

Brane world cosmological solution to the gravitino problem

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We investigate the thermal production of gravitinos in the context of the brane world cosmology. Since the expansion law is modified from the one in the standard cosmology, the Boltzmann equation for the gravitino production is altered. We find that the late-time gravitino abundance is proportional to the “transition temperature”, at which the modified expansion law in the brane world cosmology is connecting with the standard one, rather than the reheating temperature after inflation as in the standard cosmology. This means that, even though the reheating temperature is very high, we can avoid the overproduction of gravitinos by taking the transition temperature low enough. Therefore, the gravitino problem can be solved.

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Several particles, which only gravitationally couple to the ordinary matters, appear in supergravity models (some models inspired by string theories). Since their couplings are Planck suppressed, their lifetime is quite long as estimated to be

$$\tau \simeq \frac{M_{Pl}^2}{m^3} \simeq 10^5 \left(\frac{1 \text{ TeV}}{m} \right)^3 \text{ sec}, \quad (1)$$

where m is the mass of the particles, and $M_{Pl} \simeq 1.2 \times 10^{19}$ GeV is the Planck mass. It is normally assumed that the scale of m is of order of the electroweak scale which is obtained through supersymmetry breaking in supergravity models. These particles with such a long lifetime often cause a serious problem in cosmology. The so-called “gravitino problem” is a famous example [1]. If the gravitino decays after big bang nucleosynthesis (BBN), its energetic daughters would destroy the light nuclei by photo-dissociation and, as a result, upset the successful prediction of BBN. As is well known, to avoid this problem the upper bound (depending on the gravitino mass) is imposed on the reheating temperature after inflation.

Here let us see a little more detail of the gravitino problem. Gravitinos have been produced by thermal scatterings in plasma since the reheating stage after inflation. The Boltzmann equation relevant to this gravitino production process is given by

$$\frac{dn_{3/2}}{dt} + 3Hn_{3/2} = \langle \sigma_{\text{tot}} v \rangle n_{\text{rad}}^2, \quad (2)$$

where H is the Hubble parameter, $n_{3/2}$ and n_{rad} are the number densities of gravitino and relativistic species in thermal bath, respectively, and $\langle \sigma_{\text{tot}} v \rangle$ denotes the thermal

average of the total gravitino production cross section times the relative velocity. In the standard cosmology, the Hubble parameter obeys the Friedmann equation,

$$H^2 = \frac{8\pi G}{3} \rho, \quad (3)$$

with the energy density $\rho = \frac{\pi^2}{30} g_* T^4$ in the radiation dominated universe. Here, G is the Newton’s gravitational constant, and $g_*(T)$ is the total number of the massless degrees of freedom. It is useful to rewrite Eq. (2) as

$$\frac{dY_{3/2}}{dT} = - \frac{s \langle \sigma_{\text{tot}} v \rangle}{HT} Y_{\text{rad}}^2, \quad (4)$$

where $Y_{3/2} \equiv n_{3/2}/s$ and $Y_{\text{rad}} \equiv n_{\text{rad}}/s$ with s being the entropy density. Note that the r.h.s. of this equation is almost independent of temperature T , since $\langle \sigma_{\text{tot}} v \rangle \simeq M_{Pl}^{-2}$, $H \propto T^2$, $s \propto T^3$, etc. Thus we find that the late-time gravitino abundance is proportional to the reheating temperature T_R and approximately described as [2],

$$Y_{3/2} \simeq 10^{-11} \left(\frac{T_R}{10^{10} \text{ GeV}} \right). \quad (5)$$

This abundance should be small in order for the gravitino decay products not to destroy the light elements successfully synthesized during BBN, thus we obtain the upper bound on the reheating temperature such as [3,4]

$$T_R \leq 10^6 - 10^7 \text{ GeV}, \quad (6)$$

for the gravitino mass $m_{3/2} = \mathcal{O}(10^2 \text{ GeV})$, for example. In the recent analysis [4] the effect of the hadronic processes also has been taken into account.

This stringent constraint is problematic in the inflationary cosmology. The reheating temperature provided by typical inflation models such as the chaotic inflation or the hybrid inflation is estimated to be much larger than the upper bound of Eq. (6) [5]. Therefore there exists a gap

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between the natural reheating temperature for the inflation models and its upper bound of Eq. (6). It seems not so easy to construct an inflation model which can, simultaneously, provide a reheating temperature low enough to satisfy this upper bound and be consistent with the present cosmological observations, the density fluctuations, etc.¹ Some ideas to avoid the gravitino problem have been proposed such as the thermal inflation [8] and a model to forbid the radiative decay of gravitinos [9].

Recently, the brane world models have been attracting a lot of attention as a novel higher dimensional theory. In these models, it is assumed that the standard model particles are confined on a “3-brane” while gravity resides in the whole higher dimensional spacetime. The model first proposed by Randall and Sundrum [10], the so-called “RS II” model, is a simple and interesting one, and its cosmological evolutions have been intensively investigated [11]. In the model, our 4-dimensional universe is realized on the 3-brane with a positive tension located at the ultraviolet (UV) boundary of a five dimensional Anti de-Sitter spacetime. In this setup, the Friedmann equation for a spatially flat spacetime is found to be

$$H^2 = \frac{8\pi G}{3} \rho \left(1 + \frac{\rho}{\rho_0} \right), \quad (7)$$

where

$$\rho_0 = 96\pi G M_5^6, \quad (8)$$

with M_5 being the five dimensional Planck mass, and the four-dimensional cosmological constant has been tuned to be zero. Here we have omitted the term so-called “dark radiation”, since this term is severely constrained by big bang nucleosynthesis [12]. The second term proportional to ρ^2 is a new ingredient in the brane world cosmology and lead to a nonstandard expansion law.

Note that according to Eq. (7) the evolution of the early universe can be divided into two eras. At the era where $\rho/\rho_0 \gg 1$ the second term dominates and the expansion law is nonstandard (brane world cosmology era), while at the era $\rho/\rho_0 \ll 1$ the first term dominates and the expansion of the universe obeys the standard expansion law (standard cosmology era). In the following, we call a temperature defined as $\rho(T_i)/\rho_0 = 1$ “transition temperature”, at which the evolution of the early universe changes from the brane world cosmology era into the standard one.² Using the transition temperature, we rewrite Eq. (7) into the form

¹Some inflation models with low reheating temperature has been proposed in the context of the brane world cosmology [6,7], where some issues related to this letter are also discussed.

²We can find a lower bound on the transition temperature such as $T_i > \mathcal{O}(1 \text{ TeV})$ according to the discussion in [10] concerning the precision measurements of the Newton’s gravitational law in submillimeter range.

$$H^2 = \frac{8\pi G}{3} \rho \left[1 + \left(\frac{T}{T_i} \right)^4 \right] = H_{\text{st}}^2 \left[1 + \left(\frac{T}{T_i} \right)^4 \right], \quad (9)$$

where H_{st} is the Hubble parameter in the standard cosmology (Eq. (3)).

This modification of the expansion law at a high temperature ($T > T_i$) leads some drastic changes for several cosmological issues. In fact, for example, an enhancement of the thermal relic abundance of dark matter has been recently pointed out [13]. In this letter, we investigate the brane cosmological effect for the gravitino production process. Since the Friedmann equation is modified into Eq. (9), the solution of the Boltzmann equation of Eq. (4) can be modified, which leads us to a different consequence for the gravitino problem.

Before proceeding our discussion, let us specify our setup. Our brane world model is the supersymmetric extended RS II model [14]. The cosmological solution of this extended model is the same as that in the nonsupersymmetric model, since the Einstein’s equation belongs to the bosonic part. As is well known in the RS II model, the zero-mode graviton is localized around the UV brane, while the graviton Kaluza-Klein (KK) modes are likely localized around the infrared (IR) brane (in the RS II model, the IR brane is located at the infinite boundary). This feature is the key to recover almost four-dimensional Einstein gravity even with the infinite fifth dimensional radius, namely, the idea of the “alternative to compactification”. If the supersymmetry is manifest, all the component fields in a supergravity multiplet take the same configurations in the fifth dimensional direction. Thus the zero-mode gravitino is localized around the UV brane, and as an approximation we regard it as a field residing on the UV brane. Suppose that the SUSY is broken on the UV brane. The zero-mode gravitino obtains its mass through the super-Higgs effect as in four-dimensional supergravity, while KK gravitinos would remain the same, since supersymmetry is broken only locally on the UV brane in the infinite fifth dimensional coordinate. As discussed above, we treat the zero-mode gravitino as the field on the UV brane (the brane on which all the supersymmetric standard model fields reside) and apply the above Boltzmann equation for the gravitino production. On the other hand, the plasma on the UV brane (our brane) can, in general, excite the KK gravitinos³. Since the KK gravitinos take different configurations in the fifth dimensional direction and couple differently to the fields on our brane, their production processes will obey different types of Boltzmann equations. It is very interesting and challenging to analyze the KK gravitino produc-

³The KK graviton can also be emitted from the plasma on the UV brane and contribute to the dark radiation term in the Friedmann equation. This issue has been studied in [15], and the resultant dark radiation is found to be consistent with the BBN constraint.

tion process. We leave this important issue for future works.

Now let us analyze the thermal production of the gravitino in the brane world cosmology. The Boltzmann equation we analyze takes the form of

$$\frac{dY_{3/2}}{dT} = -\frac{s\langle\sigma_{\text{tot}}v\rangle}{H_{\text{st}}T}Y_{\text{rad}}^2 \times \frac{1}{\sqrt{1 + (\frac{T}{T_i})^4}}. \quad (10)$$

The prefactor in the r.h.s. is the same as the one in the standard cosmology. Suppose that the reheating temperature after inflation is much higher than the transition temperature. Then we integrate the Boltzmann equation from the reheating temperature $T_R (\gg T_i)$ in the brane world cosmology era to a low temperature $T_{\text{low}} (\ll T_i)$ in the standard cosmology era. First let us integrate the Boltzmann equation approximately in such a way that

$$-\int_{T_R}^{T_{\text{low}}} \frac{dT}{\sqrt{1 + (\frac{T}{T_i})^4}} \sim \int_{T_i}^{T_R} dT \left(\frac{T_i}{T}\right)^2 + \int_{T_{\text{low}}}^{T_i} dT \sim 2T_i. \quad (11)$$

We will see this result is a good approximation from the following analytic formula we will show. Note that the reheating temperature T_R in the formula of the gravitino abundance in the standard cosmology (see Eq. (5)) is replaced by the transition temperature, namely, $T_R \rightarrow 2T_i$. Therefore we can avoid overproduction of gravitinos by

$$T_i \leq 10^6 - 10^7 \text{ GeV} \quad (12)$$

independently of the reheating temperature. This is our main result in this letter. Needless to say, if $T_i \gg T_R$ we can reproduce the result in the standard cosmology.

For completeness, we show analytic formulas of the solution of the Boltzmann equation as follows:

$$\begin{aligned} Y_{3/2}(T_{\text{low}}) &= \int_{T_{\text{low}}}^{T_R} \frac{s\langle\sigma_{\text{tot}}v\rangle}{H_{\text{st}}T} Y_{\text{rad}}^2 \frac{dT}{\sqrt{1 + (\frac{T}{T_i})^4}} \\ &\simeq \frac{s\langle\sigma_{\text{tot}}v\rangle}{H_{\text{st}}T} Y_{\text{rad}}^2 \Big|_{T=T_R} T_i \left[Z_R F\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -Z_R^4\right) \right], \end{aligned} \quad (13)$$

where $F(\alpha, \beta, \gamma; z) = {}_2F_1(\alpha, \beta, \gamma; z)$ is the Gauss' hypergeometric function, and Z_R is defined as $Z_R = T_R/T_i (\gg 1)$. By using the relations,

$$\begin{aligned} F(\alpha, \beta, \gamma; z) &= (1-z)^{-\alpha} F\left(\alpha, \gamma - \beta, \gamma; \frac{z}{z-1}\right), \\ F(\alpha, \beta, \gamma; 1) &= \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)}, \end{aligned} \quad (14)$$

we obtain

$$Z_R F\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -Z_R^4\right) \rightarrow \frac{\Gamma(5/4)\Gamma(1/4)}{\Gamma(1)\Gamma(1/2)} \simeq 1.85 \quad (15)$$

for $Z_R \gg 1$. This result shows that Eq. (11) is actually a good approximation.

If the reheating is completed in the brane world cosmology era as we have assumed, the reheating temperature becomes different from the one in the standard cosmology due to the nonstandard expansion law. Here let us estimate the reheating temperature and check that our assumption, $T_R \gg T_i$, can be satisfied. Comparing the total decay width of an inflaton (Γ_I) and the Hubble parameter such as

$$\Gamma_I^2 = H^2 = \frac{8\pi G}{3} \rho(T_R) \left[1 + \left(\frac{T_R}{T_i}\right)^4 \right] \simeq \frac{T_R^8}{M_P^2 T_i^4}, \quad (16)$$

where $M_P = M_{Pl}/\sqrt{8\pi}$ is the reduced Planck mass, we find

$$T_R \simeq T_i \left(\frac{M_P}{T_i}\right)^{1/4} \left(\frac{\Gamma_I}{T_i}\right)^{1/4}. \quad (17)$$

Therefore our assumption can be satisfied if $(M_P \Gamma_I / T_i^2)^{1/4} \gg 1$. It is interesting to see a relation between the reheating temperature in the brane world cosmology and the standard one. In the standard cosmology, the reheating temperature is estimated as $T_R^{(s)} \simeq \sqrt{M_P \Gamma_I}$. Substituting this into Eq. (17), the reheating temperature of the brane world cosmology is described as

$$T_R \simeq T_i \left(\frac{T_R^{(s)}}{T_i}\right)^{1/2}. \quad (18)$$

Recall that the gap between $T_R^{(s)}$ and the upper bound of Eq. (6), namely, $T_R^{(s)} \gg 10^6 - 10^7 \text{ GeV}$ causes the gravitino problem originally. As discussed above, Eq. (6) is replaced by Eq. (12) in the brane world cosmology. Therefore the gap between the temperatures, on the contrary, justifies our assumption $T_R \gg T_i$ due to $T_R^{(s)} \gg T_i$.

In summary, we investigate the abundance of gravitinos in the framework of inflationary brane world cosmology. The point is that the expansion law is modified in the brane world cosmology and this modification can lead to different results for various issues in the standard cosmology. We have found that the late-time gravitino abundance is proportional to the transition temperature independently of the reheating temperature, if the reheating temperature is higher than the transition temperature. Therefore the gravitino problem disappears by assigning a transition temperature low enough. We also have checked that consistency of our assumption, $T_i \ll T_R$.

Finally we give some comments. A necessary condition to solve the gravitino problem by this manner is that inflation must occur in the brane world cosmology era as we have assumed. In order to verify our strategy toward a

solution of the gravitino problem, it is important to explore whether the inflation can take place during the brane world cosmology era [16]. Furthermore, the detailed study of density perturbation generated from inflation models in the brane world cosmology is an important future issue [17,18]. Also, as mentioned above, the issue of the KK gravitino production should be investigated in future.

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- [1] M. Y. Khlopov and A. D. Linde, *Phys. Lett. B* **138**, 265 (1984); J. R. Ellis, J. E. Kim, and D. V. Nanopoulos, *Phys. Lett. B* **145**, 181 (1984).
 - [2] M. Kawasaki and T. Moroi, *Prog. Theor. Phys.* **93**, 879 (1995).
 - [3] R. H. Cyburt, J. R. Ellis, B. D. Fields and K. A. Olive, *Phys. Rev. D* **67**, 103521 (2003).
 - [4] M. Kawasaki, K. Kohri, and T. Moroi, *astro-ph/0402490*.
 - [5] For a review of inflation, see, e.g., A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990); K. A. Olive, *Phys. Rep.* **190**, 307 (1990); D. H. Lyth and A. Riotto, *Phys. Rep.* **314**, 1 (1999).
 - [6] A. Mazumdar, *Phys. Rev. D* **64**, 027304 (2001); *Nucl. Phys. B* **597**, 561 (2001).
 - [7] M. C. Bento, N. C. Santos, and R. G. Felipe, *Phys. Rev. D* **69**, 123513 (2004).
 - [8] D. H. Lyth and E. D. Stewart, *Phys. Rev. D* **53**, 1784 (1996).
 - [9] T. Asaka and T. Yanagida, *Phys. Lett. B* **494**, 297 (2000).
 - [10] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
 - [11] For a review, see, e.g., D. Langlois, *Prog. Theor. Phys. Suppl.* **148**, 181 (2003).
 - [12] K. Ichiki, M. Yahiro, T. Kajino, M. Orito, and G. J. Mathews, *Phys. Rev. D* **66**, 043521 (2002).
 - [13] N. Okada and O. Seto, *Phys. Rev. D* **70**, 083531, (2004); T. Nihei, N. Okada, and O. Seto, *hep-ph/0409219*.
 - [14] T. Gherghetta and A. Pomarol, *Nucl. Phys. B* **586**, 141 (2000); R. Altendorfer, J. Bagger, and D. Nemeschansky, *Phys. Rev. D* **63**, 125025 (2001); A. Falkowski, Z. Lalak, and S. Pokorski, *Phys. Lett. B* **491**, 172 (2000); M. Zucker, *Phys. Rev. D* **64**, 024024 (2001); T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **108**, 203 (2002).
 - [15] A. Hebecker and J. March-Russell, *Nucl. Phys. B* **608**, 375 (2001); D. Langlois, L. Sorbo, and M. Rodriguez-Martinez, *Phys. Rev. Lett.* **89**, 171301 (2002).
 - [16] R. Maartens, D. Wands, B. A. Bassett, and I. Heard, *Phys. Rev. D* **62**, 041301 (2000).
 - [17] H. Kodama, A. Ishibashi, and O. Seto, *Phys. Rev. D* **62**, 064022 (2000); S. Mukohyama, *Phys. Rev. D* **62**, 084015 (2000); C. van de Bruck, M. Dorca, R. H. Brandenberger, and A. Lukas, *Phys. Rev. D* **62**, 123515 (2000); K. Koyama and J. Soda, *Phys. Rev. D* **62**, 123502 (2000).
 - [18] D. Langlois, R. Maartens, and D. Wands, *Phys. Lett. B* **489**, 259 (2000); S. Tsujikawa and A. R. Liddle, *J. Cosmol. Astropart. Phys.* 03 (2004) 001.