An Accurate Method for Approximating the Interference Statistics of DS/CDMA Cellular Systems with Power Control over Frequency-Selective Fading Channels

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***Abstract*— This letter proposes an approximation method by characteristic function (AM-CF) method to approximate the distribution of interference in DS/CDMA cellular systems. This method considers the effects of frequency-selective multipath fading; it also assumes perfect power control and a rectangular/sinc chip waveform. The AM-CF method can yield results that fit the Monte Carlo simulation results more accurately than the conventional standard Gaussian approximation method.**

***Index Terms*— CDMA, frequency-selective fading, interference statistics, power control.**

I. Introduction

INTERFERENCE statistics of a DS/CDMA (direct-sequence/code-division multiple-access) system are essential to the understanding of the system's dynamics. Approximating interference statistics has received a lot of attention. In the literature, the most widely used method is the SGA (standard Gaussian approximation) method. Although the SGA is easy to use and applicable to a complicated circumstance, e.g. the cellular system over the frequency-selective fading channel in [1], it is known that the SGA is not very accurate [2]. In order to improve accuracy, many other methods have been proposed, such as the improved Gaussian approximation (IGA) method [2], the simplified IGA method [3], and the characteristic function method [4]. These methods have better accuracy, however they are only applicable to the limited circumstance of a single cell system over the AWGN channel. Therefore, an approximation method that has better accuracy and can be applied to a complicated circumstance is still desirable. This letter proposes an approximation method by characteristic function (AM-CF) to approximate the distribution of the MAI (multiple access interference) signals in DS/CDMA cellular systems. The method considers the effect of a frequency-selective multipath fading channel; it also assumes perfect power control and a rectangular/sinc chip waveform. Using this method, the distribution of the MAI signals is more accurately approximated.

II. System Model

The system under consideration is a DS/CDMA cellular system with N_B base stations (BSs) and N_M mobile stations (MSs) per cell, and with frequency-selective, slowly fading multipath channels. The power gain of each resolvable path p on a link from any MS m to any BS h is statistically divided into long-term fading L_{mh} and short-term fading $S_{mh,p}$. For simplicity, $\{S_{mh,p}\}$ are assumed to be Rayleigh fadings that have an independently and identically distributed (iid) exponential pdf with mean $1/K$ and variance $1/K^2$, where K is the number of resovable paths on a link. It is also assumed that binary phase-shift keying modulation and optimal RAKE receivers are employed. For an MS M communicating with BS H, the decision statistics $Z_M[k]$ at the k-th data symbol (the symbol duration is T), normalized with respect to $T \cdot \sqrt{S_{MH}[k]}$, is obtained in four parts [5] by

$$Z_M[k] = r_M[k] + I[k] + I_S[k] + n[k] \quad (1)$$

where $S_{MH}[k] \triangleq \sum_{p=1}^{K} S_{MH,p}[k]$ is a unit-mean gamma RV with variance $1/K$, and $r_M[k]$, $I[k]$, $I_S[k]$, and $n[k]$ denote the desired signal, the MAI interference, the multipath interference, and the noise, respectively. Notably, $I_S[k]$ and $n[k]$ are usually negligible as compared to $I[k]$ and will be ignored hereafter [1].

We consider a time interval over which the short-term fading varies while the long-term fading is constant. And the perfect strength-based power control is further assumed to be such that the transmission power of MS M is $P_M[k] = Q_0/(L_{MH} \cdot S_{MH}[k])$, where Q_0 is the desired received power level. The term $r_M[k] = d_M[k] \cdot \sqrt{Q_0}$, where $d_M[k]$ represents the MS M's data sequence that has values $\{\pm 1\}$ with equal probability. The term $I[k]$ is obtained by

$$\begin{aligned} I[k] \;=\; & \sum_{p=1}^{K} \sqrt{\frac{S_{MH,p}[k]}{S_{MH}[k]}} \sum_{m \neq M} \sum_{p'=1}^{K} \sqrt{P_m[k] L_{mH} S_{mH,p'}[k]} \\ & \cdot \alpha_{M,p}^{(m,p')}[k] \cdot \cos\left(\theta_{M,p}^{(m,p')}[k]\right) \end{aligned} \quad (2)$$

where $\alpha_{M,p}^{(m,p')}[k]$ and $\theta_{M,p}^{(m,p')}[k]$ are RVs related to the spreading code cross-correlation and the phase difference of the signal paths, respectively. The term $\alpha_{M,p}^{(m,p')}[k]$ is given by

$$\begin{aligned} \alpha_{M,p}^{(m,p')}[k] = \frac{1}{T} \int_{kT+(p+\tau_M)T_c}^{(k+1)T+(p+\tau_M)T_c} & d_m\left(t - (p' + \tau_m)T_c\right) \cdot \\ & c_m\left(t - (p' + \tau_m)T_c\right) \cdot c_M\left(t - (p + \tau_M)T_c\right) dt \end{aligned} \quad (3)$$

where T_c is the chip duration, $c_m(t)$ is MS m's spreading code signal, and τ_m reflects the different timing due

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to asynchronous transmission and has a uniform distribution over $[0, 1)$. For the case of random spreading codes with a rectangular chip waveform and an even processing gain G, it is known that $E(\alpha_{M,p}^{(m,p')}[k]) = 0$ and $var(\alpha_{M,p}^{(m,p')}[k]) = 2G/3$ [6]. On the other hand, the pdf of $\alpha_{M,p}^{(m,p')}[k]$ equals to that of $(\tau \sum_{i=1}^{G} d_m[i] \cdot c_m[i] \cdot c_M[i]/G + (1-\tau) \sum_{i=1}^{G} d_m[i] \cdot c_m[i] \cdot c_M[i+1]/G)$, where τ is a uniform RV over $[0, 1)$, and $c_m[i]$ is the random spreading code. It has the same pdf as that of $(\tau A + (1-\tau)B)$ where A and B are iid discrete RVs having identical pdf of $\Pr(A = x) = \Pr(B = x) = f_b((x+1)G/2; G), x = -1, -1+2/G, ..., 1-2/G, 1$, where $f_b(k; n) \triangleq C_k^n/2^n$ denotes a binomial probability with binomial coefficient C_k^n. After derivation, the pdf $f_\alpha(x)$ of $\alpha_{M,p}^{(m,p')}[k]$ has a closed-form solution given by

$$f_\alpha(x) = \begin{cases} g_n \cdot \delta(0), & |x| = \frac{2n}{G}, \quad n \in \{0, 1, \cdots, \frac{G}{2}\}, \\ h_n, & \frac{2(n-1)}{G} < |x| < \frac{2n}{G}, n \in \{1, \cdots, \frac{G}{2}\}, \end{cases} \tag{4}$$

where $g_n = f_b^2(n + G/2; G)$ and

$$h_n = G \sum_{i=-G/2}^{n-1} \sum_{j=n}^{G/2} \frac{f_b\left(i + G/2; G\right) \cdot f_b\left(j + G/2; G\right)}{j - i}. \tag{5}$$

The term $\theta_{M,p}^{(m,p')}[k]$ has a uniform distribution over $[0, 2\pi)$, and therefore the $\cos(\theta_{M,p}^{(m,p')}[k])$ is a zero-mean RV with variance 1/2, and its pdf, denoted by $f_\varphi(x)$, is given by

$$f_\varphi(x) = \frac{1}{\pi\sqrt{1 - x^2}}, |x| < 1. \tag{6}$$

III. The AM-CF Method

The distribution of the MAI signal in (2) is too complicated to be computed directly. We propose an AM-CF method to estimate the distribution of the MAI signal, which is composed of the intra-cell and other-cell interference signal. To help make things clearer, the time index k is neglected hereafter.

A. Approximating the Statistics of the Intra-cell Interference

From (2), the intra-cell interference signal I_I is given by

$$\begin{aligned} I_I = \sum_{p=1}^{K} \sqrt{\frac{S_{MH,p}}{S_{MH}}} \sum_{\substack{m \in MS^{(H)} \\ m \neq M}} \sum_{p'=1}^{K} \sqrt{\frac{Q_0 \cdot S_{mH,p'}}{S_{mH}}} \cdot \\ \alpha_{M,p}^{(m,p')} \cdot \cos\left(\theta_{M,p}^{(m,p')}\right) \end{aligned} \tag{7}$$

where $MS^{(H)}$ denotes the set of MSs communicating with BS H. Based on the statistics of $\{\alpha_{M,p}^{(m,p')}\}$, $\{\theta_{M,p}^{(m,p')}\}$, and $\{S_{mh,p}, \forall(m,h) \neq (M,H)\}$, the mean and variance of I_I are obtained to be zero and $Q_0 \cdot (N_M - 1)/(3G)$, respectively. The terms within the summation in (7) are obviously not iid RVs. Therefore, it is very difficult to derive its pdf. The AM-CF method modifies the profile of $\{S_{mh,p}\}$ by removing $\{S_{mh,p}, p = 2, \cdots, K\}$ and letting $S_{mh,1} = \sum_{p=1}^{K} S_{mh,p}$, and further treating terms within the summation in (7) as mutually independent RVs. By doing so, the overall characteristic function is obtained by simply multiplying the characteristic function of each term. As will be shown later, based on the AM-CF method, the pdf of I_I can be approximated without significant distortion by the pdf of I_I^* which is defined as

$$I_I^* \triangleq \sum_{i=1}^{N_M - 1} \sqrt{Q_0} \cdot \alpha_i^{'} \cdot \varphi_i^{'}. \tag{8}$$

The terms $\{\alpha_i^{'}, \varphi_i^{'}\}$ are mutually independent RVs, and the pdfs of $\alpha_i^{'}$ and $\varphi_i^{'}$ have the forms given in (4) and (6), respectively. For the case of an even G and a rectangular chip waveform, the pdf of $\chi_i^{'} = \alpha_i^{'} \cdot \varphi_i^{'}$ has a closed-form solution, denoted by $f_{\chi'}(x)$, given by

$$\begin{aligned} f_{\chi'}(x) &= \int f_\alpha(s) \cdot f_\varphi\left(\tfrac{x}{s}\right) \tfrac{1}{|s|} ds \\ &= \begin{cases} g_0 \cdot \delta(x), x = 0, \\ \frac{2h_n}{\pi} \log \frac{n + \sqrt{n^2 - G^2 \cdot x^2/4}}{G \cdot x/2} + \\ \sum_{m=n+1}^{G/2} \frac{2h_m}{\pi} \log \frac{m + \sqrt{m^2 - G^2 \cdot x^2/4}}{(m-1) + \sqrt{(m-1)^2 - G^2 \cdot x^2/4}} + \\ \sum_{m=n}^{G/2} \frac{g_m G}{\pi\sqrt{m^2 - (G \cdot x/2)^2}}, \\ \frac{2(n-1)}{G} < |x| \leq \frac{2n}{G}, n \in \{1, 2, \cdots, \frac{G}{2}\}. \end{cases} \end{aligned} \tag{9}$$

The mean and variance of $\chi_i^{'}$ are zero and $1/(3G)$, respectively. Notably, the mean and variance of I_I^* are the same as the mean and variance of I_I, respectively. Since I_I^* is a sum of iid RVs, we have $F_{I_I^*}(\omega) = (F_{\chi'}(\sqrt{Q_0} \cdot \omega))^{N_M - 1}$, where $F_{\chi'}(\omega) = \mathcal{F}(f_{\chi'}(x))$ is the characteristic function of the pdf $f_{\chi'}(x)$, and $\mathcal{F}(\cdot)$ represents the Fourier transform. Accordingly, the pdf of I_I^* is obtained by $f_{I_I^*}(x) = \mathcal{F}^{-1}(F_{I_I^*}(\omega))$, where $\mathcal{F}^{-1}(\cdot)$ denotes the inverse Fourier transform.

B. Approximating the Statistics of the Other-cell Interference

From (2), the other-cell interference signal I_O is given by

$$\begin{aligned} I_O = \sum_{p=1}^{K} \sqrt{\frac{S_{MH,p}}{S_{MH}}} \sum_{\substack{m \in MS^{(h)} \\ h \neq H}} \sum_{p'=1}^{K} \sqrt{\frac{Q_0 \cdot L_{mH} \cdot S_{mH,p'}}{L_{mh} \cdot S_{mh}}} \\ \cdot \alpha_{M,p}^{(m,p')} \cdot \cos\left(\theta_{M,p}^{(m,p')}\right). \end{aligned} \tag{10}$$

The mean and variance of I_O are zero and $\sum_{m \in MS^{(h)}, h \neq H} (L_{mH}/L_{mh}) \cdot K/(K-1) \cdot Q_0/(3G)$, respectively. By dividing the average other-cell interference power $var(I_O)$, from the $N_B - 1$ cells, into N equal quantities, the AM-CF method estimates the pdf of I_O by the pdf of I_O^* which is defined as

$$I_O^* \triangleq \sum_{i=1}^{N} \sqrt{Q_0 \cdot L^{''} \cdot \frac{S_{1,i}}{S_{2,i}}} \cdot \alpha_i^{''} \cdot \varphi_i^{''}. \tag{11}$$

The parameter N is artificially introduced and $L^{''}$ is given by $L^{''} \triangleq (\sum_{m \in MS^{(h)}, h \neq H} L_{mH}/L_{mh})/N$ so that the mean and variance of I_O^* equal to those of I_O. Once the N is chosen and assume $var(I_O)$ is measurable, the term $L^{''}$ can be calculated according to its definition. By this way, the problem of measuring each L_{mh}, which is practically not easy to do, is bypassed. Another purpose of parameter N is to tune the distribution of I_O^*. The $\{S_{1,i}, S_{2,i}\}$ are iid RVs with the same distributions as that of S_{MH}, and the pdf of $\beta_i^{''} \triangleq \sqrt{S_{1,i}/S_{2,i}}$, denoted by $f_\beta(x)$, is obtained by

$$\begin{aligned} f_\beta(x) &= \int f_S\left(s \cdot x^2\right) \cdot f_S(s) \cdot 2s \cdot x \cdot ds \\ &= 2\frac{\Gamma(2K) \cdot x^{2K-1}}{\Gamma^2(K) \cdot (1+x^2)^{2K}} \end{aligned} \tag{12}$$

where $f_S(x)$ denotes the pdf of S_{MH}. The pdfs of $\alpha_i^{''}$ and $\varphi_i^{''}$ have the form given by (4) and (6), respectively, and the pdf of $\chi_i^{''} \triangleq \alpha_i^{''} \cdot \varphi_i^{''}$ is the same as the $f_{\chi'}(x)$ given by (9). Consequently, the pdf of $\psi_i = \beta_i^{''} \cdot \chi_i^{''}$ has an identical pdf, denoted by $f_\psi(x)$, which is obtained by $f_\psi(x) = \int_{-1}^{+1} f_{\chi'}(s) \cdot f_\beta(x \cdot s^{-1}) \cdot |s|^{-1} ds$. Since I_O^* is a sum of iid RVs, $\sqrt{Q_0 \cdot L''} \cdot \psi_i$, we have $F_{I_O^*}(\omega) = (F_\psi(\sqrt{Q_0 \cdot L''} \cdot \omega))^N$, where $F_\psi(x)$ is the characteristic function of the pdf $f_\psi(x)$.

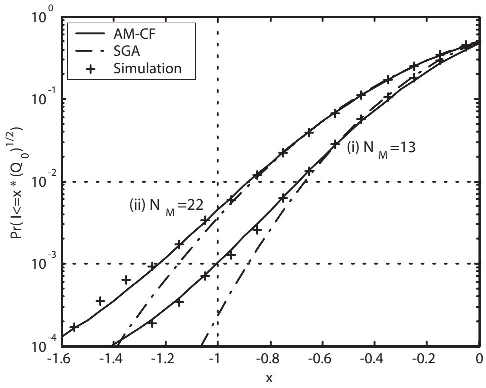
C. Approximating the Statistics of the MAI Interference

Since the intra-cell and other-cell interference signals are independent under a strength-based power control, the pdf of the MAI signal I can be approximated by $f_I(x) = \mathcal{F}^{-1}((F_{\chi'}(\sqrt{Q_0} \cdot \omega))^{N_M - 1} \cdot (F_\psi(\sqrt{Q_0 \cdot L''} \cdot \omega))^N)$. Note that the BER is obtained by $BER = \Pr(I < -\sqrt{Q_0})$.

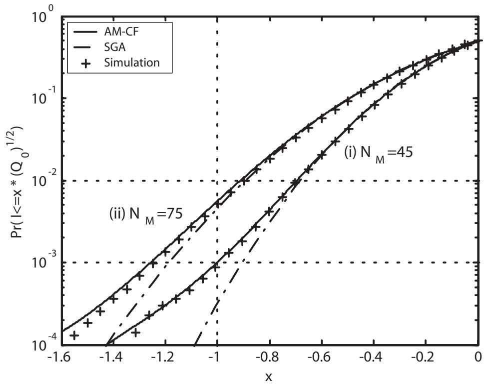
IV. Results and Discussions

Fig. 1(a) shows the cdf curves of the MAI signals within the central cell using AM-CF and SGA methods, as well as the Monte Carlo simulation results based on (2), (3), and (6) with 10^6 samples and $(N_B, G, K) = (19, 128, 2)$. The model of the long-term fading is the same as that in [5] with the propagation loss of 3.5 and the log-normal shadow fading standard deviation of 8dB, and each MS chooses its serving BS based on measured pilot power [5]. The MS number N_M is 13 (22) and the parameter N is chosen as 20 (34) for case (i) (case (ii)) where the corresponding BER is around 10^{-3} $(5 \cdot 10^{-2})$. It is found that with proper N, AM-CF curves fit the simulation results better than SGA curves. For example, in case (i), BER based on the AM-CF method deviates from the simulation result by 4%, while BER based on the SGA method deviates by as high as 73%. The reason why the SGA does not work well is highly related to the power control. Under a perfect strength-based power control, the MS's transmission power might become pretty high due to a deep fade, which will induce a high other-cell interference power to others and therefore a shallow falloff of the pdf of I_O. In the proposed AM-CF method, as shown in (11), I_O^* contains the factor $1/S_{2,i}$, which reflects the power control effect, so that the AM-CF could perform better than the SGA. Also notice that the AM-CF seems not very sensitive to the parameter N.

Fig. 1(b) shows results of the situation which is the same as those in Fig. 1(a) except that the chip waveform is changed to a sinc one. The N_M is 45 (75) and the N is chosen as 80 (130) for case (i) (case (ii)). The Eqs. (8) and (11) in AM-CF method remain the same and the factor $\alpha_{M,p}^{(m,p')}$ can be numerically calculated according to (3). Results show that the AM-CF curves still fit the simulation results better than SGA curves. For example, in case (i), BER based on the AM-CF method deviates from the simulation result by 8%, while BER based on the SGA method deviates by as high as 64%. Note that, the system attains a dramatic gain in capacity by using sinc waveform. It is because the sinc waveform is the optimal one that minimizes interference and improves the capacity [7].



(a) Rectangular chip waveform



(b) Sinc chip waveform

Fig. 1. Comparison of cdf curves of interference signals.

It can be believed that the rationale of the AM-CF method is applicable to more realistic conditions considering, such as short-term fadings with non-equal average power, power control error, power control period, power control step, power control command delay, MS velocity, etc. However further systematic work is needed to study these extensions of the AM-CF method. Moreover, the determination of the parameter N is still an issue which needs further study.

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