44.71%	22.95%
0.4	32.18%	10.97%	5.69%	45.43%	24.06%
0.8	32.79%	11.85%	5.72%	46.14%	25.17%
1.0	33.41%	12.73%	5.74%	46.86%	26.29%

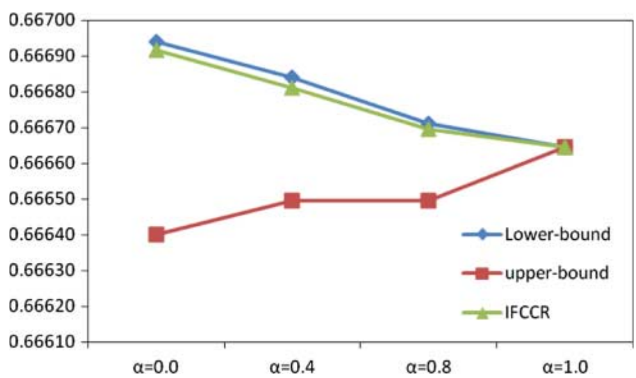

Figure 5. Efficiency scores for DMU 13 by the FDEA model (Kao and Liu 2000) and by the IFCCR model.

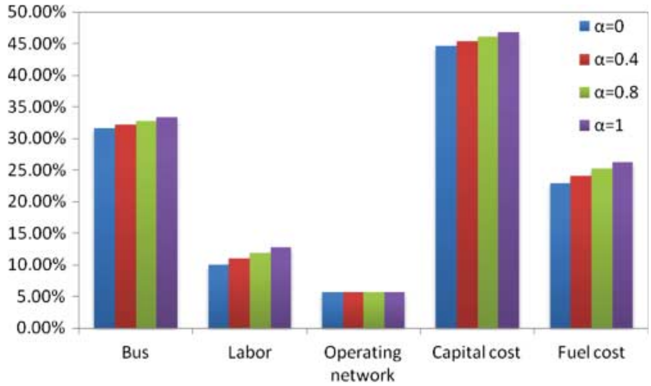

Figure 6. Slack values of input variables for DMU 13 under different α values by the IFCCR model.

more amounts of its inputs. Hence, a pessimistic decision maker may choose a larger α value by which the inefficient DMUs will be improved more remarkably, and vice versa. With this procedure, the decision maker can easily determine how to improve the inefficient DMUs' performance in a context containing crisp and fuzzy input/output measures. With flexible settings of α values, the proposed IFDEA models can facilitate the managers to make more flexible and correct decisions, based on informative and useful evaluation results.

7. Concluding remarks

Previous FDEA models have separately determined the lower- and upper-bound efficiency scores under various α-cut levels by using subjective ranking methods to find the crisp evaluation results. This can often lead to unreasonable frontiers – with lower-bound efficiency scores greater than upper-bound efficiency scores. This paper contributes two IFDEA models, IFCCR and IFBCC, that have successfully overcome this problem. The proposed IFDEA models can determine crisp evaluation scores under various α-levels with CRS and VRS technologies. In addition, the proposed IFDEA models can easily determine the slack values for both lower- and upper-bound input/output variables simultaneously. With the computed slack values under various α-cut levels, the associated fuzzy values for input variables can be determined to achieve efficiency. The numerical example has illustrated that the proposed IFDEA models are more generalised and with greater simplicity than an existent FDEA model. The case study has also demonstrated that the proposed IFDEA modelling approach can satisfactorily evaluate the relative efficiency for DMUs with a portion of qualitative variables measured with vagueness.

This study inevitably has some limitations which call for further exploration. First, the proposed IFDEA models are to determine the efficiency score under a pre-specified α-level. In practice, however, it might be difficult for a decision maker to preset the α-level. Therefore, one may further elaborate the IFDEA models to determine the efficiency scores by simultaneously considering all possible α-levels. Second, more comparisons with other existent FDEA models deserve further studies to test the superiority robustness of the proposed IFDEA models. Turning to the empirical applications in bus transport evaluation, aside from passenger satisfaction, other qualitative data such as driver attitudes, vehicle comfort or amenity, and passenger complaints may also affect the overall performance of services. Therefore, in the future study, conducting a survey on such qualitative data before applying the proposed IFDEA models will make the performance evaluation more holistic.

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