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A QUARTER-WAVE PLATE USING SUBSTRATE-MODE HOLOGRAMS

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Substrate-mode holograms

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SUMMARY : Based on the phase shifts due to the total internal reflection and the grating diffraction, a new type of quarter-wave plate using substrate-mode holograms is presented. A sample of this device was fabricated to demonstrate its function.

Lame quart d'onde avec des hologrammes en mode substrat

RÉSUMÉ : A partir des transitions de phase résultant de la réflexion interne totale et de la diffraction, un nouveau type de lame 4 avec des hologrammes en mode substrat est présenté. Un exemplaire de l'appareil a été fabriqué pour démontrer sa fonction.

INTRODUCTION

A quarter-wave plate is a commonly used polarization component. It is normally made with birefringent materials such as calcite, quartz or mica, and is expensive. And Cescato *et al.* [1] presented the holographic quarter-wave plate using the fact that there is birefringence as the light passes through the grating. Although it can also function as that of a conventional quarter-wave plate does, its efficiency is limited due to high order diffractions of this surface relief grating.

Some papers [2, 3] reported that optical interconnections using substrate-mode holograms with dichromated gelatin (DCG) recording material have many merits such as easy fabrication, low cost, high diffraction efficiency, compactness of monolithic structure and easily used etc. As the light wave passes through the substrate-mode holograms, there are two grating diffractions and at least one total internal reflection. They introduce phase shifts of p-polarization relative to s-polarization. Based on these phenomena, a new type of quarter-wave plate is presented. In addition, a sample was fabricated and its function was demonstrated.

PRINCIPLE

The architecture of this new type of quarter-wave plate using substrate-mode holograms is depicted in *figure 1*. It consists of two transmission-type phase volume gratings with identical structures and a substrate. The input collimated wave is normally incident on the grating G1 at A under the Bragg condition, and it can be diffracted by G1 into the substrate with high diffraction efficiency. The diffraction angle is designed so that it is larger than the critical angle in the substrate, therefore the diffracted wave is totally reflected at B and toward G2. This wave is again totally reflected at C, and the wave from C satisfies the Bragg condition of G2. The propagation direction of the reflected wave is also designed to be in parallel to that of the wave diffracted by G1. Because the structure of G2 is the

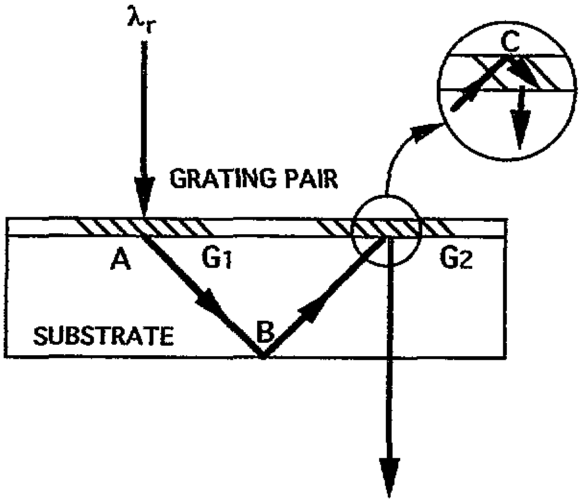

FIG. 1. — *The architecture of a quarter-wave plate using substrate-mode holograms.*

same as that of G1, the diffracted wave of G2 will be parallel to the input wave, that is, the output wave passes normally through the substrate. The detail of the wave-propagation in G2 is shown in the upper right circle of *figure 1*.

Now, the influences due to the grating diffraction are considered. If d is the grating thickness, λ_r is the wavelength of the incident wave in free space, n_1 is the amplitude of the refractive index modulation, and θ_d is the diffraction angle in the phase volume grating, then the diffraction efficiencies of a transmission-type volume grating for s- and p-polarizations under Bragg condition can be written as [4]

$$\eta_s = \sin^2 \frac{\pi n_1 d}{\lambda_r \sqrt{\cos \theta_d}}, \tag{1}$$

and

$$\eta_p = \sin^2 \frac{\pi n_1 d \sqrt{\cos \theta_d}}{\lambda_r}, \tag{2}$$

respectively. Due to the polarization difference between s- and p-fields, their refractive indices in the grating are slightly different. The difference is given as [5],

$$\Delta n = n_{\|} - n_{\perp} \approx \frac{n_1^2}{n_e}; \tag{3}$$

where $n_{\|}$ and $n_{\perp}$ are the refractive indices of p- and s-polarizations respectively, and n_e is the average refractive index of the recording material.

Next, the influences due to the total internal reflection (TIR) are considered. Although the reflectivity is 1, there are phase shifts for s- and p-polarizations. Supposing TIR occurs at the interface between two media whose refractive indices are n_a and n_b respectively, and $n_a > n_b$, then the phase shift of p-polarization relative to s-polarization can be expressed as [6],

$$\delta = 2 \tan^{-1} \left[\frac{\cos \theta \sqrt{\sin^2 \theta - \left(\frac{n_b}{n_a} \right)^2}}{\sin^2 \theta} \right] \tag{4}$$

where θ is the incident angle of TIR.

Based on the facts described above, the total relative phase shifts between p- and s-polarizations as the light passes through the substrate-mode holograms are

$$\Delta = \frac{4 \pi \Delta n d}{\lambda_r} + \delta_1 + \delta_2 ; \tag{5}$$

where δ_1 and δ_2 are the relative phase shifts due to TIRs at B and C, and they can be obtained from Eq. (4). If the device using substrate-mode holograms is so designed that the following conditions

$$\eta_s = \eta_p , \tag{6}$$

and

$$\Delta = \frac{\pi}{2}, \tag{7}$$

are satisfied, it is obvious that this device functions as a quarter-wave plate. That is, n_1, d, θ_d, n_e and the refractive index n_s of the substrate should be carefully chosen in fabricating a quarter-wave plate for λ_r. Among these parameters, d, n_e and n_s can be directly obtained from the specifications of the recording material. n_1 and θ_d can be calculated by introducing the values of d, n_e, n_s and λ_r into Eqs. (1)-(5) under the restricted conditions of Eq. (6) and Eq. (7).

FABRICATION AND RESULT

In this paper, a quarter-wave plate for 780 nm was given as an example. The gelatin reduced from Kodak 649F photographic plates and the simplified processing technique presented by Gerogekutty and Liu [7] were used for prepapring DCG recording material. Substituting $\lambda_r = 780$ nm, $d = 17$ μm, $n_e = 1.54$, and $n_s = 1.517$ into Eqs. (1)-(7), $\theta_d = 45°$ and $n_1 = 0.0226$ were obtained. We used an argon-ion laser with 488 nm for fabricating the device. In the K-vector diagram [8] shown in *figure 2*, K_{r1} and K_{r2} are the wavevectors of the reconstruction and diffracted waves respectively at a wavelength $\lambda_r = 780$ nm, and K_{c1} and K_{c2} are the wavevectors of the construction reference wave and object wave respectively at a wavelength 488 nm. From the geometrical relations in *figure 2*, $\theta_{c1} = 8.6°$ and $\theta_{c2} = 36.2°$ were obtained, that is,

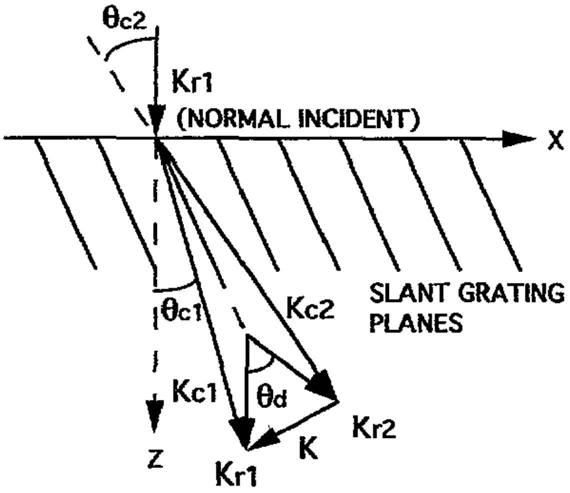

FIG. 2. — *The K-vector diagram of short wavelength recording for long wavelength reconstruction.*

13.4° and 65.4° in free space, respectively. An automatic high-precision pulse rotation system with resolution 0.005° (model numbers PS-θ-125 and CPC-3DN) manufactured by Japan Chuo Precision Industrial Co., Ltd. was introduced into the optical setup, so the above angular relations could be satisfied. The distance between G1 and G2 was 2.06 mm, and the thickness of the substrate was 1 mm. The diffraction efficiency of this sample was 92 %.

To demonstrate its function, a linearly polarized light with vibration plane + 45° relative to the + x axis was chosen as the input as shown in *figure 3*, and we found that the output was a right-circular light.

Retardance Measurement

In order to measure the relative phase shift of this sample, a simple experimental setup used by Enger and Case [9], as shown in *figure 4*, was employed. If a linearly polarized light with vibration phane + 45° relative to the x-axis incidents normally on the sample then the transmitted wave can be written as :

$$E = \begin{bmatrix} T_{\perp} \\ T_{\parallel} \exp(\mathrm{i}\Delta) \end{bmatrix} \exp[\mathrm{i}(kz - wt)] \,, \qquad (8)$$

where $T_{\perp}$ and $T_{\parallel}$ are the transmitted wave amplitudes of s- and p-polarization, respectively. Supposing the transmission axis of the analyzer is at angle α relative to x-axis, the wave behind the analyzer is

$$E = \begin{bmatrix} \cos\alpha & 0 \\ 0 & \sin\alpha \end{bmatrix} \begin{bmatrix} T_{\perp} \\ T_{\parallel} \exp(\mathrm{i}\Delta) \end{bmatrix} \times \times \exp[\mathrm{i}(kz - wt)] \,. \qquad (9)$$

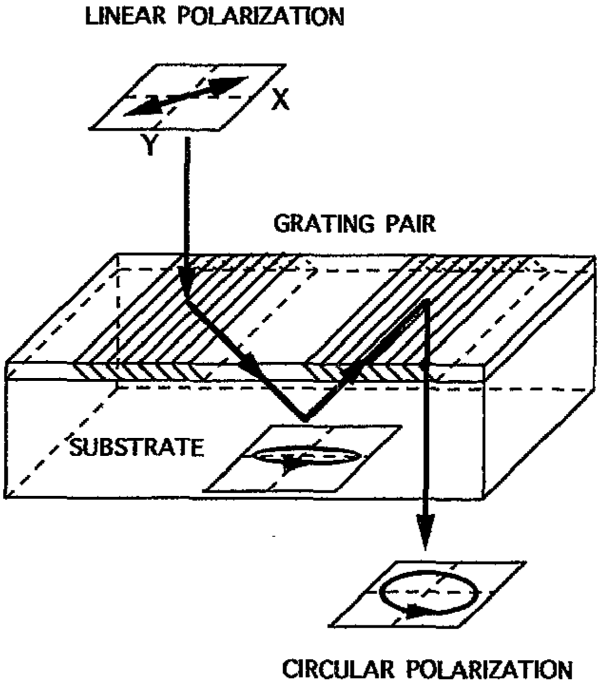

FIG. 3. — *Setup for demonstrating the function of the sample.*

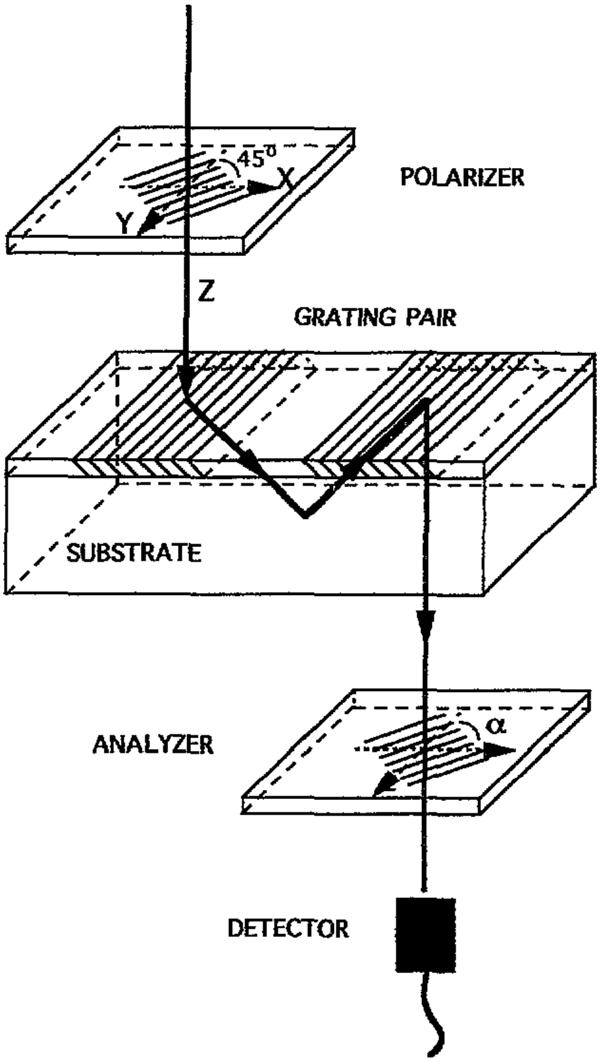

FIG. 4. — *Setup for measuring the relative phase shift.*

The intensity measured by the detector is

$$I_\alpha = |E \cdot E^*|^2 = T_\perp^2 \cos^2\alpha + T_\parallel^2 \sin^2\alpha + 2\,T_\perp\, T_\parallel \cos\alpha \sin\alpha \cos\Delta\,. \quad (10)$$

Let I_M and I_m be maximum and minimum intensities respectively as the analyzer is rotated in on cycle, and $I - \frac{\pi}{4}$ be the intensity as $\alpha = -\frac{\pi}{4}$, we define

$$M_1 = \frac{I - \frac{\pi}{4}}{I_M + I_m}\,, \quad (11)$$

and

$$M_2 = \frac{I_M - I_m}{I_M + I_m}\,. \quad (12)$$

Substituting Eq. (10) into Eqs. (11) and (12), we obtain the following equations :

$$M_1 = \frac{1}{2} - \left(\frac{R}{R^2+1}\right)\cos\Delta\,, \quad (13)$$

and

$$M_2 = \cos\theta\left(\frac{R^2-1}{R^2+1}\right) - 4\left(\frac{R}{R^2+1}\right)\cos\Delta\sin\theta\,, \quad (14)$$

where

$$\tan\theta = \left(\frac{2R}{R^2-1}\right)\cos\Delta\,, \quad (15)$$

and

$$R = \frac{T_\parallel}{T_\perp}\,. \quad (16)$$

The relative phase shifts Δ and R can be obtained by solving numerically Eqs. (13) and (14).

As the sample was measured with the above technique, we obtained $M_1 = 0.5036$ and $M_2 = 0.0161$. Then, $\Delta = 90.4°$ and $R = 1.022$ were calculated. It is obvious that the quarter-wave plate using the substrate-mode holograms has the same characteristics as those of the ideal quarter-wave plate.

DISCUSSION

The solutions of n_1 and θ_d mentioned above are obtained with a feasible condition of $\eta_s = \eta_p > 0.9$. They might have many solutions only under the restricted conditions of Eqs. (6) and Eq. (7). The value of the refractive index modulation n_1 depends on the exposure, and it is difficult to fabricate DCG [10] with $n_1 > 0.1$. The quarter-wave plate for other wavelengths can be fabricated with the same procedures.

There is a lateral shift between the incident light and the emergent light as this device is used. But they are parallel and perpendicular to the device, and it is very easy to introduce this device into a conventional optical system.

CONCLUSION

Based on the phase shifts due to the grating diffraction and the total internal reflection, a new type of quarter-wave plate using substrate-mode holograms with dichromated gelatin recording material is presented. This quarter-wave plate has all the merits as those of the substrate-mode holograms for optical interconnections. In addition, a sample of this new type of quarter-wave plate was fabricated and its function was demonstrated.

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