The Journal of Chemical Physics

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Citation: The Journal of Chemical Physics **140**, 214108 (2014); doi: 10.1063/1.4880416
View online: http://dx.doi.org/10.1063/1.4880416
View Table of Contents: http://scitation.aip.org/content/aip/journal/jcp/140/21?ver=pdfcov
Published by the AIP Publishing

Fluctuation-induced transport of two coupled particles: Effect of the interparticle interaction

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(Received 31 March 2014; accepted 18 May 2014; published online 5 June 2014)

We consider a system of two coupled particles fluctuating between two states, with different interparticle interaction potentials and particle friction coefficients. An external action drives the interstate transitions that induces reciprocating motion along the internal coordinate x (the interparticle distance). The system moves unidirectionally due to rectification of the internal motion by asymmetric friction fluctuations and thus operates as a dimeric motor that converts input energy into net movement. We focus on how the law of interaction between the particles affects the dimer transport and, in particular, the role of thermal noise in the motion inducing mechanism. It is argued that if the interaction potential behaves at large distances as x^{α}, depending on the value of the exponent α, the thermal noise plays a constructive ($\alpha > 2$), neutral ($\alpha = 2$), or destructive ($\alpha < 2$) role. In the case of $\alpha = 1$, corresponding piecewise linear potential profiles, an exact solution is obtained and discussed in detail. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4880416]

I. INTRODUCTION

Directed transport of Brownian particles induced by unbiased nonequilibrium fluctuations is now a well established phenomenon in various nonlinear systems. Substantial progress has been made in understanding this phenomenon over the past two decades (see Refs. 1–3 for a comprehensive review). Much of the motivation behind studies in this area comes from the challenge of uncovering the operational mechanisms of molecular motor proteins[4] and ion pumps,[5] involved in regulating diverse cellular functions. Transport of biological motors in the molecularly crowded environment of living cells can be either normal or anomalous depending on the cargo size.[6] Another source of motivation arises from the need to build artificial motors and machines of molecular[7] and nano[8] dimensions. Among a vast number of other promising applications,[2] it is worth noting novel methods for particle sorting at the nanoscale.[9]

The early studies were mainly concerned with searching for directed motion scenarios of a point-like particle without internal degrees of freedom. Research over the past 10–15 years has focused on the emergence of directed motion in more complex systems. The interplay between different dynamic modes, nonlinearity, and noise is an intriguing topic to investigate. For motile objects with internal structure, experimental evidence[10] and theoretical arguments[11–13] indicate that the intrinsic properties of such objects, as well as the coordination and interaction between their subunits, play a prominent role in various motion inducing mechanisms. In particular, it was demonstrated experimentally[14] that multidomain motor proteins can move significantly faster than their individual subunits, which also operate as motors.

A dimer is one of the simplest objects, where the internal degree of freedom may play a prominent role in the emergence of directional motion. Dimeric models are particularly relevant to studies of intracellular transport.[4] A conventional motor protein consists of two globular domains (referred to as "heads") joined by a coiled coil α-helical domain, each of which can be attached to filaments of the cytoskeleton and hydrolyze adenosine triphosphate. Bio-inspired artificial dimeric nanowalkers capable to walk directionally along extended tracks have been attracted considerable attention as prototypes of real nanomechanical devices.[15] Two possible mechanisms of the coordinated motion of the motor heads during their walking along the track have been proposed: hand-over-hand,[16] where the motor domains alternate between trailing and leading positions, and inchworm,[17] where one motor domain is always ahead of the other. Noise-induced transport of two coupled particles has been considered in several different ways.[16–30] Most of them are based on rectifying Brownian motion by periodic, asymmetric (ratchet)

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potential in which the dimer moves.[16–18] Other approaches, in which the necessary spatial symmetry breaking is provided by an asymmetry inherent to the system itself rather than to its environment, are also available.[12] In this way, the directed transport can be achieved in a symmetric potential[19,20] and, moreover, without any effective potential. The latter scenario has been naturally implemented by considering various systems with broken friction symmetry.[21–27] A combined effect of a spatially periodic potential, whose amplitude is time-periodically modulated, and friction asymmetry has been demonstrated in Ref. 28. The rectification mechanism in protein translocation through a pore into a cell may be effected by binding intracellular particles.[31]

Interaction between coupled particles has a strong effect on dynamic properties of dimeric motors.[13,20,29,30] The origin of such interaction is, generally speaking, unknown. However, one may speculate that the coiled coil domain (or another agent joining the dimer heads), as well as an implicit influence of one head on the other due to their attachment to the track, coordinate the dynamics of the heads thus leading to an effective interaction between them. Additionally, the dimer heads can interact via many non-covalent interactions, including electrostatic, dipole, van der Waals, etc. Finally, the effective interaction incorporates effects due to the dimension reduction of the full three-dimensional dimer dynamics to an approximate one-dimensional description. Clearly, it is a formidable task to take all these factors into account. In theoretical modeling, one typically postulates that the particles are coupled linearly, which is a gross oversimplification. There are several attempts to go beyond the linear coupling approximation,[20,29,30] which convincingly demonstrate that a non-linear coupling gives rise to much richer dynamic behavior of the dimer, with many effects unmodeled in the simplified representation of linearly coupled particles. A first example is two coupled identical particles moving along a periodic symmetric substrate potential. In the deterministic limit, a harmonic external driving may lead to spontaneous symmetry breaking in the form of a permanent directed motion of the dimer, provided the interparticle interaction potential is nonconvex.[20] Another example is furnished by considering transport of two coupled particles moving in a flashing ratchet potential and comparing the results for Lennard-Jones and spring-type interactions between the particles.[29] A study of two particles coupled nonlinearly through a bistable potential on a periodically rocked ratchet has shown[30] that with this type of interaction the particles are allowed to alternate in the lead, operating in a hand-over-hand manner, that cannot be obtained in models with linear coupling.

In this paper, we continue to explore the effect of the interparticle interaction on the dimer transport. In so doing, we make use of a simple two-state model, consisting of two coupled Brownian particles walking along a linear track. The particles can exist in two conformational states, with different interparticle interaction potentials and particle friction coefficients. External perturbations cause transitions between the states, so reciprocating motion along the internal coordinate (the interparticle distance) emerges. The motion of overall system is coupled to the internal motion. Thus by rectifying the internal motion (by asymmetric friction fluctuations), the dimer moves unidirectionally, functioning as a two-headed motor. As a conventional combustion motor, this motor is composed of a reciprocating engine (represented by the dynamics of the internal degree of freedom) capable to convert part of the energy supplied by the noise source into reciprocating mechanical motion and a symmetry-breaking mechanism (based on asymmetric friction fluctuations resulting from transitions between the conformational states). In the initial version of this model,[21] a linear elastic coupling between the particles is considered.

Here we exploit another formulation of the model,[24] focusing on the effect of the interparticle interaction. We concentrate on the operation of the reciprocating engine whose performance is quantified by the reciprocating velocity. Our main result is that if the interaction potential behaves as x^{α}, thermal noise has a positive, neutral, or negative effect on this velocity depending on the value of exponent α.

The outline of the paper is as follows. To make the paper self-contained, in Sec. II we briefly review the major points, which are needed for understanding the model and in further calculations (for a more detailed description see our previous papers[24–26]). Then, using different approaches, we consider the engine operation in the low (Sec. III) and high (Sec. IV) switching frequency regimes. Section V presents an exact solution for the reciprocating velocity in the case of piecewise linear potential profiles. Finally, we discuss and summarize our findings in Sec. VI.

II. THE MODEL

Consider a system composed of two coupled overdamped particles moving at temperature T along a linear track. The system can exist in one of two conformational states, $\sigma = +$ and $\sigma = -$. The discrete variable $\sigma(t)$ denotes the potential of the interparticle interaction, $U_\sigma(x = x_2 - x_1)$, where x_1 and x_2 are the particle positions along the track, and the particle friction coefficients, $\zeta_1(\sigma)$ and $\zeta_2(\sigma)$. As the particles approach each other, an increasingly strong repulsion between them will appear, so the particles cannot overtake one another (inchworm mode). We also assume that the potentials $U_+(x)$ and $U_-(x)$ diverge as $x \to \infty$.

An external signal triggers transitions between the states. The dynamics of $\sigma(t)$ (which synchronizes the potential and friction fluctuations) is modeled by a dichotomic Markov process

$$\boxed{+} \underset{\gamma_-}{\overset{\gamma_+}{\rightleftharpoons}} \boxed{-}, \tag{1}$$

where $\gamma_\pm$ are the transition rate constants. The system dynamics is governed by two coupled Langevin equations,[32]

$$\zeta_i(\sigma)\frac{dx_i(t)}{dt} = -\frac{\partial U_\sigma(x_2 - x_1)}{\partial x_i} + \sqrt{2\zeta_i(\sigma)T}\,\xi_i(t), \quad i = 1, 2, \tag{2}$$

together with the rate equation (1). The thermal noise is modeled by uncorrelated standardized Gaussian white noises $\xi_1(t)$ and $\xi_2(t)$, $\overline{\xi_i(t)} = 0$, $\overline{\xi_i(t)\xi_k(s)} = \delta_{ik}\delta(t - s)$, where the overbar indicates the average over the thermal noise. The problem becomes separable by introducing the relative coordinate $x = x_2 - x_1$ and the center-of-mass coordinate $X = (x_1 + x_2)/2$.

The first from the two stochastic equations written in terms of the new variables is simply the Langevin equation

$$\zeta_\sigma \dot{x} = -U'_\sigma(x) + \sqrt{2\zeta_\sigma T}\xi(t), \tag{3}$$

where a dot and a prime denote derivatives with respect to time and position, the effective friction coefficient ζ_σ is defined by the relation $\zeta_\sigma^{-1} = \zeta_1^{-1}(\sigma) + \zeta_2^{-1}(\sigma)$ and $\xi(t)$ is a Gaussian white noise with zero mean and the correlation $\overline{\xi(t)\xi(s)} = \delta(t-s)$. Equation (3) jointly with Eq. (1) describes the internal dynamics, that is Brownian motion in a fluctuating confining potential (this is why, in what follows, the term "Brownian particle" refers to the internal coordinate x). The internal dynamics proceeds independently of the system dynamics. The corresponding equation for the time evolution of joint probability densities $\rho_\sigma(x, t), \sigma = +, -$, for finding the Brownian particle in state σ near point x at time t, reads

$$\frac{\partial \rho_\sigma(x,t)}{\partial t} = -\frac{\partial J_\sigma(x,t)}{\partial x} - \sigma[\gamma_+\rho_+(x,t) - \gamma_-\rho_-(x,t)], \tag{4}$$

where

$$J_\sigma(x,t) = -\frac{1}{\beta\zeta_\sigma} e^{-\beta U_\sigma(x)} \frac{\partial}{\partial x}[e^{\beta U_\sigma(x)}\rho_\sigma(x,t)] \tag{5}$$

is the probability current along the x-coordinate in state σ and $\beta = 1/T$. The average velocity of the motion in state σ is defined as

$$v_\sigma(t) = \int_{-\infty}^{\infty} J_\sigma(x,t)\,dx. \tag{6}$$

The second from the two stochastic equations written in terms of x and X relates the system velocity $\dot{X}$ to the internal velocity $\dot{x}$, pointing to the fact that the system and the internal dynamics are coupled. At long times, the velocities converge to their steady state values, $v_\pm(t) \to v_\pm$ and $\dot{X} \to V$. In this regime, the net current is identically zero, $J_+(x) + J_-(x) \equiv 0$, as it must be for bounded motion, so the Brownian particle exhibits back and forth movement, keeping the absolute value of the average velocity fixed, $v \equiv |v_\pm|$. The relation between the system velocity and the velocity of internal motion, averaged over the noises, reads[24,26]

$$V = \epsilon v, \qquad \epsilon = \frac{\zeta_1(+)\zeta_2(-) - \zeta_1(-)\zeta_2(+)}{[\zeta_1(+) + \zeta_2(+)][\zeta_1(-) + \zeta_2(-)]}. \tag{7}$$

Thus, an external perturbation does not directly affect the system variable X, but excites only the internal degree of freedom x, thus generating back and forth motion along x coordinate, on scales large compared to those of thermal fluctuations. The reciprocating motion is rectified by friction fluctuations, that is controlled by the rectification coefficient ϵ. As a result, the system works as a two-headed motor. The principal components of the motor are the engine converting input energy from an external source into the reciprocating motion and a symmetry-destroying mechanism. In contrast to the commonly used flashing ratchet,[16–18] where an asymmetric periodic potential is required to produce directed transport, the model at hand relies on rectification due to broken friction symmetry and hence needs no effective potential interaction between a motor and its track. Several symmetry-breaking scenarios based on the friction asymmetry have been suggested and discussed in the literature.[21–27] All of them do not involve the interaction between the dimer heads. On the contrary, this interaction plays an important role in the operation and performance of the reciprocating engine. So the considered problem is reduced to the analysis of the reciprocating engine.

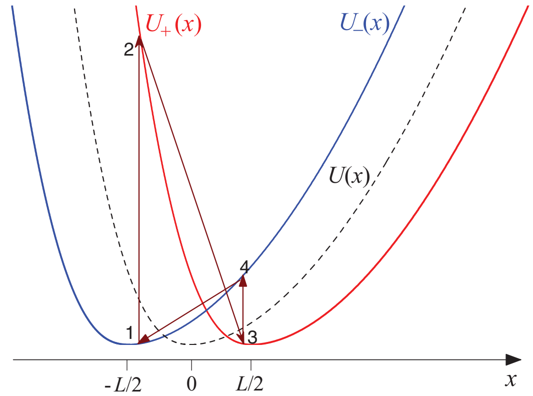

FIG. 1. Scheme of the reciprocating engine. Brownian particle is moving in the potential that fluctuates between two profiles $U_+(x)$ and $U_-(x)$ (solid lines), which are identical but shifted (by a distance $\pm L/2$) copies of the potential $U(x)$ (dashed line). The arrows represent the working cycle of the reciprocating engine.

A. Reciprocating engine

Consider the steady state dynamics of a Brownian particle fluctuating between two states $\sigma = +$ and $\sigma = -$ (see Eq. (1)), with different confining potential profiles, $U_+(x)$ and $U_-(x)$. This is the model of a nanoscale reciprocating engine.[24] The present paper places an emphasis on the role played by the confining potential in the motion-inducing mechanism. In order to make the analysis more clear and compact, we will restrict ourselves to the simplest case of a fully symmetric model, where $\gamma_+ = \gamma_- \equiv \gamma$ and $\zeta_+ = \zeta_- \equiv \zeta$.[33] We also assume the potential profiles $U_+(x)$ and $U_-(x)$ to be identical but shifted (by a distance $\pm L/2$ along the x axis) copies of the potential $U(x)$ (see Fig. 1), i.e.,

$$U_+(x) = U(x - L/2), \qquad U_-(x) = U(x + L/2). \tag{8}$$

It is convenient to introduce the normalized-to-unity probability densities $p_\sigma(x) = 2\rho_\sigma(x)$, $\sigma = +, -$, describing the particle position distribution in state σ. In terms of $p_\pm(x)$, the master equation (4) takes the form

$$J'_\sigma(x) = -\sigma\gamma[p_+(x) - p_-(x)]/2, \tag{9}$$

where

$$J_\sigma(x) = -\frac{1}{2\beta\zeta} e^{-\beta U_\sigma(x)} \frac{d}{dx}[e^{\beta U_\sigma(x)} p_\sigma(x)] \tag{10}$$

is the stationary probability current in state σ. The quantity of interest, that is the reciprocating velocity v defined by Eq. (6), can be expressed as

$$v = \gamma\,\Delta/2. \tag{11}$$

Here the notations $\Delta \equiv (\langle x\rangle_+ - \langle x\rangle_-)$ and $\langle\cdots\rangle_\sigma \equiv \int_{-\infty}^{\infty}\cdots\; p_\sigma(x)\,dx$ have been introduced for future convenience.

When the potential $U(x)$ is parabolic, $U(x) = k_2x^2$ (which implies a linear coupling between the particles in the dimer) an exact solution for v is easily obtained. It is straightforward to show[24] that in this case

$$v = \frac{\gamma L}{2}\frac{1}{1+\gamma\zeta/k_2}. \tag{12}$$

Equation (12) combined with Eq. (7) yields the expression for the motor velocity V, which coincides with that obtained by Fogedby *et al.*[23] within the model of linearly coupled particles.[21] In the presence of an external load force, the motor velocity is monotonically decreasing with increasing load. The velocity-force dependence and the stalling condition, which are important operational characteristics of any motor, have been found within the Mogilner *et al.* model.[21,23]

The temperature-independent velocity in Eq. (12) implies no contribution to the reciprocating motion from the thermal noise. This is a consequence of the linear coupling approximation: external and thermal noises are not coupled. In what follows, we show that nonlinear interaction between the dimer heads leads to noise coupling and the temperature-dependent velocity.

III. LOW FREQUENCY REGIME

Consider the engine described above operating in the low frequency regime. In this case the state's mean residence time γ^{-1} is large compared with other characteristic times in the problem. As a starting point, it is expedient to rewrite Eqs. (9) and (10) in the form

$$p_\sigma(x) = p_{\sigma,0}(x)\{1+\sigma[\Phi_\sigma(x) - \langle\Phi_\sigma(x)\rangle_{\sigma,0}]\}, \tag{13}$$

where $p_{\sigma,0}(x)$ is the equilibrium probability density for the particle position in the potential $U_\sigma(x)$, $p_{\sigma,0}(x) = e^{-\beta U_\sigma(x)}/Z$, $Z = \int_{-\infty}^{\infty} e^{-\beta U(x)}\,dx$, $\langle\cdots\rangle_{\sigma,0} \equiv \int_{-\infty}^{\infty}\cdots\; p_{\sigma,0}(x)\,dx$, and

$$\Phi_\sigma(x) = \beta\gamma\zeta Z\int_{-\infty}^{x} d\xi\, e^{\beta U_\sigma(\xi)}\int_{-\infty}^{\xi} d\xi_1\,[p_+(\xi_1) - p_-(\xi_1)]. \tag{14}$$

In this transformation we have made use of the facts that (i) the probability density and the probability current vanish as $|x|\to\infty$, (ii) $p_\sigma(x)$ is normalized to unity, and (iii) $\int_{-\infty}^{\infty} e^{-\beta U_\pm(x)} = Z$ in view of Eq. (8). Then the quantity Δ, determining the reciprocating velocity in Eq. (11), reads

$$\Delta = (\langle x\rangle_{+,0} - \langle x\rangle_{-,0}) + \langle(x - \langle x\rangle_{+,0})\Phi_+(x)\rangle_{+,0} + \langle(x-\langle x\rangle_{-,0})\Phi_-(x)\rangle_{-,0}. \tag{15}$$

In the low frequency regime, transitions between states represent the slow component of the system dynamics, while the relaxation within the states constitutes the fast part of the dynamics. So at steady state the system is close to a local equilibrium, in the sense that the particle distribution in state σ ($\sigma = +,-$) can be described, to a quite good approximation, by the equilibrium probability density $p_{\sigma,0}(x)$. The effect of interstate transitions can be regarded as a small perturbation to the equilibrium situation. Based on perturbation theory, the first order corrections to $p_{\pm,0}(x)$ and to Δ are obtained by approximating $\Phi_\pm(x)$ in Eq. (14) with $\Phi_{\pm,0}(x)$, in which the unknown probability densities $p_+(x)$ and $p_-(x)$ are replaced by the equilibrium functions $p_{+,0}(x)$ and $p_{-,0}(x)$, respectively. Then, in view of the symmetry condition in Eq. (8) that implies $p_{\pm,0}(x) = p_0(x\mp L/2) = e^{-\beta U(x\mp L/2)}/Z$, Eq. (14) can be approximated as follows:

$$\Phi_\sigma(x) \simeq \Phi_{\sigma,0}(x) = -\beta\gamma\zeta\int_{-\infty}^{x} d\xi\, e^{\beta U(\xi-\sigma L/2)}\int_{\xi-L/2}^{\xi+L/2} e^{-\beta U(\xi_1)}d\xi_1. \tag{16}$$

We consider high enough temperatures so that the potential variation along the length L is small (compared to the thermal energy T) and can be neglected. Then $\Phi_{\sigma,0}(x)$ in Eq. (16) may be written in the following approximated form: $\Phi_{\sigma,0}(x) \simeq -\beta\gamma\zeta L\,x + \text{const}$. Combining this with Eqs. (11), (15), and the identity $\langle x\rangle_{\pm,0} = \pm L/2 + \langle x\rangle_0$ gives eventually the reciprocating velocity in the low frequency regime

$$v \simeq \gamma L[1 - 2\beta\gamma\zeta(\langle x^2\rangle_0 - \langle x\rangle_0^2)]/2, \tag{17}$$

where $\langle\cdots\rangle_0 \equiv \int_{-\infty}^{\infty}\cdots\; p_0(x)\,dx$. Thus here the velocity is directly related to the variance of the particle's position distribution determined by the potential $U(x)$ and the temperature.

To proceed further, an explicit form of the potential is required. Let the potential function $U(x)$ be symmetric

$$U(x) = k_\alpha|x|^\alpha, \tag{18}$$

with k_α characterizing the steepness of the potential and the exponent $\alpha \geq 1$. The variance $(\langle x^2\rangle_0 - \langle x\rangle_0^2)$ is easily calculated. Substituting the result into Eq. (17) yields the desired solution, which can be expressed as follows:

$$v \simeq \mathfrak{v}\omega[1 - A\varepsilon^{(\alpha-2)/\alpha}\omega], \tag{19}$$

where $\mathfrak{v} = k_\alpha L^{\alpha-1}/(2\zeta)$ is the characteristic velocity associated with a particle sliding in the potential $U(x)$ and $A = 2^{\alpha-1}\Gamma(3/\alpha)/\Gamma(1/\alpha)$ with $\Gamma(x)$ the gamma function. Equation (19) is written in terms of two appropriate dimensionless parameters,

$$\varepsilon = \beta k_\alpha(L/2)^\alpha, \qquad \omega = \frac{\gamma\zeta}{k_\alpha L^{\alpha-2}}, \tag{20}$$

which have a clear physical meaning. The former compares the potential variation along the length $L/2$ with the thermal energy T, while the latter compares the characteristic sliding time in the potential with the typical switching time. In the particular case of $\alpha = 2$, $A = 1$ and the approximate solution (19) coincides, as it should be, with the low-frequency asymptotics of the exact solution for the parabolic potential, given in Eq. (12).

The second term in square brackets in Eq. (19) represents the first order correction. In the regime under consideration, $\omega \ll \epsilon \ll 1$, the correction is small as compared to unity. However, this correction has a qualitative effect on the velocity because, with it, new physical factors come into play. Indeed, the first order correction is always negative. Generally, its magnitude depends on the temperature. However, if the potential is parabolic, $\alpha = 2$, the correction (and hence the velocity v) is temperature independent implying, in agreement with the previous work,[24,26] no contribution to the reciprocating velocity from thermal noise. In order to exhibit nonlinear effects, it is natural to choose this case as the reference case for comparison. If the exponent $\alpha > 2$, the correction term becomes smaller (compared to the reference case) and the velocity increases with temperature. On the contrary, if $\alpha < 2$, the negative correction is greater and the velocity decreases with temperature. Thus, a nonparabolicity of the potential leads to a coupling between external and thermal noises, so that thermal noise is involved as a component of the mechanism by which the motion occurs. Depending on the value of the exponent α, the thermal noise plays a constructive ($\alpha > 2$), neutral ($\alpha = 2$), or destructive role ($\alpha < 2$) in the operative mechanism.

Our last remark in this section concerns the effect of asymmetry of the potential. Consider an asymmetric potential

$$U(x) = \begin{cases} k_{\alpha_r} x^{\alpha_r}, & \text{for } x > 0 \\ k_{\alpha_l} |x|^{\alpha_l}, & \text{for } x < 0 \end{cases}. \tag{21}$$

For the sake of definiteness we suppose that

$$\alpha_l \geq \alpha_r \geq 1. \tag{22}$$

Then the velocity, Eq. (17), can be written as

$$v \simeq \gamma L \left[1 - 2A_1 \beta^{(\alpha_r-2)/\alpha_r} k_{\alpha_r}^{-2/\alpha_r} \gamma\zeta\right]/2, \tag{23}$$

where

$$A_1 = \frac{\Gamma(3/\alpha_r)}{\Gamma(1/\alpha_r)} \frac{1+B_3(l_l/l_r)^3}{1+B_1 l_l/l_r} - \left[\frac{\Gamma(2/\alpha_r)}{\Gamma(1/\alpha_r)} \frac{1-B_2(l_l/l_r)^2}{1+B_1 l_l/l_r}\right]^2, \tag{24}$$

$B_i = \Gamma(i/\alpha_l)/\Gamma(i/\alpha_r)$ (for $i = 1, 2, 3$), and $l_j = \alpha_j(\beta k_{\alpha_j})^{-1/\alpha_j}$ (for $j = r, l$). Note that the coefficient A_1 is positive, $A_1 > 0$, and in fact temperature independent, in view of the smallness of β and condition (22). Obviously, when $k_{\alpha_r} = k_{\alpha_l} = k_\alpha$ and $\alpha_r = \alpha_l = \alpha$, Eq. (23) is simplified into Eq. (19). As Eq. (23) indicates, the temperature dependence of the velocity is controlled by the relatively gradual branch of the potential (characterized in our case by the exponent α_r).

IV. HIGH FREQUENCY REGIME

Consider the engine operating at high switching rates, where the state's mean residence time γ^{-1} is the shortest characteristic time scale in the overdamped regime, i.e., in particular $\omega \gg 1$. When $\gamma \to \infty$, the particle jumps from one state to the other many times before being moved on an appreciable distance along x axis. This suggests that in the high frequency limit the particle position distributions in states + and − approach to each other. As follows from Eq. (9), in the zero order approximation (with respect to γ^{-1}) one can neglect the difference between the distributions and take $p_+(x) \simeq p_-(x) \simeq p_\infty(x)$. Then, using the condition of zero net steady state current, $J_+(x) + J_-(x) = 0$, and Eq. (10), an equation for $p_\infty(x)$ is derived:

$$p'_\infty(x) + \beta U_{\rm eff}(x) p_\infty(x) = 0,$$
$$U_{\rm eff}(x) = [U_+(x) + U_-(x)]/2. \tag{25}$$

Thus, in this regime, the local equilibrium in the effective potential $U_{\rm eff}(x)$ is established:

$$p_\infty(x) = e^{-\beta U_{\rm eff}(x)} \Big/ \int_{-\infty}^{\infty} e^{-\beta U_{\rm eff}(x)} dx. \tag{26}$$

The particle actually feels the effective potential, since it is not capable of following the rapid potential changes. In this way, we find, using Eqs. (10), (25) and the notation $\langle\cdots\rangle_\infty \equiv \int_{-\infty}^{\infty} \cdots\ p_\infty(x)\, dx$, that the reciprocating velocity, Eq. (6), can be written as

$$v \simeq \frac{1}{4\zeta} \langle U'_-(x) - U'_+(x)\rangle_\infty. \tag{27}$$

We consider the state potential profiles, which are identical but shifted copies, as stated in Eq. (8). We also consider the case of high temperatures, where the potential variation along the length $L/2$ can be neglected because it is small as compared to T, i.e., $\epsilon \ll 1$. With these assumptions, (i) the effective potential in Eq. (25) is reduced to $U(x)$, (ii) $p_\infty(x)$ in Eq. (26) can be approximated by the equilibrium probability density for the particle position in the potential $U(x)$, $p_\infty(x) \simeq p_0(x) = e^{-\beta U(x)} / \int_{-\infty}^{\infty} e^{-\beta U(x)} dx$, and (iii) the difference $U'_-(x) - U'_+(x)$ in Eq. (27) can be approximated by $LU''(x)$. As a result, Eq. (27) is simplified, yielding

$$v \simeq \frac{L}{4\zeta} \beta \langle U'^2(x)\rangle_0. \tag{28}$$

To proceed further, we first use the symmetric potential given in Eq. (18). Then it follows from Eq. (28) that the leading term of the asymptotic expansion of v for $\omega \gg 1$ is ω-independent and given by

$$v \simeq \mathcal{A}\mathfrak{v}\varepsilon^{(2-\alpha)/\alpha}, \tag{29}$$

with $\mathfrak{v} = k_\alpha L^{\alpha-1}/(2\zeta)$ and $\mathcal{A} = 2^{1-\alpha}\alpha^2\Gamma(2 - 1/\alpha)/\Gamma(1/\alpha)$. In the particular case of $\alpha = 2$, $\mathcal{A} = 1$ and the ϵ-independent estimate of the velocity given in Eq. (29) coincides, as it should be, with the high-frequency asymptotics of the exact solution for the parabolic potential, Eq. (12). As Eq. (29) indicates, the velocity grows with temperature if the exponent $\alpha > 2$. But if $\alpha < 2$, v goes to zero as $T \to \infty$. Thus, the cooperation between the external and internal fluctuations is manifested differently, depending on the exponent α that characterizes the potential profile. Just as in the low-frequency regime, thermal noise enhances (weakens) the system response when $\alpha > 2$ ($\alpha < 2$) or contributes nothing into the motion inducing mechanism when $\alpha = 2$.

In order to demonstrate the effect of the asymmetry, let us take the potential to be of the form given by Eq. (21), with exponents α_r and α_l satisfying condition (22). Then it follows from Eq. (28) that

$$v \simeq \mathcal{A}_1 \frac{L}{4\zeta} k_{\alpha_l}^{1/\alpha_l} k_{\alpha_r}^{1/\alpha_r} \beta^{(2-\widetilde{\alpha})/\widetilde{\alpha}}, \tag{30}$$

where $\widetilde{\alpha}^{-1} = (\alpha_l^{-1} + \alpha_r^{-1})/2$ and

$$\mathcal{A}_1 = \alpha_l \alpha_r \frac{\Gamma(2-1/\alpha_l)}{\Gamma(1/\alpha_r)} \frac{1+\mathcal{B}_1 l_r/l_l}{1+\mathcal{B}_2 l_r/l_l}, \tag{31}$$

with $\mathcal{B}_1 = \Gamma(2-1/\alpha_r)/\Gamma(2-1/\alpha_l)$, $\mathcal{B}_2 = \Gamma(1/\alpha_l)/\Gamma(1/\alpha_r)$, and $l_j = \alpha_j(\beta k_{\alpha_j})^{-1/\alpha_j}$ (for $j = r,\ l$). The coefficient $\mathcal{A}_1$ is in fact temperature independent, in view of the smallness of β and condition (22). For the symmetric potential, Eqs. (30) and (31) are simplified into Eq. (29). As Eq. (30) indicates, both the left and right branches of the potential contribute on the same footing to the temperature dependence of the velocity, which is in contrast to that seen in the low-frequency regime (see the last paragraph in Sec. III).

V. V-SHAPED POTENTIAL

In this section a remarkably simple case of $\alpha = 1$ is considered, where the identical [but shifted, see Eq. (8)] symmetrical potential profiles $U_+(x)$ and $U_-(x)$ have V-shaped form:

$$U(x) = k_1|x|. \tag{32}$$

So stated, the problem is amenable to an exact solution.

To seek the solution, it is convenient to introduce new variables:

$$q(x) = [p_+(x) + p_-(x)]/2, \quad r(x) = [p_+(x) - p_-(x)]/2. \tag{33}$$

In view of the problem symmetry, $q(-x) = q(x)$ and $r(-x) = -r(x)$. Additionally, the function $q(x)$ is normalized to unity, $\int_{-\infty}^{\infty} q(x)\,dx = 1$. In terms of the new variables, the desired reciprocating velocity, Eq. (6), reads

$$v = \gamma \int_{-\infty}^{\infty} x r(x)\,dx. \tag{34}$$

As follows from Eqs. (9) and (10), the functions $q(x)$ and $r(x)$ satisfy the system of differential equations:

$$\begin{aligned} &\beta u'(x)q(x) + \beta\phi'(x)r(x) + q'(x) = 0,\\ &r''(x) + \beta[\phi'(x)q(x)]' + \beta[u'(x)r(x)]' = 2\beta\gamma\zeta r(x), \end{aligned} \tag{35}$$

where $u(x) = [U_+(x) + U_-(x)]/2$ and $\phi(x) = [U_+(x) - U_-(x)]/2$. Additionally, $q(x)$ and $r(x)$ must satisfy the boundary conditions, requiring vanishing the probability densities as $|x|$ goes to infinity, as well as the matching conditions, which provide continuity of the probability densities and currents at $x = \pm L/2$. The first one of Eq. (35) simply reflects the fact that the net probability current for the reciprocating motion takes the zero value at any point x, while the second one is obtained by summing up Eq. (9) with $\sigma = +$ and $\sigma = -$.

Equations (35) are greatly simplified due to the particularly simple form of the potential profiles in the present case [see Eq. (32)]. Indeed the potential fluctuations are felt by the particle only in the region $|x| < L/2$, where the average force is zero, so we can write

$$\begin{aligned} u'(x) &= \begin{cases} k_1\,\mathrm{sgn}(x), & \text{for} |x| > L/2 \\ 0, & \text{for} |x| < L/2 \end{cases},\\ \phi'(x) &= \begin{cases} 0, & \text{for} |x| > L/2 \\ -k_1, & \text{for } |x| < L/2 \end{cases}. \end{aligned} \tag{36}$$

This enables us to solve the problem exactly. The straightforward but tedious computation yields the final result

$$v = \mathfrak{v} \frac{\coth\sqrt{\varepsilon(\varepsilon+\omega)} + z - (1-z^2)/\sqrt{\varepsilon(\varepsilon+\omega)}}{(1+\varepsilon^{-1})[\coth\sqrt{\varepsilon(\varepsilon+\omega)} + z] + \omega^{-1}[\coth\sqrt{\varepsilon(\varepsilon+\omega)} + \sqrt{\varepsilon/(\varepsilon+\omega)}]}, \tag{37}$$

where $\mathfrak{v} = k_1/(2\zeta)$ is the characteristic velocity, $\varepsilon = \beta k_1 L/2$ and $\omega = \gamma\zeta L/k_1$ are the relevant dimensionless parameters [see Eq. (20)], and $z = 2\sqrt{\varepsilon+\omega}/(\sqrt{\varepsilon} + \sqrt{\varepsilon+4\omega})$ is their combination. Although the model is extremely simple, the expression for the velocity in this equation is rather involved because of the coupling between the external and thermal noises.

Upon variation of ε (representing the inverse temperature) from zero to infinity, the velocity given by Eq. (37) grows monotonously from zero to $\mathfrak{v}\omega/(\omega+1)$ at any nonzero value of ω (representing the frequency of the potential switching). Note that, even without thermal noise, externally driven transitions between states cause the reciprocating motion. Moreover, the velocity v reaches its maximum value just at $T = 0$. Thus in the case of $\alpha = 1$ thermal noise plays a destructive role, that is in qualitative agreement with the conclusions drawn from the general analysis of the limiting cases of $\omega \ll 1$ and $\omega \gg 1$ (see Secs. III and IV). It follows from Eq. (37) that at low temperatures, where ε is the largest parameter in the problem, $\varepsilon \gg 1$ and $\varepsilon \gg \omega$, the ratio $v/\mathfrak{v}$ can be approximated as follows:

$$v/\mathfrak{v} \simeq \frac{\omega}{\omega+1}\left(1 - \frac{\omega}{\omega+1}\varepsilon^{-1}\right). \tag{38}$$

At high temperatures, where ε is the smallest parameter in the problem, $\varepsilon \ll 1$ and $\varepsilon \ll \omega$, the velocity vanishes as

$$v/\mathfrak{v} \simeq \varepsilon(1 - \sqrt{\varepsilon/\omega}). \tag{39}$$

Upon variation of ω, the velocity, Eq. (37), exhibits a behavior similar to that seen above for varying ε: it increases monotonically from zero at $\omega = 0$ to $\mathfrak{v}\varepsilon/(\varepsilon + 1)$ as $\omega \to \infty$. In the low-frequency regime, where ω is the smallest parameter of the problem, $\omega \ll 1$ and $\omega \ll \varepsilon$, Eq. (37) is approximately reduced to

$$v/\mathfrak{v} \simeq \omega\left\{1 - \omega\left[1 + \frac{1}{\varepsilon}\left(1 + \frac{1 - e^{-2\varepsilon}}{2\varepsilon}\right)\right]\right\}. \quad (40)$$

Furthermore, if the temperature is high enough, more specifically if $\omega \ll \varepsilon \ll 1$, this equation, as it should be, takes the form of Eq. (19), with $\alpha = 1$ and, respectively, $A = 2$. In the opposite limiting case, where ω is the largest parameter, $\omega \gg 1$ and $\omega \gg \varepsilon, \varepsilon^{-1}$, it follows that

$$v/\mathfrak{v} \simeq \frac{\varepsilon}{1+\varepsilon}\left[1 - \frac{1+2\varepsilon}{2(1+\varepsilon)\omega}\right]. \quad (41)$$

At high temperatures, $\varepsilon \ll 1$, the leading term of ε-expansion of this equation coincides, as it should be, with the estimate given in Eq. (29), with $\alpha = 1$ and, respectively, $\mathcal{A} = 1$.

Thus the results for the particular case $\alpha = 1$ corroborate and illustrate the general conclusions drawn from the analysis of the low and high frequency regimes (see Secs. III and IV).

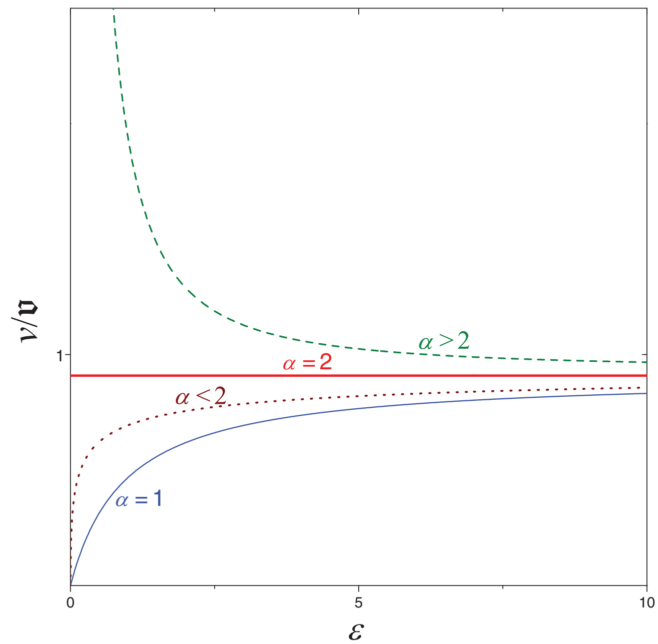

FIG. 2. The ratio $v/\mathfrak{v}$ as a function of the dimensionless inverse temperature ε for different values of the exponent α and $\omega = 10$. The thick solid line, $\alpha = 2$, corresponds to the exact solution given by Eq. (12) and the thin solid line, $\alpha = 1$, corresponds to the exact solution given by Eq. (37). The dotted (dashed) line schematically represents the case of $\alpha < 2$ ($\alpha > 2$).

VI. DISCUSSION AND CONCLUSIONS

The results presented above, including the approximate estimates given by Eqs. (19) and (29), as well as the exact solutions given by Eqs. (12) and (37), agree qualitatively in that the exponent α characterizing the potential behavior determines the effect of thermal noise on the system response. The role of the thermal noise is constructive, neutral, or destructive when $\alpha > 2$, $\alpha = 2$, or $\alpha < 2$, respectively. This observation, well illustrated by Fig. 2, is the main point of our paper. Note that the reciprocating motion exists even at $T = 0$. At high temperatures (and $\alpha \neq 2$) the effect of thermal noise is manifested differently in the two limiting cases $\omega \to 0$ and $\omega \to \infty$. In the low frequency regime, the contribution of the noise into the reciprocating velocity v is represented by a small correction, see Eq. (19). In the high frequency regime, the temperature determines the leading term of the asymptotic behavior of v, as Eq. (29) shows.

The interplay (and in particular the constructive cooperation) between thermal (equilibrium) and external noises is a subject of long-standing and continuing interest, especially in studies of fluctuation-induced transport.[1–3,34] A distinct advantage of our model is that it allows qualitatively different manifestations of the noise coupling to be demonstrated in a unified manner by changing the interparticle interaction.

On the other hand, some care should be exercised in the interpretation of the observations made. In particular, we have shown that the velocity v takes a nonzero value as the transition rate $\gamma \to \infty$. However, in this regime the reciprocation motion evolves on the micro- rather than on the nanoscale, so it cannot be rectified and used to produce directed motion. Thus in the high frequency regime our results give an intermediate (rather than limiting) asymptotic behavior of the dimer drift velocity.

Additionally, an obvious shortcoming of the model is that the dynamics for switching between competing states is assumed independent of particle spatial position and thermal noise, which is correct only for systems switchable by large (on the nanoscale) forces. A more general model, which incorporates the spatial dependence of the system reactivity, has been proposed and analyzed in Refs. 25 and 26. Of course, the generalized model exhibits a more complicated behavior, but its simplified version exploited here suffices for our purposes to reveal the effect of the potential on the motion-inducing mechanism. The approximation used is the price we have paid to make the problem analytically treatable and physically transparent.

Summing up, we have investigated the effect of the interparticle interaction on active transport of a dimer. As a model, we have examined a system of two coupled particles fluctuating between two states, with different interaction potentials and particle friction coefficients. Externally driven interstate transitions induce reciprocating motion along the internal coordinate (the interparticle distance) that is rectified to a direct current by asymmetric friction fluctuations. With such formulation, the dimer motor consists of the engine that converts nonequilibrium fluctuations into reciprocating motion and the symmetry breaking mechanism. We have focused on the reciprocating engine that is confined Brownian motion in the fluctuating potential (see Fig. 1). The confining potential represents the interaction between the dimer heads, which is assumed to behave at large distances as x^α, $\alpha \geq 1$. The engine operation has been analyzed in the two limiting regimes of the low and high frequency of potential switching. The results of this approximate analysis, Eqs. (19) and (29), together with the exact solution obtained in the case of $\alpha = 1$, Eq. (37), as well as the exact solution for the parabolic potential, Eq. (12), lead to the main conclusion of this work that thermal noise comes into play with nonlinearity of the interparticle interaction. More specifically, while thermal noise contributes nothing into the motion inducing mechanism when $\alpha = 2$, it enhances (weakens) the system response when $\alpha > 2$ ($\alpha < 2$) (see Fig. 2). When the potential is asymmetric, noticeable observation is that in the high-frequency regime both the left and right branches of the potential contribute to the temperature dependence of the velocity on the same footing, while in the low-frequency regime this dependence is controlled by the relatively gradual branch of the potential. Through this study, we learn how the interparticle interaction affects the dimer transport and, in particular, the role of thermal noise in the motion-inducing mechanism.

ACKNOWLEDGMENTS

This work was supported by Academia Sinica. Yu.A.M. thanks V.Yu. Zitserman and E.N. Govorun for helpful discussions and the Russian Foundation for Basic Research for partial support (Grant No. 14-03-00343). V.M.R. thanks the Complex Program for Basic Research of the National Academy of Sciences of Ukraine for partial support (Grant No. 3/14-H). Yu.A.M. and V.M.R. gratefully acknowledge the kind hospitality received from the Institute of Atomic and Molecular Sciences.

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