Knowledge-Based Systems 58 (2014) 58–65

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

A simple utility function with the rules-verified weights for analyzing the top competitiveness of WCY 2012

Yu-Chien Ko [a], Hamido Fujita [b,*], Gwo-Hshiung Tzeng [c,d]

[a] Department of Information Management, Chung Hua University, 707, Sec.2 Wufu Road, Hsinchu 30012, Taiwan
[b] Software and Information Science, Iwate Prefectural University, Takizawa, Japan
[c] Graduate Institute of Urban Planning, College of Public Affairs, National Taipei University, 151, University Rd., San Shia District, New Taipei City 23741, Taiwan
[d] Institute of Management of Technology, National Chiao Tung University, Ta-Hsuch Road, Hsinchu 300, Taiwan

ARTICLE INFO

Article history:
Received 29 May 2013
Received in revised form 10 October 2013
Accepted 13 October 2013
Available online 24 October 2013

Keywords:
Uncertainty reduction
Evidential weight based on preferences (EWP)
Utility
Induction
World Competitiveness Yearbook (WCY)

ABSTRACT

Uncertainty always causes hesitations during decision-making. The uncertainty reduction however is not available through simple operations and easy interpretation. This research solves this problem by proposing an evidential weight based on preferences (EWP). The key technique of EWP is the integration of the roughness measures of an induction rule to reduce noises in doubts. The utilities composed of the derived weight from EWP are empirically used on World Competitiveness Yearbook 2012 to analyze the benchmarking nations. This case study shows European welfare nations (Denmark, Finland, Norway, and Sweden) focus on the long term strategic competitiveness while Asian tiger nations (Singapore, Hong Kong, Korea, and Taiwan) are energetic on short term surviving competitiveness.
© 2013 Elsevier B.V. All rights reserved.

1. Introduction

Evidence and inference are the fundamentals of decision-making. Uncertainty is always a challenge to both of these elements. In the theoretical frameworks like Fig. 1, the evidential weight, as proposed by Keynes, is based on the probability relations to express the rational belief about the importance and relevance between a primary proposition (premise) and a secondary proposition (conclusion) [1,2]. However, the uncertainty concerns such as the incomplete information for the probability judgment [3], probability unreliability [4–6], and suspicious conduct over all probabilities [7], can cause hesitation in decision-making. These often happen when there is difficulty of consistent interpretation between the relevance and importance of the propositions. Moreover, this inconsistency is normally an effect of the increase or decrease of evidential probabilities. For instance, an increased number of evidences may subsequently increase noise, which cannot raise the importance of the premises. Or, a high relevance might trade off the importance, and vice versa, thus making the interpretation ineffective. These inconsistency problems should be resolved through uncertainty reduction. Recently, the roughness [8], fuzziness [9], statistical reasoning [10], extended from the theory of evidence [6,7], are used to reduce various uncertainties. However, they have not been able to get rid of the uncertainty in the evidential weight, thus cannot provide a good consistency between importance and relevance to explain implications.

This research aims to propose an evidential weight based on preferences (EWP) with a reduced uncertainty. The technique of the roughness theory is used to approximate a weight having a consistent relevance and importance. The derived weights based on preferences are further designed to formulate a simple utility function (*SUF*) for analyzing the top competitiveness. The utility of *SUF* is the product of the derived weight and an observation value, thus different from Keeney and Raiffa's [11]. To achieve this goal, a methodology is designed by the roughness theory which can induce rules indifferently thus making EWP indifferent from each other. The resultant EWP is then standardized as the rules-verified weight for utility. Empirically, a competitiveness study about Asian Tigers nations (Singapore, Hong Kong, Korea, and Taiwan) and European welfare nations (Denmark, Finland, Norway, and Sweden) is used for illustration. The difference between these two groups is interpreted relevantly and importantly by our proposed utilities.

Roughness is the key concept to solve the uncertainty of evidential weight. Rough sets theory (RST) extended the theory of evidence [6,7] to present the vagueness of approximations with the rough membership function (the accuracy rate) in 1995–1997 [12–14]. Later, RST proposed a certainty measure and a coverage

* Corresponding author. Tel.: +81 6646316.
E-mail addresses: eugene@chu.edu.tw (Y.-C. Ko), issam@iwate-pu.ac.jp (H. Fujita), ghtzeng@mail.ntpu.edu.tw, ghtzeng@cc.nctu.edu.tw (G.-H. Tzeng).

0950-7051/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.knosys.2013.10.017

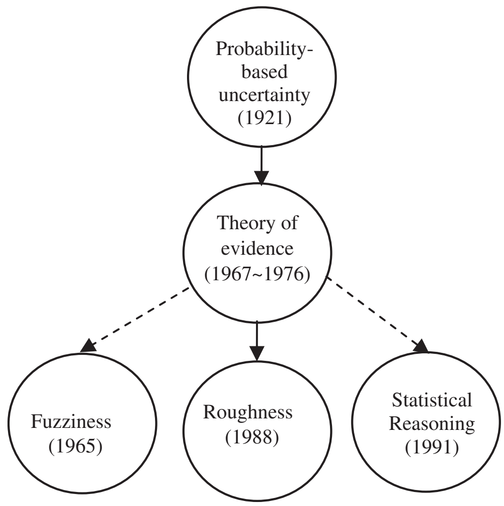

Fig. 1. Probability-based uncertainty.

measure for the induction rules in 2002 [15]. The uncertainty definition for an induction rule was almost complete then. However, these three separated measures cannot identify a unique weight to consistently explain the evidential relevance and importance. Consequently, the dominance-based rough set approach (DRSA) was developed after RST to consider classification, sorting, choice, and ranking problems, and to specify noise as the imprecise relevance [16]. The noise is something like a sample (any objects; or in this paper, nations), which has an inconsistency between its premise and conclusion. It usually cannot be avoided and hard to control in the real world. The consistency level which is best to explain the importance and relevance of the premise is still non-deterministic [17]. The uncertainty reduction in evidential weight becomes more difficult when the problems of measures integration and consistency level influence each other.

The characteristics of the benchmarking nations (the top ten or the upper half in competitiveness) can reveal the competitiveness strategies that stakeholders are interested in. With the aforementioned uncertainty, applying the evidential weight to analyze the benchmarking nations has challenges summarized as the followings:

- A simple utility function composed of evidential weights has not been successfully derived for competitiveness analysis. Generally, the utility knowledge can sufficiently help determine the key competitiveness characteristics. Especially, the evidential weight and the observation can make utilities more illustrative. *World Competitiveness Yearbook* (WCY), however, assumes that every criterion performs equally and operates in a simple linear formula as Eq. (1) [18].

$$f(x) = \sum_{j=1}^{m} w_j r_{x,j} \tag{1}$$

  where w_j is a weight of criterion q_j, m is the number of criteria, x represents a nation, $r_{x,j}$ represents a value of criterion q_j with respect to nation x. Finally, $f(x)$ is the competitiveness score of nation x. In the academic researches [19,20], the equal weights are criticized. After empirical testing, they claimed that the weights of WCY cannot be equal. A modified function becomes necessary and important.
- The evidential weights are still vague or unreliable for utilities [3,5,12–17]. Especially that they cannot clearly interpret the relevance and importance consistently.

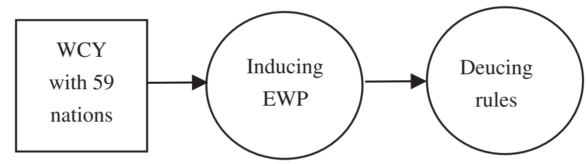

Fig. 2. I-EWP for the benchmarking nations.

To overcome the above challenges, a methodology for classification of the benchmarking nations named I-EWP (the induction of EWP), is designed in Fig. 2. I-EWP extends RST and DRSA to reduce uncertainty in the relevance and importance by integrating the roughness measures. I-EWP can be processed by Lingo 12 empirically.

I-EWP has two stages. Stage I solves EWP by considering the certainty, coverage, and accuracy rates of RST and DRSA. These are presented in Section 2.3, and redesigned for the top competitiveness in Section 3.2. Stage II proposes *SUF* with the derived weight from I-EWP to classify the benchmarking nations. This *SUF* is verified with the classification results. Empirically, all nations and criteria of WCY 2012 are included to avoid subjective bias.

This paper has two main parts. The first part presents the design and implementation of I-EWP, the details of which are described in Section 3. The second part applies the rules-verified weight to a case study about European welfare nations and Asian tiger nations. Most of these nations belong to the top level but have different styles of competitiveness. Their difference is hard to distinguish because their competitiveness is close to one another. Therefore, the weighted utilities are aggregated to distinguish their competitiveness. This issue will be discussed in Section 5.

The remainder of this paper is organized as follows: Section 2 reviews the evidential weights. Section 3 presents the design and implementation of I-EWP. Section 4 addresses application results of I-EWP, and Section 5 presents discussions on the proposed utilities and the case study. Finally, concluding remarks are stated to close the paper.

2. Literature review

The related theories of evidential weights are presented in this section; Section 2.2 is about the evidential weight; Section 2.3 is about DRSA and RST. The dataset of this research is described next.

2.1. Research dataset

International Institute for Management Development (IMD) annually publishes WCY, a well-known report which ranks and analyzes how a nation's environment can create and develop sustainable enterprises [21,22]. WCY is a product cooperating with

Table 1
4 factors and 20 criteria of WCY-IMD 2012.

Economic performance		Business efficiency	
q_1	Domestic economy	q_{11}	Productivity and efficiency
q_2	International trade	q_{12}	Labor market
q_3	International investment	q_{13}	Finance
q_4	Employment	q_{14}	Management practices
q_5	Prices	q_{15}	Attitudes and values
Government Efficiency		**Infrastructure**	
q_6	Public finance	q_{16}	Basic infrastructure
q_7	Fiscal policy	q_{17}	Technological infrastructure
q_8	Institutional framework	q_{18}	Scientific infrastructure
q_9	Business legislation	q_{19}	Health and environment
q_{10}	Societal framework	q_{20}	Education

fifty-four partner institutes worldwide. Its ranking considers broad perspectives by gathering the latest and most relevant data on the subject and by analyzing the policy consequence. The dataset include 59 nations, 4 consolidated factors, and 20 criteria in Table 1 [18].

2.2. The evidential weight

The evidential weight was proposed by Keynes in 1921 to express the degree of relevance in terms of probability. The main idea claims that the doubtful arguments relevant to decision should be considered quantitatively instead of Logic only. Its application requires considering not only the knowledge of decision makers but also circumstances for induction [23], thus can estimate the evidential relevance like goodness and risk [3,24]. However, the evolution of the evidential weight is criticized as below.

- The numerical indeterminateness of probability. Mathematical expectation of probability cannot always possibly determine which alternative ought to be chosen [3].
- The uncertainty from Bayesian measure. Bayesian's assumption that a single probability measure over states can represent belief is challenged with unreliability. The reason is that only partial information is available, thus Bayesian measure can be epistemic and cannot be fit for decision [5].
- The incomplete information for decision. This is explained by a stopping problem, wherein there is difficulty in finding a rational principle to decide when or where to stop the process of acquiring information in forming a probability judgment [25].

The above weaknesses, which illustrate the uncertainty of evidential weights, have not been solved. In the literatures, there are three paradigms intending to solve this uncertainty and give judgment information. The first is the roughness, presented in Section 2.3. The second is the fuzziness proposed in 1965 which is not included in this paper due to incomplete definition about the evidential weights. The third is the statistical reasoning which has the techniques of analyzing randomness, imprecision, and vagueness [26,27]. Roughness concept in decision making is suggested as the most promising candidate for a unified theory to solve the uncertainty [28]. Therefore, the roughness theory is chosen in this paper to analyze the evidential weight.

2.3. DRSA and RST

DRSA is a powerful technique of relational structure and can induce conditional preferences for classification, sorting, choice, and ranking [16]. The induced preferences for the ranking can imply the evidences to achieve the dominance class. There are four parts to illustrate this concept. First is the ranking unions: $Cl_t^{\geqslant}$ (the upward union of classes which includes objects ranked at least tth) and $Cl_t^{\leqslant}$ (the downward union of classes which includes objects ranked less than tth), where Cl is a cluster set containing preference-ordered classes Cl_t, $t \in T$ and $T = \{1,2,\ldots,n\}$. The formulations for the above statement can be expressed as $Cl = \{Cl_1, \ldots, Cl_t, \ldots, Cl_n\}$, $Cl_1 = \{y \in U{:}y$ is ranked in the top position$\}$, $Cl_2 = \{y \in U{:}y$ is ranked in the second position$\},\ldots,$ and $Cl_n = \{y \in U{:}y$ is ranked in the bottom position$\}$ where U is a set with decision- makers' preference orders and n is the number of preference-ordered classes. For all s, $t \in T$ and $s \geqslant t$ (rank of $s \geqslant$ rank of t), every object in Cl_s is preferred to be at least as good as any of object in Cl_t. The upward union is constructed as $Cl_t^{\geqslant} = \cup_{s \geqslant t} Cl_s$ for $s \geqslant t$; inversely, the downward union as $Cl_t^{\leqslant} = \cup_{s<t} Cl_s$ for $s < t$. A representation of the upward union, called the dominating set, can rely on a set of criteria, P. It follows the dominance principle of requiring each chosen object at least as good as object x in all considered criteria of P. The granules of a dominating set based on P can be viewed as the granular cones in the criteria value space. Vice versa, the dominated set for the downward union follows the dominance principle and has the granules in the opposite direction. These cones are named as P-dominating and P-dominated sets [25], respectively.

Second is about the dominance sets. For instance, object y dominates object x with respect to a criteria set P (denotation $yD_P x$). A dominance set means an important set. Given x, $y \in U$ and P, the dominance sets are formulated as:

$$P - \text{dominating set}: \ D_P^+(x) = \{y \in U, yD_P x\}$$
$$P - \text{dominated set}: \ D_P^-(x) = \{y \in U, xD_P y\}$$

where x, $y \in Cl$, $y \succsim_q x$ for $D_P^+(x), x \succsim_q y$ for $D_P^-(x)$, and all $q \in P$.

Third is about the use of relevant evidences to explain the ranking unions with conditional preferences. For instance of assigning objects into P-dominating set, evidences have two types. One is called consistent evidence, i.e., objects can be properly assigned into $D_P^+(x)$ and $Cl_t^{\geqslant}$. The other is inconsistent evidence, i.e., objects assigned in $Cl_t^{\geqslant}$ possibly violate the dominance principle of $D_P^+(x)$. In other word, this inconsistent evidence is not a member of a dominating set but assigned to the upward union. Therefore, inconsistent evidence is the major part making induction degenerate. According to the dominance consistency, there are three approximations defined for relevant evidences.

$$\underline{P}(Cl_t^{\geqslant}) = \{x \in U : D_P^+(x) \subseteq Cl_t^{\geqslant}\}, \overline{P}(Cl_t^{\geqslant}) = \bigcup_{x \in Cl_t^{\geqslant}} D_P^+(x)$$
$$Bnp(Cl_t^{\geqslant}) = \overline{P}(Cl_t^{\geqslant}) - \underline{P}(Cl_t^{\geqslant})$$
$$\underline{P}(Cl_t^{\leqslant}) = \{x \in U, D_P^-(x) \subseteq Cl_t^{\leqslant}\}, \ \overline{P}(Cl_t^{\leqslant}) = \bigcup_{x \in Cl_t^{\leqslant}} D_P^-(x),$$
$$Bnp(Cl_t^{\leqslant}) = \overline{P}(Cl_t^{\leqslant}) - \underline{P}(Cl_t^{\leqslant})$$

where $t = 1, 2, \ldots, n,\quad Bnp(Cl_t^{\geqslant})$ and $Bnp(Cl_t^{\leqslant})$ are P-doubtful regions. Objects in P-doubtful regions are inconsistent. In a simple word, $\underline{P}(Cl_t^{\geqslant})$ requires the largest union of P-dominating sets to be properly included in $Cl_t^{\geqslant}$. $\overline{P}(Cl_t^{\geqslant})$ requires the smallest union of P-dominating sets to contain all elements of $Cl_t^{\geqslant}$ while allowing some inconsistent objects.

Finally, the following is about the three measures related to the evidential weight.

- Accuracy rate (AR) [14,29]
  The accuracy rate presents the ratio of the proper assignment to the possible assignment. Two typical accuracy rates (α) are listed as:

$$\alpha(Cl_t^{\geqslant}) = \frac{|\underline{P}(Cl_t^{\geqslant})|}{|\overline{P}(Cl_t^{\geqslant})|} = \frac{|\underline{P}(Cl_t^{\geqslant})|}{|U| - |\underline{P}(Cl_t^{\leqslant})|}$$
$$\alpha(Cl_t^{\leqslant}) = \frac{|\underline{P}(Cl_t^{\leqslant})|}{|\overline{P}(Cl_t^{\leqslant})|} = \frac{|\underline{P}(Cl_t^{\leqslant})|}{|U| - |\underline{P}(Cl_t^{\geqslant})|}$$

  The symbol α is used to present 'a ratio of the cardinalities of P-lower approximation to those of P-upper approximation, i.e., the degree of the properly classifying approximation relative to the possibly classified approximation'.
- Coverage rates (CR) [15,29]
  The coverage rate expresses 'the probability of objects in the P-lower approximation relatively belonging to the corresponding union of decision classes', defined by Pawlak and Greco. There are two typical coverage rates (CR) for the upward unions $Cl_t^{\geqslant}$ and the downward union $Cl_t^{\leqslant}$, which are formulated as follows:

$$CR(Cl_t^{\geqslant}) = \frac{|\underline{P}(Cl_t^{\geqslant})|}{|Cl_t^{\geqslant}|}, CR(Cl_t^{\leqslant}) = \frac{|\underline{P}(Cl_t^{\leqslant})|}{|Cl_t^{\leqslant}|}$$

- Certainty rate (Cer) [15]
  A certainty rate of RST is formulated as:

$$\mathrm{Cer}(\phi,\psi)=\frac{\mathrm{Card}\|\phi\cap\psi\|}{\mathrm{Card}\|\phi\|}\ \text{for}\ \phi\to\psi$$

where ϕ and ψ are sets for condition and conclusion. Card $|\cdot|$ means the number of elements in a set. In a reverse way to explain the certainty rate, the ratio can be used to express the degree of the noise within the condition for implication.

Saaty (2001) proposed that pair-wise comparisons and inductions can be formulated as ratios, and then transformed the comparisons into the priority of criteria, or the criteria weights [30]. He also mentioned that the ratios represent how much more or less a criterion is as compared to another, and that its application can determine how close the criteria are. Furthermore, he emphasized that ratio operations are independent from irrelevant alternatives. Thus the ratio scales derived from different scales (criteria) can be implemented mathematically to generate a characteristic ratio with invariance. Based on these theories, a multiplication of ratios can express the quality of induction. These ratio operations can be further used to solve the evidential uncertainty, as mentioned next section.

3. I-EWP and The proposed utility

I-EWP is designed to reduce the uncertainty in EWP through the induction process for each criterion. Because the induction processes are independent from each other, the roughness measures are also independent between any two criteria. Thus, the product of the roughness measures can exist in an indifference curve distinguished by preference orders. There are four parts in this section. Firstly, the data set for this research is presented next. Section 3.2 presents the uncertainty reduction of I-EWP and the proposed utilities for classifying the benchmarking nations. The validation of driving EWP is presented in Section 3.3. A simple example of I-EWP is illustrated in Section 3.4.

3.1. Dataset

The dataset of this research is collected from WCY 2012, which adopts all criteria and nations, i.e., 20 criteria and 59 nations (objects shown by x or z). The ranking union for the dominating competitiveness includes the top ten or the upper half nations: Canada, Germany, Hong Kong, Norway, Qatar, Singapore, Sweden, Switzerland, Taiwan, and USA. Alternatively, the upper half has 29 nations.

3.2. I-EWP and the induced utilities

The roughness measures for an induction rule are defined with the coverage, accuracy, and certainty rates which are related to the evidential weight. RST has techniques such as a deterministic approach to express the vagueness with approximations (1982) [31], the probabilistic rough set model dealing with the information uncertainty (1988) [8], linking the belief function and plausibility of evidence theory to the lower and upper approximations (1996) [13,14], and the complete definition of the roughness measures for an induction rule (2002) [15]. Alternatively, DRSA extended the roughness theory to ranking relevance (1995) [32]. Later, DRSA provided evidential measures for the ranking union (2001) [29] and for the variable-consistency (2007) [17]. However, the variable-consistency measure does not specify the best level for uncertainty reduction. Up to now, all these measures individually perform well while only one measure cannot clearly specify uncertainty nor reduce uncertainty well. Therefore, an integration design is proposed in the followings.

The design of I-EWP for an induction rule has three parts. The first part is based on DRSA to approximate the optimal classification for the benchmarking nations. The evidential relevance and importance for the ranking union will be solved here. The second is based on RST to approximate the minimum uncertainty (or maximum certainty) for the evidential relevance and importance. The third integrates the first and the second parts to approximate an optimal classification with uncertainty reduction. The resulting value is assigned as EWP in Fig. 3. This EWP will be standardized to replace the weight in Eq. (1) for presenting a simple utility function different from Keeney's (1976).

I-EWP is implemented from the induction rule, $q^{\geqslant}_{j,t'}\to Cl^{\geqslant}_{t}$, as presented in Definition 2. There are six definitions for completing the implementation. Definition 1 presents the competitiveness DRSA. Definition 2 explains the induction rule of competitiveness. Definition 3 talks about the induction evidences. Definition 4 is about the measures based on the induction evidences. Definition 5 discusses the quality classification. And finally, Definition 6 discusses the calculation of the rules-verified weights by standardized EWP.

Definition 1 (*The competitiveness DRSA*). *DRSA* $=(U,Q,f,R,Cl^{\geqslant}_{t})$, where $U=\{y|y=1,\ldots,n\}$, $Q=\{q_1,q_2,\ldots,q_m\}$, $f{:}U\times Q\to R$, R is a ranking set, $R\in\{1\text{th},2\text{th},\ldots,n\text{th}\}$, $Cl^{\geqslant}_{t}$ is a ranking union having nations at least t, and t is a rank place like 10th.

Definition 2. An induction rule of competiveness $q^{\geqslant}_{j,t'}\to Cl^{\geqslant}_{t}$ represents how a criterion $q_j$supports nations to achieve the top t positions where $q^{\geqslant}_{j,t'},(q^{\geqslant}_{j,t'}=\bigcup_{s\geqslant t'}q_{j,s})$, is also a ranking union containing the top t' positions with respect to q_j. This rule associates the ranking evidences to a ranking union, which is independent to addition or removal of other criteria. Our design can be conceptualized as in Fig. 4.

Definition 3 (*The induction evidences (objects)*). Under the induction rule $q^{\geqslant}_{j,t'}\to Cl^{\geqslant}_{t}$, there are two approximations defined with boundaries $\underline{x}$ and $\bar{x}$ where $\underline{x}\in Cl^{\geqslant}_{t},\bar{x}\in Cl^{\geqslant}_{t}$, and the rank of $\underline{x}$ is always higher than or equal to that of $\bar{x}$. $\underline{x}$ is assumed as the boundary of the important evidences and $\bar{x}$ as the boundary of the relevant evidences. These two types of evidences are defined as:

Important evidences: $D^{+}_{P}(\underline{x})$.

Relevant evidences:$D^{+}_{P}(\bar{x})$.

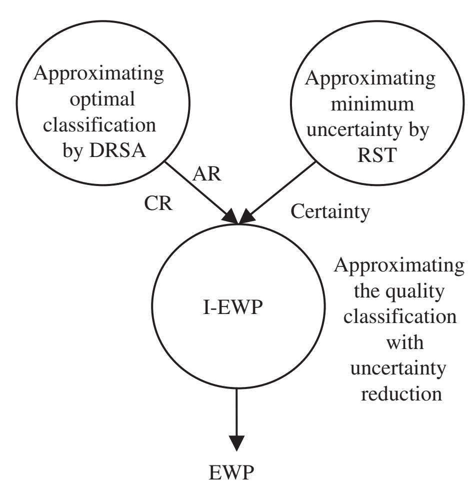

Fig. 3. The concept of I-EWP design.

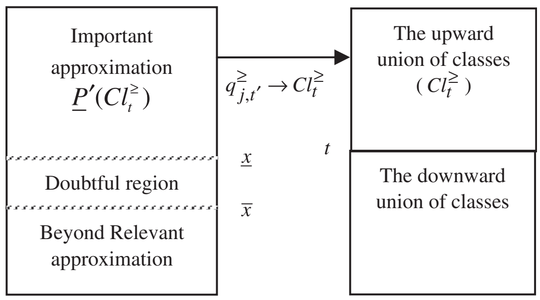

Fig. 4. Approximations based on the induction evidences.

The important evidences belong to the upper part of the relevant evidences in Fig. 4. The approximations based on the induction evidences are defined as:

Important approximation: $\underline{P}'(Cl_t^{\geq}) = D_P^+(\underline{x}) \cap Cl_t^{\geq}$.

Relevant approximation: $\overline{P}'(Cl_t^{\geq}) = D_P^+(\bar{x})$.

Doubtful region: $D_P^+(\bar{x}) - D_P^+(\underline{x})$.

Important approximation contains the important evidences belonging to the ranking union. It is same as the lower approximation of DRSA. Relevant approximation contains the evidences above the boundary $\bar{x}$ and requires that $\bar{x}$ belongs to the ranking union. Doubtful region contains the evidences that are relevant but not important. The noise in this area is dissimilar to the important evidence, and is called distinguished noise. Therefore, the noise within the approximations is defined as:

Undistinguished noises: $D_P^+(\underline{x}) - \underline{P}'(Cl_t^{\geq})$.

Distinguished noises: $D_P^+(\bar{x}) - D_P^+(\underline{x}) - \overline{P}(Cl_t^{\geq})$.

The distinguished noises are objects away from the important evidences, and normally located in the doubtful region. The undistinguished noises are together with the important evidences and cannot be separated by objective methods.

Obviously, the more evidences in $\underline{P}'(Cl_t^{\geq})$ the more important P is; the more noise in $\overline{P}'(Cl_t^{\geq})$ the less relevant P is. Due to the impact of noises, $\underline{x}$ and $\bar{x}$ are non-deterministic priori. Therefore, $\underline{x}$ and $\bar{x}$ are presented as slash lines in Fig. 4. They can be specified by approximating the optimal classification with the minimum distinguished noises.

Definition 4 (*Measures of the induction evidences*). Three measures related to the evidential weight of Fig. 4 are defined below.

- Evidence-accuracy rate (α') [14,29]
  An accuracy rate presents the ratio of 'Important approximation' to 'Relevant approximation', i.e., the degree of the properly classified evidence relative to the possibly relevant evidences, and is defined as:
  $$\alpha' = \frac{|\underline{P}'(Cl_t^{\geq})|}{|\overline{P}'(Cl_t^{\geq})|}$$
  $\alpha'$for a logical implication represents the degree of necessary condition of 'Important approximation' in the relevant evidences.
- Evidence-coverage rates (CR') [15,29]
  A coverage rate expresses the ratio of 'Important approximation' relatively belonging to the ranking union, and is defined as:
  $$CR' = \frac{|\underline{P}'(Cl_t^{\geq})|}{|Cl_t^{\geq}|}$$

CR' for a logical implication represents the degree of sufficient condition that 'Important approximation' influences the ranking union.

- Evidence-certainty rate (Cer') [15]
  A certainty rate expresses the ratio of objects in 'Important approximation' relatively belonging to the important evidences: $\mathrm{Cer}' = \frac{|\underline{P}'(Cl_t^{\geq})|}{|D_P^+(\underline{x})|}$ where $|\cdot|$ means the number of evidences in a set. Cer' represents the degree of reliability of $\underline{P}'(Cl_t^{\geq})$.

Definition 5 (*The quality classification rate (EWP)*). The classification rate for $q_{j,t'}^{\geq} \to Cl_t^{\geq}$ needs to consider both sufficient and necessary conditions. The product of CR' and α' will be a unique value on an indifference curve, which originates from the product of sufficient and necessary ratios for the indifferent induction rules. The induction measures are independent to addition or removal of other criteria. The product values thus can be used for preference orders. Further, the quality of classification needs have the reliability concern. According to the logical implication, a quality classification can be formulated as:

Quality classification ⇔ Minimum uncertainty.

'Quality classification if and only if minimum uncertainty' can be processed by mathematics to get a unique value on an indifference curve. Therefore, the quality classification rate based on evidential weight can be formulated below.

Model I: Solving EWP_j

$$\text{Max } EWP_j = \mathrm{Cer}' \times CR' \times \alpha'$$
$$\textbf{s.t. } P = \{q_j\}$$
$$\mathrm{Cer}' = \frac{|\underline{P}'(Cl_t^{\geq})|}{|D_P^+(\underline{x})|}, \quad CR' = \frac{|\underline{P}'(Cl_t^{\geq})|}{|Cl_t^{\geq}|}, \quad \alpha' = \frac{|\underline{P}'(Cl_t^{\geq})|}{|\overline{P}'(Cl_t^{\geq})|},$$

Model I will approximate a unique value, EWP_j, to consistently enlighten the relevance and importance of criterion q_j supporting nations to achieve the benchmarking positions. EWP_jcan be used as a weight like a slope in Fig. 5. Fig. 5 also illustrates how noise in the doubtful region is reduced by Model I. This process cuts nations into yes or no supporting evidences when approximating the quality classification. The vagueness in the doubtful region will diminish due to the optimal solution. The noise in 'Important evidences' will be counted as imprecision to the classification. The ranking position of $\underline{x}$ will becomes higher as much as possible to reduce the noise of 'Important evidences'. The ranking position of $\bar{x}$ also becomes highest to reduce distinguished noises. When approximating the optimal solution, $\underline{x}$ and $\bar{x}$ will be adjusted to the same position, and EWP_j is solved as the slope of Fig. 5.

Definition 6 (*The rules-verified weights*). EWP_j can be standardized by Eq. (2) to get the rules-verified weights.

$$w_j = \frac{EWP_j}{\sum_{j=1}^{m} EWP_j} \tag{2}$$

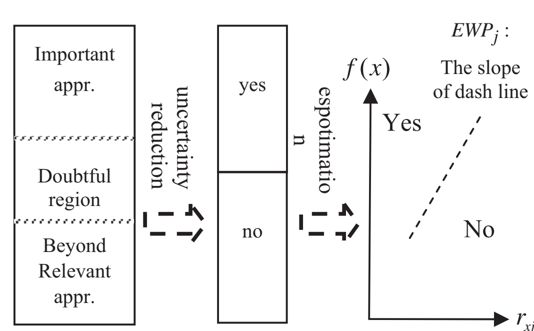

Fig. 5. The process of uncertainty reduction.

where w_j is within a range 0–1 thus each criterion can show its relevance and importance consistently relative to others. The rules-verified weights have two merits. First, the weighs get rid of distinguished noise in the doubtful region. Second, each weight explains the relevance and importance consistently. The rules-verified weight will further function in the utility discussed in the following.

3.3. Validation of the proposed utilities

SUF is designed by substituting w_j of Eq. (1) with w_j of Eq. (2). Then, the deduction rules constructed with the utilities become able to classify the benchmarking nations. For instance,

'*if* $f(x) \geqslant h$ *then* $x \in$ the top ten nations' claims a boundary utility h to separate the benchmarking and non-benchmarking nations. h can be solved with constrains: $\min\{f(x)|x \in \text{the top ten nations}\} = h$ and $\max\{f(x)|x \notin \text{the top ten nations}\} < h$. *SUF* can be proved as below.

Proof. Let

P1: The benchmarking and non-benchmarking nations;
P2: $(w_1, \ldots, w_m)$ is a tuple of the rules-verified weights and $SUF(x) = \sum_{j=1}^{m} w_j r_{x,j}$;
P3: The utility classification,

$$\underset{j=1,x\in Cl_t^{\geqslant}}{SUF(x)} \geqslant \underset{j=1,z\notin Cl_t^{\geqslant}}{SUF(z)}$$

where $Cl_t^{\geqslant}$ is the set of the benchmarking nations.

According to the syllogism,

$\because P1 \rightarrow P2$ can be proved by Model I
$\because P2 \rightarrow P3$ can be proved by the classification results
$\therefore P1 \rightarrow P3$ is proved to be true

The validation of $P2 \rightarrow P3$ will be fulfilled in R1 and R2 based on Table 2 of Section 4.1. Therefore, *SUF* is proved true for the benchmarking nations. □

3.4. An example of the rules-verified weight

An example here illustrates how a criterion classifies objects with an induction rule. By approximating the maximum 'Important approximation' and minimum 'distinguished noise', the slope of Fig. 5 can be solved as EWP_j for a criterion q_j. The dataset of this example has $U = \{x_1,x_2,x_3,x_4\}$, $Q = \{q_1,q_2\}$, $R_{q1} = R_{q2} = \{1,2,3,4\}$, these values, $1, \ldots, 4$, are ranks, $Cl_t^{\geqslant} = \{Cl_1, Cl_2, Cl_3, Cl_4\}$, and the ranking union is $\{Cl_1, Cl_2\}$.

As seen in the experiment results, the cells with bold values are the 'Important approximations' when EWP_1 are optimally solved for $q_{1,t'}^{\geqslant} \rightarrow Cl_2^{\geqslant}$ and EWP_2 for $q_{2,t'}^{\geqslant} \rightarrow Cl_2^{\geqslant}$. Obviously, both $q_{1,t'}^{\geqslant} \rightarrow Cl_2^{\geqslant}$ and $q_{2,t'}^{\geqslant} \rightarrow Cl_2^{\geqslant}$ have quality classification. I-EWP diminishes vagueness and explains the relevance and importance of criteria consistently by a weight value only. The inconsistency problem between relevance and importance is solved here.

Table 2
An example of EWP.

	q_1	q_2	$Cl_t^{\geqslant}$	Induction rules and EWP_j
x_1	**1**	**1**	$\mathbf{Cl_1}$	$q_{1,t'}^{\geqslant} \rightarrow Cl_2^{\geqslant}$
x_2	**2**	3	$\mathbf{Cl_2}$	$EWP_1 = 1 \times 1 \times 1 = 1$ for q_1
x_3	3	2	Cl_3	$q_{2,t'}^{\geqslant} \rightarrow Cl_2^{\geqslant}$
x_4	4	4	Cl_4	$EWP_2 = 1 \times 0.5 \times 1 = 0.5$ for q_2 $EWP_j = \text{Cer}' \times CR' \times \alpha'$

4. Application results

The application results have two parts. The first includes the rules-verified weights and the deduction rules, which are constructed from *SUF* to illustrate the classification of multiple criteria. The second is about the aggregated utilities for illustrating economic performance, government efficiency, business efficiency, and infrastructure. Therefore, stakeholders can catch points for policy making.

4.1. The resulted weights and induction rules

The rules-verified weights for the top ten and the upper half levels in 2012 are solved and presented in Table 3 where the bold figures represent the highest weights.

The deduction rules based on *SUF* for the top ten and the upper half nations are obtained as:

R1:

$$\begin{aligned}
&\textit{if } f(x) \geqslant 67.85 \textit{ then } x \in \text{ the top ten nations}\\
&\text{Cer}' = 1, CR' = 1, \alpha' = 1\\
&SUF(x) = 0.08 \times r_{x,1} + 0.05 \times r_{x,2} + \cdots + 0.03 \times r_{x,20}
\end{aligned}$$

R2:

$$\begin{aligned}
&\textit{if } f(x) \geqslant 60.64 \textit{ then } x \in \text{ the upper half nations}\\
&\text{Cer}' = 1, CR' = 1, \alpha' = 1\\
&SUF(x) = 0.04 \times r_{x,1} + 0.03 \times r_{x,2} + \cdots + 0.06 \times r_{x,20}
\end{aligned}$$

Obviously, R1 and R2 successfully classify the benchmarking nations and prove that *SUF* really exists for the benchmarking nations. The proposed *SUF* utilities are thus proved true by our rules-verified weights. Another example of the aggregated utilities is illustrated below.

4.2. The aggregated utilities

The aggregated utilities of *SUF* can consistently compare the relevance and importance of economics, government, business, and infrastructure for competitiveness. This can be formulated by using Eq. (3) and by setting criteria value as one. The results are presented in Table 4. As shown in the table, the infrastructure plays as more important for the upper half nations and the business efficiency as a more important factor for the top ten nations.

$$f_P(x) = \sum_{j\in P}^{|P|} w_j \times (1) \tag{3}$$

Another application of *SUF* gives a utilities pattern of the top ten nations for the competitiveness factors as seen in Fig. 6. Results show that the business efficiency has highest relevance and importance

Table 3
The rules-verified weights (w_j) for WCY 2012.

For the top level				For the upper half level			
w_1	**0.08**	w_{11}	**0.08**	w_1	0.04	w_{11}	0.06
w_2	0.05	w_{12}	0.02	w_2	0.03	w_{12}	0.04
w_3	0.05	w_{13}	**0.08**	w_3	0.05	w_{13}	0.06
w_4	0.02	w_{14}	**0.08**	w_4	0.04	w_{14}	0.06
w_5	0.02	w_{15}	0.05	w_5	0.02	w_{15}	0.05
w_6	0.07	w_{16}	0.03	w_6	0.03	w_{16}	0.06
w_7	0.03	w_{17}	0.07	w_7	0.02	w_{17}	0.06
w_8	0.06	w_{18}	0.04	w_8	**0.08**	w_{18}	0.06
w_9	0.05	w_{19}	0.05	w_9	0.06	w_{19}	0.06
w_{10}	0.04	w_{20}	0.03	w_{10}	0.06	w_{20}	0.06

Table 4
The aggregated utilities of WCY 2012.

For the top ten level		For the upper half	
$w_{Economic}$	0.23	$w_{Economic}$	0.18
$w_{Government}$	0.25	$w_{Government}$	0.25
$w_{Business}$	0.30	$w_{Business}$	0.26
$w_{Infrastructure}$	0.22	$w_{Infrastructure}$	0.30

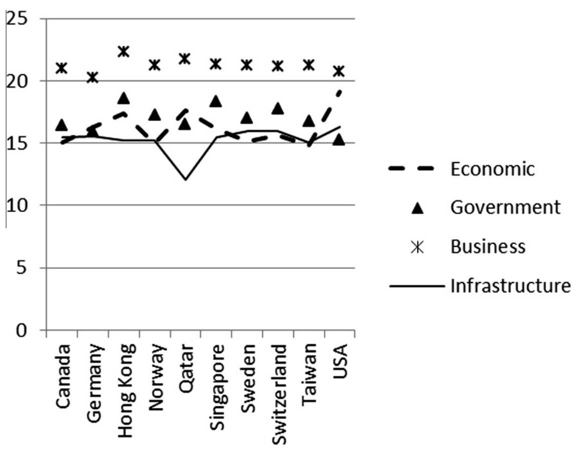

Fig. 6. A competitiveness pattern of the top ten 2012.

Table 5
Comparison among the related techniques.

Weakness	Bayesian weights	Regression weights	Rules-verified weights
Evidential relevance	1	0	1
Evidential importance	0	0	1
Uncertainty reduction	0	1	1
Total advantages	1	1	3

than the other factors. Further, eight nations of the top ten, Canada, Germany, Hong Kong, Norway, Singapore, Switzerland, and Taiwan had their government efficiency as their second important factor. Exceptions are USA and Qatar. Obviously, the aggregated utilities of government are the second important and relevant for the top ten nations. Details of Fig. 6 below thus verify Table 4.

4.3. Achievements of I-EWP methodology

This research has accomplished three achievements. First, the evidential weight which can keep relevance and importance consistent is solved by Model I. Second, *SUF* with the rules-verified weights is effective for the benchmarking nations. Third, the competitiveness patterns can be formed by the aggregated utilities. These findings are further discussed in Section 5.

5. Discussions on EWP and the case study

This section has two parts. One is about the technique discussion. The other is a case study about implications on European welfare nations and Asian tiger nations.

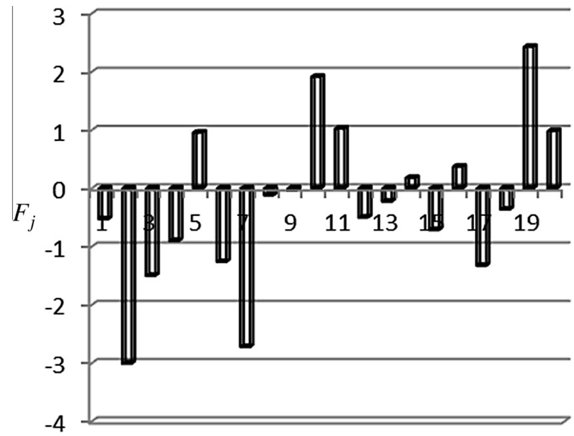

Fig. 7. The relative advantages between European welfare nations and Asian tiger nations.

5.1. Technique discussion

The technique discussion has four stages, the goal, methodology, applications, and comparison. This research aims to improve the uncertainty of evidential weights by integrating the roughness measures and proposing the use of *SUF* with our derived weights.

I-EWP methodology identifies the induction evidences, defines the measures of the induction evidences, integrates the inductions measures, reduces the distinguished noise to the minimum by approximating the maximum 'Important approximation', and applies the utility function with the derived weights for classification. These derived weights are used as priori knowledge for deduction thus making them different from the regression weights which are initiated with unknown or non-deterministic variables [33–35]. *SUF*, which has a known priori, obviously is easier and simpler than the general regression.

The comparison among the rules-verified weights, the regression weights, and Bayesian weights are enlisted in Table 5 below. Their comparison conditions are fairly constructed with the same dataset. Their priori and posterior information are the following:

- The regression weights are posterior of unknown or non-deterministic variables.
- Bayesian weights are posterior of data.
- The rules-verified weights are posterior of preferences.

The comparisons show Bayesian weight considers evidential relevance but does not consider the evidential importance and uncertainty reduction for propositions. In the case of regression weights, which are solved from the evaluation of expectations and observations, they are good at uncertainty reduction but do not directly consider the evidential relevance and importance. Only the rules-verified weights have all these three merits.

5.2. The case study

The case study about the relative advantages between European welfare nations and Asian tiger nations is based on the utilities of *SUF*. The aggregated utilities can provide an easy and simple vision to interpret the relative advantages by Eq. (4), which is formulated as:

$$F_j = \sum_{x \in welfare}^{4} w_j r_{xj} - \sum_{y \in tigers}^{4} w_j r_{yj} \tag{4}$$

The application results of Eq. (4) on the case study are presented in Fig. 7 wherein the positive values mean European nations have advantages over Asian tiger nations and the negative values mean Asian tiger nations have the advantages. According to Fig. 7, users can tell Asian tiger nations have more advantages (13 of 20) than those of European welfare nations. As a whole, they are more competitive. In details, European welfare nations are better in price (q_5), societal framework (q_{10}), productivity and efficiency (q_{11}), management practices (q_{14}), basic infrastructure (q_{16}), health and environment (q_{19}), and education (q_{20}). Generally, developing these advantages requires a long time. Especially that societal framework (q_{10}), management practices (q_{14}), health and environment (q_{19}), and education (q_{20}) all influence future generations. Alternatively, Asian tiger nations had thirteen criteria performing better than European welfare nations. The major advantages are international trade (q_2), international investment (q_3), employment (q_4), public finance (q_6), fiscal policy (q_7), attitude (q_{15}), and technology infrastructure (q_{17}). Asian tiger nations seem to have energetic and aggressive attitudes. The benefits of these achievements can be seen in a shorter time. We would like to give a comment 'European welfare nations focus more on a long term strategy in competitiveness while Asian tiger nations are more energetic on short-term surviving competitiveness'. In the future there are two issues deserving of further exploration. First, criteria relationship can be discovered and visualized for competitiveness. Second, more methods can be available to interpret competitiveness. It is our great expectation that all these explorations can be achieved and collected in the knowledge-based systems in the near future.

6. Concluding remarks

This research supposes that the distinguished noise in the doubtful region can cause bigger uncertainty to the evidential weight. In this research an evidential weight based on preferences (EWP) is induced by reducing distinguished noise, and standardized to a rules-verified weight. The derived weights are used to formulate a simple utility function (*SUF*) for analyzing the top competitiveness. In our design, the rules-verified weight keeps evidential relevance and importance consistent thus capable of correct interpretation. There are fourth achievements in this paper. First, the epistemic uncertainty originated in 1921 is improved in probability relation. Second, the utility function proposed in 1973 and 1976 is improved with our derived weights and proved correct for classification. Third, the roughness measures for evidential weight from 1988–2007 are integrated. Fourth, the economic, government, business, and infrastructure of WCY 2012 can be compared by the proposed utilities. The business efficiency plays the most important role in the top competitiveness level while the infrastructure is the most important one in the upper half level.

The case study between two competitiveness patterns shows that European welfare nations adopt a long term strategy for future generations. They gain advantages over Asian tiger nations in societal framework (q_{10}), management practices (q_{14}), basic infrastructure (q_{16}), health and environment (q_{19}), and education (q_{20}). On the other hand, Asian tiger nations are energetic and focus on the short term surviving competitiveness. They are good at international trade (q_2), international investment (q_3), employment (q_4), public finance (q_6), fiscal policy (q_7), attitude (q_{15}), and technology infrastructure (q_{17}).

References

[1] J.M. Keynes, The weight of argumens, in: A Treatise on Probability, Macmillan, London, 1921.
[2] J.M. Keynes, The meaning of probability, in: A Treatise on Probability, Macmillan, London, 1921.
[3] A. Feduzia, On Keynes's conception of the weight of evidence, Journal of Economic Behavior & Organization 76 (2) (2010) 338–351.
[4] P. Gardenfors, N.-E. Sahlin, Unreliable probabilities, risk taking and decision making, in: P. Gardenfors, N.-E. Sahlin (Eds.), Decision, Probability and Utility, Cambridge University Press, Cambridge, 1988, pp. 313–334.
[5] A.P. Dempster, Upper and lower probabilities induced by a multivalued mapping, Annals of Mathematical Statistics 38 (1967) 325–339.
[6] G. Shafer, A Mathematical Theory of Evidence, Princeton University Press, Princeton, NJ, 1976.
[7] J.M. Keynes, The application of probability to conduct, in: A Treatise on Probability, Macmillan, London, 1921.
[8] Z. Pawlak, Rough sets: probabilistic versus deterministic approach, International Journal of Man-Machine Studies 29 (1988) 81–95.
[9] L.A. Zadeh, Fuzzy sets, Information and Control 8 (3) (1965) 338–353.
[10] P. Walley, Statistical Reasoning with Imprecise Probabilities, Chapman and Hall, London, 1991.
[11] R.L. Keeney, H. Raiffa, Assessment of an additive value function, Decision with Multiple Objectives – Preferences and Value Tradeoffs, Cambridge University Press, 1976, 1993.
[12] Z. Pawlak, Rough sets, Communications of the ACM 38 (11) (1995) 88–95.
[13] Z. Pawlak, Rough sets, rough relations and rough functions, Fundamenta Informaticae – Special Issue: Rough Sets Archive 27 (2-3) (1996) 103–108.
[14] Z. Pawlak, Rough set approach to knowledge-based decision support, European Journal of Operational Research 99 (1) (1997) 48–57.
[15] Z. Pawlak, Rough sets, decision algorithms and Bayes' theorem, European Journal of Operational Research 136 (1) (2002) 181–189.
[16] S. Greco, B. Matarazzo, R. Slowinski, Extension of the rough set approach to multicriteria decision support, INFOR 38 (3) (2000) 161–193.
[17] J. Blaszczynski, S. Greco, R. Slowinski, Multi-criteria classification – a new scheme for application of dominance-based decision rules, European Journal of Operational Research 181 (3) (2007) 1030–1044.
[18] IMD, World Competitiveness Yearbook 2012, Institute Management Development, Lausanne, Switzerland, 2012. <http://worldcompetitiveness.com/OnLine/App/Index.htm>.
[19] M. Oral, H. Chabchoub, An estimation model for replicating the rankings of the World Competitiveness Report, International Journal of Forecasting 13 (1997) 527–537.
[20] M. Oral, H. Chabchoub, On the methodology of World Competitiveness Report, European Journal of Operational Research 90 (3) (1996) 514–535.
[21] US President's Commission on Industrial Competitiveness, Global Competition: The new reality, US Government Printing Office, Washington, DC, 1985.
[22] M.E. Porter, The Competitive Advantage of Nations, The Free Press, MacMillan, New York, 1990.
[23] J.M Keynes, The foundations of statistical inference, in: A Treatise on Probability, Macmillan, London, 1921.
[24] M.E. Brady, J.M. Keynes's theoretical approach to decision making under condition of risk and uncertainty, The British Journal for the Philosophy of Science 44 (1993) 357–376.
[25] A. Feduzia, On Keynes's conception of the weight of evidence, Journal of Economic Behavior & Organization 76 (2) (2010) 338–351.
[26] R. Coppi, Management of uncertainty in statistical reasoning: the case of regression analysis, International Journal of Approximate Reasoning 47 (3) (2008) 284–305.
[27] P. Walley, Statistical Reasoning with Imprecise Probabilities, Chapman and Hall, London, 1991.
[28] P. Walley, Towards a unified theory of imprecise probability, International Journal of Approximate Reasoning 24 (2000) 125–148.
[29] S. Greco, B. Matarazzo, R. Slowinski, Rough set theory for multicriteria decision analysis, European Journal of Operational Research 129 (1) (2001) 1–47.
[30] T.L. Saaty, Decision making hierarchies, in: The Analytic Network Process, second ed., RWS Publications, 2001.
[31] Z. Pawlak, Rough sets, International Journal of Information and Computer Sciences 11 (1982) 145–172.
[32] S. Greco, B. Matarazzo, R. Slowinski, Rough Set Approach to Multi-Attribute Choice and Ranking Problems, ICS Research Report 38/95, Warsaw University of Technology, Warsaw, 1995;
S. Greco, B. Matarazzo, R. Slowinski, in: G. Fandel, T. Gal (Eds.), Proceedings of the Twelfth International Conference on Multiple Criteria Decision Making, Hagen, Germany, Springer, Berlin, 1997, pp. 318–329.
[33] D.R. Anderson, D.J. Sweeney, T.A. Williams, Multiple regression, in: Statistics for Business and Economics, 11th ed., South Western Cengage Learning, 2011.
[34] B. Ari, H.A. Güvenir, Clustered linear regression, Knowledge-Based Systems 15 (3) (2002) 169–175.
[35] H.A. Guvenir, I. Uysal, Regression on feature projections, Knowledge-Based Systems 13 (4) (2000) 207–214.