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The Cournot production game with multiple firms under an ambiguous decision environment

Jr-Fong Dang [a], I-Hsuan Hong [b,*,1], Jing-Ming Lin [a]

[a] Department of Industrial Engineering & Management, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan
[b] Institute of Industrial Engineering, National Taiwan University, 1 Section 4 Roosevelt Road, Taipei 106, Taiwan

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ABSTRACT

The recent economic recession has added more uncertainty to industries' decision processes regarding production quantity. Moreover, due to the nature of ambiguous information, manufacturers often fail to achieve precise assessments of the parameters of market demand, production cost functions, etc. This paper develops the Cournot production game with multiple firms in an ambiguous decision environment, where the form of ambiguity is described by a set of fuzzy parameters. Our model applies the weighted center of gravity method (WCoG) to defuzzify the fuzzy profit function considering firms' control parameters. The resulting outcomes are in the form of matrix representations. We also analyze the effect of firms' control parameters on outcomes. The results indicate that a firm's fuzzy profit function plays an important role in economic interpretation. To investigate the effect of parameter perturbations on firms' outcomes, we also conduct the sensitivity analysis.

1. Introduction

A competitive global economy presents opportunities for new approaches to solve for the production quantity of decision makers who are forced to operate in ambiguous environments. Intuitively, the decisions of multiple agents will affect the payoffs of others. Thus, the conventional Cournot game is one of several methods commonly applied to analyze precise scenarios when exact model parameters are available. Liang et al. [20] categorize these as "precision-based models" because all data are required to be precise. Yet, real-world decision makers typically are hampered due to a lack of data and/or the imprecision of the available information concerning the behavior of other decision makers (see [17]), and both conditions make it difficult to apply the Cournot game to real problems. Therefore, the excess of capacity observed in the recent economic recession motivated us to develop a Cournot game with ambiguous parameters.

Employing the available game-theoretical models for decision making can be difficult due to the uncertainty of data in the form of randomness and ambiguity. Even though the literature has proposed many stochastic game-theoretical models [3,11,15,27,30], these models only consider the probabilistic type of uncertainty. In practice, however, the probability distribution may not be available or the limited number of data points cannot provide exact estimates of a manufacturer's variable cost since the procurement costs can fluctuate over time. Thus, the fuzzy sets theory pioneered by Zadeh [36] becomes useful because it can mathematically model the vagueness and impression of human cognitive processes, e.g., the phrase "around x dollars", to describe a cost that can be regarded as a fuzzy number $\tilde{x}$.

* Corresponding author. Tel.: +886 2 3366 9507; fax: +886 2 2372 5856.
E-mail address: ihong@ntu.edu.tw (I-H. Hong).
[1] The corresponding author holds a joint appointment with the Department of Mechanical Engineering, National Taiwan University.

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Unlike the literature which considers only zero-sum games, Dang and Hong [10] indicate that there are two streams of fuzzy games: fuzzy matrix games and fuzzy non-cooperative games. The matrix game can be solved by the linear programming method based on the duality (see [4,28,29]). Maeda [22,23] studies two-person zero-sum games and bimatrix games with fuzzy payoffs and applies α-cut, possibility, and necessity theories to introduce two concepts of the equilibrium derived by the mathematical programming. Liu and Kao [21] obtain the upper and lower bound values of a matrix game by utilizing a pair of two-level mathematical programming.

Yao and Wu [31], who probably initiated the fuzzy non-cooperative game involving fuzzy data, apply the ranking method transforming fuzzy numbers for comparison to defuzzify the demand and supply functions so that consumer surplus and producer surplus can be calculated in a conventional manner. Their method of transforming fuzzy numbers to crisp values is also utilized to construct the monopoly model [6]. Yao and Wu [32] discuss the best price of two mutual complementary merchandises in a fuzzy sense. Yao and Chang [33] obtain the optimal quantity for maximizing the profit function whose parameters are fuzzy numbers. Yao and Shih [34] derive the membership function of the profit function when the optimal quantity occurs. Liang et al. [20] propose a duopoly model considering only fuzzy costs to obtain the optimal quantity of each firm. Dang and Hong [10], who highlight an unreasonable occurrence of a negative equilibrium quantity, and the limited flexibility for modification of the ranking method in fuzzy modeling in previous studies, propose a fuzzy Cournot game with rigorous definitions ensuring a positive equilibrium quantity and with a controlling mechanism. However, for model simplicity, Dang and Hong [10] restrict themselves to constant parameters of the controlling mechanism and to a duopoly production game. Furthermore, Guo [12] initially proposes a one-shot decision approach to solve for the Cournot equilibrium, where the solution procedure can be separated in two steps. In the first step, a decision maker can be categorized as passive, normal or active attitude depicted by the satisfaction level that relates to his/her own profits. At the second step, a solution procedure is then performed to seek for the Cournot equilibrium quantity, where the difference between the possibility of the demand uncertainty and the satisfaction level is within a pre-specified tolerance. Guo et al. [13] further extend the method proposed in Guo [12] to a duopoly market with asymmetric possibilistic information describing the demand uncertainty only known by one firm. This paper differs from (Guo [12] and Guo et al. [13]) in deriving the Cournot equilibrium quantity with less computational manipulations which simplify the analysis of the model. Furthermore, the proposed model in this paper comprehensively considers the uncertainty resulted from demand and cost functions.

This paper makes two contributions to the literature. First, we introduce a method solving for the equilibrium quantity of each competing firm in a competitive market with multiple firms, where the demand and cost functions are characterized by the form of ambiguity described by a set of fuzzy parameters, and the weighted center of gravity (WCoG) proposed by Bender and Simonovic [5] is used to defuzzify a firm's profit function into a crisp value. For simplicity, we assume that a firm's demand function and cost function take the form of linearity with fuzzy parameters, in order to obtain managerial insights with less analytical complexity (see [10,33]). Second, we investigate the impact of the perturbation of uncertainty on the resulting outcomes. For instance, we note that the fuzzy profit function and firms' control parameters play key roles in analyzing the perturbation of equilibrium quantity.

The remainder of this paper is organized as follows. In Section 2, we introduce the concepts and definitions of the proposed model. Section 3 addresses the Cournot production game and presents the proposed method to solve for the equilibrium quantity of each firm in an ambiguous environment where multiple firms exist in a competitive market. In Section 4, we analyze the resulting outcomes and discuss several valuable managerial insights.

2. Preliminary

This section presents the fuzzy sets theory and weighted center of gravity (WCoG) which are integral to this paper.

2.1. Fuzzy sets theory

The fuzzy sets theory initiated by Zadeh [36] attempts to analyze and to solve problems with a source of ambiguity called fuzziness. In the following, we introduce the definitions and notations of triangular fuzzy numbers, the extension principle, and the WCoG method.

2.1.1. Triangular fuzzy numbers

A popular type of fuzzy numbers is the triangular type because it is easy to handle arithmetically and has intuitive interpretation [9,29,30]. Dağdeviren and Yüksel [9] indicate that using triangular fuzzy numbers has proven efficient for calculating a decision making problem. Petrovic et al. [25] and Giannoccaro et al. [14] show that triangular fuzzy numbers are the most suitable for modeling market demand in a fuzzy sense (see [1,2,16,24] for other applications of triangular fuzzy numbers). The membership function $\mu_{\widetilde{A}}(x)$ of a triangular fuzzy number $\widetilde{A}$ can be defined by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-m_A+l_A}{l_A}, & m_A - l_A \leqslant x \leqslant m_A \\ \frac{m_A+r_A-x}{r_A}, & m_A \leqslant x \leqslant m_A + r_A \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where $\widetilde{A}$ is represented as a triplet $(m_A - l_A, m_A, m_A + r_A)$, and m_A, l_A and r_A are the apex, left, and right spreads of the fuzzy number $\widetilde{A}$, respectively. The upper and lower bounds of $\widetilde{A}$ can be derived by the apex, right, and left spreads, i.e., the upper bound of $\widetilde{A}$ equals $m_A + r_A$, and the lower bound of $\widetilde{A}$ equals $m_A - l_A$.

2.1.2. Extension principle

Let "$\odot$" be any binary operation $\oplus$ and $\otimes$ between two fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$. Based on the extension principle, the membership function of $\widetilde{A} \odot \widetilde{B}$ is defined by

$$\mu_{\widetilde{A}\odot\widetilde{B}}(z) = \sup_{x \circ y} \min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(y)\}$$

where "$\odot = \oplus$ or $\otimes$" corresponds to the operation "$\circ = +$ or $\times$". This result helps us to derive the membership function of a fuzzy number.

2.2. Weighted center of gravity

Among the many ranking methods proposed (see [7,26]), the center of gravity (COG) ranking method, also known as the centroid method [35] is commonly used to obtain the centroid of a fuzzy number because of its straightforward geometrical interpretation. However, the COG method is inappropriate to distinguish two fuzzy sets that may have the same centroid, but greatly differ in degree of fuzziness. In this case, the WCoG method is more useful because of flexibility [5]

$$\text{WCoG} = \frac{\int g(x)\mu(x)^k dx}{\int \mu(x)^k dx}, \tag{2}$$

where $g(x)$ is the horizontal component of the area under scrutiny and $\mu(x)$ is the membership function. We can define $g(x)$ to relieve the problem in which the COG method cannot distinguish two fuzzy numbers having the same center but different spreads. For example, a segmental design of $g(x)$, where $g(x)$ puts zero weights on the left spread and positive weights on the right spread provides the flexibility to distinguish two fuzzy numbers having the same center but different spreads.

In addition, Bender and Simonovic [5] indicate that the value of k is a control parameter representing the decision maker's preference. The definition of the control parameter k is a geometrical notion, which affects the shape of the membership function. The membership function with $k < 1 (k > 1)$ behaves as a concave (quasiconcave) function, whereas the membership function with $k = 1$ represents the degeneracy case of triangular fuzzy numbers. The different weighted centers can be derived from the different values of the control parameter k.

3. The model

As mentioned, it is almost impossible to find the exact economic assessment of data for parameters' estimation in the real world. In this section, we apply the fuzzy sets theory to solve for the equilibrium quantity of multiple firms, given fuzzy demand and fuzzy cost functions.

3.1. The Cournot game with multiple firms in a fuzzy decision environment

Let q_i denote the production quantity of firm i, $i = 1, \ldots, n$. Consider the general fuzzy profit function of firm i, $\tilde{\pi}_i$, as

$$\tilde{\pi}_i = \widetilde{P}(Q)q_i - \widetilde{TC}_i(q_i), \; i = 1, \cdots, \; n \tag{3}$$

where $\widetilde{P}(Q)$ is the fuzzy inverse demand function, $\widetilde{TC}_i(q_i)$ is the fuzzy cost of firm i, and Q is the total quantity in the market, namely $Q = \sum_{i=1}^{n} q_i$.

The fuzziness of $\widetilde{P}(Q)$ and $\widetilde{TC}_i(q_i)$ result from the fuzzy parameters of the inverse demand function and the cost function. From (3) it is clear that if $\widetilde{P}(Q)$ or $\widetilde{TC}(q_i)$ is unbounded, then $\tilde{\pi}_i$ is possibly unbounded as well. Thus, we require that both $\widetilde{P}(Q)$ and $\widetilde{TC}(q_i)$ are bounded. We utilize (2) to defuzzify the fuzzy profit function into a crisp value. The weighted center of the fuzzy profit function can be stated as

$$\text{WCoG}(\tilde{\pi}_i) = \frac{\int_{\pi_i^L}^{\pi_i^U} g(\pi_i)\mu(\pi_i)^k d\pi_i}{\int_{\pi_i^L}^{\pi_i^U} \mu(\pi_i)^k d\pi_i}, \tag{4}$$

where $g(\pi_i)$ is the horizontal component of the area under scrutiny (see [5]) and $\mu(\pi_i)$ is the membership function of firm i's profit function. As mentioned, the membership function can be described as a decision maker's subjective perception. The value of k would affect the shape and value of the decision maker's membership function as shown in Fig. 1. In this paper, we define the parameter k_i as the control parameter of firm i. For example, suppose that there is a fuzzy number $\tilde{\pi}$ as shown in Fig. 1 where three points, A, B and C, represent the three different levels of membership function value at the same point of π. As we can see, point A is a higher value of the membership function value than points B and C if the decision maker determines $k < 1$. In other words, one can apply different values of k to reflect the membership function value. A decision maker

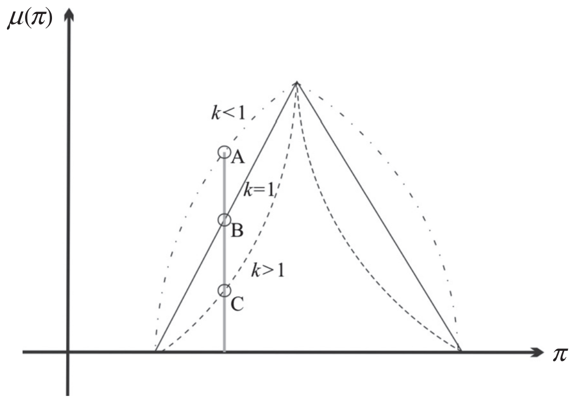


Fig. 1. The shape of the membership function given different values of control parameter.

can determine $k > 1$ if he/she perceives a low membership function value of the extreme cases of the profit function. In practice, a conservative decision maker may decide the control parameter k greater than 1 because he/she may not perceive a high level of membership function value of the extreme cases, whereas an aggressive decision maker may determine the control parameter k less than 1 because he/she may perceive a high level of membership function value of the extreme cases.

Based on Dang and Hong [10], decision makers utilizing (2) to defuzzify the fuzzy profit function must check whether $\int_{\pi_i^L}^{\pi_i^U} g(\pi_i)d\pi_i$ *is* or *is not* bounded. Thus, the best response functions of firm i can be obtained by optimizing each firm's profit functions with respect to each firm's decision variable q_i and assuming the competitors' quantities q_{-i} as given, where q_{-i} denotes the vector of firms' production quantities except firm i's quantity. Hence, firm i's best response function is

$$R^i(q_{-i}) = \arg\max_{q_i} \ \text{WCoG}(\tilde{\pi}_i), \ \ i = 1, \ldots, n. \tag{5}$$

We can now obtain each firm's equilibrium quantity by simultaneously solving the first-order condition obtained by letting each firm's best response function equal zero. In addition, we can derive the membership function $\mu(\pi_i)$ via the extension principle in Zadeh [36]. Note that by setting the different controlling mechanisms of $g(x)$ and k we can adapt the proposed method to fit the different criteria of decision makers or markets.

3.2. Linear model

Next, we develop the Cournot production game with triangular fuzzy numbers as the parameters. Initially, we introduce the conventional Cournot game and generalize it in the fuzzy business environment. Given the linear inverse demand function

$$P(Q) = a - bQ, \ 0 \leqslant Q \leqslant \frac{a}{b} \tag{6}$$

where $a, b > 0$ are given numbers and $P(Q)$ is the unit price. The total cost function of firm i, $i = 1, \ldots, n$, denoted by $TC_i(q_i)$, is stated as

$$TC_i(q_i) = c_i + d_i q_i \tag{7}$$

where c_i denotes the fixed cost of firm i's production and d_i represents its variable cost of production. Thus, firm i's profit function is given by

$$\pi_i = P(Q) \cdot q_i - TC_i(q_i) = (a - bQ)q_i - TC_i(q_i). \tag{8}$$

Consider the situation where all parameters are fuzzy numbers, i.e., our model assigns a zero to the left and right spreads of a fuzzy number for a crisp value. In particular, we let $\widetilde{P}(Q) = \tilde{a} - \tilde{b}Q$, $\widetilde{TC}_i(q_i) = \tilde{c}_i + \tilde{d}_i q_i$, and $\tilde{\pi}_i = \widetilde{P}(Q) \cdot q_i - \widetilde{TC}_i(q_i)$ where $\widetilde{P}(Q)$, $\widetilde{TC}_i(q_i)$, and $\tilde{\pi}_i$ represent the fuzzy price, fuzzy cost function of firm i, and fuzzy profit function of firm i, $i = 1, \ldots, n$, respectively. As mentioned, this paper uses triangular fuzzy numbers because they are the most suitable for modeling market demand (see [14,25]). In this case, all parameters are nonnegative triangular fuzzy numbers; in other words, the lower bounds of fuzzy numbers in the following are greater than or equal to zero

$$\begin{aligned} \tilde{a} &= (a - l_a, a, a + r_a), \\ \tilde{b} &= (b - l_b, b, b + r_b), \\ \tilde{c}_i &= (c_i - l_{c_i}, c_i, c_i + r_{c_i}), \\ \tilde{d}_i &= (d_i - l_{d_i}, d_i, d_i + r_{d_i}). \end{aligned} \tag{9}$$

To derive the lower bound of the price, we substitute the lower bound of a, namely $a - l_a$, and the upper bound of b, namely $b + r_b$, into (6). Also, the upper bound of the price can be derived by substituting the upper bound of a, namely $a + r_a$, and the lower bound of b, namely $b - l_b$, into (6). Hence, by the extension principle, we have

$$\widetilde{P}(Q) = (a - bQ - (l_a + r_bQ), a - bQ, a - bQ + (r_a + l_bQ)). \tag{10}$$

Similarly, $\widetilde{TC}_i(q_i)$ and $\tilde{\pi}_i$ can be obtained in (7) and (8). Note that $\widetilde{P}(Q)$, $\widetilde{TC}_i(q_i)$, and $\tilde{\pi}_i$ are also triangular fuzzy numbers due to the nature of the extension principle. Noting that a firm may overestimate or underestimate the economic situation concerning price fluctuation in the market, it follows that the weighted center of the fuzzy profit function with the right spread greater than the left spread locates on the right side of the apex of the fuzzy profit function. Similarly, the weighted center of the fuzzy profit function with the right spread less than the left locates on the left side of the apex of the fuzzy profit function. These results produce two definitions: the firm having the fuzzy profit function with the right spread greater than the left spread is in the *optimistic case*, and the firm having the right spread less than the left spread is in the *pessimistic case*.

Defining the controlling mechanism follows the condition developed in Dang and Hong [10] before applying (2) to defuzzify the fuzzy profit function into a crisp value. Let $g(x) = x$, which is a common assumption in Chen and Chen [8]. Let k_i denote firm i's control parameter. The membership function of firm i's profit function can be derived by utilizing the arithmetic operations on the parameters' α-cuts. As a result, firm i's profit function is a triangular fuzzy number (see [18]). Thus, $\mathrm{WCoG}(\tilde{\pi}_i)$ can be algebraically calculated as

$$\mathrm{WCoG}(\tilde{\pi}_i) = \frac{\int_{\pi_i^L}^{\pi_i^U} \pi_i \mu(\pi_i)^{k_i} d\pi_i}{\int_{\pi_i^L}^{\pi_i^U} \mu(\pi_i)^{k_i} d\pi_i} = \frac{\int_{\pi_i^L}^{\pi_i^A} \pi_i \mu(\pi_i)^{k_i} d\pi_i + \int_{\pi_i^A}^{\pi_i^U} \pi_i \mu(\pi_i)^{k_i} d\pi_i}{\int_{\pi_i^L}^{\pi_i^A} \mu(\pi_i)^{k_i} d\pi_i + \int_{\pi_i^A}^{\pi_i^U} \mu(\pi_i)^{k_i} d\pi_i}, i = 1, \ldots, n. \tag{11}$$

The membership function of firm i's fuzzy profit function, $\mu\ (\pi_i)$, can be derived by substituting its apex, π_i^A, left spread $\pi_i^A - \pi_i^L$ and right spread $\pi_i^U - \pi_i^A$ into (1). As a result, (11) can be rewritten as

$$\begin{aligned}
\mathrm{WCoG}(\tilde{\pi}_i) &= \frac{\int_{\pi_i^L}^{\pi_i^A} \pi_i \left(\frac{\pi_i - \pi_i^L}{\pi_i^A - \pi_i^L}\right)^{k_i} d\pi_i + \int_{\pi_i^A}^{\pi_i^U} \pi_i \left(\frac{\pi_i - \pi_i^U}{\pi_i^A - \pi_i^U}\right)^{k_i} d\pi_i}{\int_{\pi_i^L}^{\pi_i^A} \left(\frac{\pi_i - \pi_i^L}{\pi_i^A - \pi_i^L}\right)^{k_i} d\pi_i + \int_{\pi_i^A}^{\pi_i^U} \left(\frac{\pi_i - \pi_i^U}{\pi_i^A - \pi_i^U}\right)^{k_i} d\pi_i} \\
&= \frac{\frac{1}{(\pi_i^A - \pi_i^L)^{k_i}} \int_{\pi_i^L}^{\pi_i^A} \pi_i (\pi_i - \pi_i^L)^{k_i} d\pi_i + \frac{1}{(\pi_i^A - \pi_i^U)^{k_i}} \int_{\pi_i^A}^{\pi_i^U} \pi_i (\pi_i - \pi_i^U)^{k_i} d\pi_i}{\frac{1}{(\pi_i^A - \pi_i^L)^{k_i}} \int_{\pi_i^L}^{\pi_i^A} (\pi_i - \pi_i^L)^{k_i} d\pi_i + \frac{1}{(\pi_i^A - \pi_i^U)^{k_i}} \int_{\pi_i^A}^{\pi_i^U} (\pi_i - \pi_i^U)^{k_i} d\pi_i}, \qquad i = 1, \ldots, n.
\end{aligned} \tag{12}$$

In order to simplify the integral $\int_{\pi_i^L}^{\pi_i^A} \pi_i (\pi_i - \pi_i^L)^{k_i} d\pi_i$, denote s being $\pi_i - \pi_i^L$. Then, $d\pi_i = ds$. Substituting these into $\int_{\pi_i^L}^{\pi_i^A} \pi_i (\pi_i - \pi_i^L)^{k_i} d\pi_i$, we have $\int_0^{\pi_i^A - \pi_i^L} (s + \pi_i^L) s^{k_i} ds$. Similarly, by letting $\pi_i - \pi_i^U = t$ and $d\pi_i = dt$, the integral $\int_{\pi_i^A}^{\pi_i^U} \pi_i (\pi_i - \pi_i^U)^{k_i} d\pi_i$ can be simplified as $\int_{\pi_i^A - \pi_i^U}^{0} (t + \pi_i^U) t^{k_i} dt$. As a result, (12) can be rearranged as

$$\begin{aligned}
\mathrm{WCoG}(\tilde{\pi}_i) &= \frac{\frac{1}{(\pi_i^A - \pi_i^L)^{k_i}} \int_0^{\pi_i^A - \pi_i^L} (s + \pi_i^L) s^{k_i} ds + \frac{1}{(\pi_i^A - \pi_i^U)^{k_i}} \int_{\pi_i^A - \pi_i^U}^{0} (t + \pi_i^U) t^{k_i} dt}{\frac{1}{(\pi_i^A - \pi_i^L)^{k_i}} \int_{\pi_i^L}^{\pi_i^A} (\pi_i - \pi_i^L)^{k_i} d\pi_i + \frac{1}{(\pi_i^A - \pi_i^U)^{k_i}} \int_{\pi_i^A}^{\pi_i^U} (\pi_i - \pi_i^U)^{k_i} d\pi_i} = \frac{\frac{(\pi_i^A - \pi_i^L)(\pi_i^L + \pi_i^A + k_i \pi_i^A)}{(1+k_i)(2+k_i)} + \frac{(\pi_i^U - \pi_i^A)(\pi_i^U + \pi_i^A + k_i \pi_i^A)}{(1+k_i)(2+k_i)}}{\frac{\pi_i^A - \pi_i^L}{1+k_i} + \frac{\pi_i^U - \pi_i^A}{1+k_i}} \\
&= \frac{\pi_i^L + k_i \pi_i^A + \pi_i^U}{2 + k_i}, \ i = 1, \ldots, \ n.
\end{aligned} \tag{13}$$

Thus, the partial derivative of $\mathrm{WCoG}(\tilde{\pi}_i)$ can be stated as

$$\begin{aligned}
\frac{\partial \mathrm{WCoG}(\tilde{\pi}_i)}{\partial q_i} = \frac{1}{2 + k_i} ((2a - l_a + r_a + ak_i) - 2(2b - l_b + r_b + bk_i) q_i - (2b - l_b + r_b + bk_i) \\
\sum_{\substack{j=1 \\ j \neq i}}^{n} q_j - (2d_i - l_{d_i} + r_{d_i} + d_i k_i)).
\end{aligned} \tag{14}$$

By letting (14) equal zero, we have the first-order condition of firm i, $i = 1, \ldots, n$. The equilibrium quantity of firm i follows by simultaneously solving the n first-order conditions. For notational simplicity, the weighted centers of fuzzy parameters are

$$\begin{aligned}
a^c(k_i) &= \frac{2a - l_a + r_a + ak_i}{2 + k_i}, \\
b^c(k_i) &= \frac{2b - l_b + r_b + bk_i}{2 + k_i}, \ i = 1, \cdots, \ n. \\
d_i^c(k_i) &= \frac{2d_i - l_{d_i} + r_{d_i} + d_i k_i}{2 + k_i}.
\end{aligned} \tag{15}$$

According to (15), (14) can be rearranged as (16)

$$-2b^c(k_i)q_i - b^c(k_i)\sum_{\substack{j=1\\ j\neq i}}^{n} q_j + \left(a^c(k_i) - d_i^c(k_i)\right) = 0,\ i = 1, \cdots,\ n. \tag{16}$$

The second-order conditions (17) allow us to ensure the concavity of $\mathrm{WCoG}(\tilde{\pi}_i)$.

$$\frac{\partial^2 \mathrm{WCoG}(\tilde{\pi}_i)}{\partial (q_i)^2} = -2b^c(k_i) < 0,\ i = 1, \ldots, n. \tag{17}$$

To further simplify (16), we introduce an $n \times n$ matrix

$$\mathbf{A} \equiv \begin{bmatrix} -2b^c(k_1) & -b^c(k_1) & \cdots & -b^c(k_1) \\ -b^c(k_2) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -b^c(k_{n-1}) \\ -b^c(k_n) & \cdots & -b^c(k_n) & -2b^c(k_n) \end{bmatrix}_{n\times n},$$

the column vectors

$$\mathbf{q} \equiv \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}_{n\times 1} \quad \text{and} \quad \mathbf{B} \equiv \begin{bmatrix} -(a^c(k_1) - d_1^c(k_1)) \\ -(a^c(k_2) - d_2^c(k_2)) \\ \vdots \\ -(a^c(k_n) - d_n^c(k_n)) \end{bmatrix}_{n\times 1},$$

and rewrite the n first-order conditions as

$$\mathbf{Aq} = \mathbf{B}. \tag{18}$$

We can now obtain the equilibrium quantity of each firm by solving the system $\mathbf{Aq} = \mathbf{B}$ with an inverse matrix $\mathbf{A}^{-1}$. That is, $\mathbf{q}^* = \mathbf{A}^{-1}\mathbf{B}$. Based on Larson et al. [19], we can derive $\mathbf{A}^{-1}$ by

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \mathrm{adj}\mathbf{A}. \tag{19}$$

Note that the determinant of $\mathbf{A}$, $\det\mathbf{A}$, and the adjacent matrix of $\mathbf{A}$, $\mathrm{adj}\mathbf{A}$, can be obtained by $\mathbf{A}$. From (16), we have

$$\det \mathbf{A} = (-1)^n(n+1),$$

and $\mathrm{adj}\mathbf{A}$ can be expressed as

$$\mathrm{adj}\mathbf{A} = \begin{bmatrix} (-1)^{n-1} n \prod\limits_{\substack{i=1\\ i\neq 1}}^{n} b^c(k_i) & (-1)^n \prod\limits_{\substack{i=1\\ i\neq 2}}^{n} b^c(k_i) & \cdots & (-1)^n \prod\limits_{\substack{i=1\\ i\neq n}}^{n} b^c(k_i) \\ (-1)^n \prod\limits_{\substack{i=1\\ i\neq 1}}^{n} b^c(k_i) & (-1)^{n-1} n \prod\limits_{\substack{i=1\\ i\neq 2}}^{n} b^c(k_i) & \cdots & (-1)^n \prod\limits_{\substack{i=1\\ i\neq n}}^{n} b^c(k_i) \\ \vdots & \begin{matrix} (-1)^n \prod\limits_{\substack{i=1\\ i\neq 2}}^{n} b^c(k_i) \\ \vdots \end{matrix} & \ddots & \begin{matrix} \vdots \\ (-1)^n \prod\limits_{\substack{i=1\\ i\neq n}}^{n} b^c(k_i) \end{matrix} \\ (-1)^n \prod\limits_{\substack{i=1\\ i\neq 1}}^{n} b^c(k_i) & (-1)^n \prod\limits_{\substack{i=1\\ i\neq 2}}^{n} b^c(k_i) & \cdots & (-1)^{n-1} n \prod\limits_{\substack{i=1\\ i\neq n}}^{n} b^c(k_i) \end{bmatrix}_{n\times n.}$$

Given each firm's control parameter, k_i, we have

$$\begin{bmatrix} q_1^* \\ q_2^* \\ \vdots \\ q_n^* \end{bmatrix} = \begin{bmatrix} \dfrac{(n+1)\left(a^c(k_1)-d_1^c(k_1)\right)\prod\limits_{\substack{i=1\\ i\neq 1}}^{n} b^c(k_i)-\sum\limits_{r=1}^{n}((a^c(k_r)-d_r^c(k_r))\prod\limits_{\substack{i=1\\ i\neq r}}^{n} b^c(k_i))}{(n+1)\prod\limits_{i=1}^{n} b^c(k_i)} \\ \dfrac{(n+1)\left(a^c(k_2)-d_2^c(k_2)\right)\prod\limits_{\substack{i=1\\ i\neq 2}}^{n} b^c(k_i)-\sum\limits_{r=1}^{n}((a^c(k_r)-d_r^c(k_r))\prod\limits_{\substack{i=1\\ i\neq r}}^{n} b^c(k_i))}{(n+1)\prod\limits_{i=1}^{n} b^c(k_i)} \\ \vdots \\ \dfrac{(n+1)\left(a^c(k_n)-d_N^c(k_n)\right)\prod\limits_{\substack{i=1\\ i\neq n}}^{n} b^c(k_i)-\sum\limits_{r=1}^{n}((a^c(k_r)-d_r^c(k_r))\prod\limits_{\substack{i=1\\ i\neq r}}^{n} b^c(k_i))}{(n+1)\prod\limits_{i=1}^{n} b^c(k_i)} \end{bmatrix}_{n\times 1}. \tag{20}$$

To ensure a nonnegative equilibrium quantity of firm i, we impose the condition that $q_i \geqslant 0$, $i = 1, \ldots, n$. Assumption 1 follows from this condition.

Assumption 1

$$(n+1)\left(a^c(k_i)-d_i^c(k_i)\right)\prod_{\substack{j=1\\ j\neq i}}^{n} b^c(k_j)-\sum_{r=1}^{n}((a^c(k_r)-d_i^c(k_r))\prod_{\substack{j=1\\ j\neq r}}^{n} b^c(k_j))>0$$

In addition, (20) is the same as the equilibrium quantity of the conventional Cournot game when the spreads of all fuzzy parameters are equal to zero. Furthermore, the equilibrium market demand can be derived by

$$Q^* = \sum_i^n q_i^* = \frac{\sum_i^n\left(\left(a^c(k_i)-d_i^c(k_i)\right)\prod\limits_{\substack{j=1\\ j\neq i}}^{n} b^c(k_j)\right)}{(n+1)\prod\limits_{i=1}^{n} b^c(k_i)}. \tag{21}$$

The proposed model with the parameter k offers decision makers flexibility in describing their control parameters. If a decision maker intends to remove k from the proposed model, he/she could define $k = 1$, which is a special case of our current model.

The equilibrium quantity of each firm as shown in (20) is a solution to a system of n first-order conditions. Thus, the issue of uniqueness is of interest. In this paper, we derive the unique equilibrium quantity because $\det\mathbf{A} = (-1)^n(n+1)$ is greater than zero based on Cramer's Rule [19]. Furthermore, (20) indicates that if $\prod_{i=1}^n b^c(k_i) \neq 0$, the equilibrium quantity of each firm would not be unbounded.

4. Economic analysis of the control parameter *k* and fuzzy parameters

Below, we map the resulting solutions derived in Section 3 to understand each firm's control parameter in a fuzzy business environment. Usually, decision makers in the market will conjecture similar predictions about the economic reality. For instance, most likely, they will predict low consumption of products in a recession and adjust their quantity of production accordingly. In other words, all of them may have the same control parameter.

4.1. A potential entrant's incentive to enter a competitive market

To investigate the influence of a potential entrant on the equilibrium quantity, we assume that each incumbent's and potential entrant's control parameter k_i, $i = 1, \ldots, n$ are identical, namely, $k_i = k$ for all i. Substituting $k_i = k$ into (20), we have

$$q_i^* = \frac{a^c(k) - nd_i^c(k) + \sum_{\substack{j=1 \\ j \neq i}}^{n} d_j^c(k)}{(n+1)b^c(k)} \quad i = 1, \cdots, n. \tag{22}$$

The equilibrium quantity of a new entrant, firm $n+1$, can be derived in (20)

$$q_{n+1}^* = \frac{a^c(k) - (n+1)d_{n+1}^c(k) + \sum_{j=1}^{n} d_j^c(k)}{(n+2)b^c(k)}. \tag{23}$$

To ensure a nonnegative equilibrium quantity of firm $n+1$, we impose the condition that $q_{n+1} \geqslant 0$. Assumption 2 follows from this condition.

Assumption 2

$$d_{n+1}^c(k) < \frac{a^c(k) + \sum_{j=1}^{n} d_j^c(k)}{n+1}$$

Our first insight is that an entrant needs to drive its production cost at least as low as the right-hand side of the inequality in Assumption 2, yet, entrants typically incur a high production cost in the beginning. As a result, Assumption 2 states that a low production variable cost can be viewed as a barrier to entry.

Proposition 1. *When a potential entrant enters a competitive market, the equilibrium quantity of each firm will decrease, but the equilibrium market demand will increase.*

Proof. To present the perturbation on firm i's equilibrium quantity as an entrant enters the competitive market, we calculate the difference between the equilibrium quantities of firm i in the original and new markets. For notational simplicity, let $q_i^*(n)$ and $q_i^*(n+1)$ denote the equilibrium quantity of firm i in the original market consisting of n firms and in the new market consisting of $n+1$ firms, respectively. Therefore

$$q_i^*(n) - q_i^*(n+1) = \frac{a^c(k) - (n+1)d_{n+1}^c(k) + \sum_{i=1}^{n} d_i^c(k)}{b^c(k)(2 + 3n + n^2)}. \tag{24}$$

The denominator of (24), $b^c(k)(2 + 3n + n^2)$, is positive because this paper assumes that the terms $b^c(k)$ and n are positive. In addition, the nominator of (24) is positive because of Assumption 2, meaning that the equilibrium quantity of firm i decreases as a new entrant enters the competitive market. Similarly, by (21) we have

$$Q^*(n) - Q^*(n+1) = -\left(\frac{a^c(k) - (n+1)d_{n+1}^c(k) + \sum_{i=1}^{n} d_i^c(k)}{(n+2)b^c(k)} \right), \tag{25}$$

where $Q^*(n)$ and $Q^*(n+1)$ denote the equilibrium market demand in the original market consisting of n firms and in the new market consisting of $n+1$ firms. Obviously, the nominator of (25) and (21) is the same, which leads to the conclusion that the equilibrium market demand increases as a new entrant enters the competitive market. This completes the proof. □

4.2. Effect of a firm's control parameter k

Next, we analyze the behavior of each firm in the optimistic and pessimistic cases related to the location of the weighted center of the fuzzy profit function. For comparative purposes, we perturb firm i's control parameter, k_i, with all of the other firms' control parameters remaining in the same level, k. This assumption allows us to conduct the sensitivity analysis with less computational burden. Thus, we rewrite (20) as

$$q_i^* = \frac{nb^c(k)\left[a^c(k_i) - d_i^c(k_i)\right] - b^c(k_i)\left[(n-1)a^c(k) - \sum_{\substack{j=1 \\ j \neq i}}^{n} d_j^c(k)\right]}{(n+1)b^c(k_i)b^c(k)} \tag{26}$$

and

$$q_j^* = \frac{nb^c(k_i)\left[a^c(k) - d_j^c(k)\right] - b^c(k)\left[a^c(k_i) - d_i^c(k_i)\right] - b^c(k_i)\left[(n-2)a^c(k) - \sum_{\substack{j'=1\\ j'\neq i,j}}^{n} d_{j'}^c(k)\right]}{(n+1)b^c(k_i)b^c(k)}, \; j = 1,\ldots,n, j \neq i. \tag{27}$$

Proposition 2. *An increase in firm i's control parameter k_i leads to the following.*

(*i*) *Given that the left and right spreads of each parameter are identical, the change in firm i's control parameter k_i has no influence on the equilibrium quantity.*
(*ii*) *In the optimistic case, an increase in firm i's control parameter k_i will result in a decrease in its equilibrium quantity; in the pessimistic case, an increase in firm i's control parameter k_i will result in an increase in its equilibrium quantity.*
(*iii*) *If an increase in firm i's control parameter k_i results in an increase in its equilibrium quantity, the equilibrium quantities of the other firms will decrease.*

Proof.

(*i*) The partial derivative of (26) with respect to k_i is

$$\frac{\partial q_i^*}{\partial k_i} = \frac{nb(l_a - r_a) - n(a - d_i)(l_b - r_b) - nb(l_{d_i} - r_{d_i})}{(n+1)(2b - l_b + r_b + bk_i)^2}. \tag{28}$$

If each fuzzy parameter has the same spreads, $l_a = r_a$, $l_b = r_b$ and $l_{d_i} = r_{d_i}$, there is no change in firm i's equilibrium quantity due to a perturbation of k_i.

(*ii*) The denominator of (28), $(n + 1)(2b - l_b + r_b + bk_i)^2$, is positive because the terms, $n + 1$ and $(2b - l_b + r_b + bk_i)^2$, are positive. Suppose that the fuzzy profit function is $l_a < r_a$, $l_b > r_b$, and $l_{d_i} > r_{d_i}$, which leads to the optimistic case; obviously, the numerator is negative. As a result, firm i decreases its equilibrium quantity as k_i increases. Similarly, firm i is in the pessimistic case, given that the fuzzy profit function is $l_a < r_a$, $l_b < r_b$, and $l_{d_i} < r_{d_i}$, resulting in a positive sign of (28). Thus, firm i increases its equilibrium quantity as k_i increases.

(*iii*) The partial derivative of (27) with respect to k_i gives

$$\frac{\partial q_j^*}{\partial k_i} = \frac{-b(l_a - r_a) + (a - d_i)(l_b - r_b) + b(l_{d_i} - r_{d_i})}{(n+1)(2b - l_b + r_b + bk_i)^2}. \tag{29}$$

From (29), it is clear that the change in firm j's equilibrium quantity negatively relates to the change in firm i's equilibrium quantity because the coefficients of $l_a - r_a$, $l_b - r_b$, and $l_{d_i} - r_{d_i}$ change signs in (28). This completes the proof. □

From (28) and (29) we note that the equilibrium quantity of firm i increases or decreases in k_i at a faster rate than that of firm j due to the coefficients of $l_a - r_a$, $l_b - r_b$, and $l_{d_i} - r_{d_i}$.

Proposition 3. *An increase in firm i's control parameter, k_i, will result in a decrease in its weighted fuzzy profit function in the optimistic case and an increase in its weighted fuzzy profit function in the pessimistic case.*

Proof. The partial derivative of (11) with respect to k_i is

$$\frac{\partial \mathrm{WCoG}(\tilde{\pi}_i)}{\partial k_i} = \frac{2\pi_i^A - (\pi_i^L + \pi_i^U)}{(2 + k_i)^2}. \tag{30}$$

As mentioned, if firm i's weighted center of the fuzzy profit function locates on the right side of the apex, firm i is in the optimistic case; i.e., an increase in its control parameter, k_i, results in a decrease in $\mathrm{WCoG}(\tilde{\pi}_i)$, since $2\pi_i^A - (\pi_i^L + \pi_i^U) = (\pi_i^A - \pi_i^L) - (\pi_i^U - \pi_i^A) < 0$. On the contrary, if firm i is in the pessimistic case, an increase in k_i results in an increase in $\mathrm{WCoG}(\tilde{\pi}_i)$, since $2\pi_i^A - (\pi_i^L + \pi_i^U) = (\pi_i^A - \pi_i^L) - (\pi_i^U - \pi_i^A) > 0$. This completes the proof. □

To demonstrate the impact of firm i's control parameter on the resulting outcomes, two examples consider the optimistic and pessimistic cases. For simplicity, assume a competitive market with three firms. In the optimistic case of firm 1, we consider the Cournot production game with the following setups. Suppose that $\tilde{a} = (23, 24, 27)$, $\tilde{b} = (0.8, 1, 1.1)$, $\tilde{d}_1 = (2, 4, 5)$,

$\tilde{d}_2 = (2,3,5)$, and $\tilde{d}_3 = (3,5,6)$. In the pessimistic case of firm 1, let the fuzzy parameters be $\tilde{a} = (23,25,26)$, $\tilde{b} = (0.8,0.9,1.2)$, $\tilde{d}_1 = (2,3,6)$, $\tilde{d}_2 = (2,3,5)$, and $\tilde{d}_3 = (3,5,6)$. Using our proposed model, plot the equilibrium quantity of firm 1 in Fig. 2 for both cases as k_1 varies. In Fig. 2, the solid line is the equilibrium of firm 1 in the optimistic case and the dotted line is the equilibrium quantity of firm 1 in the pessimistic case. Observe that the equilibrium quantity of firm 1 decreases in k_1 in the optimistic case, but increases in k_1 in the pessimistic case because the control parameter affects the weighted center of the fuzzy profit function.

In Fig. 3, we analyze the total equilibrium market demand obtained for the cases as k_1 varies, and conclude that the total equilibrium market demand increases in k_1 in the pessimistic case, but decreases in k_1 in the optimistic case. In Fig. 4, we analyze the equilibrium quantity of firm 2, given firm 1's control parameter. In Fig. 4a, the dotted line is the equilibrium quantity of firm 1 and the solid line is the equilibrium quantity of firm 2 as k_1 varies in the optimistic case. In Fig. 4b, the dotted line is the equilibrium quantity of firm 1 and the solid line is the equilibrium quantity of firm 2 as k_1 varies in the pessimistic case. In Fig. 4a, we observe that the equilibrium quantity of firm 1 decreases in k_1 in the optimistic case, but the equilibrium quantity of firm 2 increases in k_1. In Fig. 4b, we observe that the equilibrium of firm 1 increases in k_1, but the equilibrium quantity of firm 2 decreases in k_1. In other words, the change in the equilibrium quantity of firm 1 negatively relates to the change in the equilibrium quantity of firm 2.

4.3. Effect of fuzzy parameters

As discussed earlier, fuzzy parameters can be defuzzified into a crisp value representing the weighted center of the associated fuzzy parameter. We now look at the impacts of weighted centers on the resulting equilibrium quantity and total market demand.

Considering the weighted center of fuzzy parameter $\tilde{a}$ as $k_i = 1$, $i = 1, \ldots, n$, it is trivial having the weighted center of $\tilde{a}$ in (31).

$$a^c = \frac{1}{3}\{(a - l_a) + a + (a + r_a)\} \tag{31}$$

Similarly, the weighted centers of the other fuzzy parameters can be written as

$$\begin{aligned} b^c &= \frac{1}{3}\{(b - l_b) + b + (b + r_b)\}, \\ d_i^c &= \frac{1}{3}\{(d_i - l_{d_i}) + d_i + (d_i + r_{d_i})\},\ i = 1, \ldots, n. \end{aligned} \tag{32}$$

Substituting (31) and (32) into (20), the equilibrium quantity of firm i can be rewritten as

$$q_i^* = \frac{(n+1)(a^c - d_i^c) - \sum_{r=1}^{n}(a^c - d_r^c)}{(n+1)b^c}. \tag{33}$$

Note that all possible perturbations in fuzzy parameters can be represented in the right-hand side of (31) and (32) to reveal changes in the weighted centers of fuzzy parameters. Eq. (33) indicates that the equilibrium quantity is a function of the weighted center of fuzzy parameters. In other words, the change in the equilibrium quantity due to any perturbation in fuzzy parameters can be explored in (33). As an example, assuming that the right spread of fuzzy parameter $\tilde{a}$, namely r_a, increases, then a^c will be increasing because of the positive coefficient with r_a in (31). As a result, the equilibrium quantity increases as a^c increases. Furthermore, by deriving the total market demand written as

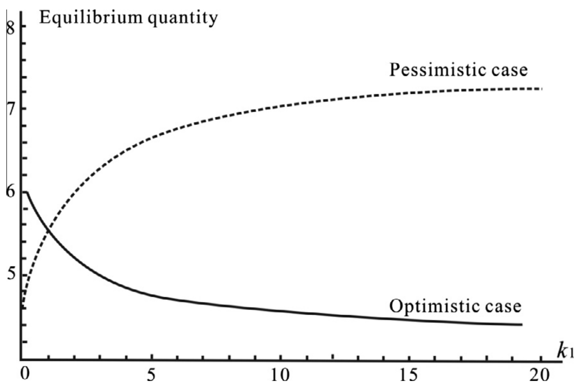


Fig. 2. The equilibrium quantity of firm 1 for different values of k_1.

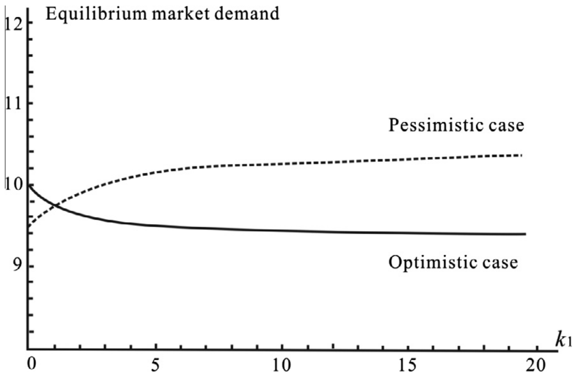
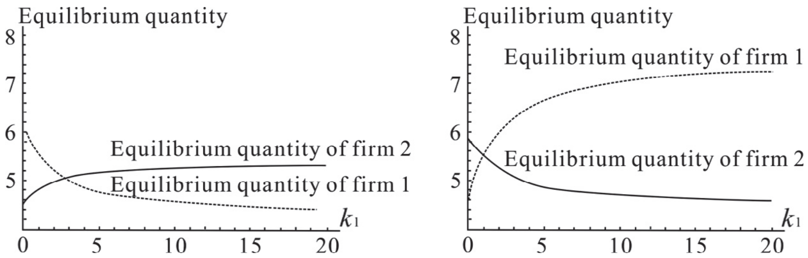


Fig. 3. The total equilibrium market demand for different values of k_1.



(a) The equilibrium quantity of firm 1 in the optimistic case

(b) The equilibrium quantity of firm 1 in the pessimistic case

Fig. 4. The equilibrium quantity for different values of k_1.

$$Q^* = \sum_i^n q_i^* = \frac{\sum_i^n (a^c - d_i^c)}{(n+1)b^c}, \tag{34}$$

we can now conduct the sensitivity analysis of the total market demand and equilibrium quantity of firm i, $i = 1, \ldots, n$. Taking the partial derivative of (33) and (34) with respect to a^c, we have

$$\frac{\partial q_i^*}{\partial a^c} = \frac{1}{(n+1)b^c}, \tag{35}$$

$$\frac{\partial Q^*}{\partial a^c} = \frac{n}{(n+1)b^c}. \tag{36}$$

Similarly, we also take the partial derivative of (33) and (34) with respect to the other weighted centers of fuzzy parameters. Following Assumption 1, the resulting equilibrium quantity is positive and implies that the numerator of (33), $(n+1)(a^c - d_i^c) - \sum_{r=1}^n (a^c - d_r^c)$ is also positive. Obviously, the total market demand is positive due to the positive quantities of individual firms; therefore, the numerator of (34) $\sum_i^n (a^c - d_i^c)$ is positive. Table 1 summarizes these results, for example, $\frac{n}{(n+1)b^c}$ is listed in the upper-left cell. We also make two observations:

Observation 1. The total market demand, Q, increases in a^c, but decreases in b^c, d_i^c and d_j^c.

Observation 2. The equilibrium quantity of firm i, q_i, increases in a^c and d_j^c, but decreases in b^c and d_i^c.

Table 1
Partial derivatives of outcomes with respect to different weighted centers.

	a^c	b^c	d_i^c	d_j^c
q_i	$\frac{1}{(n+1)b^c}$	$\frac{-(n+1)\left[(n+1)(a^c-d_i^c)-\sum_{r=1}^n (a^c-d_r^c)\right]}{[(n+1)b^c]^2}$	$\frac{-n}{(n+1)b^c}$	$\frac{1}{(n+1)b^c}$
Q	$\frac{n}{(n+1)b^c}$	$\frac{-(n+1)\left[\sum_i^n (a^c-d_i^c)\right]}{[(n+1)b^c]^2}$	$\frac{-1}{(n+1)b^c}$	$\frac{-1}{(n+1)b^c}$

Note that the equilibrium quantity of firm i increases in d_j^c, but the equilibrium quantity of firm j decreases in d_j^c at a faster rate; thus an increase in d_j^c would lead to a decrease in Q.

4.4. Elasticity of equilibrium quantity

Economists usually measure responsiveness by examining elasticity. Therefore, the own-elasticity of the equilibrium quantity of firm i with respect to its own control parameter can be written as

$$E_{k_i,q_i^*} = (\partial q_i^*/\partial k_i)/(q_i^*/k_i). \tag{37}$$

Similarly, we define the cross-elasticity of the equilibrium quantity of firm j with respect to firm i's control parameter as

$$E_{k_i,q_j^*} = \left(\partial q_j^*/\partial k_i\right)/\left(q_j^*/k_i\right). \tag{38}$$

By substitution of (20), (28) and (29), we can obtain (37) and (38). Furthermore, the own-elasticity is twice the cross-elasticity as shown below

$$\left|\frac{E_{k_i,q_i^*}}{E_{k_i,q_j^*}}\right| = 2. \tag{39}$$

As a result, the perturbation of firm i's control parameter on its equilibrium is more sensitive than on other firms' control parameters.

5. Conclusions

The conventional Cournot game is inadequate to explain or describe a real-world situation when an ambiguous environment hampers an accurate assessment of the relevant data. Moreover, probability distributions may be unavailable or difficult to estimate due to the paucity of data, poor record-keeping, etc. Thus, fuzzy sets theory is the most appropriate tool when the uncertain parameters cannot be described by probability distributions.

In this paper, we propose the Cournot production game for solving the fuzziness aspect of demand and cost uncertainty. We present a solution procedure to solve for the equilibrium quantity of each firm in a competitive market and indicate the condition to ensure the uniqueness and existence of the equilibrium quantity. For simplicity, we assume that the demand and cost functions of firms behave in a linear form and triangular fuzzy parameters. The linearity assumption commonly adopted in the literature helps us obtain the equilibrium quantities in the form of a matrix representation.

We conduct a sensitivity analysis to examine the effect of the perturbation of the control parameter on firms' outcomes including the equilibrium quantity, the total market demand, and the weighted center of the fuzzy profit function. We observe that the equilibrium quantity of each firm varies with its control parameter at a faster rate than with other competitors' control parameters. We find that the fuzzy profit function plays a key role in analyzing the effect of the control parameter. We define the optimistic and pessimistic cases based on the location of the weighted center of the fuzzy profit function to discuss the change in the equilibrium quantity due to the perturbation of the control parameter. The results of our model show that the control parameter has different influences on equilibrium quantity as we consider the optimistic and pessimistic cases of each firm. We also analyze the effects of parameter perturbations on firms' outcomes including the equilibrium quantity and total market demand. It is worth mentioning that the resulting equilibrium quantity of each firm varies with its own parameters at a faster rate than with its competitors' parameters.

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