EFFECT OF FLEXIBILITY ON IMPEDANCE FUNCTIONS FOR CIRCULAR FOUNDATION

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ABSTRACT: This paper investigates the effect of foundation rigidity on impedance functions for a circular foundation on a viscoelastic soil medium. In addition to vertical and rocking impedances, the paper also investigates the influence on coupling impedance for horizontal and rocking motions of foundation and horizontal impedance. To generate impedance functions for flexible foundation, a substructure technique is used. For the substructure of the flexible foundation, classical plate theory with neglecting inertial force is employed to obtain the deformation of the foundation due to the interaction stress. For the substructure of the soil medium, the technique, which can deal with wave equations in cylindrical coordinates with arbitrarily prescribed boundary conditions, is employed to obtain the displacement field in the soil medium due to the interaction stresses. Then, with the help of the variational principle, the displacement continuity condition of both substructures is imposed to generate the impedances for the flexible foundation. To demonstrate the effectiveness of the presented procedure, comparison with some previous numerical results is made. Selected numerical results of the presented procedure are presented. Also, comparison between results with and without the assumption of relaxed stress condition is given in order to show the significance of the assumption.

INTRODUCTION

A reliable way to perform dynamic analysis of soil-structure interaction is to use the substructure method in which the soil medium is treated as a continuum body, and structure itself is modeled by the finite element method. Therefore, obtaining the impedance, which represents the resistance of soil medium to the vibration of structure, is an important step for the interaction analysis.

A large amount of research work has been done for obtaining the impedance in recent decades. However, in most of the research work the assumption of rigid foundation is made. For example, Lysmer (1965) used the analytic solution for constant normal ring traction on half-space medium to generate a vertical compliance function (inverse of impedance function) for a rigid circular plate; and Luco and Westmann (1971) calculated all the compliance functions for a rigid circular plate on a half-space medium by reducing Fredholm integral equations to algebraic equations using the finite difference method. Liou et al. (1991; 1992) developed a technique to decompose the prescribed tractions on half-space and layered half-space media, which can match with general solutions of wave equations in cylindrical coordinates, to generate all the impedance functions for a rigid circular plate rigidly welded on soil medium.

Only a small amount of research work investigating the effect of foundation flexibility on impedance functions is reported in literature. Lin (1978) employed Fredholm integral equations to obtain the vertical and rocking

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impedance functions for thin plate with thick walled cylinder on viscoelastic half-space medium. Krenk and Schmidt (1981) applied Green's influence function of half-space and the plate theory of Reissner (1975) and Mindlin (1951) to solve the vibration problems of the plate on half-space medium. Whittaker and Christiano (1982) used a discretization scheme to find the elastic solutions for flexible plate and half-space medium independently, and then enforced the compatibility condition to solve the foundation-soil interaction problems. Iguchi and Luco (1982) employed the idea similar to Lin's (1978) to calculate the dynamic responses of flexible circular foundation with rigid core on layered viscoelastic half-space medium. Rajapakse (1989) used Green's function of half-space and classical plate theory to calculate the dynamic responses of flexible annular plate on viscoelastic half-space medium.

For the studies regarding the effect of plate flexibility as aforementioned, the assumption of relaxed contact stress condition, in which contact shear stresses between foundation and soil medium is ignored, is always made. Therefore, only the influence of foundation rigidity on vertical and rocking impedance can be investigated. However, in addition to vertical and rocking impedance, coupling impedance for rocking and horizontal motions of foundation and horizontal impedance are also influenced by foundation rigidity, if the flexible foundation is assumed to be rigidly welded to the soil medium. Therefore, investigating this influence is also one of the major purposes of this paper.

In this paper, the contact stresses (interaction stresses) between the foundation plate and soil medium is assumed to be piecewise linear in the r-direction of cylindrical coordinates for each Fourier component with respect to azimuth (variable θ). To deal with the foundation-soil system, the substructure concept is applied. For the substructure of foundation, the classical plate theory with ignorance of inertial force is employed to solve the problem of the foundation plate subjected to the assumed unknown contact stress. For the soil medium with prescribed unknown contact stresses, the technique, developed by Liou (1989, 1991, 1992), is employed to decompose the prescribed unknown contact stresses in order to match with the general solution of wave equations in cylindrical coordinates. Then, the condition of displacement continuity for the foundation plate and soil medium is imposed through variational principle to obtain the unknown intensities of piecewise linear contact stresses and the impedance functions for the foundation plate.

Some selected numerical results of the proposed procedure are presented in the paper. To demonstrate the effectivenss of the proposed procedure, some comparison with previous work reported by Iguchi and Luco (1982) is made. For vertical and rocking impedances, the results with and without the assumption of relaxed-contact-stress condition are given in the same figures in order to show the discrepancy between the two results. For the case of foundation rigidly attached to soil medium, the coupling and horizontal impedances is also influenced by the rigidity of the foundation. The results of coupling and horizontal impedances for different foundation rigidities are also given.

ANALYSIS OF FOUNDATION-SOIL SYSTEM

For dynamic analysis of a foundation-soil system, the motion of the system is assumed to be harmonic with respect to time, and the time harmonic

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variation ($e^{i\omega t}$) will be omitted in the derivations, explanations and mathematical manipulations in the paper for convenience. In the foundation-soil interaction system, the foundation is assumed to be rigid to axial deformation and flexible to curvature deformation with neglecting shear deformation and inertial force. The soil medium can be viscoelastic half-space or layered half-space, and the contact condition between foundation and soil can be smooth or rigidly welded.

To obtain the impedance functions for vibrations of flexible foundation, substructure technique is employed. The contact stresses (interaction stresses) between foundation and soil can be expressed in terms of Fourier components with respect to the azimuth (θ coordinate in cylindrical coordinates). For each Fourier component of contact stresses, piecewise linear distribution in the $r-$ direction of cylindrical coordinates is assumed. Let contact area be a circle with radius a_o. The radius a_o can be divided into m equal intervals as $b = a_o/m$. The piecewise linear stress model for each Fourier component can then be expressed as follows:

$$\bar{\tau}_{rz} = \sum_{j=1}^{m-1} h_j(r)q_j + h_o(r)q_o + h_m(r)q_m = \mathbf{h^Tq} \tag{1a}$$

$$\bar{\sigma}_{zz} = \sum_{j=1}^{m-1} h_j(r)p_j + h_o(r)p_o + h_m(r)p_m = \mathbf{h^Tp} \tag{1b}$$

$$\bar{\tau}_{\theta z} = \sum_{j=1}^{m-1} h_j(r)s_j + h_o(r)s_o + h_m(r)s_m = \mathbf{h^Ts} \tag{1c}$$

where

$$h_j(r) = 1 + \frac{r - jb}{b}; \quad \text{if} \quad (j - 1)b \leq r \leq jb \quad \text{and} \quad 1 \leq j \leq m \tag{1d}$$

$$h_j(r) = 1 - \frac{r - jb}{b}; \quad \text{if} \quad jb \leq r \leq (j + 1)b \quad \text{and} \quad 0 \leq j \leq m - 1 \tag{1e}$$

$$h_j(r) = 0; \quad \text{otherwise} \tag{1f}$$

and q_j, p_j, and s_j = unknown intensities of interaction stresses at node j for $\bar{\tau}_{rz}$, $\bar{\sigma}_{zz}$, and $\bar{\tau}_{\theta z}$, respectively.

Now, substructure technique is applied. For the substructure of soil medium with prescribed tractions in (1), Liou et al. (1989, 1991, 1992) have developed a technique to decompose the prescribed tractions of (1) in order to match with the general solutions of three dimensional wave equations in cylindrical coordinates reported by Sezawa (1929). According to Liou's reports, the corresponding displacement vector of soil medium at the surface ($z = 0$) contact with foundation can be expressed in terms of the contact stress vector of (1) for each Fourier component as follows:

$$\begin{Bmatrix} u_r(r) \\ u_z(r) \\ u_\theta(r) \end{Bmatrix} = -\int_0^\infty \mathbf{JQD}\, dk\mathbf{P} \cos n\theta \tag{2}$$

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where contact stress vector $\mathbf{P}^{\mathrm{T}} = (\mathbf{q}^T, \mathbf{p}^T, \mathbf{s}^T)$ in which vectors $\mathbf{q}$, $\mathbf{p}$, and $\mathbf{s}$ are shown in (1); $\mathbf{J} = 3 \times 3$ Bessel function matrix; $\mathbf{Q} = 3 \times 3$ transfer matrix for the soil medium; $3 \times 3\,(m + 1)$ matrix $\mathbf{D}$ contains the integrations involving Bessel functions with respect to variable r, and k = azimuthal wave number. Matrices $\mathbf{J}$, $\mathbf{Q}$, and $\mathbf{D}$ are defined in the papers by Liou et al. (1991, 1992) for both layered and half-space soil medium.

Let traction vector $\bar{\mathbf{t}}_o = (\bar{\tau}_{rz}, \bar{\sigma}_{zz}, \bar{\tau}_{\theta z})^T \cos n\theta$ and be defined in (1). If one applies variational principle to the substructure of soil medium for each Fourier component, the following equation can be obtained:

$$\delta W = \int_0^{a_o} \int_0^{2\pi} \delta \bar{\mathbf{t}}_o^T \mathbf{u}_o r\, d\theta\, dr = -\left(\frac{2\pi}{\pi}\right) \delta \mathbf{P}^T \int_0^{a_o} \mathbf{H} \int_0^{\infty} \mathbf{JQD} r\, dk\, dr \mathbf{P}$$

$$= -\left(\frac{2\pi}{\pi}\right) \delta \mathbf{P}^T \int_0^{\infty} \left(\int_0^{a_o} \mathbf{HJ} r\, dr \right) \mathbf{QD}\, dk \mathbf{P}$$

$$= -\left(\frac{2\pi}{\pi}\right) \delta \mathbf{P}^T \int_0^{\infty} \mathbf{D}^{\mathrm{T}} \mathbf{QD} k\, dk \mathbf{P} = \left(\frac{2\pi}{\pi}\right) \delta \mathbf{P}^T \mathbf{K}_1 \mathbf{P} \quad (3)$$

where $3(m + 1) \times 3$ matrix

$$\mathbf{H} = \begin{bmatrix} h & 0 & 0 \\ 0 & h & 0 \\ 0 & 0 & h \end{bmatrix} \quad (3a)$$

and vectors $\mathbf{h}$ and $\mathbf{u}_o$ are defined in (1) and (2), respectively. The coefficients 2π and π in (3) come from the integrals $\int_0^{2\pi} \cos^2 n\theta\, d\theta$ or $\int_0^{2\pi} \sin^2 n\theta\, d\theta$, which are not explicitly expressed in the formulations. In the derivation of (3), the relationship $\int_0^{a_o} \mathbf{HJ} r\, dr = k\mathbf{D}^{\mathrm{T}}$ is shown in the paper by Liou et al. (1992). Also, the matrix $\mathbf{K}_1 = -\int_0^{\infty} \mathbf{D}^{\mathrm{T}}\mathbf{QD} k\, dk$ is symmetric as shown in the paper by Liou et al. (1992).

Numerically, most computational effort in the preceding brief derivation is spent on calculating matrix $\mathbf{D}$ in which numerical integrations are involving Bessel functions with respect to variable r. The matrix $\mathbf{D}$ is independent of excitation frequency and complexity of properties of soil medium. The changes of excitation frequency or soil properties are only reflected in matrix $\mathbf{Q}$ in (3). Therefore, one can generate several matrices $\mathbf{K}_1$ of (3) for different excitation frequencies simutaneously, if one reserves the storage space in the computer for the several matrices $\mathbf{K}_1$. This feature of calculating $\mathbf{K}_1$ in (3) can save a lot of computational cost for generating impedance functions for soil-structure interaction analysis.

Consider the substructure of foundation. For the analysis of foundation plate subjected to the contact stress assumed in (1), classical plate theory is employed with neglecting inertial force. Since the foundation is assumed to be rigid to axial stress, only traction component $\bar{\sigma}_{zz}$ in (1) has an effect on the deformation of the foundation plate. The governing equation of the foundation plate for nth Fourier component in cylindrical coordinates can be written as

$$\bar{D}\nabla^2\nabla^2 w(r) = q(r) \quad (4a)$$

where

$$w(r)\cos n\theta = [\bar{u}_z(r) - \Delta_z(r)]\cos n\theta; \qquad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{n^2}{r^2} \quad (4b)$$

where $\bar{D}$ = plate rigidity; $\Delta_z(r)\cos n\theta$ = displacement field or rigid body motion; $\bar{u}_z(r)\cos n\theta$ = total displacement field; and $q(r) = \bar{\sigma}_{zz}(r)$ for the nth Fourier component in which $\bar{\sigma}_{zz}$ is defined in (1). Also, note that the variation with respect to θ ($\cos n\theta$) has been omitted in (4). By observing (4a), one can conclude that displacement for rigid body motion can occur only for vertical motion ($n = 0$) with $\Delta_z(r) = \Delta_o$ and rocking motion ($n = 1$) with $\Delta_z(r)\cos\theta = \Delta_1 r\cos\theta$.

To solve (4), one can treat $\bar{\sigma}_{zz}(r_p)\,dr_p$ as concentrated loading applied at the location $r = r_p$ to find the deformation shape of the plate and then integrate with respect to r_p to obtain the total deformation of the plate in terms of the unknown intensities at the nodal ring of piecewise linear stress model (1). For the application of concentrated loading $\bar{\sigma}_{zz}(r_p)\,dr_p$ at $r = r_p$, the displacement field of the plate can be written as follows:

$$dw_1(r) = F_n(r); \qquad \text{for } r < r_p \tag{5a}$$

$$dw_2(r) = f_n(r); \qquad \text{for } r_p < r < a_o \tag{5b}$$

where

$$F_o(r) = C_{10} + C_{20}r^2 + C_{30}\ln(r) + C_{40}r^2\ln(r); \qquad \text{for } n = 0 \tag{5c}$$

$$F_1(r) = C_{11} + C_{21}r^3 + C_{31}r^{-1} + C_{41}r\ln(r); \qquad \text{for } n = 1 \tag{5d}$$

where a_o = radius of the plate and the expressions for $f_n(r)$ are similar to that for $F_n(r)$ except that the coefficients are different. To determine the coefficients, one can use the boundary conditions of the plate and the continuity conditions at $r = r_p$. The final solution of the displacement field of the plate subjected to the assumed piecewise linear traction in (1) can be obtained by superimposing the integrations of (5a–b) with respect to r_p. This leads to the following equation:

$$w(r) = [\mathbf{1}]^T\mathbf{R}(r)\mathbf{p} \tag{6a}$$

where $(m + 1) \times (m + 1)$ matrix

$$\mathbf{R}(r) = \begin{bmatrix} R_{oo}(r) & \cdots & R_{om}(r) \\ \vdots & \ddots & \vdots \\ R_{mo}(r) & \cdots & R_{mm}(r) \end{bmatrix} \tag{6b}$$

in which R_{ij} = displacement field for $(i - 1)b < r < (i + 1)b$ due to the traction $h_j(r)$ defined in (1a); vector $[\mathbf{1}] = (1, 1, \ldots, 1)^T$; and stress intensity vector $\mathbf{p}$ is defined in Eq. (1b).

The total displacement $\bar{u}_z(r)\cos n\theta$ of the plate for the nth Fourier component can be obtained by including the displacement of rigid body motion defined in (4).

$$\bar{u}_z(r)\cos n\theta = [\Delta_z + w(r)]\cos n\theta$$

$$= \bar{u}_z^r(r)\cos n\theta + [\mathbf{1}]^T\mathbf{R}(r)\mathbf{p}\cos n\theta = \bar{u}_z^r(r)\cos n\theta + \bar{u}_z^f(r)\cos n\theta \tag{7}$$

For the other two displacement components $\bar{u}_r$ *and* $\bar{u}_\theta$ of the foundation plate, only the displacement of rigid body motion is accounted, since the foundation plate is assumed to be rigid for in-plane motions.

Let displacement vector be expressed as $\bar{\mathbf{u}}_o = \bar{\mathbf{u}}_o^r + \bar{\mathbf{u}}_o^f = [(\bar{u}_r, \Delta_z, \bar{u}_\theta)^T + (0, w, 0)^T]\cos n\theta$, in which $\mathbf{u}_o^r$ represents displacement of rigid body motion of foundation and $\mathbf{u}_o^f$ represents flexibility deformation of foundation. The virtual work done due to the variation of traction in (1) can be obtained as follows:

$$\delta W = \int_0^{a_o}\int_0^{2\pi} \delta \bar{\mathbf{t}}_o^T \bar{\mathbf{u}}_o r\, d\theta\, dr = -\left(\frac{2\pi}{\pi}\right)\left[\delta \mathbf{P}^T \int_0^{a_o} \mathbf{H}\bar{\mathbf{u}}_o' r\, dr + \delta \mathbf{p}^T \int_0^{a_o} \mathbf{h}[\mathbf{1}]\mathbf{R}(r) r\, dr \mathbf{p}\right] = \left(\frac{2\pi}{\pi}\right)(\delta \mathbf{P}^T \mathbf{B}_1 \boldsymbol{\Delta} + \delta \mathbf{p}^T \mathbf{K}_2 \mathbf{p}) \quad (8)$$

where vector $\boldsymbol{\Delta}$ contains amplitudes of n_o components of rigid body motion; $\mathbf{B}_1 = 3(m+1)Xn_o$ matrix; and $\mathbf{K}_2 = (m+1) \times (m+1)$ matrix.

Now, the continuity of displacements for both substructures of soil medium and foundation plate is imposed through equating (3) to the modified (8). The following equations are obtained:

$$\left(\frac{2\pi}{\pi}\right)\mathbf{K}_1\mathbf{P} = \left(\frac{2\pi}{\pi}\right)(\mathbf{B}_1\boldsymbol{\Delta} + \bar{\mathbf{K}}_2\mathbf{P}) \quad (9)$$

or

$$\left(\frac{2\pi}{\pi}\right)(\mathbf{K}_1 - \bar{\mathbf{K}}_2)\mathbf{P} = \left(\frac{2\pi}{\pi}\right)\mathbf{K}\mathbf{P} = \left(\frac{2\pi}{\pi}\right)\mathbf{B}_1\boldsymbol{\Delta} \quad (9a)$$

or

$$\mathbf{V} = \left(\frac{2\pi}{\pi}\right)\mathbf{B}_1\boldsymbol{\Delta} \quad (9b)$$

where vector $\mathbf{V}$ = generalized displacements at the nodal rings of the assumed piecewise-linear-stress model. To match with the dimensions of matrix $\mathbf{K}_1$ in (3), matrix $\mathbf{K}_2$ in (8) is expanded to the dimensions of $3(m+1) \times 3(m+1)$ and expressed as $\bar{\mathbf{K}}_2$ in (9). The $\bar{\mathbf{K}}_2$ is simply defined as follows:

$$\bar{\mathbf{K}}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{K}_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (9c)$$

Eq. (9*b*) gives the relationship between the nodal generalized displacements of the assumed stress model of (1) and the displacement amplitudes of rigid body motion. To obtain the corresponding force-stress relationship between forces for rigid body motion and stress intensities of the piecewise-linear-stress model, the reciprocal theorem can be used. This leads to the following equation:

$$\mathbf{F} = \left(\frac{2\pi}{\pi}\right)\mathbf{B}_1^T\mathbf{P} \quad (10)$$

where vector $\mathbf{F}$ = generalized forces for rigid body motion. Substituting $\mathbf{P} = \mathbf{K}^{-1}\mathbf{B}_1\boldsymbol{\Delta}$ from (9*a*) into (10) yields

$$\mathbf{F} = \left(\frac{2\pi}{\pi}\right)\mathbf{B}_1^T\mathbf{K}^{-1}\mathbf{B}_1\boldsymbol{\Delta} = \mathbf{I}\boldsymbol{\Delta} \quad (11)$$

The matrix $\mathbf{I}$ is the impedance function matrix corresponding to amplitudes

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of the rigid body motion of the foundation. In the foregoing equations, note that matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ can be proved symmetric using reciprocal theorem. Therefore, impedance matrix $\mathbf{I}$ is symmetric too. This concludes the derivations of impedance matrix for circular flexible foundation.

NUMERICAL ANALYSIS

Two types of foundation plates on viscoelastic half-space medium shown in Fig. 1 are selected to calculate impedance functions. One of the two types is a circular foundation with a rigid core. Therefore, the connection condition between the rigid core and flexible annular plate is rigid. The other type is a flexible circular plate with a thin walled cylinder connected to its rim. The connection condition with the thin walled cylinder is assumed to be a hinge. The contact condition between the foundation plate and soil medium can be smooth or rigidly welded. Hysteretic damping with damping ratio $\xi = 0.05$ is assumed in the half-space medium. After some extensive study, $m = 20$ for the piecewise linear stress model in (1) can give accurate results. Therefore, $m = 20$ is chosen for the calculation of results presented in this paper.

To investigate the influence of foundation flexibility, the relative rigidities of the foundation $\alpha = (\bar{D}/G_o a_o^3)$ are selected to be ∞, 10, 1, 0.1, and 0.01, in which $\bar{D}$ is plate rigidity; G_o is real part of the complex shear modulus of soil medium; and a_o is radius of foundation. All of the numerical results presented in the figures are nondimensionalized by a_o and G_o, and the vibration frequencies are nondimensionalized by a_o and the real part of shear wave velocity $\Re(c_s)$ in half-space medium.

To demonstrate the effectiveness of the proposed procedure, comparison of the rocking impedance function with the previous results reported by Iguchi and Luco (1982), in which Poisson ratios for soil and foundation are 0.4 and 0.167, respectively and radius of rigid core is a quarter of the radius of the foundation ($C = 0.25$ in Fig. 1), are made in Fig. 2 for $\alpha = 1$, 0.1, and 0.01. In the figures, dashed lines show the results by the proposed procedure and solid lines show the results by Iguchi and Luco (1982). From the figures, one observes that both results match pretty well to each other.

Figs. 3–6 show the results of impedance functions for a circular foundation with a rigid core. In the example, the Poisson ratios of the soil medium and foundation plate are 0.33 and 0.21, respectively, and the radius of the rigid core is one fourth of the radius of the foundation ($C = 0.25$ in Fig. 1). In the figures, solid lines show the results for rigidly welded contact condition and dashed lines show the results for smooth contact condition. If smooth

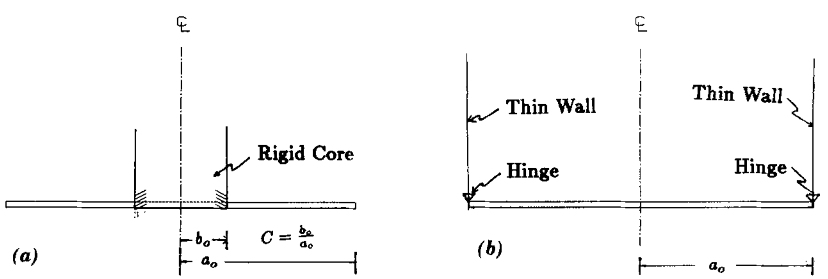

FIG. 1. (a) Foundation with Rigid Core; (b) Foundation with Thin-Walled Cylinder

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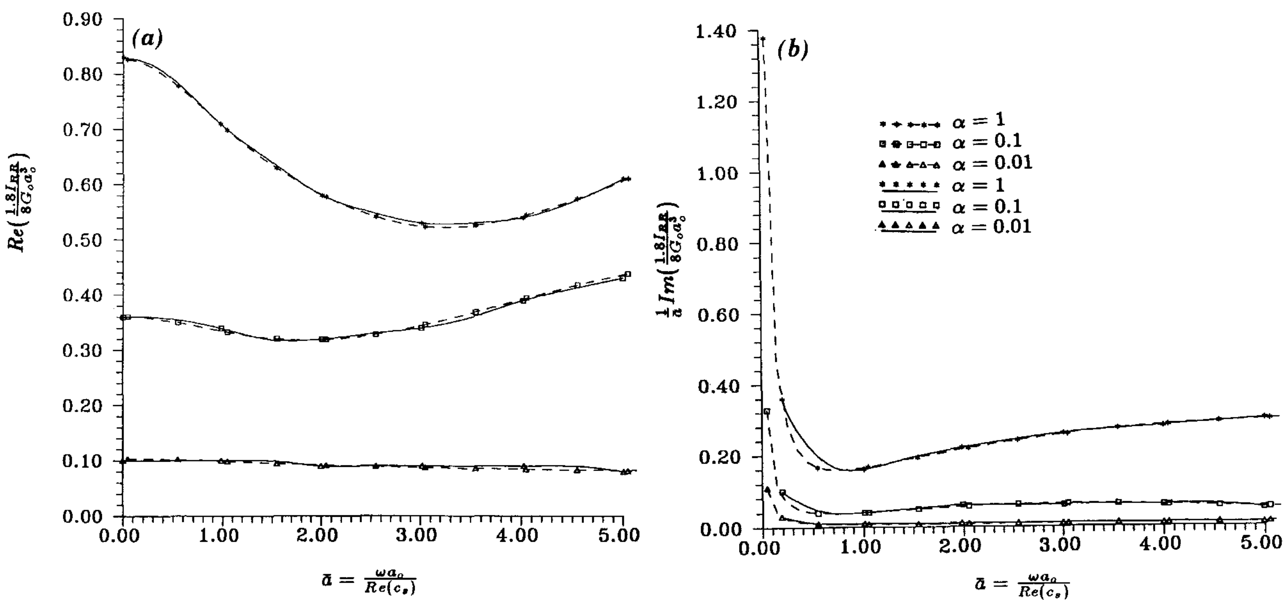

FIG. 2. Comparison of Nondimensionalized Rocking Impedance

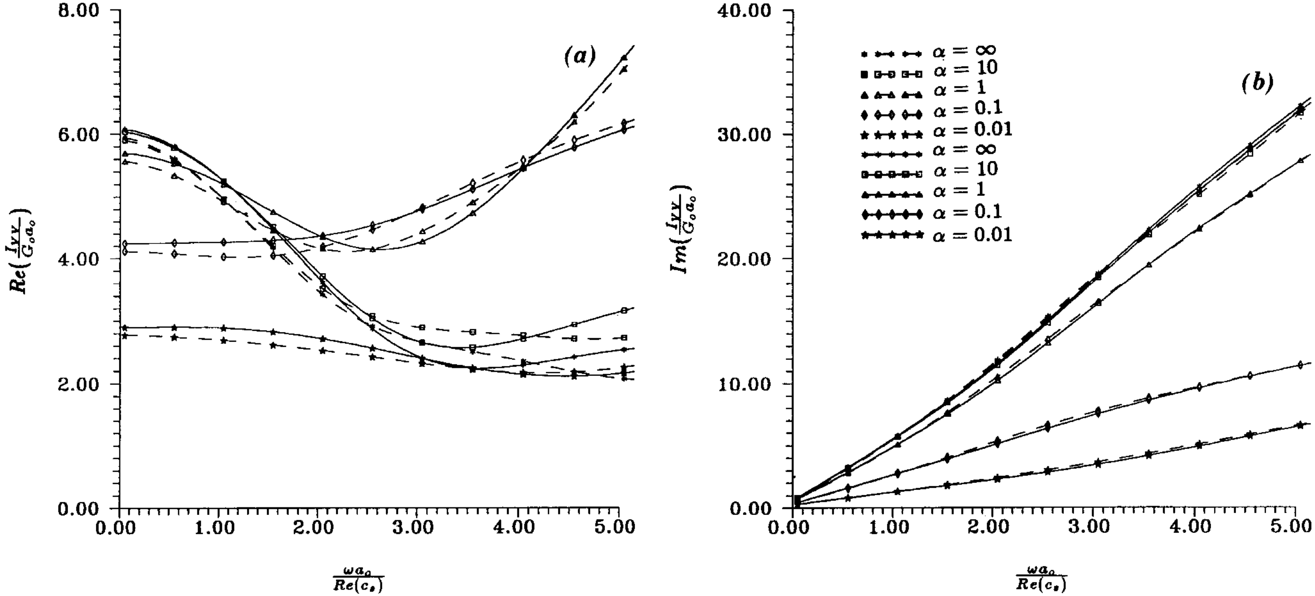

FIG. 3. Nondimensional Vertical Impedance

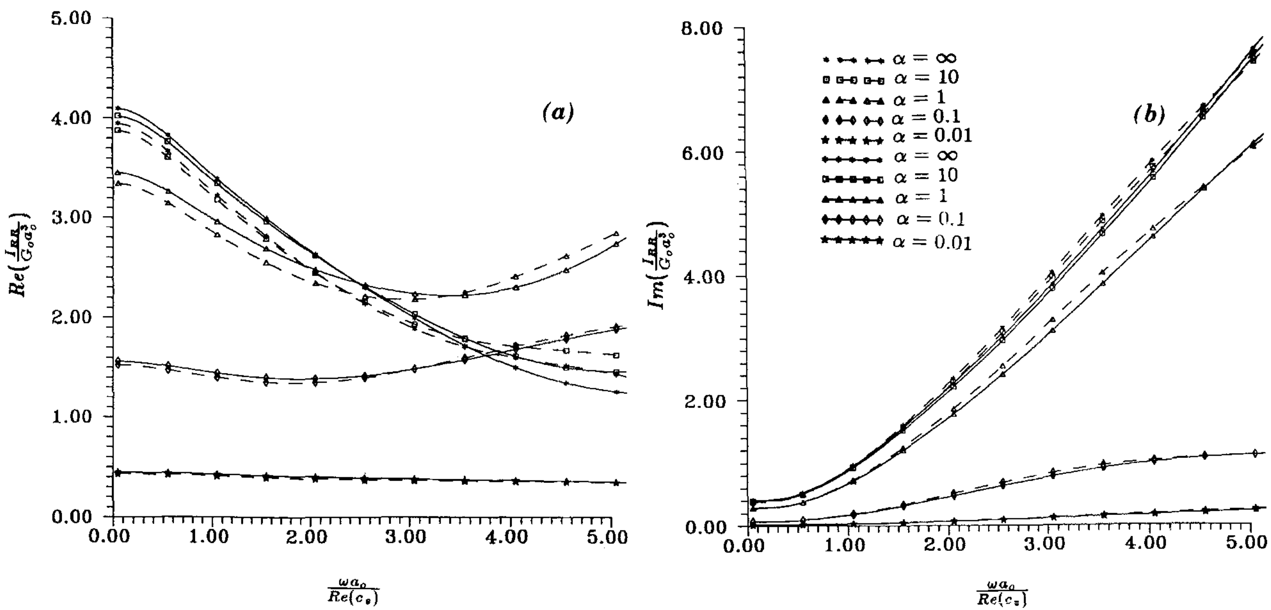

FIG. 4. Nondimensionalized Rocking Impedance

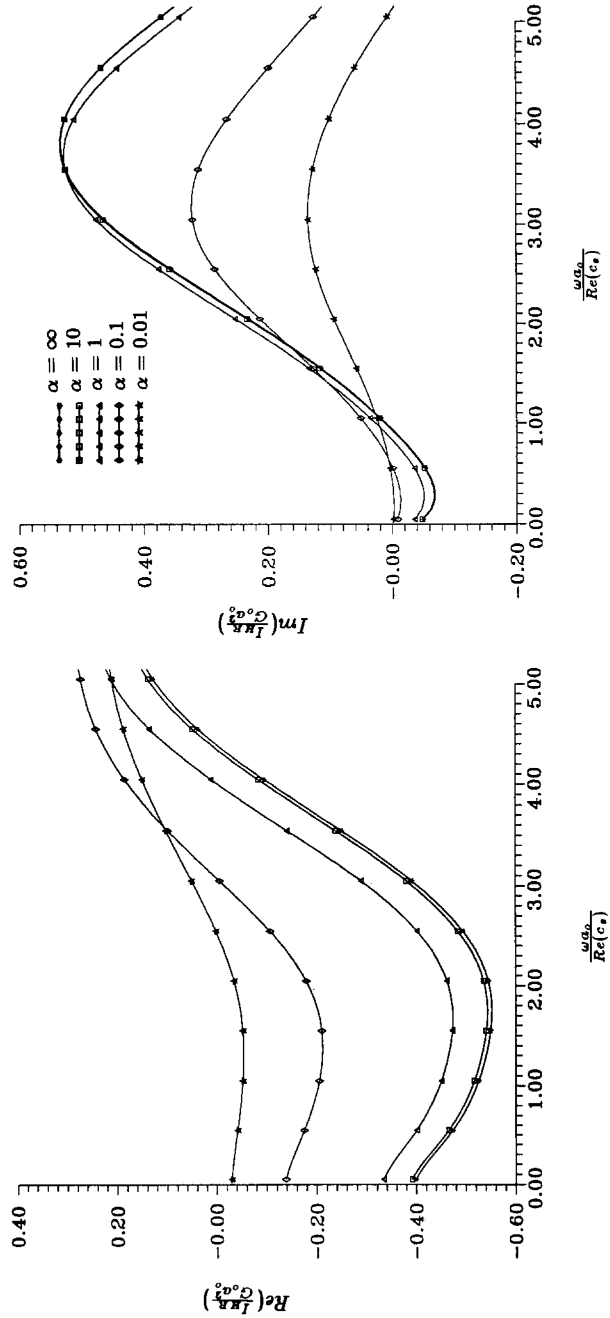

FIG. 5. Nondimensionalized Coupling Impedance

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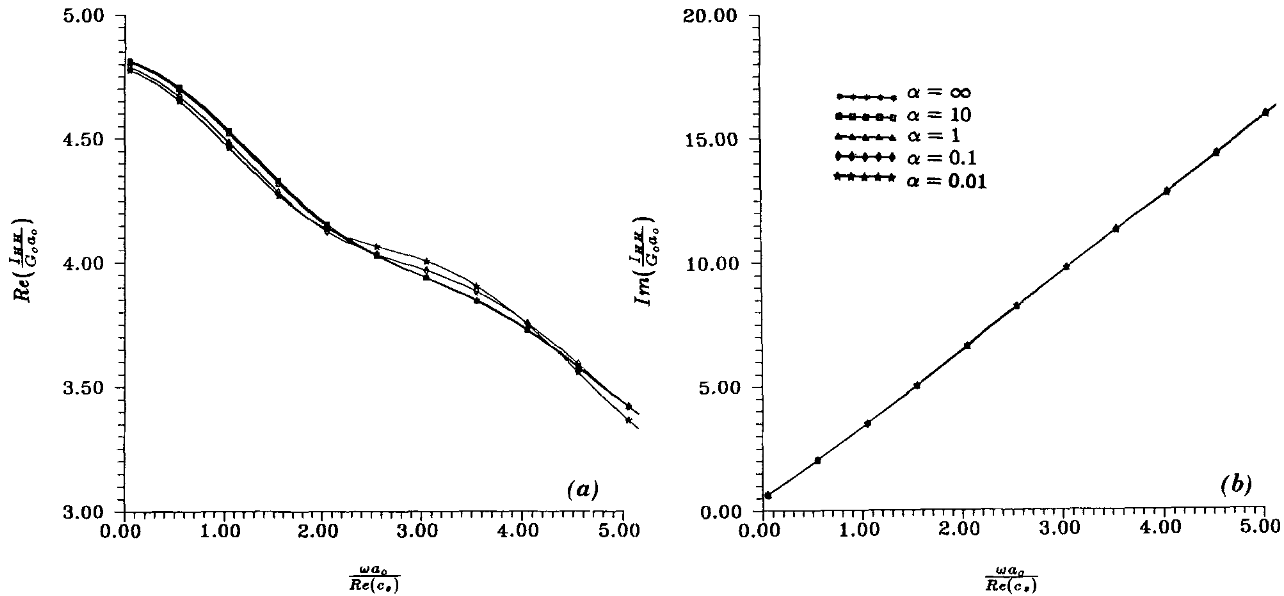

FIG. 6. Nondimensionalized Horizontal Impedance

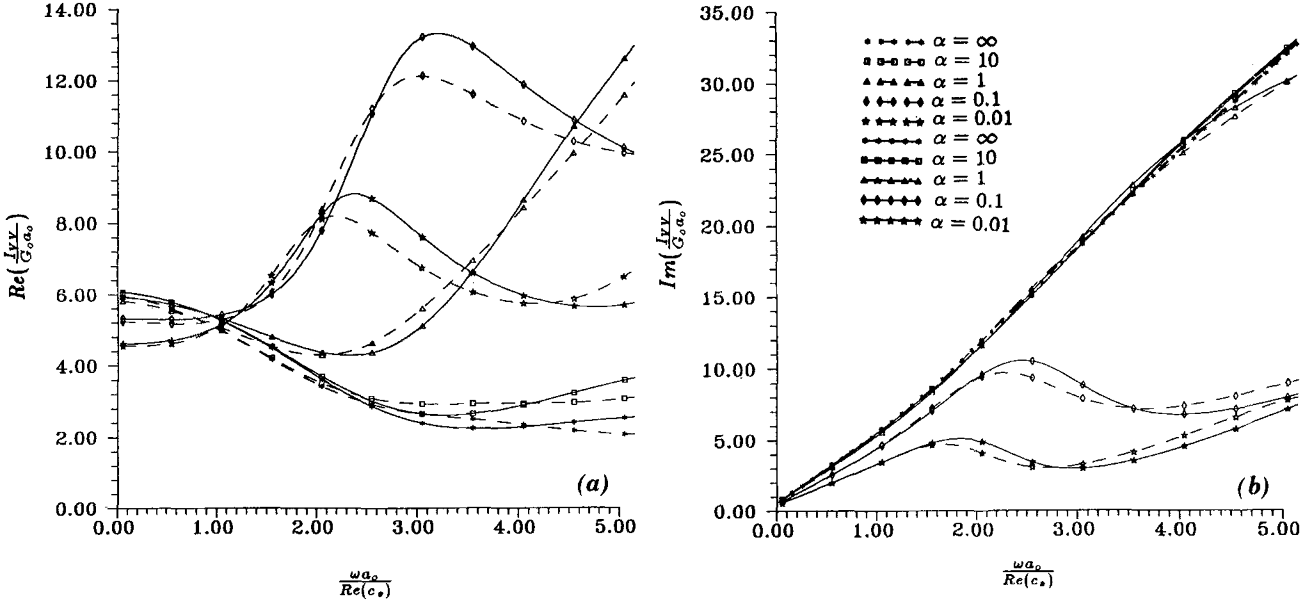

FIG. 7. Nondimensionalized Vertical Impedance

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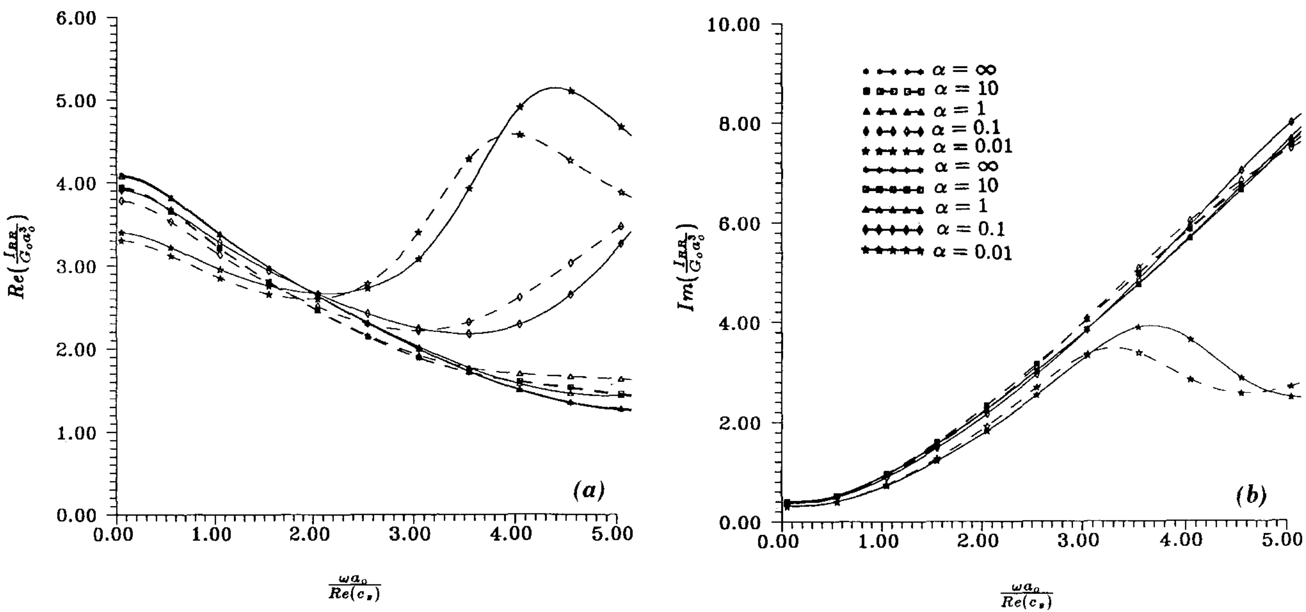

FIG. 8. Nondimensionalized Rocking Impedance

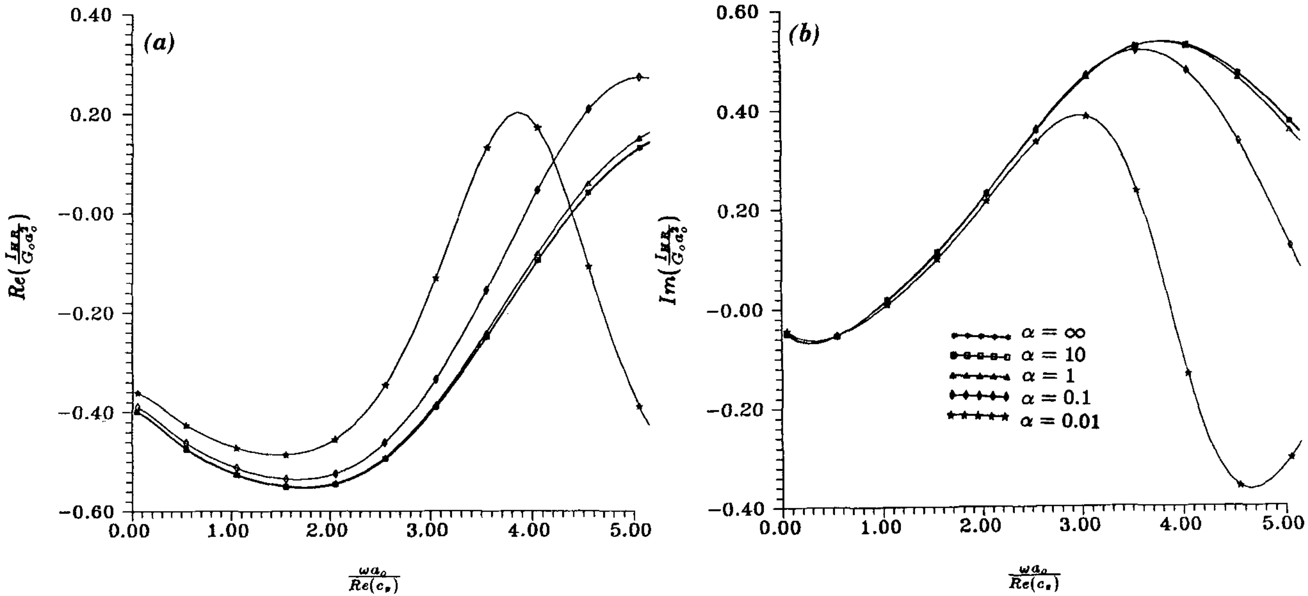

FIG. 9. Nondimensionalized Coupling Impedance

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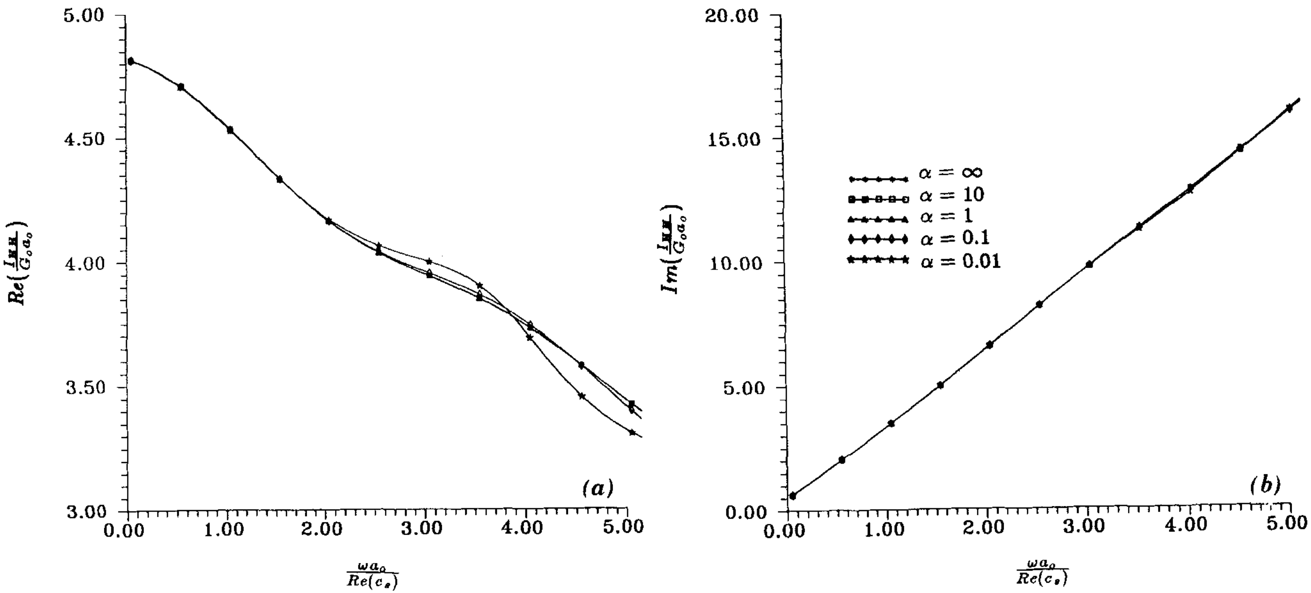

FIG. 10. Nondimensionalized Horizontal Impedance

contact condition is assumed, there is no effect of foundation rigidity on coupling impedance for rocking and horizontal motions and horizontal impedance. Therefore, Figs. 5 and 6 for coupling and horizontal impedances only show the results for rigidly welded contact condition.

From Figs. 3 and 4, one can find that foundation rigidity does have a strong influence on vertical and rocking impedance functions except the relative rigidity of foundation $\alpha \geq 10$. Also, one can observe some discrepancy between the real parts of the results for smooth and rigidly welded contact conditions. The discrepancy is diminishing as the foundation rigidity is getting smaller. Figs. 5 and 6 give coupling and horizontal impedance foundation. By examining both figures, one can conclude that foundation rigidity has a stronger effect on the coupling impedance function and has a smaller influence on the horizontal impedance function except that the foundation rigidity is small (saying $\alpha < 0.01$).

Figs. 7–10 show the results of vertical, rocking, coupling, and horizontal impedance functions, respectively, for the case of flexible foundation with a thin-walled cylinder connected to its rim. Similar phenomenons to that, from the case of the flexible foundation with a rigid core, can be observed. However, a larger discrepancy between two contact conditions (smooth and rigidly welded) is observed while compared to that of the flexible foundation with a rigid core. Also, if one compares Figs. 7–9 with the corresponding Figs. 3–5, one can find that the influence behaviors of foundation rigidity are totally different. This means that not only the foundation rigidity influences impedance functions, but also types of supporting system of structure has strong effect on impedance functions.

After some numerical investigations have been done, conclusions can be drawn as follows:

The foundation can be assumed rigid if the relative rigidity of foundation α is greater than 10.

For a very flexible foundation ($\alpha < 0.01$), the effect of foundation rigidity on the horizontal impedance function may be important.

The contact condition between the foundation and surrounding soil is important if some degree of precision is wanted to be attained.

The type of superstructure connected to the foundation is also important for calculating impedance functions for soil-structure interaction analysis.

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