



A simulation analysis for the re-solving issue of the network revenue management problem



Kuancheng Huang*, Chia-Yi Lin

Department of Transportation and Logistics Management, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan

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ABSTRACT

The classic dynamic programming approach is not applicable to the airline network revenue management (RM) problem of a practical size due to the curse of dimensionality. Many heuristic methods, including the most popular bid-price control approach, generate the approximate control decisions based on various static formulations, which need to be re-solved to take into account the dynamic features of the problem. By a simulation experiment, this study examines the re-solving issue of the bid-price method and tests a new method, the parameterized function approach, in which no problem-resolving is required. Based on the results, the parameterized function approach is found to be a promising alternative. As for the bid-price control approach, a high re-solving frequency is needed for a good result.

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1. Introduction

Revenue Management (RM), also referred to as Yield Management (YM), has become a common practice in the airline industry ever since American Airlines successfully applied several RM techniques to raise its revenue. Based on certain demand forecasting techniques and optimization models, RM has been found to be very effective in generating extra revenue by dealing with the diversified and uncertain demand, given a fixed capacity. It is very difficult for any major airline nowadays to operate profitably without RM, given that, according to most estimates, the revenue gain from applying RM is about 4%–5%, which is comparable to many airlines' total profitability in a good year (Talluri and van Ryzin, 2004). In addition, RM has been successfully extended to some other industries. For example, a similar result of 1%–8% has been reported for the improvement in profits in the hotel sector (Jones, 2000). Nonetheless, how to realize the basic concept of RM, i.e., selling the right product to the right customer at the right price, remains a challenge.

Due to the current hub-and-spoke operation, the focus of RM research has shifted from the traditional single-leg version to the network version. The problem complexity and the associated computational load make it impossible to derive the optimal control for a problem of practical size. The mainstream approaches, such as bid price and virtual nesting, have some limitations such as the inaccuracy, due to the suboptimal nature, and the interruption

in operations, due to the problem-resolving needed for correcting the deficiencies of static models.

The key purpose of this study is to examine the impact of the update frequency for one network RM approach, the most popular bid-price control approach. The control performance in general should be better when the bid prices are updated more frequently. However, it is necessary to achieve a balance between the improved performance and the negative effects (such as the interruption in operations as well as the computational effort required for re-solving the static problem). This study has performed a simulation experiment to examine the re-solving issue of the bid-price approach. In addition to the bases provided by the optimal control and the FCFS (first-come-first-served) policy, the results are compared with a new method, the approach based on the parameterized function for revenue approximation (Huang and Liang, 2011), in which no problem-resolving is needed.

The remainder of this paper is organized as follows. The second section provides the problem background and reviews the related literature. The bid-price method, the method based on the parameterized function for revenue approximation and the framework of the simulation experiment are presented in the third section. The numerical experiment is described in the fourth section. Finally, the findings of this study are summarized and conclusions are drawn in the final section.

2. Background and literature review

Researchers have studied various kinds of seat inventory control problems for airlines. Weatherford and Bodily (1992) provide a very

* Corresponding author. Tel.: +886 3 5731928.

E-mail address: kchuang@cc.nctu.edu.tw (K. Huang).

general approach to categorize the nature of RM problems, and McGill and van Ryzin (1999), Talluri and van Ryzin (2004) and Chiang et al. (2007) serve as an excellent reference of the research literature on RM. The literature review of this study focuses on the RM problem within the network context.

In an airline network, a fare class of an origin–destination pair (later referred to as an ODF) can utilize the seats of multiple legs, and a seat on a leg is usually shared by multiple ODFs. The network RM problem incorporating this network feature and the dynamic characteristics of the demand can be formulated as the following DP model (Talluri and van Ryzin, 2004).

$$V_t(\mathbf{x}) = P_t^0 V_{t-1}(\mathbf{x}) + \sum_{j=1}^J P_t^j \max(V_{t-1}(\mathbf{x} - \mathbf{S}^j) + F_j, V_{t-1}(\mathbf{x})) \quad (1)$$

where $P_t^0 = 1 - \sum_{j=1}^J P_t^j$

- t : indices of decision periods ($t = 0 \dots T$, assuming $t = 0$ is the period of flight departure, and $t = T$ is the beginning of the booking process.)
- j : indices of ODFs ($j = 1 \dots J$)
- P_t^j : probability of the booking request for ODF j in period t
- F_j : revenue of ODF j
- i : indices of legs ($i = 1 \dots I$)
- \mathbf{S} : an incident matrix ($I \times J$), representing the relationship between the ODFs and the legs. Its entry s_{ij} is equal to 1 if ODF j uses leg i ; otherwise, it is 0.
- \mathbf{S}^j : the j th column vector of \mathbf{S} , representing the legs used by ODF j .
- x_i : the number of available seats on leg i , and the vector \mathbf{x} represents the available seats on all legs.
- $V_t(\mathbf{x})$: expected revenue given the available seats on the legs \mathbf{x} in period t

The Bellman equation of the DP model (1) shows how to evaluate the expected revenue given the arrival information of the demands in a recursive manner. With the boundary condition $V_0(\mathbf{x}) = 0$ at the end of the booking process (flight departure), the objective is to maximize the expected revenue $V_T(\mathbf{C})$ given \mathbf{C} seats available on the legs (*i.e.*, the system capacity) at the beginning of the booking process.

The optimal control policy that results in the maximum expected revenue can be generated by (2) based on the two terms inside the max function of (1). For each period t given the available seats on the legs \mathbf{x} , a booking request of ODF j should be accepted if its revenue is larger than the expected revenue decrease due to state change (*i.e.*, the opportunity cost) in period ($t-1$). The computational load to evaluate the expected revenue for (1) and then to generate the optimal control policy based on (2) for the entire state space is intractable for most practical problems. Thus, an approximate algorithm with a manageable computational load and acceptable solution quality is usually used.

$$F_j \geq V_{t-1}(\mathbf{x}) - V_{t-1}(\mathbf{x} - \mathbf{S}^j) \quad (2)$$

The most popular approach for the network RM problem is the bid-price control approach, as it is intuitive and easy to implement (Escudero et al., 2013). A bid price is attached to each leg, and a booking request for a fare class of an origin–destination pair is accepted if its revenue is greater than the sum of the bid prices of the used legs. The key issue of most bid-price based algorithms is to find a suitable set of bid prices, which is supposed to depend on the number of seats available on the legs and

the number of time periods left before departure. Williamson (1992) set the bid prices as the dual prices of the leg capacity constraints in a static linear programming (LP) model, in which the demand patterns of the ODFs are replaced by the point estimations regarding the remaining periods as a whole. Thus, frequent updates of bid prices during the booking process are generally required. The other issue associated with Williamson (1992) was that the stochastic feature of the demand was overlooked in the deterministic LP model. Many researchers have focused their efforts on the sophisticated algorithms used to generate better bid prices to address the dynamic and/or stochastic feature of the problem. For some recent works, please refer to Adelman (2007), Topaloglu (2008), Akan and Ata (2009), Ball and Queyranne (2009), Topaloglu (2009), Kunnumkal and Topaloglu (2010), and Escudero et al. (2013). In particular, more and more network RM models (*e.g.*, Meissner and Strauss, 2012) have taken into account the choice behavior of customers, which is a feature not considered in this study.

Nonetheless, the classic bid-price method based on a static and deterministic LP model, such as Williamson (1992), is still widely used in practice (Chen and Homem-de-Mello, 2010), and re-solving the problem to update the bid prices remains an important issue for implementing the network RM control. In general, the performance should be improved if the bid prices are updated more frequently, as the actual situation of the demand and seat availability can be taken into account in a timely manner. However, this intuitive speculation needs to be examined and it would be better if it were supported by some numerical analysis. Even more importantly, it is necessary to find a balance between the improved control accuracy and the negative effects.

Cooper (2002), the first to address the re-solving issue of the bid-price control method, showed that re-solving does not necessarily lead to a better result based on a very simple single-leg example. From the aspect of the general control-algorithm approach, Secomandi (2008) established sufficient conditions under which re-solving does not worsen the performance of the control policy. In addition, the counter-intuitive example in Cooper (2002) was re-visited in a numerical experiment, in which eight control policies were compared. Chen and Homem-de-Mello (2010) dealt with the original multi-stage stochastic network RM problem by means of an approach where they solved a sequence of two-stage stochastic programming (SP) problems with simple recourse. Their theoretical results show that solving more successive two-stage SP problems can never result in a reduction in expected revenue. In addition, they also proposed a heuristic method to determine the re-solving schedule, in which the updates are not evenly spaced within the booking horizon. Recently, Jasin and Kumar (2012) derived an upper bound for the expected revenue loss of various re-solving control policies, when compared with the optimal control, and further designed two re-solving schedules with bounded asymptotic revenue loss.

Huang and Liang (2011) have developed a new control method for the network RM problem, in which the dynamic decision is based on the parameterized functions, which approximate the expected revenues for the entire state space in terms of seat availability. As the parameters of the functions for all time periods can be estimated in advance, no update is required for this approach. Given this special advantage, this method is also tested in the simulation experiment to serve as a basis for performance evaluation, in addition to the optimal control and the FCFS (first-come-first-served) policy. Meanwhile, beyond the purpose of examining the bid-price control method in terms of the re-solving frequency, this study also aims to evaluate the applicability of the method based on parameterized functions in Huang and Liang (2011) to the network RM problem.

3. Simulation framework

In order to examine the re-solving issue of the static bid-price control approach, this study designed a simulation experiment. The bid-price control is presented in the first sub-section, and the approach based on the parameterized function is presented in the second sub-section. Finally, the simulation procedures for these two types of control methods are provided in the third sub-section.

3.1. Bid-price control

Suppose the bid price for leg i is denoted by $\mu_i(t, \mathbf{x})$, in general a function of the time period and the state of the seat availability. As in Williamson (1992), a booking request for ODF j is accepted only if its fare is higher than the sum of the associated bid prices, according to (3).

$$F_j \geq \sum_{i \in S} \mu_i(t, \mathbf{x}) \quad (3)$$

As for how the bid prices are determined, Williamson (1992) developed a deterministic linear program model as (4) to (6), in which the decision variable is the allocated tickets (sales) of the ODFs. The objective (4) is to maximize the total revenue subject to the capacities of the legs in (5) and the estimated mean demand of the ODFs in (6). In particular, the focus of this LP model is not on the solution (the allocation) or the objective (revenue). The key purpose is to set the dual price of the capacity constraint (5) as the bid price of the corresponding leg.

$$\text{Maximize } \sum_j F_j y_j \quad (4)$$

$$\text{s.t. } \sum_j s_{ij} y_j \leq x_i \quad \forall i \in I \quad (5)$$

$$y_j \leq E[D_j] \quad \forall j \in J \quad (6)$$

- y_j : the sale (allocation) of ODF j
- D_j : demand of ODF j with $E[D_j]$ as the mean value

As the average values of the ODF demand for the whole booking horizon are used, the above LP model is likely to suffer from overlooking both the stochastic and dynamic features of the problem. In order to reflect the changes over time in terms of demand and seat availability, re-solving the LP model is required so as to generate the new bid prices, which under an ideal situation can represent the opportunity cost (or the value) of the leg seats. The key concern here is how often the bid-prices should be updated. Is it always the case that the more frequently they are updated, the better?

3.2. Parameterized function control

The expected revenue function $V_t(\mathbf{x})$ in (1) contains some special features due to the nature of RM problems. First, it is monotonically increasing with respect to the number of leg seats \mathbf{x} , as more seats always generate more revenue. Second, the marginal benefit (revenue contribution) of a seat on a leg is diminishing, and the expected revenue function should reach a fixed value if the available seats on the legs are sufficient to accommodate all possible booking requests. These features provide the basis to choose the parameterized function $g_t(\mathbf{x})$ defined in (7)–(8) to approximate the expected revenue function $V_t(\mathbf{x})$ in (1), as proposed in Huang and Liang (2011).

$$g_t(\mathbf{x}) = A_t \left(1 - \sum_{i=1}^I b_t^i e^{-a_t^i x_i} \right) \quad (7)$$

$$A_t = \sum_{j=1}^J \left(F_j \sum_{\tau=1}^t P_\tau^j \right) \quad (8)$$

- $g_t(\mathbf{x})$: parameterized function to approximate the expected revenue given the available seats on the legs \mathbf{x} in period t

The function is horizontally asymptotic to the parameter A_t , which is computed by (8) to represent the maximum possible revenue, given the possible booking requests from period t to the end of the booking process, by assuming that there are an infinite number of leg seats. As for the other two leg-specific parameters (a_t^i and b_t^i), they are used to model the rising characteristic of the approximate function with respect to the number of seats x_i on the leg. Their values can be determined by the DP-based procedure specified by (9) and (10).

The procedure used to estimate the parameters of the approximate function has a structure similar to the original DP formulation of the network RM problem, which suffers from an enormous amount of points in the state space. However, for the purpose of curve fitting, not many data points are actually required. Thus, the concept of sampling is introduced, and only a limited number of points in the state space are evaluated to determine the parameters of the functions for revenue approximation. The related computation can be represented by (9) and (10), and the whole procedure is illustrated in Fig. 1.

$$W_t(\mathbf{x}_t^h) = P_t^0 g_{t-1}(\mathbf{x}_t^h) + \sum_{j=1}^J P_t^j R_t(F_j, g_{t-1}(\mathbf{x}_t^h)) \quad (9)$$

for $h = 1, \dots, H$

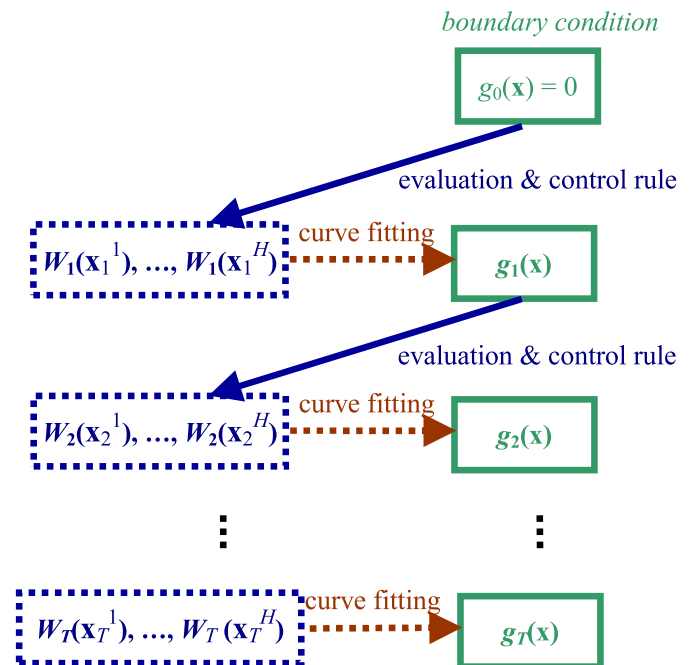


Fig. 1. Generation of the Parameterized Functions by Sampling.

$$R_t(F_j, g_{t-1}(\mathbf{x}_t^h)) = \begin{cases} F_j + g_{t-1}(\mathbf{x}_t^h - \mathbf{S}^j) & \text{if } F_j \geq g_{t-1}(\mathbf{x}_t^h) - g_{t-1}(\mathbf{x}_t^h - \mathbf{S}^j) \\ g_{t-1}(\mathbf{x}_t^h) & \text{otherwise} \end{cases} \quad (10)$$

- \mathbf{x}_t^h : one sampling point h in the state space in period t , and H is the total number of sampling points.
- $W_t(\mathbf{x}_t^h)$: expected revenue for the sampling point \mathbf{x}_t^h under the control rule of $g_{t-1}(\mathbf{x})$
- $R_t(F_j, g_{t-1}(\mathbf{x}))$: revenue function at period t evaluated on the basis of $g_{t-1}(\mathbf{x})$ and under the control rule of $g_{t-1}(\mathbf{x})$

Given the boundary condition $g_0(\mathbf{x}) = 0$, for each time period t , a limited number (H) of data points in the state space are randomly sampled. For each sampling point, the expected revenue is evaluated based on the parameterized function of the previous stage, $g_{t-1}(\mathbf{x})$, as shown in (9) and (10). Once all of the sampling points are evaluated, the parameterized function for the current stage, $g_t(\mathbf{x})$, can be established by any curve fitting technique to determine the values of the parameters of all legs (a_i^t and $b_i^t, \forall i$).

Once the recursive process of (9) and (10) is completed from $t = 1$ to T , the parameterized function for each time period $g_t(\mathbf{x})$ is available. The control rule for all possible situations (the whole state space) can then be determined by (11), which is similar to the optimal control specified by (2). As the computation of (7) and (11) is nominal for today's computers, the accept/deny decision can be made as soon as a booking request during the booking process is received based on the pre-determined and stored parameters. Thus, no problem-resolving and parameter update are required, unless it is found that the initial demand assumption has changed.

$$F_j \geq g_{t-1}(\mathbf{x}) - g_{t-1}(\mathbf{x} - \mathbf{S}^j) \quad (11)$$

3.3. Simulation procedure

The time-dependent probabilities associated with the booking requests of the various ODFs, i.e., P_t^i 's in (1), are in fact one kind of data unavailable for most airlines from the practical point of view. However, this study follows the approach in Klein (2007) as well as Chen and Homem-de-Mello (2010) to generate the booking requests based on the non-homogeneous Poisson (NHP) process.

The major steps of the simulation procedure for the bid price control can be summarized as follows:

Step 1: *Initialization*. Generate the dynamic booking requests of ODFs based on the non-homogeneous Poisson process. Set the time index to be the beginning of the booking horizon, and set the booking index to the first booking request.

Step 2: *Bid Price Update*. Solve the linear programming problem (4)–(6), based on the average of the demand for the remaining periods and the current seat availability, to derive the new bid prices if an update at the current time period is required according to the pre-specified update frequency (interval).

Step 3: *Booking Control*. Determine whether the booking request is accepted based on the seat availability and the bid prices. If it is accepted, reduce the number of the available seats of the associated leg(s).

Step 4: *Termination*. If there is no available seat on all legs, or all booking requests are handled, terminate the procedure. Otherwise, move the booking index to the next booking request, set the time index to the arrival time of the new booking request, and go to Step 2.

As for the simulation procedure for the control based on the parameterized function, the major steps are listed as follows. In general, the major difference is that no update is needed during the booking process, although some effort is needed in advance to determine the values of the parameters so as to construct the revenue approximation functions for all time periods.

Step 1: *Parameter Estimation*. Estimate the values of the parameters of the expected revenue functions for all periods based on (7)–(10), given the pre-determined dynamic demands.

Step 2: *Initialization*. Generate the dynamic booking requests of the ODFs based on the non-homogeneous Poisson process. Set the time index to be the beginning of the booking horizon, and set the booking index to the first booking request.

Step 3: *Booking Control*. Determine whether the booking request is accepted based on the current seat availability and the parameterized function of the current time period as in (11). If it is accepted, reduce the number of available seats of the associated leg(s).

Step 4: *Termination*. If there is no available seat on all legs, or all the booking requests are handled, terminate the procedure. Otherwise, move the booking index to the next booking request, set the time index to the arrival time of the new booking request, and go to Step 3.

4. Numerical experiment

4.1. Design of test problems

The numerical experiment was based on a small network shown in Fig. 2. The network and the demand arrival patterns are similar to those in Klein (2007), but the fares, the capacities, and the booking horizon have been modified. As only the one-way movement is considered (from left to right), there are 5 OD pairs in the small network. In addition, it is assumed that there are 4 fare classes, which lead the number of ODFs to be 20. The associated fares are provided in Table 1.

The capacities of the short-haul legs (A-Hub and B-Hub) are set as 10, and that of the long-haul leg (Hub-C) is 20, so the optimal control based on the DP model of (1) and (2) is solvable. The ratio of the single-leg demand to the multi-leg demand is 4:6. Among the fare classes, the shares of Y, M, B, and K (from the highest to the lowest) are based on the ratios of 1, 2, 3, and 4, respectively.

As for the dynamic characteristics of the demand, the booking requests of the four fares are generated based on the demand intensities illustrated in Fig. 3 for all origin–destination pairs. In

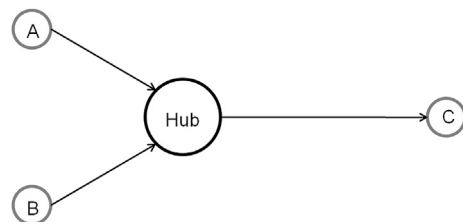


Fig. 2. Network in the numerical experiment.

Table 1
Fare information of ODFs.

	A → Hub	B → Hub	Hub → C	A → Hub → C	B → Hub → C
Y	400	400	2000	2040	2040
M	300	300	1500	1530	1530
B	200	200	1000	1020	1020
K	100	100	500	510	510

particular, in order to apply the DP model used in the optimal policy of (1) and the control based on the parameterized functions of (7)–(11), the booking horizon (18 time units) is divided into 144 time periods in a reverse fashion. That is, the period $t = 144$ is the beginning of the booking horizon, and the period $t = 1$ is the one right before departure. By adjusting the values of the parameters representing the maximum arrival rates (λ^{\max} in Fig. 3), the overall ratio of demand to supply is set at about 1.3. Thus, there is room for RM to improve the expected revenue as the flight capacities are not sufficient to serve all requests. In particular, since the high-fares booking requests tend to arrive late, the revenue loss is likely to be significant, if no RM measure is taken.

4.2. Simulation results

The generated booking requests based on the non-homogeneous Poisson process serve as the input of the simulation experiment. For the bid price control, the following five update frequencies are used: 0, 1, 3, 17, and 35. In particular, the updates of the bid prices are assumed to be evenly distributed within the booking horizon. Given that the total number of periods is 144, these update frequencies correspond to the update intervals of 144, 72, 36, 8, and 4 periods, respectively. Thus, for the example of 3

updates, the LP model of (4)–(6) is solved to derive the new bid prices in the periods where $t = 108, 72,$ and 36 .

In addition to the control based on the parameterized functions presented in Sub-section 3.2, the optimal control based on the DP model of (1) and the first-come-first-served (FCFS) policy are also tested to serve as the basis for performance evaluation. The results based on 30 simulation runs are shown in Tables 2 and 3.

From these results, it can be seen that the method based on the parameterized function can achieve performance close to that based on the optimal control policy. The gap is only 2.3%, which implies that most of the RM benefit has been caught, given that the gap between the optimal control and the FCFS policy (effectively without any RM control) is 17.1%. This result provides some support to the parameterized function method for serving as a promising alternative approach for the network RM control, in particular if its update-free advantage is further taken into account.

As for the bid-price control approach, it has been found that the update mechanism is effective in raising the expected revenue. However, the marginal return is diminishing with respect to additional updates. For example, the revenue gap is reduced from 4.1% only to 3.6% when the number of updates within the 144 periods is increased from 3 to 17. This observation is consistent with the finding in Jasin and Kumar (2012). In addition, there is a limit regarding how much the update frequency can improve the expected revenue. Although 35 updates (i.e., an update for every 4 periods) have been made, the revenue loss with respect to the optimal control can only be reduced to 3.2%, which is still more than that of the parameterized function method.

The other interesting finding concerns the variation in revenue, which is represented by the standard deviations in Table 2. In particular, the ratios of the standard deviations with respect to the optimal control for various policies are shown in Table 4. The optimal control can achieve the highest expected revenue, but its

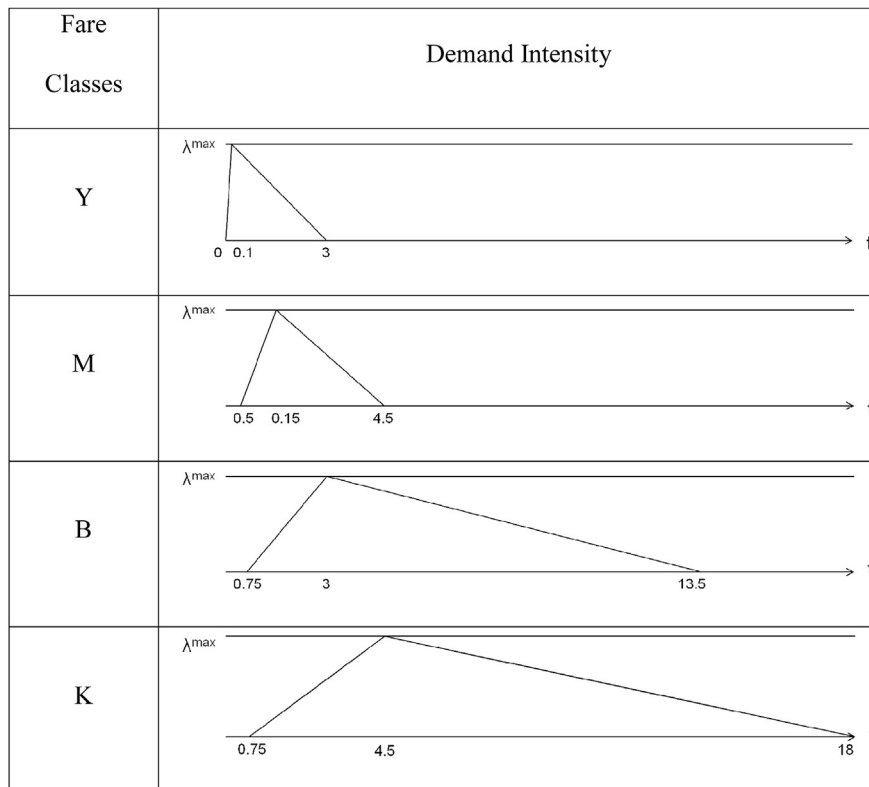


Fig. 3. Demand arrival patterns of the fare classes.

Table 2

Average revenue for various policies in the simulation.

Revenue	FCFS	Optimal DP	Parameterized function	Bid price (update frequency)				
				0	1	3	17	35
Mean	18,060	21,789	21,289	19,878	20,417	20,892	21,006	21,081
Standard deviation	2745	3611	2800	3249	3376	3602	3451	3291

revenue variation is also the highest. When compared with the series of the bid-price policies, the method based on the parameterized function results in a more stable performance, which is supported by the small standard deviation of the revenue that is close to the one based on the FCFS policy.

5. Conclusions

In view of the current popularity of the hub-and-spoke operations, the network version of the RM problem has become more important. However, due to the problem's complexity and the associated computational load, it is impossible to derive the optimal control for a problem of practical size based on the classic DP approach. The focus of this study is thus to examine the re-solving issue of the popular bid price method, in which the static linear programming problem must be solved repeatedly so as to take into account the dynamic features of the problem. This study has performed a simulation experiment to examine the impact of the re-solving frequency and has compared the results with the control method based on the parameterized function, in which no re-solving is involved.

Based on the results of the simulation experiment, it was found that the performance of the method based on the parameterized function is very good, given the results according to the optimal control and FCFS policies as the basis for comparison. In particular, if its update-free advantage is further taken into account, the parameterized function method should be a promising alternative approach for the network RM problem. As for the bid-price control method, a high re-solving frequency is needed for a similar result; otherwise, the revenue is significantly reduced. The other interesting finding was that the method based on the parameterized function can achieve a relatively stable performance in terms of revenue variation.

One major extension of this study would be to perform a large-scale simulation experiment based on the real network and data from of the airlines. In particular, more demand profiles arising from different market characteristics can be tested to derive a better understanding of the various control methods. For a problem of that scale, it is impossible to derive the optimal control based on

Table 3

Comparison with the optimal control for average revenue.

	FCFS	Parameterized function	Bid price (update frequency)				
			0	1	3	17	35
Average as a Percentage	-17.1%	-2.3%	-8.8%	-6.3%	-4.1%	-3.6%	-3.2%

Table 4

Comparison with the optimal control for revenue variation.

	FCFS	Parameterized function	Bid price (update frequency)				
			0	1	3	17	35
Standard deviation as a percentage	-24.0%	-22.5%	-10.0%	-65%	-0.2%	-4.4%	-8.9%

the DP model. However, the comparisons between the versions of the bid-price control and the alternative approaches (such as the control based on the parameterized functions) should be of great interest to the practitioners.

For most RM models, it is assumed that the request probabilities of the dynamic booking requests are available. However, this information is in reality unlikely to be available, or the level of detail is below what is required by the control models. Thus, an adaptive approach that makes the control decision based on the evolution of the demand, instead of the pre-determined demand information, should be more suitable for real-world applications. Therefore, the other extension of this study is to develop an adaptive algorithm to update the parameter estimation without the pre-determined demand information.

The last extension is to incorporate the customer choice behavior into the network RM control. In particular, it would be even better if the interaction between the RM decision of airlines and the choice behavior of customers can be taken into account. Although some theoretical models have been developed, numerical experiments with practical implications are needed to verify their applicability to real-world problems. However, as the problem's complexity is further increased due to the introduction of choice behavior, some simplifications must be made to handle the stochastic and dynamic features of the problem. Re-solving the simplified problem could remain a challenging issue when implementing network RM control.

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