Information Sciences 277 (2014) 458–480

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Pragmatical adaptive synchronization – New fuzzy model of two different and complex chaotic systems by new adaptive control

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ARTICLE INFO

Article history:
Received 31 July 2008
Received in revised form 23 March 2010
Accepted 21 February 2014
Available online 5 March 2014

Keywords:
New fuzzy model
New control Lyapunov function
New pragmatical adaptive method
Mathieu–Van der Pol system

ABSTRACT

In this paper, (1) a new fuzzy model is presented to simulate two different chaotic systems with different numbers of nonlinear terms and (2) a new adaptive approach and a new control Lyapunov function are proposed to synchronize these two different fuzzy chaotic systems and speed up the convergence of errors. By using this new model, the numbers of fuzzy rules of chaotic systems can be reduced from 2^N to $2 \times N$ and only 2 subsystems are needed, where N is the number of nonlinear terms. The fuzzy systems become much simpler. In addition, through the new fuzzy model, the new fuzzy systems are much simpler than T–S fuzzy systems (when nonlinear systems are complicated) and can be used to any other kind of application in fuzzy logic control or fuzzy modeling. Mathieu–Van der Pol system (which is called M–V system in this paper) and Quantum cellular neural networks nanosystem (which is called Q-CNN system in this paper) are used for illustrations in numerical simulation results to show the effectiveness and feasibility of our new adaptive approach and new control Lyapunov function. The T–S fuzzy modeling and traditional adaptive control are also given in Appendices B and C for comparison.

1. Introduction

In the last few years, synchronization in chaotic dynamical systems has received a great deal of interest among scientists from various fields [1–8]. The phenomenon of synchronization of two chaotic systems is fundamental in science and has a wealth of applications in technology. More and more applications of chaos synchronization were proposed. There are many control techniques to synchronize chaotic systems, such as linear and nonlinear error feedback control [9,10], active control [11,12], backstepping control method [13–15], impulsive control [16,17] and sliding mode control [18–20].

Most of those control techniques mentioned above are based on the exact knowledge of the system structure and parameters. But in practice, some or all of the system parameters are uncertain. Moreover, these parameters change from time to time. A lot of works have proceeded to solve this problem by adaptive synchronization [21–24]. In current scheme

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http://dx.doi.org/10.1016/j.ins.2014.02.128

of adaptive synchronization, traditional Lyapunov stability theorem and Barbalat lemma are used to prove that the error vector approaches zero as time approaches infinity, but the question that why those estimated parameters also approach the uncertain values remains no answer. In this article, pragmatical asymptotically stability theorem and an assumption of equal probability for ergodic initial conditions [25,26] are used to prove strictly that those estimated parameters approach the uncertain values. Moreover, traditional adaptive chaos synchronization in general is limited for the same system. In this paper, two different chaotic systems are synchronized by using adaptive control law based on pragmatical asymptotically stability theorem.

In recent years, fuzzy logic [27–30] has received much attention from the control theorists as a powerful tool for the nonlinear control. Among various kinds of fuzzy methods, Takagi–Sugeno fuzzy system is widely accepted as a tool for design and analysis of fuzzy control system [31–40]. Although this powerful model can simulate any real chaotic system, its number of fuzzy rules is based on the number of nonlinear terms. It means that if there are N nonlinear terms in a chaotic system, there will be 2^N fuzzy rules in its fuzzy model, and then there will be 2^N linear system to simulate only one chaotic system. Therefore when there are lots of nonlinear terms in a chaotic system, the problem is going to be more complicated. As a result, this paper provides a new fuzzy model to model a chaotic system with lots of nonlinear terms. By using this new fuzzy model, it becomes much simpler to synchronize two different chaotic systems with different number of nonlinear terms.

The layout of the rest of the paper is as follows. In Section 2, the theory of new fuzzy model is introduced. In Section 3, new fuzzy model of two chaotic systems are proposed. In Section 4, new Mathieu–Van der Pol system and Quantum cellular neural networks nanosystem, two simulation examples, are given for synchronization. In Section 5, conclusions are given. Pragmatical asymptotically stability theorem is enclosed in Appendix A, T–S fuzzy modeling and other kind of traditional adaptive control are discussed in Appendix B.

2. New fuzzy model theory

In system analysis and design, it is important to select an appropriate model representing a real system. As an expression model of a real plant, the fuzzy implications and the fuzzy reasoning method suggested by Takagi and Sugeno are traditionally used. The new fuzzy model is also described by fuzzy IF-THEN rules. The core of the new fuzzy model is to express each nonlinear equation in two linear sub-equations by fuzzy IF-THEN rules and then take all the first linear sub-equations to form one linear subsystem and all the second linear sub-equations to form another linear subsystem. The overall fuzzy model of the system is achieved by fuzzy blending of this two linear subsystem models. Consider a continuous-time nonlinear dynamic system as follows:

Equation i:

$$\begin{aligned}
&\text{rule } 1:\\
&\quad \text{IF } z_i(t) \text{ is } M_{i1}\\
&\quad \text{THEN } \dot{x}_i(t) = A_{i1}x(t) + B_{i1}u(t)\\
&\text{rule } 2:\\
&\quad \text{IF } z_i(t) \text{ is } M_{i2}\\
&\quad \text{THEN } \dot{x}_i(t) = A_{i2}x(t) + B_{i2}u(t)
\end{aligned} \tag{2.1}$$

where

$$x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$$
$$u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$$

$i = 1, 2, \ldots, n$ (n is the number of nonlinear terms). M_{i1}, M_{i2} are fuzzy sets, A_i, B_i are column vectors and $\dot{x}_i(t) = A_{ij}x(t) + B_{ij}u(t),\ j = 1, 2$, is the output from the first and the second IF-THEN rules. Given a pair of $(x(t), u(t))$ and take all the first linear sub-equations to form one linear subsystem and all the second linear sub-equations to form another linear subsystem, the final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \begin{bmatrix} M_{11} \\ M_{21} \\ \vdots \\ M_{i1} \end{bmatrix}^T \begin{bmatrix} A_{11}x(t) + B_{11}u(t) \\ A_{21}x(t) + B_{21}u(t) \\ \vdots \\ A_{i1}x(t) + B_{i1}u(t) \end{bmatrix} + \begin{bmatrix} M_{12} \\ M_{22} \\ \vdots \\ M_{i2} \end{bmatrix}^T \begin{bmatrix} A_{12}x(t) + B_{12}u(t) \\ A_{22}x(t) + B_{22}u(t) \\ \vdots \\ A_{i2}x(t) + B_{i2}u(t) \end{bmatrix} \tag{2.2}$$

Note that:

for each equation i:

$$\sum_{j=1}^{2} M_{ij}(z_i(t)) = 1$$
$$M_{ij}(z_i(t)) \geqslant 0, \quad i = 1, 2, \ldots, n \text{ and } j = 1, 2$$

Via the new fuzzy model, the final form of the fuzzy model becomes very simple. The new model provides a much more convenient approach for fuzzy model research and fuzzy application. The simulation results of complicated chaotic systems are discussed in next section.

3. New fuzzy model of chaotic systems

New fuzzy models of Q-CNN system and M–V system are shown in this section.

3.1. Fuzzy modeling of Q-CNN system

Quantum-CNN system is [29]:

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2 \\ \dot{x}_2 = -w_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 \\ \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4 \\ \dot{x}_4 = -w_2(x_3 - x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 \end{cases} \tag{3.1}$$

where a_1, a_2, w_1 and w_2 are the parameters. When $a_1 = 6.8, a_2 = 4.3, w_1 = 4.7$ and $w_2 = 3.9$ and initial states is $(0.1, 0.5, 0.1, 0.5)$, the system is chaotic as shown in Fig. 1.

Step of fuzzy modeling:

Step 1:

Assume that $\sqrt{1-x_1^2}\sin x_2 \in [-Z_1, Z_1]$ and $Z_1 > 0$, then the first equation of (3.1) can be exactly represented by new fuzzy model as following:

$$\text{Rule 1}: \text{ IF } \sqrt{1-x_1^2}\sin x_2 \text{ is } M_{11}, \text{ THEN } \dot{x}_1 = -2a_1Z_1 \tag{3.2}$$

$$\text{Rule 2}: \text{ IF } \sqrt{1-x_1^2}\sin x_2 \text{ is } M_{12}, \text{ THEN } \dot{x}_1 = 2a_1Z_1 \tag{3.3}$$

where

$$M_{11} = \frac{1}{2}\left(1 + \frac{\sqrt{1-x_1^2}\sin x_2}{Z_1}\right), \quad M_{12} = \frac{1}{2}\left(1 - \frac{\sqrt{1-x_1^2}\sin x_2}{Z_1}\right)$$

and $Z_1 = 0.8$. M_{11} and M_{12} are fuzzy sets of the first equation of (3.1) and $M_{11} + M_{12} = 1$.

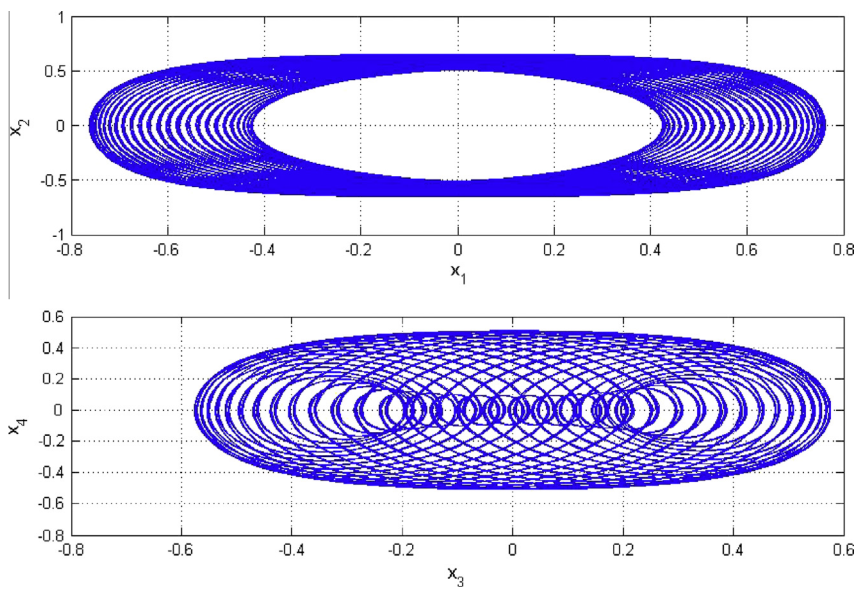

Fig. 1. Chaotic behavior of quantum cellular neural networks nano system.

Step 2:

Assume that $\cos x_2/\sqrt{1-x_1^2} \in [-Z_2, Z_2]$ and $Z_2 > 0$, then the second equation of (3.1) can be exactly represented by new fuzzy model as following:

$$\text{Rule 1: IF } \cos x_2/\sqrt{1-x_1^2} \text{ is } M_{21}, \text{ THEN } \dot{x}_2 = -w_1(x_1 - x_3) + 2a_1x_1Z_2 \tag{3.4}$$

$$\text{Rule 2: IF } \cos x_2/\sqrt{1-x_1^2} \text{ is } M_{22}, \text{ THEN } \dot{x}_2 = -w_1(x_1 - x_3) - 2a_1x_1Z_2 \tag{3.5}$$

where

$$M_{21} = \frac{1}{2}\left(1 + \frac{\cos x_2/\sqrt{1-x_1^2}}{Z_2}\right), \quad M_{22} = \frac{1}{2}\left(1 - \frac{\cos x_2/\sqrt{1-x_1^2}}{Z_2}\right)$$

and $Z_2 = 1.6$. M_{21} and M_{22} are fuzzy sets of the second equation of (3.1) and $M_{21} + M_{22} = 1$.

Step 3:

Assume that $\sqrt{1-x_3^2}\sin x_4 \in [-Z_3, Z_3]$ and $Z_3 > 0$, then the third equation of (3.1) can be exactly represented by new fuzzy model as following:

$$\text{Rule 1: IF } \sqrt{1-x_3^2}\sin x_4 \text{ is } M_{31}, \text{ THEN } \dot{x}_3 = -2a_2Z_3 \tag{3.6}$$

$$\text{Rule 2: IF } \sqrt{1-x_3^2}\sin x_4 \text{ is } M_{32}, \text{ THEN } \dot{x}_3 = 2a_2Z_3 \tag{3.7}$$

where

$$M_{31} = \frac{1}{2}\left(1 + \frac{\sqrt{1-x_3^2}\sin x_4}{Z_3}\right), \quad M_{32} = \frac{1}{2}\left(1 - \frac{\sqrt{1-x_3^2}\sin x_4}{Z_3}\right)$$

and $Z_3 = 0.5$. M_{31} and M_{32} are fuzzy sets of the third equation of (3.1) and $M_{31} + M_{32} = 1$.

Step 4:

Assume that $\cos x_4/\sqrt{1-x_3^2} \in [-Z_4, Z_4]$ and $Z_4 > 0$, then the fourth equation of (3.1) can be exactly represented by new fuzzy model as following:

$$\text{Rule 1: IF } \cos x_4/\sqrt{1-x_3^2} \text{ is } M_{41}, \text{ THEN } \dot{x}_4 = -w_2(x_3 - x_1) + 2a_2x_3Z_4 \tag{3.8}$$

$$\text{Rule 2: IF } \cos x_4/\sqrt{1-x_3^2} \text{ is } M_{42}, \text{ THEN } \dot{x}_4 = -w_2(x_3 - x_1) - 2a_2x_3Z_4 \tag{3.9}$$

where

$$M_{41} = \frac{1}{2}\left(1 + \frac{\cos x_4/\sqrt{1-x_3^2}}{Z_4}\right), \quad M_{42} = \frac{1}{2}\left(1 - \frac{\cos x_4/\sqrt{1-x_3^2}}{Z_4}\right)$$

and $Z_4 = 1.3$. M_{41} and M_{42} are fuzzy sets of the fourth equation of (3.1) and $M_{41} + M_{42} = 1$.

Here, we call (3.2), (3.4), (3.6) and (3.8) the first liner subsystem under the fuzzy rules and (3.3), (3.5), (3.7) and (3.9) the second liner subsystem under the fuzzy rules.

The first linear subsystem is

$$\begin{cases} \dot{x}_1 = -2a_1Z_1 \\ \dot{x}_2 = -w_1(x_1 - x_3) + 2a_1x_1Z_2 \\ \dot{x}_3 = -2a_2Z_3 \\ \dot{x}_4 = -w_2(x_3 - x_1) + 2a_2x_3Z_4) \end{cases} \tag{3.10}$$

The second linear subsystem is

$$\begin{cases} \dot{x}_1 = 2a_1Z_1 \\ \dot{x}_2 = -w_1(x_1 - x_3) - 2a_1x_1Z_2 \\ \dot{x}_3 = 2a_2Z_3 \\ \dot{x}_4 = -w_2(x_3 - x_1) - 2a_2x_3Z_4) \end{cases} \tag{3.11}$$

The final output of the fuzzy Quantum-CNN system is inferred as follows and the chaotic behavior of fuzzy system is shown in Fig. 2.

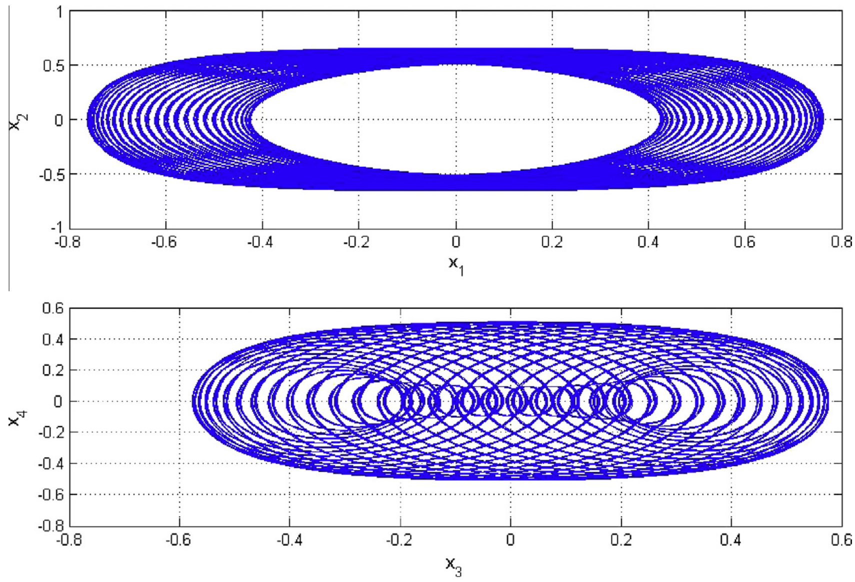

Fig. 2. Chaotic behavior of fuzzy quantum cellular neural networks nano system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} M_{11} \\ M_{21} \\ M_{31} \\ M_{41} \end{bmatrix}^T \begin{bmatrix} -2a_1Z_1 \\ -w_1(x_1-x_3)+2a_1x_1Z_2 \\ -2a_2Z_3 \\ -w_2(x_3-x_1)+2a_2x_3Z_4 \end{bmatrix} + \begin{bmatrix} M_{12} \\ M_{22} \\ M_{32} \\ M_{42} \end{bmatrix}^T \begin{bmatrix} 2a_1Z_1 \\ -w_1(x_1-x_3)-2a_1x_1Z_2 \\ 2a_2Z_3 \\ -w_2(x_3-x_1)-2a_2x_3Z_4 \end{bmatrix} \tag{3.12}$$

The T–S fuzzy model of Q-CNN system is also discussed in Appendix B for comparison which chaotic behaviors are shown in Fig. 3. Comparing Figs. 1–3, obviously, via using new fuzzy model, not only the number of fuzzy rules in Quantum-CNN system can be reduced from 2^4 to 2×4, but also the simulation results are perfectly the same to the original chaotic behavior of the Quantum-CNN system.

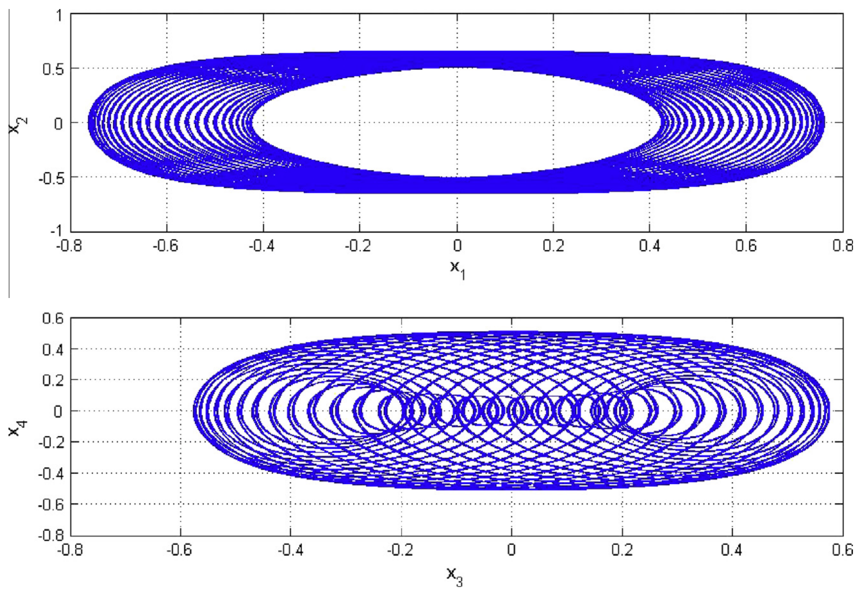

Fig. 3. Chaotic behavior of T–S fuzzy Q-CNN system.

3.2. Fuzzy modeling of M–V system

For M–V system:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -(a + by_3)y_1 - (a + by_3)y_1^3 - cy_2 + dy_3 \\ \dot{y}_3 = y_4 \\ \dot{y}_4 = -ey_3 + f(y_4 - y_3^2 y_4) + gy_1 \end{cases} \tag{3.13}$$

where *a, b, c, d, e, f, g* are the parameters. This system exhibits chaos as shown in Fig. 3 when the parameters of system are $a = 10, b = 3, c = 0.4, d = 70, e = 1, f = 5, g = 0.1$ and the initial states of system are $(y_{10}, y_{20}, y_{30}, y_{40}) = (0.1, -0.5, 0.1, -0.5)$.

Step of fuzzy modeling:

Before modeling, the right hand side of second equation of (3.13) can be divided into part A, $-(a + by_3)y_1 - cy_2 + dy_3$, and part B, $-(a + by_3)y_1^3$, i.e. $\dot{y}_2 = A + B$, where A and B are the index of these two parts.

Step 1:

Assume that $y_1 \in [-W_1, W_1]$ and $W_1 > 0$:

$$\text{Rule 1}: \text{ IF } y_1 \text{ is } N_{A21}, \text{ THEN } A = -(a + by_3)W_1 - cy_2 + dy_3 \tag{3.14}$$

$$\text{Rule 2}: \text{ IF } y_1 \text{ is } N_{A22}, \text{ THEN } A = (a + by_3)W_1 - cy_2 + dy_3 \tag{3.15}$$

where

$$N_{A21} = \frac{1}{2}\left(1 + \frac{y_1}{W_1}\right), \quad N_{A22} = \frac{1}{2}\left(1 - \frac{y_1}{W_1}\right)$$

and $W_1 = 6$. N_{A21} and N_{A22} are fuzzy sets of the second equation of (3.11) and $N_{A21} + N_{A22} = 1$.

Assume that $y_1^3 \in [-W_2, W_2]$ and $W_2 > 0$:

$$\text{Rule 1}: \text{ IF } y_1^3 \text{ is } N_{B21}, \text{ THEN } B = -(a + by_3)W_2 \tag{3.16}$$

$$\text{Rule 2}: \text{ IF } y_1^3 \text{ is } N_{B22}, \text{ THEN } B = (a + by_3)W_2 \tag{3.17}$$

where

$$N_{B21} = \frac{1}{2}\left(1 + \frac{y_1^3}{W_2}\right), \quad N_{B22} = \frac{1}{2}\left(1 - \frac{y_1^3}{W_2}\right)$$

and $W_2 = 140$. N_{B21} and N_{B22} are fuzzy sets of the second equation of (3.13) and $N_{B21} + N_{B22} = 1$.

The second equation of (3.13) can be exactly represented by new fuzzy model as following:

$$\text{Rule 1}: \text{ IF } y_1 \text{ is } N_{A21} \text{ and IF } y_1^3 \text{ is } N_{B21}, \text{ THEN } \dot{y}_2 = N_{A21}A + N_{B21}B \tag{3.18}$$

$$\text{Rule 2}: \text{ IF } y_1 \text{ is } N_{A22} \text{ and IF } y_1^3 \text{ is } N_{B22}, \text{ THEN } \dot{y}_2 = N_{A22}A + N_{B22}B \tag{3.19}$$

Step 2:

Assume that $y_3y_4 \in [-W_3, W_3]$ and $W_2 > 0$, then the fourth equation of (3.13) can be exactly represented by new fuzzy model as following:

$$\text{Rule 1}: \text{ IF } y_3y_4 \text{ is } N_{41}, \text{ THEN } \dot{y}_4 = -ey_3 + f(y_4 - W_3y_3) + gy_1 \tag{3.20}$$

$$\text{Rule 2}: \text{ IF } y_3y_4 \text{ is } N_{42}, \text{ THEN } \dot{y}_4 = -ey_3 + f(y_4 + W_3y_3) + gy_1 \tag{3.21}$$

where

$$N_{41} = \frac{1}{2}\left(1 + \frac{y_3y_4}{W_2}\right), \quad N_{42} = \frac{1}{2}\left(1 - \frac{y_3y_4}{W_2}\right)$$

and $W_2 = 1.6$. N_{41} and N_{42} are fuzzy set of the fourth equation of (3.13) and $N_{41} + N_{42} = 1$.

Here, we call (3.18) and (3.20) the first liner subsystems under the fuzzy rules and (3.19) and (3.21) the second liner subsystems under the fuzzy rules:

The first linear subsystem is

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = N_{A21}(-(a + by_3)W_1 - cy_2 + dy) + N_{B21}(-(a + by_3)W_2) \\ \dot{y}_3 = y_4 \\ \dot{y}_4 = -ey_3 + f(y_4 - W_3y_3) + gy_1 \end{cases} \tag{3.22}$$

The second linear subsystem is

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = N_{A22}((a+by_3)W_1 - cy_2 + dy_3) + N_{B22}((a+by_3)W_2) \\ \dot{y}_3 = y_4 \\ \dot{y}_4 = -ey_3 + f(y_4 + W_3y_3) + gy_1 \end{cases} \tag{3.23}$$

The final output of the new Mathieu–Van der Pol system is inferred as follows:

$$\begin{aligned} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} &= \begin{bmatrix} N_{11} \\ N_{21} \\ N_{31} \\ N_{41} \end{bmatrix}^T \begin{bmatrix} y_2 \\ N_{A21}(-(a+by_3)W_1 - cy_2 + dy) + N_{B21}(-(a+by_3)W_2 \\ y_4 \\ -ey_3 + f(y_4 - W_3y_3) + gy_1 \end{bmatrix} \\ &+ \begin{bmatrix} N_{12} \\ N_{22} \\ N_{32} \\ N_{42} \end{bmatrix}^T \begin{bmatrix} y_2 \\ N_{A22}((a+by_3)W_1 - cy_2 + dy_3) + N_{B22}((a+by_3)W_2 \\ y_4 \\ -ey_3 + f(y_4 + W_3y_3) + gy_1 \end{bmatrix} \end{aligned} \tag{3.24}$$

where $N_{11} = N_{12} = N_{31} = N_{32} = 0.5$ and $N_{21} = N_{22} = 1$. The chaotic behavior of fuzzy systems is shown in Fig. 5.

The T–S fuzzy model of M–V system is also discussed in Appendix B for comparison which chaotic behaviors are shown in Fig. 6. Comparing Figs. 4–6, obviously, via using the new fuzzy model, the number of fuzzy rules in new M–V system can be reduced from 2^3 to 2×3 and the simulation results are perfectly the same to the original chaotic behavior of the M–V system.

4. Pragmatical adaptive synchronization of different fuzzy chaoticsystems via new adaptive approach

In this section, there are two cases in numerical simulation results, where Quantum-CNN system is chosen as master system, and new Mathieu–Van der Pol system is regarded as slave system. In Case I, the new adaptive control law is used to control the membership function of slave system to trace the membership function of master system. In Case II, adaptive synchronization of the slave system and master system with all unknown parameters is achieved via the new adaptive control law.

Unlike traditional Lyapunov function – quadratic from, a new control Lyapunov function is proposed as follow:

$$V(e) = \exp(ke^Te) - 1 \tag{4.1}$$

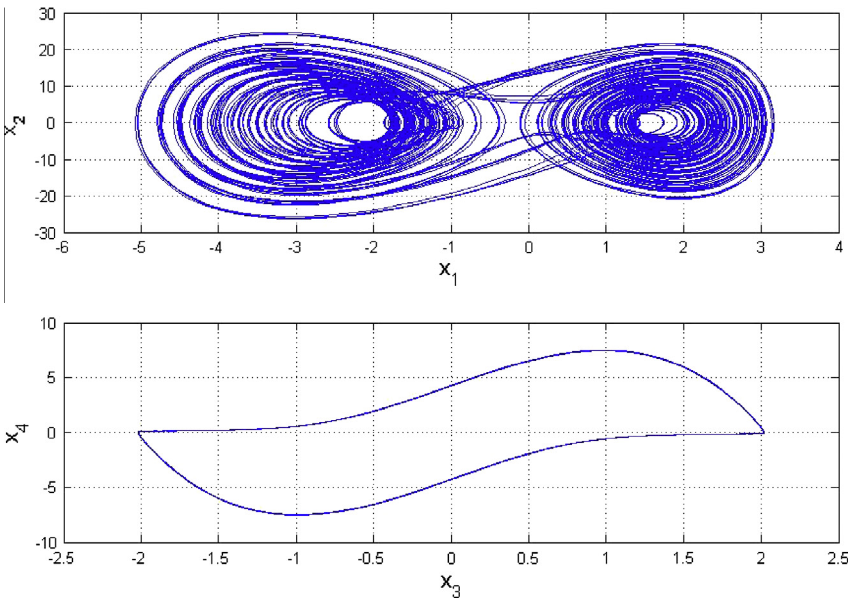

Fig. 4. Chaotic behavior of new Mathieu–Van der Pol system.

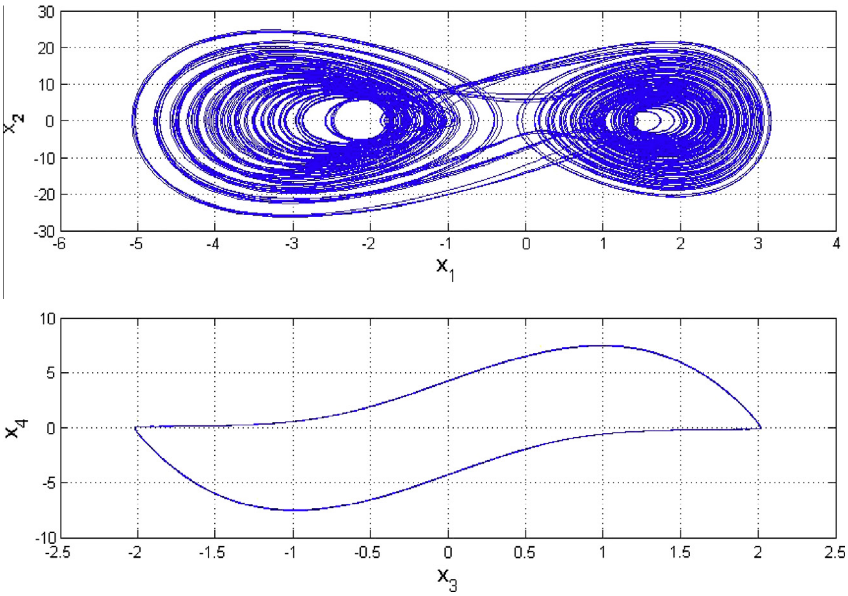

Fig. 5. Chaotic behavior of new fuzzy Mathieu–Van der Pol system.

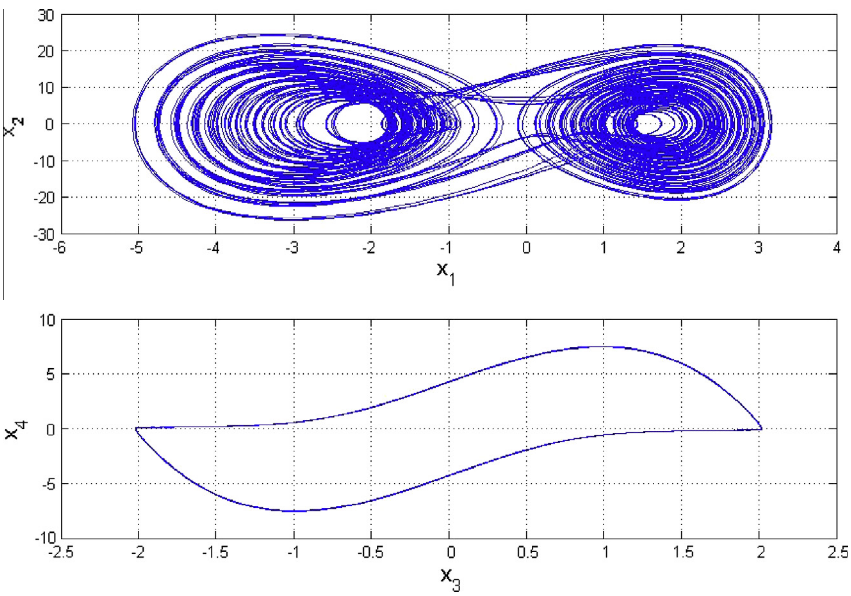

Fig. 6. Chaotic behavior of T–S fuzzy M–V system.

where *e* is the error dynamics. By designing the different forms of error states and parameters in this new control Lyapunov function, the errors of all unknown parameters are decayed exponentially and much more efficient to achieve the objective values.

Furthermore, in order to show the much better performance of our new strategy in adaptive synchronization of mismatch in parameters, the simulation results in Case II by using traditional adaptive method are also given for comparison in Appendix C.

Case I: Synchronization of different fuzzy chaotic systems with different numbers of fuzzy rules.

In order to achieve synchronization, the new fuzzy Mathieu–Van der Pol slave system (3.18) is expanded to a new form as follow:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} N_{11} \\ N_{21} \\ N_{31} \\ N_{41} \end{bmatrix}^T \begin{bmatrix} y_2 + u_{11} \\ N_{A21}(-(a + by_3)W_1 - cy_2 + dy) + N_{B21}(-(a + by_3)W_2 + u_{21} \\ y_4 + u_{31} \\ -ey_3 + f(y_4 - W_3y_3) + gy_1 + u_{41} \end{bmatrix} + \begin{bmatrix} N_{12} \\ N_{22} \\ N_{32} \\ N_{42} \end{bmatrix}^T \begin{bmatrix} y_2 + u_{12} \\ N_{A22}((a + by_3)W_1 - cy_2 + dy_3) + N_{B22}((a + by_3)W_2 + u_{22} \\ y_4 + u_{32} \\ -ey_3 + f(y_4 + W_3y_3) + gy_1 + u_{42} \end{bmatrix} \tag{4.2}$$

where

$$N_{11} = \widehat{P}_1 \times 0.5 + \widehat{Q}_1 \times S_{11}, \quad N_{12} = \widehat{P}_2 \times 0.5 + \widehat{Q}_2 \times S_{12}$$
$$N_{21} = \widehat{P}_1 \times 1 + \widehat{Q}_1 \times S_{21}, \quad N_{22} = \widehat{P}_2 \times 1 + \widehat{Q}_2 \times S_{22}$$
$$N_{31} = \widehat{P}_1 \times 0.5 + \widehat{Q}_1 \times S_{31}, \quad N_{32} = \widehat{P}_2 \times 0.5 + \widehat{Q}_2 \times S_{32}$$
$$N_{41} = \widehat{P}_1 \times \frac{1}{2}\left(1 + \frac{y_3y_4}{W_2}\right) + \widehat{Q}_1 \times S_{41}, \quad N_{42} = \widehat{P}_2 \times \frac{1}{2}\left(1 - \frac{y_3y_4}{W_2}\right) + \widehat{Q}_2 \times S_{42}$$

where $\widehat{P}_1, \widehat{P}_2, \widehat{Q}_1$ and $\widehat{Q}_2$ are estimated parameters and $(\widehat{P}_{10}, \widehat{P}_{20}, \widehat{Q}_{10}, \widehat{Q}_{20}) = (1, 1, 0, 0)$, the goals of the estimated parameters $\widehat{P}$ and $\widehat{Q}$ are 0 and 1. $S_{ij}, i = 1–4$ and $j = 1, 2$ are the fuzzy membership functions of the master system, here $S_{ij} = M_{ij}, i = 1–4$ and $j = 1, 2$.

The synchronizing processes are divided into two steps: (1) Using the first linear subsystem of the slave system in Eq. (4.2) to trace the trajectory of the first linear subsystem of the master system in (3.12). (2) Using the second linear subsystem of the slave system in Eq. (4.2) to trace the trajectory of the first linear subsystem of the master system in (3.12):

Step1: The error and error dynamics in first linear subsystem are

$$\begin{cases} e_1 = x_1 - y_1 \\ e_2 = x_2 - y_2 \\ e_3 = x_3 - y_3 \\ e_4 = x_4 - y_4 \end{cases}$$

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{y}_1 = -2a_1Z_1 - (y_2 + u_{11}) \\ \dot{e}_2 = \dot{x}_2 - \dot{y}_2 = -w_1(x_1 - x_3) + 2a_1x_1Z_2 \\ \qquad -(N_{A21}(-(a + by_3)W_1 - cy_2 + dy) \\ \qquad +N_{B21}(-(a + by_3)W_2) + u_{12}) \\ \dot{e}_3 = \dot{x}_3 - \dot{y}_3 = -2a_2Z_3 - (y_4 + u_{13}) \\ \dot{e}_4 = \dot{x}_4 - \dot{y}_4 = -w_2(x_3 - x_1) + 2a_2x_3Z_4 \\ \qquad -(-ey_3 + f(y_4 - W_3y_3) + gy_1 + u_{14}) \end{cases} \tag{4.3}$$

Choose a new control Lyapunov function of the form:

$$V_1 = \exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) - 1 + \frac{1}{2}\widetilde{P}_1^2 + \frac{1}{2}\widetilde{Q}_1^2 > 0 \tag{4.4}$$

where $k = 1.5, \widetilde{P}_1 = P_1 - \widehat{P}_1, \widetilde{Q}_1 = Q_1 - \widehat{Q}_1$ and $P_1 = 0, Q_1 = 1$.

Its time derivative through error dynamics (4.4) is

$$\begin{aligned} \dot{V}_1 &= 2k\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times (e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4) \\ &\quad + \widetilde{P}_1\dot{\widetilde{P}}_1 + \widetilde{Q}_1\dot{\widetilde{Q}}_1 \\ &= 2k\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times (e_1(-2a_1Z_1 - (y_2 + u_{11})) \\ &\quad + e_2(-w_1(x_1 - x_3) + 2a_1x_1Z_2 - (N_{A21}(-(a + by_3)W_1 - cy_2 + dy) \\ &\quad + N_{B21}(-(a + by_3)W_2) + u_{12})) + e_3(-2a_2Z_3 - (y_4 + u_{13})) \\ &\quad + e_4(-w_2(x_3 - x_1) + 2a_2x_3Z_4 - (-ey_3 + f(y_4 - W_3y_3) + gy_1 + u_{14}))) + \widetilde{P}_1\dot{\widetilde{P}}_1 + \widetilde{Q}_1\dot{\widetilde{Q}}_1 \end{aligned} \tag{4.5}$$

Choose

$$\begin{aligned}
&\dot{\widetilde{P}}_1 = -\dot{\widehat{P}}_1 = 2ke_1 \exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right) \times \widetilde{P}_1 \\
&\dot{\widetilde{Q}}_1 = -\dot{\widehat{Q}}_1 = 2ke_3 \exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right) \times \widetilde{Q}_1 \\
&u_{11} = -2a_1Z_1 - y_2 + \widehat{P}_1^2 + e_1 \\
&u_{12} = -w_1(x_1-x_3) + 2a_1x_1Z_2 - (N_{A21}(-(a+by_3)W_1 - cy_2 + dy) \\
&\qquad + N_{B21}(-(a+by_3)W_2)) + e_2 \\
&u_{13} = -2a_2Z_3 - y_4 + \widehat{Q}_1^2 + e_3 \\
&u_{14} = -w_2(x_3-x_1) + 2a_2x_3Z_4 - (-ey_3 + f(y_4 - W_3y_3) + gy_1
\end{aligned} \tag{4.6}$$

We obtain

$$\dot{V}_1 = 2k\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right) \times \left(-e_1^2 - e_2^2 - e_3^2 - e_4^2\right) < 0 \tag{4.7}$$

which is negative semi-definite function of $e_1, e_2, e_3, e_4, \widetilde{P}_1$ and $\widetilde{Q}_1$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that common origin of error dynamics (4.3) and parameter dynamics (4.6) is asymptotically stable. By pragmatical asymptotically stability theorem (see Appendix), D is a 6-manifold, $n = 6$ and the number of error state variables $p = 4$. When $e_1 = e_2 = e_3 = e_4 = 0$ and $\widetilde{P}_1, \widetilde{Q}_1$ take arbitrary values, $\dot{V}_1 = 0$, so X is of 2 dimensions, $m = n - p = 6 - 4 = 2, m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable.

Step2: The error and error dynamics in second linear subsystem are

$$\begin{cases} e_1 = x_1 - y_1 \\ e_2 = x_2 - y_2 \\ e_3 = x_3 - y_3 \\ e_4 = x_4 - y_4 \end{cases}$$

$$\begin{cases}
\dot{e}_1 = \dot{x}_1 - \dot{y}_1 = 2a_1Z_1 - (y_2 + u_{11}) \\
\dot{e}_2 = \dot{x}_2 - \dot{y}_2 = -w_1(x_1-x_3) - 2a_1x_1Z_2 - (N_{A22}((a+by_3)W_1 - cy_2 + dy) \\
\qquad\qquad + N_{B22}((a+by_3)W_2) + u_{12}) \\
\dot{e}_3 = \dot{x}_3 - \dot{y}_3 = 2a_2Z_3 - (y_4 + u_{13}) \\
\dot{e}_4 = \dot{x}_4 - \dot{y}_4 = -w_2(x_3-x_1) - 2a_2x_3Z_4 \\
\qquad\qquad -(-ey_3 + f(y_4 + W_3y_3) + gy_1 + u_{14})
\end{cases} \tag{4.8}$$

Choose a new control Lyapunov function of the form:

$$V_2 = \exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right) - 1 + \frac{1}{2}\widetilde{P}_2^2 + \frac{1}{2}\widetilde{Q}_2^2 > 0 \tag{4.9}$$

where $k = 1.5, \widetilde{P}_2 = P_2 - \widehat{P}_2, \widetilde{Q}_2 = Q_2 - \widehat{Q}_2$ and $P_2 = 0, Q_2 = 1$.

Its time derivative through error dynamics (4.4) is

$$\begin{aligned}
\dot{V}_2 &= 2k\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right) \times (e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4) + \widetilde{P}_2\dot{\widetilde{P}}_2 + \widetilde{Q}_2\dot{\widetilde{Q}}_2 \\
&= 2k\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right) \times (e_1(2a_1Z_1 - (y_2 + u_{21})) \\
&\quad + e_2(-w_1(x_1-x_3) - 2a_1x_1Z_2 - (N_{A22}((a+by_3)W_1 - cy_2 + dy) \\
&\quad + N_{B22}((a+by_3)W_2) + u_{22})) + e_3(2a_2Z_3 - (y_4 + u_{23})) \\
&\quad + e_4(-w_2(x_3-x_1) - 2a_2x_3Z_4 - (-ey_3 + f(y_4 + W_3y_3) + gy_1
\end{aligned} \tag{4.10}$$

Choose

$$\begin{aligned}
&\dot{\widetilde{P}}_2 = -\dot{\widehat{P}}_2 = 2ke_1 \exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right) \times \widetilde{P}_2 \\
&\dot{\widetilde{Q}}_2 = -\dot{\widehat{Q}}_2 = 2ke_3 \exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right) \times \widetilde{Q}_2 \\
&u_{21} = 2a_1Z_1 - y_2 + \widetilde{P}_2^2 + e_1 \\
&u_{22} = -w_1(x_1-x_3) - 2a_1x_1Z_2 - (N_{A22}((a+by_3)W_1 - cy_2 + dy) \\
&\qquad + N_{B22}((a+by_3)W_2)) + e_2 \\
&u_{23} = 2a_2Z_3 - y_4 + \widehat{Q}_2^2 + e_3 \\
&u_{24} = -w_2(x_3-x_1) - 2a_2x_3Z_4 - (-ey_3 + f(y_4 + W_3y_3) + gy_1) + e_4
\end{aligned} \tag{4.11}$$

We obtain

$$\dot V_2 = 2k\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\left(-e_1^2-e_2^2-e_3^2-e_4^2\right)<0 \tag{4.12}$$

which is negative semi-definite function of $e_1, e_2, e_3, e_4, \widetilde{P}_2$ and $\widetilde{Q}_2$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that common origin of error dynamics (4.8) and parameter dynamics (4.11) is asymptotically stable. By pragmatical asymptotically stability theorem (see Appendix), D is a 6-manifold, $n = 6$ and the number of error state variables $p = 4$. When $e_1 = e_2 = e_3 = e_4 = 0$ and $\widetilde{P}_2, \widetilde{Q}_2$ take arbitrary values, $\dot V_2 = 0$, so X is of 2 dimensions, $m = n - p = 6 - 4 = 2, m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable. After the steps 1 and 2, the two linear subsystems of the slave system can be synchronized to the two linear subsystems of the master system. It means that the chaos synchronization for these two fuzzy chaotic systems can be achieved. The simulation results are shown in Figs. 7 and 8.

Case II: Adaptive synchronization of different fuzzy chaotic systems with all unknown parameters.

In order to achieve synchronization, the new fuzzy Mathieu–Van der Pol system (3.18) is expanded to a new form as follow:

$$\begin{aligned}\begin{bmatrix}\dot y_1\\ \dot y_2\\ \dot y_3\\ \dot y_4\end{bmatrix} &= \begin{bmatrix}N_{11}\\ N_{21}\\ N_{31}\\ N_{41}\end{bmatrix}^T\begin{bmatrix}y_2+u_{11}\\ N_{A21}(-(\hat a+\hat b y_3)W_1-\hat c y_2+\hat d y)+N_{B21}(-(\hat a+\hat b y_3)W_2+u_{21}\\ y_4+u_{31}\\ -\hat e y_3+\hat f(y_4-W_3y_3)+\hat g y_1+u_{41}\end{bmatrix}\\ &\quad+\begin{bmatrix}N_{12}\\ N_{22}\\ N_{32}\\ N_{42}\end{bmatrix}^T\begin{bmatrix}y_2+u_{12}+2\hat a_1 Z_1\\ N_{A22}((\hat a+\hat b y_3)W_1-\hat c y_2+\hat d y_3)+N_{B22}((\hat a+\hat b y_3)W_2+u_{22}\\ y_4+u_{32}+2\hat a_2 Z_3\\ -\hat e y_3+\hat f(y_4+W_3y_3)+\hat g y_1+u_{42}\end{bmatrix}\end{aligned} \tag{4.13}$$

where

$$\begin{aligned}&N_{11}=\widehat{P}_1\times 0.5+\widehat{Q}_1\times S_{11},\quad N_{12}=\widehat{P}_2\times 0.5+\widehat{Q}_2\times S_{12}\\ &N_{21}=\widehat{P}_1\times 1+\widehat{Q}_1\times S_{21},\quad N_{22}=\widehat{P}_2\times 1+\widehat{Q}_2\times S_{22}\\ &N_{31}=\widehat{P}_1\times 0.5+\widehat{Q}_1\times S_{31},\quad N_{32}=\widehat{P}_2\times 0.5+\widehat{Q}_2\times S_{32}\\ &N_{41}=\widehat{P}_1\times\frac{1}{2}\left(1+\frac{y_3y_4}{W_2}\right)+\widehat{Q}_1\times S_{41},\quad N_{42}=\widehat{P}_2\times\frac{1}{2}\left(1-\frac{y_3y_4}{W_2}\right)+\widehat{Q}_2\times S_{42}\end{aligned}$$

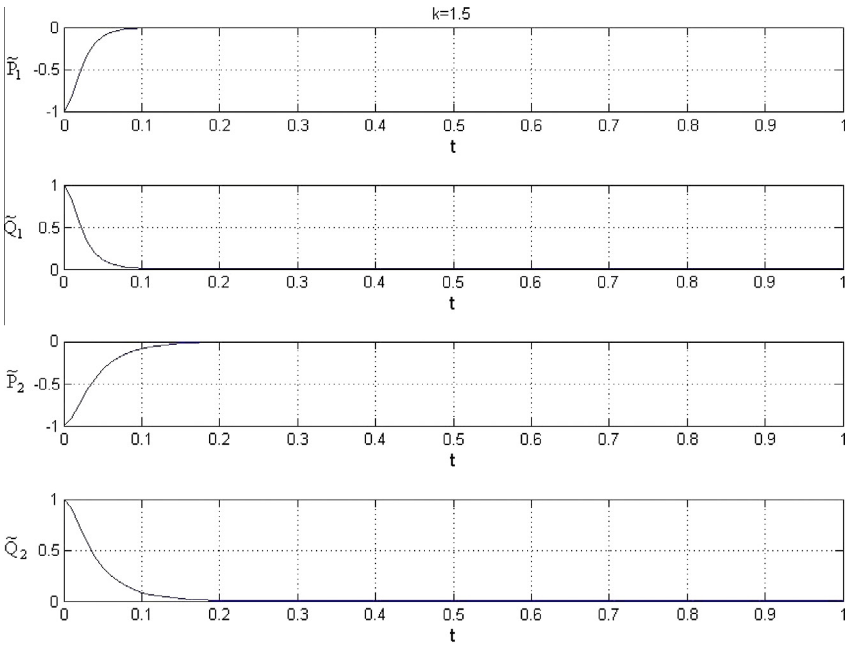

Fig. 7. Time histories of error of parameters $\widetilde{P}_1$, $\widetilde{P}_2$, $\widetilde{Q}_1$ and $\widetilde{Q}_2$ for Case I.

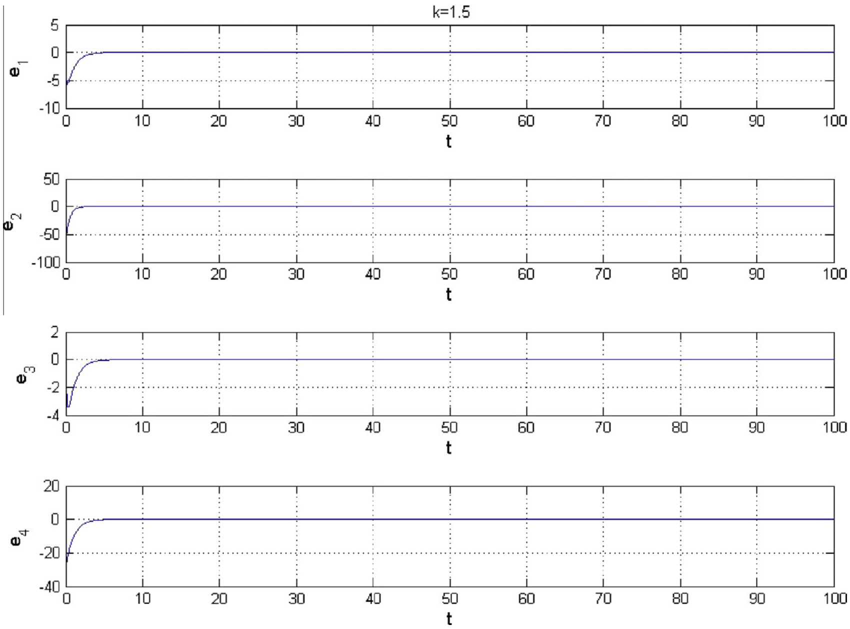

Fig. 8. Time histories of error for Case I.

where $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}, \hat{a}_1, \hat{a}_2, \hat{w}_1, \hat{w}_2$, $\widehat{P}_1, \widehat{P}_2, \widehat{Q}_1$ and $\widehat{Q}_2$ are estimated parameters and $\hat{a}_{10} = 0, \hat{a}_{20} = 0, \hat{w}_{10} = 0, w_{20} = 0$, $\hat{a}_0 = 10, \hat{b}_0 = 3, \hat{c}_0 = 0.4, \hat{d}_0 = 70, \hat{e}_0 = 1, \hat{f}_0 = 5, \hat{g}_0 = 0.1$ and $(\widehat{P}_{10}, \widehat{P}_{20}, \widehat{Q}_{10}, \widehat{Q}_{20}) = (1, 1, 0, 0)$. The goals of the estimated parameters $\widehat{P}$ and $\widehat{Q}$ are 0 and 1. $S_{ij}, i = 1–4$ and $j = 1, 2$ are the fuzzy membership functions of the master system, here $S_{ij} = M_{ij}, i = 1–4$ and $j = 1, 2$.

The synchronizing processes are divided into two steps: (1) Using the first linear subsystem of the slave system in Eq. (4.13) to trace the trajectory of the first linear subsystem of the master system in (3.12). (2) Using the second linear subsystem of the slave system in Eq. (4.13) to trace the trajectory of the first linear subsystem of the master system in (3.12):

Step1: The error and error dynamics in first linear subsystem are

$$\begin{cases} e_1 = x_1 - y_1 \\ e_2 = x_2 - y_2 \\ e_3 = x_3 - y_3 \\ e_4 = x_4 - y_4 \end{cases}$$

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{y}_1 = -2a_1 Z_1 - (y_2 + u_{11}) \\ \dot{e}_2 = \dot{x}_2 - \dot{y}_2 = -w_1(x_1 - x_3) + 2a_1 x_1 Z_2 \\ \qquad -(N_{A21}(-(\hat{a} + \hat{b}y_3)W_1 - \hat{c}y_2 + \hat{d}y) \\ \qquad +N_{B21}(-(\hat{a} + \hat{b}y_3)W_2) + u_{12}) \\ \dot{e}_3 = \dot{x}_3 - \dot{y}_3 = -2a_2 Z_3 - (y_4 + u_{13}) \\ \dot{e}_4 = \dot{x}_4 - \dot{y}_4 = -w_2(x_3 - x_1) + 2a_2 x_3 Z_4 \\ \qquad -(-\hat{e}y_3 + \hat{f}(y_4 - W_3 y_3) + \hat{g}y_1 + u_{14}) \end{cases} \tag{4.14}$$

Choose a new control Lyapunov function of the form:

$$V_1 = \exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) - 1 + \frac{1}{2}\left(\widetilde{P}_1^2 + \widetilde{Q}_1^2 + \tilde{a}_1^2 + \tilde{a}_2^2 + \tilde{w}_1^2 + \tilde{w}_2^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{e}^2 + \tilde{f}^2 + \tilde{g}^2\right) > 0 \tag{4.15}$$

where $k = 1.5, \widetilde{P}_1 = P_1 - \widehat{P}_1, \widetilde{Q}_1 = Q_1 - \widehat{Q}_1$, $\tilde{a}_1 = a_1 - \hat{a}_1, \tilde{a}_2 = a_2 - \hat{a}_2$, $\tilde{w}_1 = w_1 - \hat{w}_1, \tilde{w}_2 = w_2 - \hat{w}_2, \tilde{a} = a - \hat{a}, \tilde{b} = b - \hat{b}$, $\tilde{c} = c - \hat{c}, \tilde{d} = d - \hat{d}, \tilde{e} = e - \hat{e}, \tilde{f} = f - \hat{f}, \tilde{g} = g - \hat{g}$ and the goal of parameters:$P_1 = 0, Q_1 = 1, a_1 = 6.8, a_2 = 4.3$, $w_1 = 4.7, w_2 = 3.9$, $a = b = c = d = e = f = g = 0$.

Its time derivative through error dynamics (4.4) is

$$\begin{aligned}\dot{V}_1 &= 2k\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times(e_1\dot{e}_1+e_2\dot{e}_2+e_3\dot{e}_3+e_4\dot{e}_4)+\tilde{a}_1\dot{\tilde{a}}_1+\tilde{a}_2\dot{\tilde{a}}_2+\tilde{w}_1\dot{\tilde{w}}_1+\tilde{w}_2\dot{\tilde{w}}_2+\tilde{a}\dot{\tilde{a}}\\ &\quad+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{d}\dot{\tilde{d}}+\tilde{e}\dot{\tilde{e}}+\tilde{f}\dot{\tilde{f}}+\tilde{g}\dot{\tilde{g}}+\widetilde{P}_1\dot{\widetilde{P}}_1+\widetilde{Q}_1\dot{\widetilde{Q}}_1\\ &= 2k\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times(e_1(-2a_1Z_1-(y_2+u_{11}))+e_2(-w_1(x_1-x_3)+2a_1x_1Z_2\\ &\quad-(N_{A21}(-(\hat{a}+\hat{b}y_3)W_1-\hat{c}y_2+\hat{d}y)+N_{B21}(-(\hat{a}+\hat{b}y_3)W_2)+u_{12}))\\ &\quad+e_3(-2a_2Z_3-(y_4+u_{13}))+e_4(-w_2(x_3-x_1)+2a_2x_3Z_4\\ &\quad-(-\hat{e}y_3+\hat{f}(y_4-W_3y_3)+\hat{g}y_1+u_{14}))+\tilde{a}_1\dot{\tilde{a}}_1+\tilde{a}_2\dot{\tilde{a}}_2+\tilde{w}_1\dot{\tilde{w}}_1\\ &\quad+\tilde{w}_2\dot{\tilde{w}}_2+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{d}\dot{\tilde{d}}+\tilde{e}\dot{\tilde{e}}+\tilde{f}\dot{\tilde{f}}+\tilde{g}\dot{\tilde{g}}+\widetilde{P}_1\dot{\widetilde{P}}_1+\widetilde{Q}_1\dot{\widetilde{Q}}_1\end{aligned}\tag{4.16}$$

Choose

$$\begin{aligned}&\dot{\widetilde{P}}_1=-\dot{\widehat{P}}_1=2ke_1\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\widetilde{P}_1\\ &\dot{\widetilde{Q}}_1=-\dot{\widehat{Q}}_1=2ke_3\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\widetilde{Q}_1\\ &\dot{\tilde{a}}_1=-\dot{\hat{a}}_1=2ke_1\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{a}_1\\ &\dot{\tilde{a}}_2=-\dot{\hat{a}}_2=2ke_3\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{a}_2\\ &\dot{\tilde{w}}_1=-\dot{\hat{w}}_1=2ke_2\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{w}_1\\ &\dot{\tilde{w}}_2=-\dot{\hat{w}}_2=2ke_4\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{w}_2\\ &\dot{\tilde{a}}=-\dot{\hat{a}}=2ke_2\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{a}\\ &\dot{\tilde{b}}=-\dot{\hat{b}}=2ke_2\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{b}\\ &\dot{\tilde{c}}=-\dot{\hat{c}}=2ke_2\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{c}\\ &\dot{\tilde{d}}=-\dot{\hat{d}}=2ke_4\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{d}\\ &\dot{\tilde{e}}=-\dot{\hat{e}}=2ke_4\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{e}\\ &\dot{\tilde{f}}=-\dot{\hat{f}}=2ke_4\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{f}\\ &\dot{\tilde{g}}=-\dot{\hat{g}}=2ke_4\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\tilde{g}\end{aligned}$$

$$\begin{aligned}u_{11}&=-2a_1Z_1-y_2+\tilde{a}_1^2+\widehat{P}_1^2+e_1\\ u_{12}&=-w_1(x_1-x_3)+2a_1x_1Z_2-(N_{A21}(-(a+by_3)W_1-cy_2+dy)\\ &\quad+N_{B21}(-(a+by_3)W_2))+\tilde{w}_1^2+\tilde{a}^2+\tilde{b}^2+\tilde{c}^2+\tilde{d}^2+e_2\\ u_{13}&=-2a_2Z_3-y_4+\tilde{a}_2^2+\widetilde{Q}_1^2+e_3\\ u_{14}&=-w_2(x_3-x_1)+2a_2x_3Z_4-(-ey_3+f(y_4-W_3y_3)+gy_1)\\ &\quad+\tilde{w}_2^2+\tilde{e}^2+\tilde{f}^2+\tilde{g}^2+e_4\end{aligned}\tag{4.17}$$

We obtain

$$\dot{V}_1=2k\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times(-e_1^2-e_2^2-e_3^2-e_4^2)<0\tag{4.18}$$

which is negative semi-definite function of e_1, e_2, e_3, e_4, $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}, \hat{a}_1, \hat{a}_2, \hat{w}_1, \hat{w}_2, \widehat{P}_1$ and $\widehat{Q}_1$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that common origin of error dynamics (4.14) and parameter dynamics (4.17) is asymptotically stable. By pragmatical asymptotically stability theorem (see Appendix), D is a 17-manifold, $n = 17$ and the number of error state variables $p = 4$. When $e_1 = e_2 = e_3 = e_4 = 0$ and $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}$, $\hat{a}_1, \hat{a}_2, \hat{w}_1, \hat{w}_2, \widehat{P}_1, \widehat{Q}_1$ take arbitrary values, $\dot{V}_1 = 0$, so X is of 13 dimensions, $m = n - p = 6 - 4 = 13, m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable.

Step2: The error and error dynamics in second linear subsystem are

$$\begin{cases}e_1=x_1-y_1\\ e_2=x_2-y_2\\ e_3=x_3-y_3\\ e_4=x_4-y_4\end{cases}$$

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{y}_1 = 2a_1Z_1 - (y_2 + u_{11}) \\ \dot{e}_2 = \dot{x}_2 - \dot{y}_2 = -w_1(x_1 - x_3) - 2a_1x_1Z_2 \\ \qquad -(N_{A21}((\hat{a} + \hat{b}y_3)W_1 - \hat{c}y_2 + \hat{d}y) \\ \qquad +N_{B21}((\hat{a} + \hat{b}y_3)W_2) + u_{12}) \\ \dot{e}_3 = \dot{x}_3 - \dot{y}_3 = 2a_2Z_3 - (y_4 + u_{13}) \\ \dot{e}_4 = \dot{x}_4 - \dot{y}_4 = -w_2(x_3 - x_1) - 2a_2x_3Z_4 \\ \qquad -(-\hat{e}y_3 + \hat{f}(y_4 + W_3y_3) + \hat{g}y_1 + u_{14}) \end{cases} \tag{4.19}$$

Choose a new control Lyapunov function of the form:

$$V_2 = \exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) - 1 + \frac{1}{2}\left(\widetilde{P}_2^2 + \widetilde{Q}_2^2 + \tilde{a}_1^2 + \tilde{a}_2^2 + \tilde{w}_1^2 + \tilde{w}_2^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{e}^2 + \tilde{f}^2 + \tilde{g}^2\right) > 0 \tag{4.20}$$

where $k = 1.5, \widetilde{P}_2 = P_2 - \widehat{P}_2, \widetilde{Q}_2 = Q_2 - \widehat{Q}_2$, $\tilde{a}_1 = a_1 - \hat{a}_1, \tilde{a}_2 = a_2 - \hat{a}_2, \tilde{w}_1 = w_1 - \hat{w}_1$, $\tilde{w}_2 = w_2 - \hat{w}_2, \tilde{a} = a - \hat{a}, \tilde{b} = b - \hat{b}$, $\tilde{c} = c - \hat{c}, \tilde{d} = d - \hat{d}, \tilde{e} = e - \hat{e}$, $\tilde{f} = f - \hat{f}, \tilde{g} = g - \hat{g}$ and the goal of parameters:$P_2 = 0, Q_2 = 1, a_1 = 6.8, a_2 = 4.3$, $w_1 = 4.7, w_2 = 3.9$, $a = b = c = d = e = f = g = 0$.

Its time derivative through error dynamics (4.19) is

$$\begin{aligned} \dot{V}_2 &= 2k\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times (e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4) \\ &\quad + \tilde{a}_1\dot{\tilde{a}}_1 + \tilde{a}_2\dot{\tilde{a}}_2 + \tilde{w}_1\dot{\tilde{w}}_1 + \tilde{w}_2\dot{\tilde{w}}_2 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{e}\dot{\tilde{e}} + \tilde{f}\dot{\tilde{f}} + \tilde{g}\dot{\tilde{g}} + \widetilde{P}_2\dot{\widetilde{P}}_2 + \widetilde{Q}_2\dot{\widetilde{Q}}_2 \\ &= 2k\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times (e_1(2a_1Z_1 - (y_2 + u_{11})) \\ &\quad + e_2(-w_1(x_1 - x_3) - 2a_1x_1Z_2 - (N_{A21}((\hat{a} + \hat{b}y_3)W_1 - \hat{c}y_2 + \hat{d}y) \\ &\quad + N_{B21}((\hat{a} + \hat{b}y_3)W_2) + u_{12})) \\ &\quad + e_3(2a_2Z_3 - (y_4 + u_{13})) + e_4(-w_2(x_3 - x_1) - 2a_2x_3Z_4 \\ &\quad - (-\hat{e}y_3 + \hat{f}(y_4 + W_3y_3) + \hat{g}y_1 + u_{14})) + \tilde{a}_1\dot{\tilde{a}}_1 + \tilde{a}_2\dot{\tilde{a}}_2 + \tilde{w}_1\dot{\tilde{w}}_1 \\ &\quad + \tilde{w}_2\dot{\tilde{w}}_2 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{e}\dot{\tilde{e}} + \tilde{f}\dot{\tilde{f}} + \tilde{g}\dot{\tilde{g}} + \widetilde{P}_2\dot{\widetilde{P}}_2 + \widetilde{Q}_2\dot{\widetilde{Q}}_2 \end{aligned} \tag{4.21}$$

Choose

$$\begin{aligned} \dot{\widetilde{P}}_2 &= -\dot{\widehat{P}}_2 = 2ke_1\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \widetilde{P}_2 \\ \dot{\widetilde{Q}}_2 &= -\dot{\widehat{Q}}_2 = 2ke_3\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \widetilde{Q}_2 \\ \dot{\tilde{a}}_1 &= -\dot{\hat{a}}_1 = 2ke_1\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{a}_1 \\ \dot{\tilde{a}}_2 &= -\dot{\hat{a}}_2 = 2ke_3\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{a}_2 \\ \dot{\tilde{w}}_1 &= -\dot{\hat{w}}_1 = 2ke_2\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{w}_1 \\ \dot{\tilde{w}}_2 &= -\dot{\hat{w}}_2 = 2ke_4\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{w}_2 \\ \dot{\tilde{a}} &= -\dot{\hat{a}} = 2ke_2\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{a} \\ \dot{\tilde{b}} &= -\dot{\hat{b}} = 2ke_2\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{b} \\ \dot{\tilde{c}} &= -\dot{\hat{c}} = 2ke_2\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{c} \\ \dot{\tilde{d}} &= -\dot{\hat{d}} = 2ke_4\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{d} \\ \dot{\tilde{e}} &= -\dot{\hat{e}} = 2ke_4\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{e} \\ \dot{\tilde{f}} &= -\dot{\hat{f}} = 2ke_4\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{f} \\ \dot{\tilde{g}} &= -\dot{\hat{g}} = 2ke_4\exp\left(k(e_1^2 + e_2^2 + e_3^2 + e_4^2)\right) \times \tilde{g} \end{aligned}$$

$$\begin{aligned} u_{11} &= 2a_1Z_1 - y_2 + \tilde{a}_1^2 + \widetilde{P}_1^2 + e_1 \\ u_{12} &= -w_1(x_1 - x_3) - 2a_1x_1Z_2 - (N_{A21}((a + by_3)W_1 - cy_2 + dy) \\ &\quad + N_{B21}((a + by_3)W_2)) + \tilde{w}_1^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + e_2 \\ u_{13} &= 2a_2Z_3 - y_4 + \tilde{a}_2^2 + \widetilde{Q}_1^2 + e_3 \\ u_{14} &= -w_2(x_3 - x_1) - 2a_2x_3Z_4 - (-ey_3 + f(y_4 + W_3y_3) + gy_1) \\ &\quad + \tilde{w}_2^2 + \tilde{e}^2 + \tilde{f}^2 + \tilde{g}^2 + e_4 \end{aligned} \tag{4.22}$$

We obtain

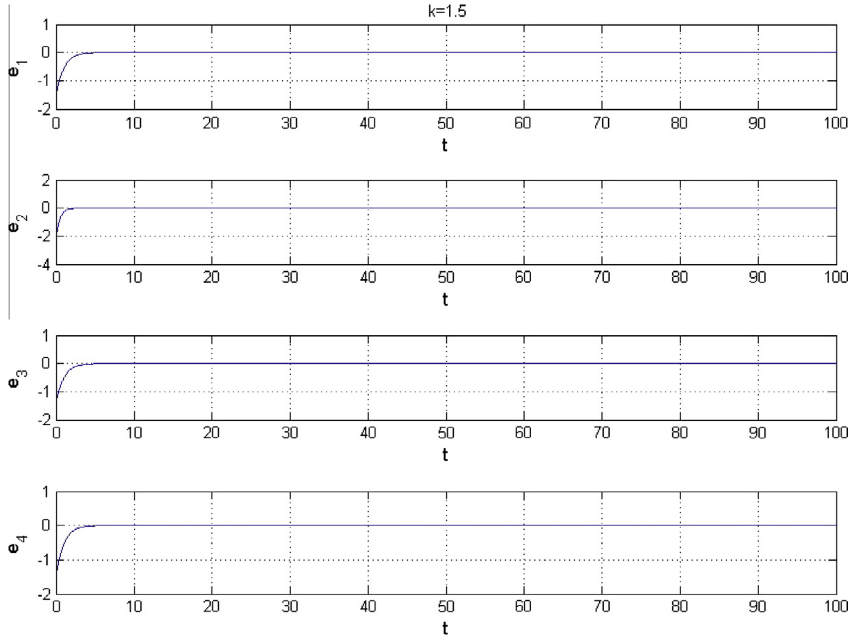

Fig. 9. Time histories of error for Case II.

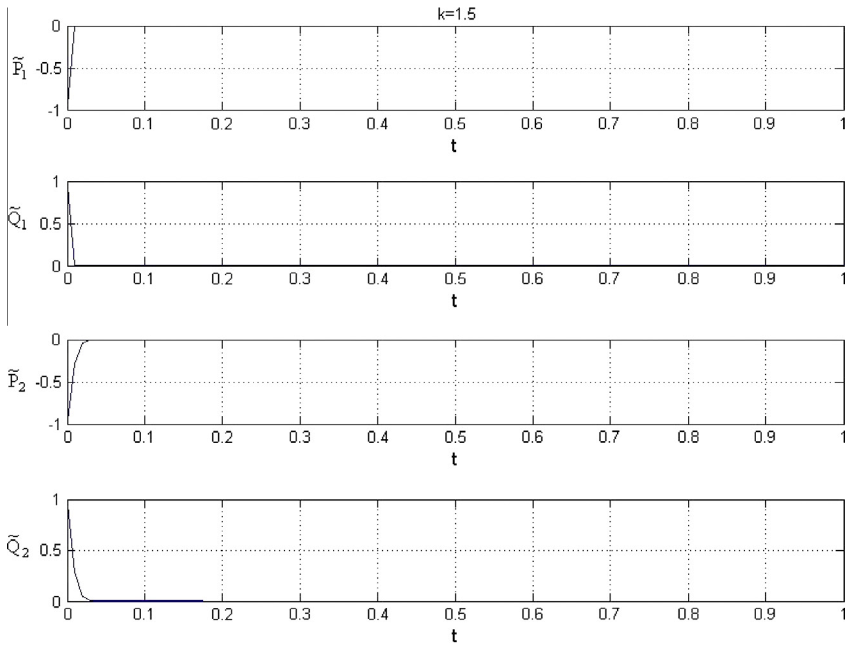

Fig. 10. Time histories of error of parameters $\widetilde{P}_1$, $\widetilde{P}_2$, $\widetilde{Q}_1$ and $\widetilde{Q}_2$ for Case II-1.

$$\dot{V}_2 = 2k\exp\left(k(e_1^2+e_2^2+e_3^2+e_4^2)\right)\times\left(-e_1^2-e_2^2-e_3^2-e_4^2\right) < 0 \tag{4.18}$$

which is negative semi-definite function of $e_1, e_2, e_3, e_4, \hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}, \hat{a}_1, \hat{a}_2, \hat{w}_1, \hat{w}_2, \widehat{P}_2$ and $\widehat{Q}_2$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain that common origin of error dynamics (4.19) and parameter dynamics (4.22) is asymptotically stable. By pragmatical asymptotically stability theorem (see Appendix), D is a 17-manifold, $n = 17$ and the number of error state variables $p = 4$. When $e_1 = e_2 = e_3 = e_4 = 0$ and $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}, \hat{a}_1, \hat{a}_2, \hat{w}_1, \hat{w}_2, \widehat{P}_2, \widehat{Q}_2$ take arbitrary values, $\dot{V}_1 = 0$, so X is of 13 dimensions, $m = n - p = 6 - 4 = 13, m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain

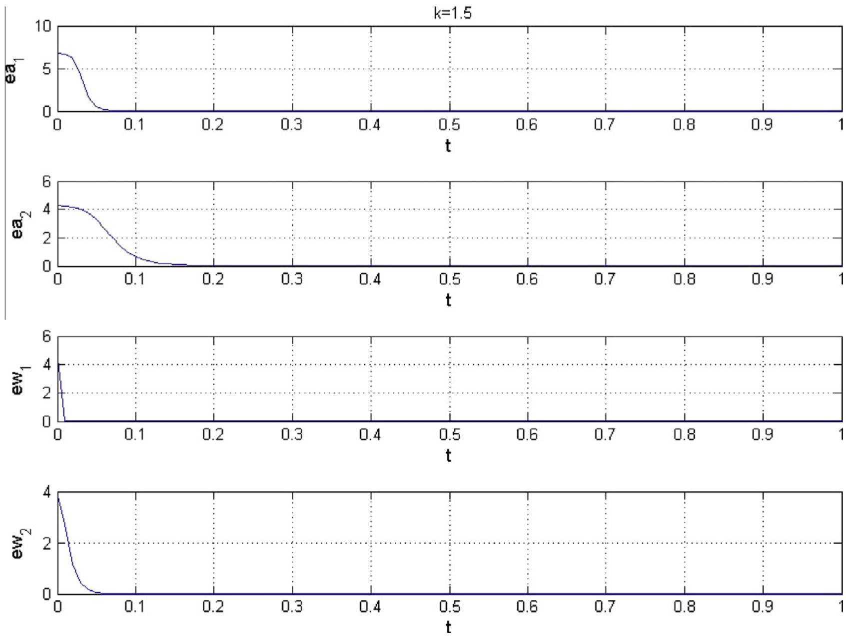

Fig. 11. Time histories of error of parameters for Case II-4.

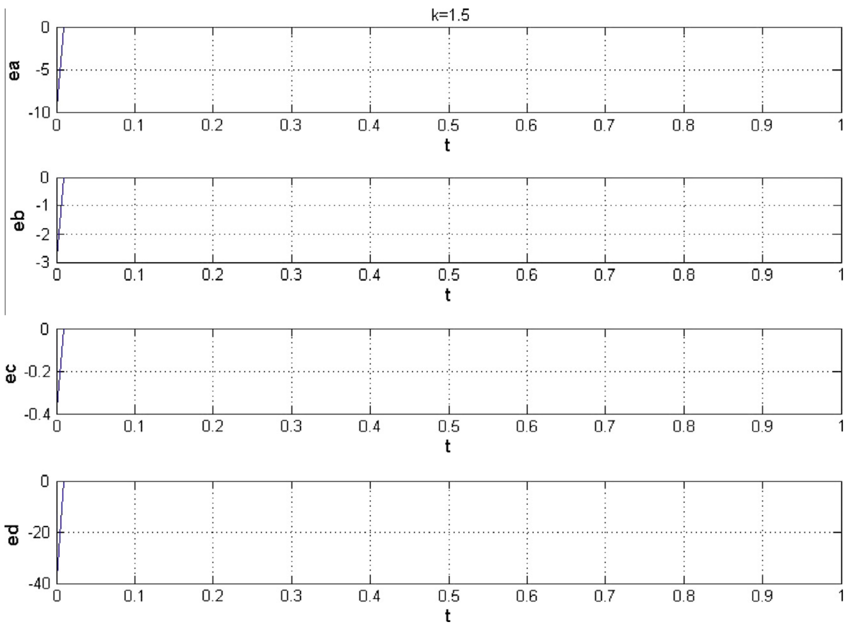

Fig. 12. Time histories of error of parameters for Case II-3.

parameters. The equilibrium point is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable. After the steps 1 and 2, the two linear subsystems of the slave system can be synchronized to the two linear subsystems of the master system. It means that the chaos synchronization for these two fuzzy chaotic systems can be achieved. The simulation results are shown in Figs. 9–13.

Comparison:

In order to show the better performance of our new adaptive approach, the simulation results which are derived in Appendix C are shown in Figs. 14–16.

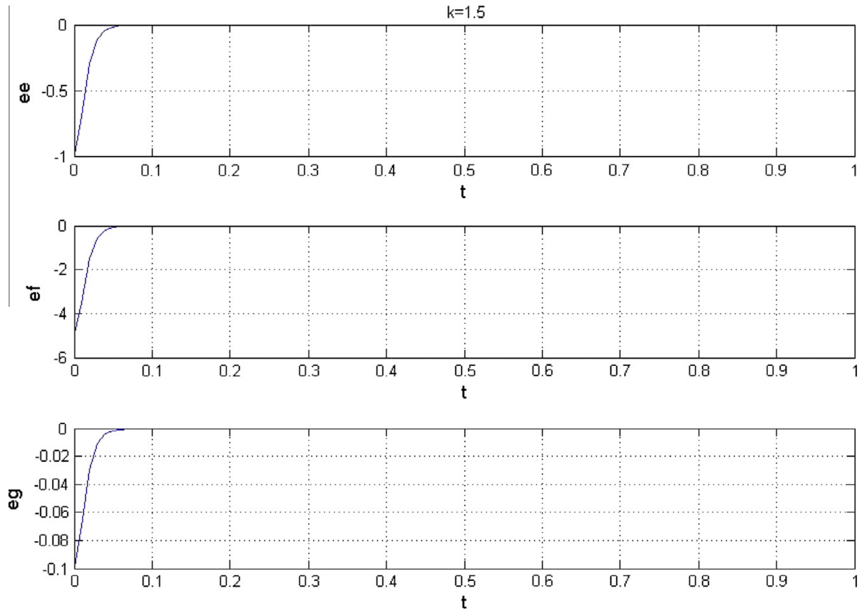

Fig. 13. Time histories of error of parameters for Case II-4.

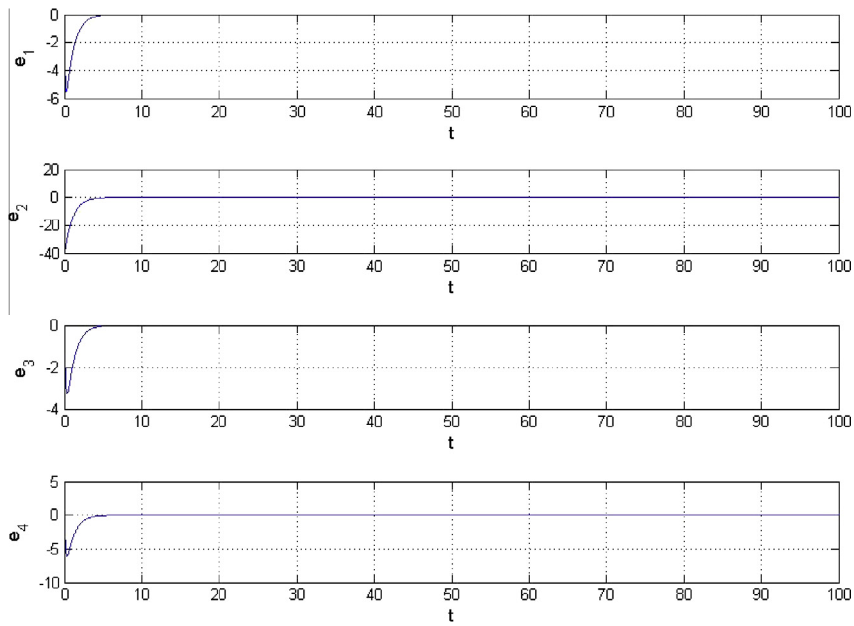

Fig. 14. Time histories of errors for Case II – traditional method.

(1) Errors of parameters $\tilde{a}_1, \tilde{a}_2, \tilde{w}_1$ and $\tilde{w}_2$:
In Figs. 11 and 15, the errors of parameters via the new adaptive approach are achieved zero points in less than 0.2 s and the traditional ones are in around 2 s.

(2) Errors of parameters $\tilde{a}, \tilde{b}, \tilde{c}$ and $\tilde{d}$:
In Figs. 12 and 16, the errors of parameters via the new adaptive approach are achieved zero points in less than 0.025 s and the traditional ones are in around 0.2 s.

(3) Errors of parameters $\tilde{a}, \tilde{a}_2, \tilde{w}_1$ and $\tilde{w}_2$:
In Figs. 12 and 16, the errors of parameters via the new adaptive approach are achieved zero points in less than 0.1 s and the traditional ones are in around 3 s.

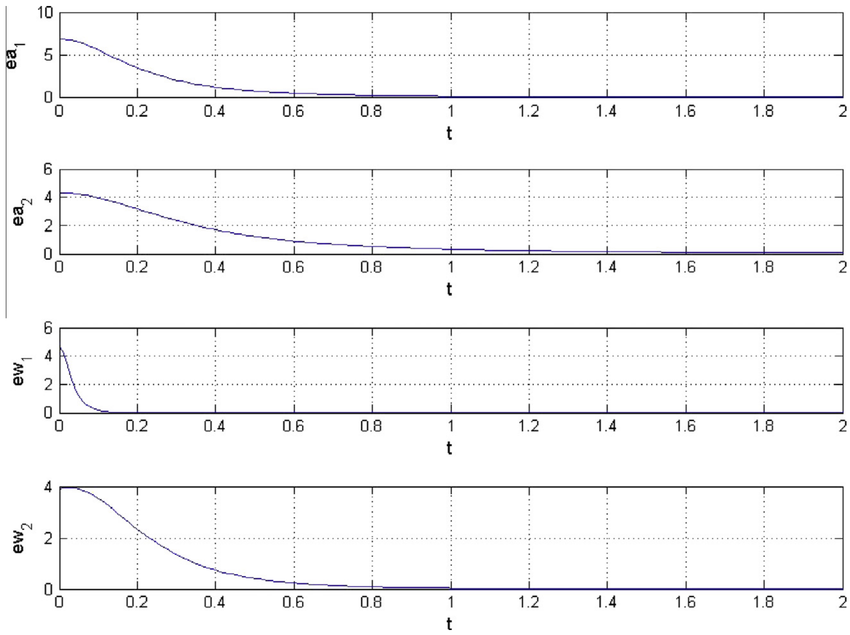

Fig. 15. Time histories of error of parameters for Case II-2 – traditional method.

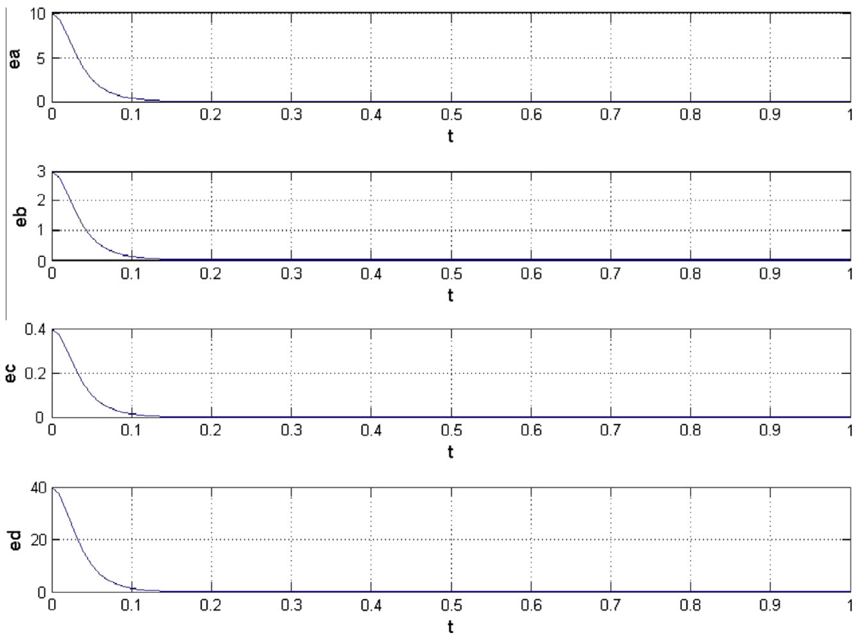

Fig. 16. Time histories of error of parameters for Case II-3 – traditional method.

As a consequence, it is obvious that adaptive synchronization via our new adaptive approach is much more efficient than traditional adaptive method.

5. Conclusions

In this paper, a new fuzzy model, a new adaptive approach and a new control Lyapunov function are proposed to efficiently achieve adaptive synchronization of two different fuzzy chaotic systems. There are two main advantages in this article: (1) Via the new fuzzy model, the fuzzy equations become a much simpler form and the numbers of fuzzy rules of chaotic systems can be reduced from 2^N to $2 \times N$, above all, the chaotic behaviors in simulation results are perfectly similar to original nonlinear system and T–S fuzzy system and (2) through the new adaptive approach, the performance of achieving adaptive synchronization is hugely improved, especially in errors of parameters. The new fuzzy model and the new adaptive control scheme presented in this paper are definitely two potential tools and can be used to various kinds of applications in fuzzy logic control or fuzzy modeling.

Acknowledgment

This research was supported by the National Science Council, Republic of China, under Grant No. NSC 96-2221-E-009-145-MY3.

Appendix A. Pragmatical adaptive control scheme

Consider the following chaotic system

$$\dot{x} = f(x, A) + u(t) \tag{A.1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ denotes a state vector, $A = [A_1, A_2, \ldots, A_m] \in R^m$ is a original coefficient vector, and f is a vector function, and $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in R^n$ is a control input vector.

The goal system which can be either chaotic or nonchaotic, is

$$\dot{y} = g(y, \widehat{B}) \tag{A.2}$$

where $y = [y_1, y_2, \ldots, y_n]^T \in R^n$ denotes a state vector, $\widehat{B} = [\widehat{B}_1, \widehat{B}_2, \ldots, \widehat{B}_P]^T \in R^p$ is a goal coefficient vector, and g is a vector function. Our goal is to design an adaptive control method and a controller $u(t)$ so that the state vector of the chaotic system (A.1) asymptotically approaches the state vector of the goal system (A.2).

The chaos control is accomplished in the sense that the limit of the error vector $e(t) = [e_1, e_2, \ldots, e_n]^T$ approaches zero:

$$\lim_{t \to \infty} e = 0 \tag{A.3}$$

where

$$e = y - x \tag{A.4}$$

From Eq. (A.4) we have

$$\dot{e} = \dot{y} - \dot{x} \tag{A.5}$$

$$\dot{e} = g(y, \widehat{B}) - f(x, A) - u(t) \tag{A.6}$$

A Lyapnuov function $V(e, \widehat{A}, \widetilde{B})$ is chosen as a positive definite function

$$V(e, \widetilde{A}, \widetilde{B}) = \frac{1}{2} e^T e + \frac{1}{2} \widetilde{A}^T \widetilde{A} + \frac{1}{2} \widetilde{B}^T \widetilde{B} \tag{A.7}$$

where $\widetilde{A} = A - \widehat{A}, \widetilde{B} = B - \widehat{B}, A$ and B are two column matrices whose elements are the original coefficients of systems (A.1) and (A.2) respectively, $\widehat{A}, \widehat{B}$ are two column matrices whose elements are the goal coefficients of systems (A.1) and (A.2) respectively.

Its derivative along any solution of the differential equation system consisting of Eq. (A.6) and update parameter differential equations for $\widetilde{A}$ and $\widetilde{B}$ is

$$\dot{V}(e) = e^T[g(y, \widehat{B}) - f(x, A) - u(t)] + \widetilde{A}\dot{\widetilde{A}} + \widetilde{B}\dot{\widetilde{B}} \tag{A.8}$$

where $u(t), \dot{\widetilde{A}}$, and $\dot{\widetilde{B}}$ are chosen so that $\dot{V} = e^T C e, C$ is a diagonal negative definite matrix, and $\dot{V}$ is a negative semi-definite function of e and parameter differences $\widetilde{A}$ and $\widetilde{B}$. In current scheme of adaptive control of chaotic motion [26–28], traditional Lyapunov stability theorem and Babalat lemma are used to prove the error vector approaches zero, as time approaches infinity. But the question, why the estimated or given parameters also approach to the uncertain or goal parameters, remains no answer. By pragmatical asymptotical stability theorem, the question can be answered strictly.

The stability for many problems in real dynamical systems is actual asymptotical stability, although may not be mathematical asymptotical stability. The mathematical asymptotical stability demands that trajectories from all initial states in the neighborhood of zero solution must approach the origin as $t \to \infty$. If there are only a small part or even a few of the initial states from which the trajectories do not approach the origin as $t \to \infty$, the zero solution is not mathematically asymptotically stable. However, when the probability of occurrence of an event is zero, it means the event does not occur actually. If the probability of occurrence of the event that the trajectries from the initial states are that they do not approach zero when $t \to \infty$, is zero, the stability of zero solution is actual asymptotical stability though it is not mathematical asymptotical stability. In order to analyze the asymptotical stability of the equilibrium point of such systems, the pragmatical asymptotical stability theorem is used.

Let X and Y be two manifolds of dimensions m and n $(m < n)$, respectively, and φ be a differentiable map from X to Y, then $\varphi(X)$ is subset of Lebesque measure 0 of Y [20]. For an autonomous system

$$\frac{dx}{dt} = f(x_1, \ldots, x_n) \tag{A.9}$$

where $x = [x_1, \ldots, x_n]^T$ is a state vector, the function $f = [f_1, \ldots, f_n]^T$ is defined on $D \subset R^n$ and $\|x\| \leqslant H > 0$. Let $x = 0$ be an equilibrium point for the system (A.9). Then

$$f(0) = 0 \tag{A.10}$$

Definition. The equilibrium point for the system (A.9) is pragmatically asymptotically stable provided that with initial points on C which is a subset of Lebesque measure 0 of D, the behaviors of the corresponding trajectories cannot be determined, while with initial points on $D - C$, the corresponding trajectories behave as that agree with traditional asymptotical stability [18,19].

Theorem. *Let $V = [x_1, \ldots, x_n]^T : D \to R_+$ be positive definite and analytic on D, such that the derivative of V through Eq. (A.9), $\dot{V}$, is negative semi-definite.*

Let X be the m-manifold consisted of point set for which $\forall x \neq 0, \dot{V}(x) = 0$ and D is a n-manifold. If $m + 1 < n$, then the equilibrium point of the system is pragmatically asymptotically stable.

Proof. Since every point of X can be passed by a trajectory of Eq. (A.9), which is one- dimensional, the collection of these trajectories, C, is a $(m + 1)$-manifold [22,23]. □

If $m + 1 < n$, then the collection C is a subset of Lebesque measure 0 of D. By the above definition, the equilibrium point of the system is pragmatically asymptotically stable.

If an initial point is ergodicly chosen in D, *the probability of that the initial point falls on the collection C is zero. Here, equal probability is assumed for every point chosen as an initial point in the neighborhood of the equilibrium point.* Hence, the event that the initial point is chosen from collection C *does not occur actually*. Therefore, under the equal probability assumption, pragmatical asymptotical stability becomes actual asymptotical stability. When the initial point falls on $D - C, \dot{V}(x) < 0$, the corresponding trajectories behave as that agree with traditional asymptotical stability because by the existence and uniqueness of the solution of initial-value problem, these trajectories never meet C.

In Eq. (A.7) V is a positive definite function of n variables, i.e. p error state variables and $n - p = m$ differences between unknown and estimated parameters, while $\dot{V} = e^T Ce$ is a negative semi-definite function of n variables. Since the number of error state variables is always more than one, $p > 1$, $m + 1 < n$ is always satisfied, by pragmatical asymptotical stability theorem we have

$$\lim_{t \to \infty} e = 0 \tag{A.11}$$

and the estimated parameters approach the uncertain parameters. The pragmatical adaptive control theorem is obtained. Therefore, the equilibrium point of the system is *pragmatically asymptotically stable. Under the equal probability assumption, it is actually asymptotically stable for both error state variables and parameter variables.*

Appendix B. T–S fuzzy model of chaotic systems

B.1. T–S fuzzy modeling of Q-CNN system

Consider the Quantum-CNN system in Eq. (3.1), if T–S fuzzy model is used for representing local linear models of Quantum-CNN system, there are going to be 16 fuzzy rules, 16 linear subsystems and 64 equations. The process of modeling is shown as follow:

T–S fuzzy model:

Assume that:

(1) $\sqrt{1 - x_1^2} \sin x_2 \in [-Z_1, Z_1]$ and $Z_1 > 0$,

(2) $\cos x_2 / \sqrt{1 - x_1^2} \in [-Z_2, Z_2]$ and $Z_2 > 0$,

(3) $\sqrt{1 - x_3^2} \sin x_4 \in [-Z_3, Z_3]$ and $Z_3 > 0$,

(4) $\cos x_4 / \sqrt{1 - x_3^2} \in [-Z_4, Z_4]$ and $Z_4 > 0$.

Then we have the following T–S fuzzy rules:

Rule 1: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{11}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{21}, $\sqrt{1-x_3^2}\sin x_4$ is M_{31} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{41}, THEN $\dot{X}=A_1X$.
Rule 2: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{11}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{21}, $\sqrt{1-x_3^2}\sin x_4$ is M_{31} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{42}, THEN $\dot{X}=A_2X$.
Rule 3: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{11}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{21}, $\sqrt{1-x_3^2}\sin x_4$ is M_{32} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{41}, THEN $\dot{X}=A_3X$.
Rule 4: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{11}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{21}, $\sqrt{1-x_3^2}\sin x_4$ is M_{32} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{42}, THEN $\dot{X}=A_4X$.
Rule 5: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{11}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{22}, $\sqrt{1-x_3^2}\sin x_4$ is M_{31} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{41}, THEN $\dot{X}=A_5X$.
Rule 6: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{11}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{22}, $\sqrt{1-x_3^2}\sin x_4$ is M_{31} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{42}, THEN $\dot{X}=A_6X$.
Rule 7: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{11}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{22}, $\sqrt{1-x_3^2}\sin x_4$ is M_{32} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{41}, THEN $\dot{X}=A_7X$.
Rule 8: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{11}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{22}, $\sqrt{1-x_3^2}\sin x_4$ is M_{32} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{42}, THEN $\dot{X}=A_8X$.
Rule 9: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{12}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{21}, $\sqrt{1-x_3^2}\sin x_4$ is M_{31} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{41}, THEN $\dot{X}=A_9X$.
Rule 10: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{12}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{21}, $\sqrt{1-x_3^2}\sin x_4$ is M_{31} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{42}, THEN $\dot{X}=A_{10}X$.
Rule 11: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{12}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{21}, $\sqrt{1-x_3^2}\sin x_4$ is M_{32} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{41}, THEN $\dot{X}=A_{11}X$.
Rule 12: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{12}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{21}, $\sqrt{1-x_3^2}\sin x_4$ is M_{32} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{42}, THEN $\dot{X}=A_{12}X$.
Rule 13: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{12}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{22}, $\sqrt{1-x_3^2}\sin x_4$ is M_{31} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{41}, THEN $\dot{X}=A_{13}X$.
Rule 14: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{12}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{22}, $\sqrt{1-x_3^2}\sin x_4$ is M_{31} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{42}, THEN $\dot{X}=A_{14}X$.
Rule 15: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{12}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{22}, $\sqrt{1-x_3^2}\sin x_4$ is M_{32} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{41}, THEN $\dot{X}=A_{15}X$.
Rule 16: IF $\sqrt{1-x_1^2}\sin x_2$ is M_{12}, $\cos x_2/\sqrt{1-x_1^2}$ is M_{22}, $\sqrt{1-x_3^2}\sin x_4$ is M_{32} and $\cos x_4/\sqrt{1-x_3^2}$ is M_{42}, THEN $\dot{X}=A_{16}X$.

Then the final output of the two cells Q-CNN system can be composed by fuzzy linear subsystems mentioned above. It is obviously an inefficient and complicated work. The final output of the T–S fuzzy Q-CNN system is shown in Fig. 12.

B.2. T–S fuzzy modeling of M–V system

Consider the M–V system in Eq. (3.1), if T–S fuzzy model is used for representing local linear models of M–V system, there are going to be 8 fuzzy rules, 8 linear subsystems and 32 equations. Here, we ignore the complicated process of T–S fuzzy modeling and show the final output of the T–S fuzzy M–V system directly in Fig. 13.

Appendix C. Traditional adaptive method

In order to show the efficiency of our new adaptive approach, the traditional adaptive method is given for comparison in this appendix.

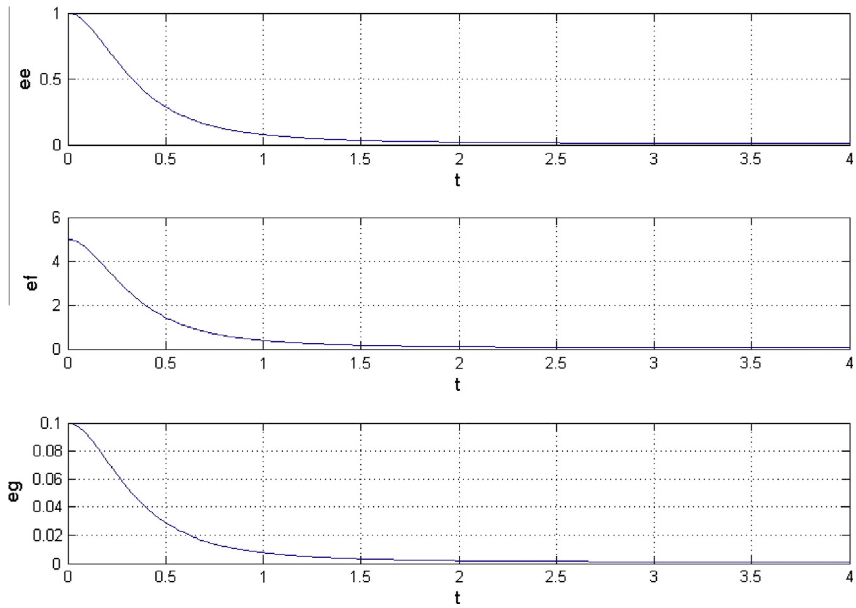

Fig. 17. Time histories of error of parameters for Case II-3 – traditional method.

Case II:
The error and error dynamics are:

$$\begin{cases} e_1 = x_1 - y_1 \\ e_2 = x_2 - y_2 \\ e_3 = x_3 - y_3 \\ e_4 = x_4 - y_4 \end{cases}$$

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{y}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2 - y_2 \\ \dot{e}_2 = \dot{x}_2 - \dot{y}_2 = -w_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 - (-(\hat{a} + \hat{b}y_3)y_1 - (\hat{a} + \hat{b}y_3)y_1^3 - \hat{c}y_2 + \hat{d}y_3) \\ \dot{e}_3 = \dot{x}_3 - \dot{y}_3 = \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4 - y_4 \\ \dot{e}_4 = \dot{x}_4 - \dot{y}_4 = -w_2(x_3 - x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 - \hat{e}y_3 + \hat{f}(y_4 - y_3^2 y_4) + \hat{g}y_1 \end{cases} \tag{C.1}$$

Choose a new control Lyapunov function of the form:

$$V_1 = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}_1^2 + \tilde{a}_2^2 + \tilde{w}_1^2 + \tilde{w}_2^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{e}^2 + \tilde{f}^2 + \tilde{g}^2) > 0 \tag{C.2}$$

where $\tilde{a}_1 = a_1 - \hat{a}_1$, $\tilde{a}_2 = a_2 - \hat{a}_2$, $\tilde{w}_1 = w_1 - \hat{w}_1$, $\tilde{w}_2 = w_2 - \hat{w}_2$, $\tilde{a} = a - \hat{a}, \tilde{b} = b - \hat{b}$, $\tilde{c} = c - \hat{c}, \tilde{d} = d - \hat{d}, \tilde{e} = e - \hat{e}$, $\tilde{f} = f - \hat{f}, \tilde{g} = g - \hat{g}$ and the goal of parameters: $a_1 = 6.8, a_2 = 4.3, w_1 = 4.7, w_2 = 3.9, a = b = c = d = e = f = g = 0$.

Its time derivative through error dynamics (C.1) is

$$\begin{aligned} \dot{V}_1 &= (e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4) + \tilde{a}_1\dot{\tilde{a}}_1 + \tilde{a}_2\dot{\tilde{a}}_2 + \tilde{w}_1\dot{\tilde{w}}_1 + \tilde{w}_2\dot{\tilde{w}}_2 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{e}\dot{\tilde{e}} + \tilde{f}\dot{\tilde{f}} + \tilde{g}\dot{\tilde{g}} + \widetilde{P}_1\dot{\widetilde{P}}_1 + \widetilde{Q}_1\dot{\widetilde{Q}}_1 \\ &= e_1\left(-2a_1\sqrt{1-x_1^2}\sin x_2 - (y_2 + u_{11})\right) \\ &\quad + e_2\left(-w_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 - (-(\hat{a} + \hat{b}y_3)y_1 - (\hat{a} + \hat{b}y_3)y_1^3 - \hat{c}y_2 + \hat{d}y_3 + u_{12})\right) \\ &\quad + e_3\left(-2a_2\sqrt{1-x_3^2}\sin x_4 - (y_4 + u_{13})\right) \\ &\quad + e_4\left(-w_2(x_3 - x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 - \left(-\hat{e}y_3 + \hat{f}(y_4 - y_3^2 y_4) + \hat{g}y_1 + u_{14}\right)\right) + \tilde{a}_1\dot{\tilde{a}}_1 + \tilde{a}_2\dot{\tilde{a}}_2 + \tilde{w}_1\dot{\tilde{w}}_1 \\ &\quad + \tilde{w}_2\dot{\tilde{w}}_2 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{e}\dot{\tilde{e}} + \tilde{f}\dot{\tilde{f}} + \tilde{g}\dot{\tilde{g}} \end{aligned} \tag{C.3}$$

Choose update law of parameters and controllers as:

$$\begin{matrix} \dot{\tilde{a}}_1 = -\dot{\hat{a}}_1 = e_1 \times \tilde{a}_1 \\ \dot{\tilde{a}}_2 = -\dot{\hat{a}}_2 = e_3 \times \tilde{a}_2 \\ \dot{\tilde{w}}_1 = -\dot{\hat{w}}_1 = e_2 \times \tilde{w}_1 \\ \dot{\tilde{w}}_2 = -\dot{\hat{w}}_2 = e_4 \times \tilde{w}_2 \\ \dot{\tilde{a}} = -\dot{\hat{a}} = e_2 \times \tilde{a} \\ \dot{\tilde{b}} = -\dot{\hat{b}} = e_2 \times \tilde{b} \end{matrix}, \quad \begin{matrix} \dot{\tilde{c}} = -\dot{\hat{c}} = e_2 \times \tilde{c} \\ \dot{\tilde{d}} = -\dot{\hat{d}} = e_2 \times \tilde{d} \\ \dot{\tilde{e}} = -\dot{\hat{e}} = e_4 \times \tilde{e} \\ \dot{\tilde{f}} = -\dot{\hat{f}} = e_4 \times \tilde{f} \\ \dot{\tilde{g}} = -\dot{\hat{g}} = e_4 \times \tilde{g} \end{matrix}$$

$$\begin{aligned} u_{11} &= -2a_1\sqrt{1-x_1^2}\sin x_2 - y_2 + \tilde{a}_1^2 + e_1 \\ u_{12} &= -w_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 - (-(\hat{a} + \hat{b}y_3)y_1 \\ &\quad - (\hat{a} + \hat{b}y_3)y_1^3 - \hat{c}y_2 + \hat{d}y_3) + \tilde{w}_1^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + e_2 \\ u_{13} &= -2a_2\sqrt{1-x_3^2}\sin x_4 - y_4 + \tilde{a}_2^2 + e_3 \\ u_{14} &= -w_2(x_3 - x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 \\ &\quad - (-\hat{e}y_3 + \hat{f}(y_4 - y_3^2 y_4) + \hat{g}y_1) + \tilde{w}_2^2 + \tilde{e}^2 + \tilde{f}^2 + \tilde{g}^2 + e_4 \end{aligned} \tag{C.4}$$

We obtain

$$\dot{V}_1 = \left(-e_1^2 - e_2^2 - e_3^2 - e_4^2\right) < 0 \tag{C.5}$$

According to the pragmatical asymptotically stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. The simulation results are shown in Figs. 14–17.

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