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Groundwater Response to Tidal Fluctuation in an Inhomogeneous Coastal Aquifer-Aquitard System

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Abstract This paper investigates tide-induced groundwater fluctuation and submarine groundwater discharge (SGD) in a leaky inhomogeneous coastal aquifer system with an upper unconfined aquifer, a lower confined aquifer, and an aquitard between them. The upper left aquifer is formed due to land reclamation. The SGD defined as the groundwater flow from land into the sea is controlled mainly by the hydraulic gradient between land and sea. An analytical expression is developed to discuss and assess the effect of inhomogeneity on the groundwater head fluctuation in the leaky aquifer system. Joint effects of aquifers' parameters such as leakage and hydraulic diffusivity on the groundwater head fluctuation and SGD are investigated. The predicted results from the analytical expression indicate that the groundwater head fluctuation in both unconfined and confined aquifers is dependent on dimensionless leakages and increases with dimensionless hydraulic diffusivity.

Keywords Coastal aquifer · Analytical solution · Tidal fluctuation · Inhomogeneity · Leakage · Hydraulic diffusivity · Submarine groundwater discharge

Notations

A	Amplitude of tide [L]
$a_1, a_2, \ldots, a_4$	Constant coefficients in Eq. 8
$a_5, a_6, \ldots, a_8$	Constant coefficients in Eq. 14
b	Constant coefficient given by Eq. 10a
bb	Constant coefficient given by Eq. 16a
$b_1, b_2, \ldots, b_4,$	Constant coefficients in Eq. 18
$b_5, b_6,$	Constant coefficients in Eq. 19
c	Constant coefficient given by Eq. 10b
cc	Constant coefficient given by Eq. 16b

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D_{11}, D_{12}	Hydraulic diffusivity of the upper aquifer [L^2/T]
D_{21}, D_{22}	Hydraulic diffusivity of the lower aquifer [L^2/T]
D_b	Dimensionless hydraulic diffusivity in the lower aquifer
D_u	Dimensionless hydraulic diffusivity in the upper aquifer
H_{11}, H_{12}, H_{21}, H_{22}	Hydraulic head in the unconfined and the confined aquifer [L]
$h_{11}(x)$, $h_{12}(x)$,$h_{21}(x)$, $h_{22}(x)$	Functions associated with hydraulic head in the unconfined and the confined aquifer [L]
H_{MSL}	Mean sea level (L)
$\overline{H}_{11}$, $\overline{H}_{12}$, $\overline{H}_{21}$, $\overline{H}_{22}$,	Normalized amplitude of groundwater head fluctuation in the unconfined and the confined aquifer []
i	$\sqrt{-1}$
l	Distance between coastline and inhomogeneous boundary
L_1, L_2	Leakage through the aquitard [1/T]
L_m	Dimensionless leakage
P	Tidal period [T]
Q_L	Submarine groundwater discharge from lower aquifer [L^2/T]
S_{11}, S_{12}	Storativity of the unconfined aquifer
S_{21}, S_{22}	Storativity of the confined aquifer
t	Time [T]
T_{11}, T_{12}	Transmissivity of the unconfined aquifer [L^2/T]
T_{21}, T_{22}	Transmissivity of the confined aquifer [L^2/T]
T, T'	Dimensionless Transmissivity of the unconfined and confined aquifers
x	Distance from coastline [L]
$\lambda_1, \lambda_2, \ldots, \lambda_4$	Constant coefficients given by Eq. 9
$\lambda_5, \lambda_6, \ldots, \lambda_8$	Constant coefficients given by Eq. 15
Φ_{11}, Φ_{12}	Phase angle in the upper aquifer
Φ_{21}, Φ_{22}	Phase angle in the lower aquifer
ω	Frequency of tide [1/T]

1 Introduction

Recently, researchers have made significant efforts to develop analytical solutions for tide induced groundwater fluctuation in coastal aquifers (e.g., Philip 1973; Parlange et al. 1984; Nielsen 1990; Jeng et al. 2002; Li and Jiao 2003a; Song et al. 2007; Chuang et al. 2012; Asadi-Aghbolaghi et al. 2012). Coastal aquifers are usually inhomogeneous due to variations in depositional and post depositional processes (Cherry et al. 2006). Some investigators focused on the development of analytical models to describe groundwater fluctuation in coastal leaky aquifer systems consisting of an upper unconfined aquifer, a lower confined aquifer, and an aquitard in between (Li et al. 2001; Jeng et al. 2002; Li and Jiao 2003a). Mostly, they considered a vertical inhomogeneity for the coastal aquifer system and assumed that the layers are homogenous and infinite extent in the landward direction. In reality, many coastal aquifers may be inhomogeneous in both horizontal and vertical directions (Guo et al. 2007; Guo et al. 2010; Chuang et al. 2010). Also, submarine groundwater discharge (SGD) in coastal aquifers has been analytically investigated by many researchers (e.g., Li and Jiao 2003b; Amir et al. 2013; Konikow et al. 2013).

Groundwater response to tidal fluctuation in a horizontal inhomogeneous aquifer had been the topic of interest for some researchers (e.g., Geng et al. 2009; Guo et al. 2010). Li and Jiao (2003a) investigated the influence of tide on the mean water-table for the case that unconfined coastal aquifers are inhomogeneous and anisotropic. The predicted result from their analytical solution has a good agreement with previous published works. They showed that the previous works such as Philip (1973), Knight (1981), Parlange et al. (1984), and Barry et al. (1996), are special cases of their solution. They showed that if the observed mean water levels in coastal areas are used for estimating the net inland recharge, the enhancing processes of sea tide on the mean groundwater levels should be taken into account. Otherwise, the net inland recharge will be overestimated. Geng et al. (2009) developed an analytical model to find groundwater head fluctuation in a single coastal confined aquifer, extending under the sea for a certain distance and overlain by a layer with different property in contrast to the aquifer. This work is an extension of the solution derived by Li et al. (2007) with considering specific storage for the capping. Their results demonstrated that neglecting the specific storage of the capping will lead to significant errors in predicting groundwater head fluctuation. Guo et al. (2010) presented an analytical model for an inhomogeneous coastal unconfined aquifer, which consists of coastal and inland zones with different hydraulic properties. The coastal zone has a limited width while the inland zone is infinite. They derived analytical solutions for both coastal and inland zones and discussed the physical behaviors. They compared their analytical solution with Jacob (1950) solution for a semi-infinite homogeneous aquifer, and concluded that the existence of the coastal zone reduces the amplitude in the inland zone by a spatially constant coefficient and increases the phase lag by a spatially constant shift. Monachesi and Guarracino (2011) presented an analytical solution for a heterogeneous coastal confined aquifer. They assumed that the hydraulic conductivity increases linearly with the inland distance. Their results showed that the time lag between sea tide and induced head fluctuation in the aquifer can be approximated with a square-root type function leading to a faster propagation of the tidal fluctuation. They compared their results with the homogeneous model provided in Jacob (1950), the linear heterogeneity produces more damped amplitudes for distances less than approximately characteristic dampening distance.

Other researchers also investigated the problems of horizontal inhomogeneity in multi-layer coastal aquifer systems (Guo et al. 2007; Chuang et al. 2010). Guo et al. (2007) derived analytical models for two coastal multi-layered aquifer systems. Model I comprises of two semi-permeable layers and a confined aquifer between them while Model II is a four-layered aquifer system including an unconfined aquifer, an upper semi-permeable layer, a confined aquifer and a lower semi-permeable layer. In each model, they assumed that a submarine outlet of the confined aquifer is covered by an outlet capping. The Li and Jiao (2001) solution which does not have an outlet capping is a special case of Guo et al. (2007) solution. Xia et al. (2007) considered a leaky aquifer system in which the unconfined aquifer terminates at the coastline, but the confined aquifer and its roof extends under the sea over a certain distance. They also considered an aquifer's submarine outlet, which is covered by a thin layer of sediment with properties dissimilar to the aquifer. They neglected the groundwater head fluctuation in unconfined aquifer and showed that some existing analytical solutions such as Jacob (1950), van der Kamp (1972), Li and Jiao (2001), and Li et al. (2007) are special cases of their solution. Chuang and Yeh (2007) derived an analytical solution to describe groundwater level fluctuation in a leaky aquifer of infinite extent under the sea. Their results showed that ignoring water table fluctuation of the unconfined aquifer will give large errors in predicting the fluctuation, time lag, and tidal influence distance of the leaky confined aquifer. Later, they also developed a similar solution for an aquifer extending a finite distance under the sea (Chuang and Yeh 2008). In both solutions, they assumed that both inland and offshore parts of aquitard and confined aquifer have different hydraulic properties. Chuang and Yeh (2007) solution would be a special case of Chuang and Yeh (2008) solution, if considering the length of the roof approach infinity in Chuang and Yeh (2008). Chuang et al. (2010) presented an analytical solution for a leaky aquifer system with a heterogeneous aquitard and underlying aquifer divided into several horizontal regions and the head in the upper unconfined aquifer is assumed constant. They found that the length and location of the discontinuous aquitards have large impacts on the amplitude and phase shift of the head fluctuation in the lower aquifer. More recently, Chuang et al. (2011) presented an analytical solution which can be considered as a generalization of most existing analytical solutions for a tidal aquifer system with single confined and leaky confined aquifers. For example, Xia et al. (2007) solution is a special case of this solution if neglecting the groundwater fluctuation in the unconfined aquifer.

This paper investigates tide-induced groundwater fluctuation in a heterogeneous leaky aquifer system with an upper inhomogeneous unconfined aquifer, a lower inhomogeneous confined aquifer, and an inhomogeneous aquitard between them. The upper left aquifer is considered as a reclaimed layer (Jiao et al. 2001). All of the layers have different hydraulic properties. An analytical model is developed to describe the groundwater head distribution and submarine groundwater discharge (SGD) in both heterogeneous unconfined and confined aquifers. The joint effects of aquifers' parameters such as leakage and hydraulic diffusivity on the head distribution are discussed and investigated via the solution of the model. The solution of the model is compared with the existing analytical solutions such as Jeng et al. (2002) and Chuang and Yeh (2007). This new solution may be helpful in assessing the impact of land reclamation on the coastal groundwater flow or the effect of submarine groundwater discharge on the ecohydrological system near the coastal regions. However, it should be noted that the present solution will fail to yield accurate result if the saturated thickness of the unconfined aquifer is not significantly greater than the tidal amplitude.

2 Mathematical Model

2.1 Problem Setup and Boundary Conditions

Figure 1 shows the schematic diagram of a coastal inhomogeneous aquifer system with a lower confined aquifer, an upper unconfined aquifer, and an aquitard in between. The upper left aquifer is a developed zone due to land reclamation. The horizontal (x) axis is positive landward and perpendicular to the coastal line while the vertical axis is parallel to the coast line. The origin of the x axis is located at the end of the aquifer system. The aquitard, confined and unconfined aquifers are heterogeneous in horizontal direction, and the interface of heterogeneity in all the layers is located at $x=l$. Assume that the flow velocity is horizontal in confined and unconfined aquifers. The leakage through the aquitards is assumed to be proportional to head differences between confined and unconfined aquifers; and the storage in aquitards is assumed negligible. In addition, the flow in the unconfined aquifer can be described by the linearized Boussinesq equation when the thickness of the unconfined aquifer is much greater than the tidal amplitude (Bear 1979; Jiao and Tang 1999; Jeng et al. 2002). The product of hydraulic conductivity and average saturated thickness of the unconfined aquifer is therefore regarded as transmissivity. Considering linearized equation for the unconfined aquifer, the governing equation describes the flow in unconfined aquifers can be written as (Bear 1979; Li et al. 2001; Jeng et al. 2002)

$$S_{11}\frac{\partial H_{11}}{\partial t}=T_{11}\frac{\partial^2 H_{11}}{\partial x^2}+L_1(H_{21}-H_{11})\,0<x<l \tag{1a}$$

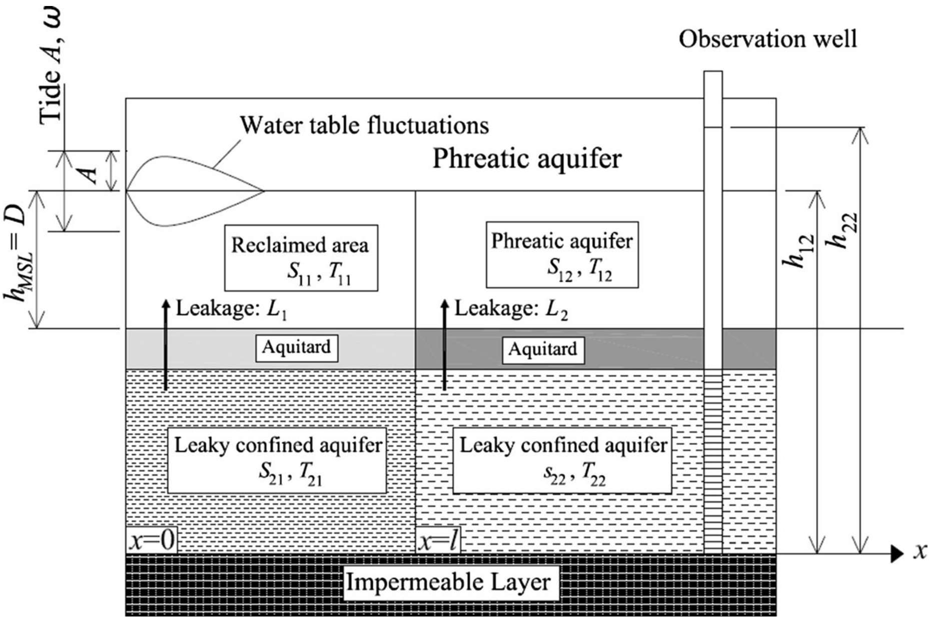

Fig. 1 Schematic diagram of a heterogeneous coastal aquifer system, comprising an unconfined aquifer, a confined aquifer, and an aquitard in between

$$S_{12}\frac{\partial H_{12}}{\partial t} = T_{12}\frac{\partial^2 H_{12}}{\partial x^2} + L_2(H_{22} - H_{12}) \ l < x \tag{1b}$$

where H_{11} and H_{12} are the hydraulic head in the unconfined aquifer; S_{11} and T_{11} are storativity and transmissivity, respectively, for $0<x<l$ and S_{12} and T_{12} are storativity and transmissivity, respectively, for $l<x$; L_1 is the specific leakage of the aquitard for $0<x<l$ and L_2 is the specific leakage of the aquitard for $l<x$. For the confined aquifer, the governing equation is

$$S_{21}\frac{\partial H_{21}}{\partial t} = T_{21}\frac{\partial^2 H_{21}}{\partial x^2} + L_1(H_{11} - H_{21}) \ 0 < x < l \tag{1c}$$

$$S_{22}\frac{\partial H_{22}}{\partial t} = T_{22}\frac{\partial^2 H_{22}}{\partial x^2} + L_2(H_{12} - H_{22}) l < x \tag{1d}$$

where H_{21} and H_{22} are hydraulic head in the confined aquifer;S_{21} and T_{21} are storativity and transmissivity, respectively, for $0<x<l$ and S_{22} and T_{22} are storativity and transmissivity, respectively, for $l<x$. The boundary conditions for the problem are

$$H_{11}(0,t) = H_{21}(0,t) = H_{MSL} + A\cos(\omega t) \tag{2a}$$

$$\left.\frac{\partial H_{12}}{\partial x}\right|_{x\to\infty} = \left.\frac{\partial H_{22}}{\partial x}\right|_{x\to\infty} = 0 \tag{2b}$$

where H_{MSL} is the mean sea level; A and ω are the amplitude and frequency of the sea tide, respectively. At the boundary of inhomogeneity (i.e., at $x=l$), the hydraulic head and flow velocity should be continuous in both confined and unconfined aquifers in all points parallel to y axis, therefore,

$$H_{11}(l,t) = H_{12}(l,t) \tag{3a}$$

$$H_{21}(l,t) = H_{22}(l,t) \tag{3b}$$

$$T_{11}\frac{\partial H_{11}(x,t)}{\partial x}\bigg|_{x=l} = T_{12}\frac{\partial H_{12}(x,t)}{\partial x}\bigg|_{x=l} \tag{3c}$$

$$T_{21}\frac{\partial H_{21}(x,t)}{\partial x}\bigg|_{x=l} = T_{22}\frac{\partial H_{22}(x,t)}{\partial x}\bigg|_{x=l} \tag{3d}$$

2.2 Analytical Solutions for Head Fluctuation

An analytical solution is developed for the governing equation (Eqs. 1a to 1d) with its associated boundary conditions (Eq. 2a, 2b, and 3a to 3d). Based on Eq. (1a), one can obtain H_{21} in terms of H_{11} as

$$H_{21} = \frac{1}{L_1}\left(S_{11}\frac{\partial H_{11}}{\partial t} - T_{11}\frac{\partial^2 H_{11}}{\partial x^2}\right) + H_{11} \tag{5}$$

Substituting Eq. 5 into Eq. 1c results in

$$\begin{aligned}&\frac{S_{11}S_{21}}{L_1}\frac{\partial^2 H_{11}}{\partial t^2} - \frac{T_{11}S_{21}}{L_1}\frac{\partial^3 H_{11}}{\partial t \partial x^2} + S_{21}\frac{\partial H_{11}}{\partial t} - \frac{T_{21}S_{11}}{L_1}\frac{\partial^3 H_{11}}{\partial t \partial x^2}\\ &+ \frac{T_{11}T_{21}}{L_1}\frac{\partial^4 H_{11}}{\partial x^4} - T_{21}\frac{\partial^2 H_{11}}{\partial x^2} + S_{11}\frac{\partial H_{11}}{\partial t} - T_{11}\frac{\partial^2 H_{11}}{\partial x^2} = 0\end{aligned} \tag{6}$$

Assume that $H_{11}=H_{MSL}+\mathrm{Re}[h_{11}(x)\exp(-i\omega t)]$, $H_{12}=H_{MSL}+\mathrm{Re}[h_{12}(x)\exp(-i\omega t)]$, $H_{21}=H_{MSL}+\mathrm{Re}[h_{21}(x)\exp(-i\omega t)]$, and $H_{22}=H_{MSL}+\mathrm{Re}[h_{22}(x)\exp(-i\omega t)]$ where $i=\sqrt{-1}$ Note that this assumption implies that the hydraulic head, a function of x and t, is separable. Substituting H_{11} into Eq. 6 yields

$$\begin{aligned}&\frac{T_{11}T_{21}}{L_1}\frac{d^4 h_{11}(x)}{dx^4} + \frac{i\omega}{L_1}(T_{11}S_{21} + T_{21}S_{11})\frac{d^2 h_{11}(x)}{dx^2} - (T_{21} + T_{11})\frac{d^2 h_{11}(x)}{dx^2}\\ &-\left(\frac{S_{11}S_{21}\omega^2}{L_1} + S_{21}i\omega + S_{11}i\omega\right)h_{11}(x) = 0\end{aligned} \tag{7}$$

The solution for Eq. 7 can be expressed as

$$h_{11} = a_1 e^{\lambda_1 x} + a_2 e^{\lambda_2 x} + a_3 e^{\lambda_3 x} + a_4 e^{\lambda_4 x} \tag{8}$$

where the constant coefficients $a_1, a_2, \ldots, a_4$ can be determined using the boundary conditions and $\lambda_1, \lambda_2, \ldots, \lambda_4$ are constant defined as

$$\lambda_{1,2,3,4} = \pm\frac{\sqrt{2}}{2}\sqrt{b \pm \sqrt{b^2 + 4c}} \tag{9}$$

with

$$b = i\omega\left(\frac{S_{21}}{T_{21}} + \frac{S_{11}}{T_{11}}\right) - \frac{L_1}{T_{11}} - \frac{L_1}{T_{21}} \tag{10a}$$

and

$$c = \frac{1}{T_{11}T_{21}}\left(S_{11}S_{21}\omega^2 + L_1 S_{21} i\omega + L_1 S_{11} i\omega\right) \tag{10b}$$

Similarly, H_{22} in terms of H_{12} can be obtained from Eq. 1b as

$$H_{22} = \frac{1}{L_2}\left(S_{12}\frac{\partial H_{12}}{\partial t} - T_{12}\frac{\partial^2 H_{12}}{\partial x^2}\right) + H_{12} \tag{11}$$

Substituting Eq. 11 into Eq. 1d yields

$$\begin{aligned}&\frac{S_{12}S_{22}}{L_2}\frac{\partial^2 H_{12}}{\partial t^2} - \frac{T_{12}S_{22}}{L_2}\frac{\partial^3 H_{12}}{\partial t \partial x^2} + S_{22}\frac{\partial H_{12}}{\partial t} - \frac{T_{22}S_{12}}{L_2}\frac{\partial^3 H_{12}}{\partial t \partial x^2} \\ &+ \frac{T_{12}T_{22}}{L_2}\frac{\partial^4 H_{12}}{\partial x^4} - T_{22}\frac{\partial^2 H_{12}}{\partial x^2} + S_{12}\frac{\partial H_{12}}{\partial t} - T_{12}\frac{\partial^2 H_{12}}{\partial x^2} = 0\end{aligned} \tag{12}$$

After substituting H_{12}=H_{MSL}+Re[$h_{12}(x)\exp(-i\omega t)$] into Eq. 12, one obtains

$$\begin{aligned}&\frac{T_{12}T_{22}}{L_2}\frac{d^4 h_{12}(x)}{dx^4} + \frac{i\omega}{L_2}\left(T_{12}S_{22} + T_{22}S_{12}\right)\frac{d^2 h_{12}(x)}{dx^2} - \left(T_{22} + T_{12}\right)\frac{d^2 h_{12}(x)}{dx^2} \\ &- \left(\frac{S_{12}S_{22}\omega^2}{L_2} + S_{22} i\omega + S_{12} i\omega\right)h_{12}(x) = 0\end{aligned} \tag{13}$$

The solution for Eq. 13 can be expressed as

$$h_{12} = a_5 e^{\lambda_5 x} + a_6 e^{\lambda_6 x} + a_7 e^{\lambda_7 x} + a_8 e^{\lambda_8 x} \tag{14}$$

where a_5, a_6, …, a_8 are constant coefficients, which can be determined using boundary conditions, and constants λ_5, λ_6, …, λ_8 can be obtained as

$$\lambda_{5,6,7,8} = \pm\frac{\sqrt{2}}{2}\sqrt{bb \pm \sqrt{bb^2 + 4cc}} \tag{15}$$

with

$$bb = i\omega\left(\frac{S_{21}}{T_{22}} + \frac{S_{12}}{T_{12}}\right) - \frac{L_2}{T_{12}} - \frac{L_2}{T_{22}} \tag{16a}$$

and

$$cc = \frac{1}{T_{12}T_{22}}\left(S_{12}S_{22}\omega^2 + L_2 S_{22} i\omega + L_2 S_{12} i\omega\right) \tag{16b}$$

The groundwater head fluctuation terminates at infinity. The positive value of the real part of λ should therefore be ignored (i.e., Re[λ_7] and Re[λ_8]>0) while the negative real part of λ is kept, indicating that $a_7=a_8=0$. Applying the boundary conditions (Eqs. 2a and 3a to 3d) to Eqs. 8 and 14 yields, respectively,

$$a_1 + a_2 + a_3 + a_4 = A \tag{17a}$$

$$\left(1-\frac{i\omega S_{11}}{L_1}\right)(a_1 + a_2 + a_3 + a_4)-\frac{T_{11}}{L_1}\left(a_1\lambda_1{}^2 + a_2\lambda_2{}^2 + a_3\lambda_3{}^2 + a_4\lambda_4{}^2\right) = A \tag{17b}$$

$$a_1e^{\lambda_1 l} + a_2e^{\lambda_2 l} + a_3e^{\lambda_3 l} + a_4e^{\lambda_4 l}-a_5e^{\lambda_5 l}-a_6e^{\lambda_6 x} = 0 \tag{17c}$$

$$\begin{aligned}&\left(1-\frac{i\omega S_{11}}{L_1}\right)\left(a_1e^{\lambda_1 l} + a_2e^{\lambda_2 l} + a_3e^{\lambda_3 l} + a_4e^{\lambda_4 l}\right)-\frac{T_{11}}{L_1}\left(a_1\lambda_1{}^2e^{\lambda_1 l} + a_2\lambda_2{}^2e^{\lambda_2 l} + a_3\lambda_3{}^2e^{\lambda_3 l} + a_4\lambda_4{}^2e^{\lambda_4 l}\right)\\&-\left(1-\frac{i\omega S_{12}}{L_2}\right)\left(a_5e^{\lambda_5 l} + a_6e^{\lambda_6 l}\right) + \frac{T_{12}}{L_2}\left(a_5\lambda_5{}^2e^{\lambda_5 l} + a_6\lambda_6{}^2e^{\lambda_6 l}\right) = 0\end{aligned} \tag{17d}$$

$$T_{11}\left(a_1\lambda_1e^{\lambda_1 l} + a_2\lambda_2e^{\lambda_2 l} + a_3\lambda_3e^{\lambda_3 l} + a_4\lambda_4e^{\lambda_4 l}\right)-T_{12}\left(a_5\lambda_5e^{\lambda_5 l} + a_6\lambda_6e^{\lambda_6 x}\right) = 0 \tag{17e}$$

$$\begin{aligned}&T_{21}\left(1-\frac{i\omega S_{11}}{L_1}\right)\left(a_1\lambda_1e^{\lambda_1 l} + a_2\lambda_2e^{\lambda_2 l} + a_3\lambda_3e^{\lambda_3 l} + a_4\lambda_4e^{\lambda_4 l}\right) - T_{22}\left(1-\frac{i\omega S_{12}}{L_2}\right)\left(a_5e^{\lambda_5 l} + a_6e^{\lambda_6 l}\right)\\&-\frac{T_{21}T_{11}}{L_1}\left(a_1\lambda_1{}^2e^{\lambda_1 l} + a_2\lambda_2{}^2e^{\lambda_2 l} + a_3\lambda_3{}^2e^{\lambda_3 l} + a_4\lambda_4{}^2e^{\lambda_4 l}\right) + \frac{T_{22}T_{12}}{L_2}\left(a_5\lambda_5{}^2e^{\lambda_5 l} + a_6\lambda_6{}^2e^{\lambda_6 l}\right) = 0\end{aligned} \tag{17f}$$

where the unknown coefficients a_1 to a_6 in Eqs. (17a) to (17f) are obtained and presented in Appendix 1, and then those coefficients in Eqs. 8 and 14 are determined.

Similarly, the head solutions for the confined aquifer can be developed as

$$h_{21} = b_1e^{\lambda_1 x} + b_2e^{\lambda_2 x} + b_3e^{\lambda_3 x} + b_4e^{\lambda_4 x} \tag{18}$$

$$h_{22} = b_5e^{\lambda_5 x} + b_6e^{\lambda_6 x} \tag{19}$$

where λ_1 to λ_6 are defined in Eqs. 9 and 15. The coefficients b_1 to b_6 can be determined by solving following equations using Mathematica software

$$b_1 + b_2 + b_3 + b_4 = A \tag{20a}$$

$$\left(1-\frac{i\omega S_{21}}{L_1}\right)(b_1 + b_2 + b_3 + b_4)-\frac{T_{21}}{L_1}\left(b_1\lambda_1{}^2 + b_2\lambda_2{}^2 + b_3\lambda_3{}^2 + b_4\lambda_4{}^2\right) = A \tag{20b}$$

$$b_1e^{\lambda_1 l} + b_2e^{\lambda_2 l} + b_3e^{\lambda_3 l} + b_4e^{\lambda_4 l} - b_5e^{\lambda_5 l} - b_6e^{\lambda_6 x} = 0 \tag{20c}$$

$$T_{21}\left(b_1\lambda_1e^{\lambda_1 l} + b_2\lambda_2e^{\lambda_2 l} + b_3\lambda_3e^{\lambda_3 l} + b_4\lambda_4e^{\lambda_4 l}\right)-T_{22}\left(b_5\lambda_5e^{\lambda_5 l} + b_6\lambda_6e^{\lambda_6 x}\right) = 0 \tag{20d}$$

$$
\begin{aligned}
&\left(1-\frac{i\omega S_{21}}{L_1}\right)\left(b_1e^{\lambda_1 l}+b_2e^{\lambda_2 l}+b_3e^{\lambda_3 l}+b_4e^{\lambda_4 l}\right)-\frac{T_{21}}{L_1}\left(b_1{\lambda_1}^2e^{\lambda_1 l}+b_2{\lambda_2}^2e^{\lambda_2 l}+b_3{\lambda_3}^2e^{\lambda_3 l}+b_4{\lambda_4}^2e^{\lambda_4 l}\right)\\
&-\left(1-\frac{i\omega S_{22}}{L_2}\right)\left(b_5e^{\lambda_5 l}+b_6e^{\lambda_6 l}\right)+\frac{T_{22}}{L_2}\left(b_5{\lambda_5}^2e^{\lambda_5 l}+b_6{\lambda_6}^2e^{\lambda_6 l}\right)=0
\end{aligned}
\tag{20e}
$$

$$
\begin{aligned}
&T_{11}\left(1-\frac{i\omega S_{21}}{L_1}\right)\left(b_1\lambda_1e^{\lambda_1 l}+b_2\lambda_2e^{\lambda_2 l}+b_3\lambda_3e^{\lambda_3 l}+b_4\lambda_4e^{\lambda_4 l}\right)-T_{12}\left(1-\frac{i\omega S_{22}}{L_2}\right)\left(b_5e^{\lambda_5 l}+b_6e^{\lambda_6 l}\right)\\
&-\frac{T_{21}T_{11}}{L_1}\left(b_1{\lambda_1}^2e^{\lambda_1 l}+b_2{\lambda_2}^2e^{\lambda_2 l}+b_3{\lambda_3}^2e^{\lambda_3 l}+b_4{\lambda_4}^2e^{\lambda_4 l}\right)+\frac{T_{22}T_{12}}{L_2}\left(b_5{\lambda_5}^2e^{\lambda_5 l}+b_6{\lambda_6}^2e^{\lambda_6 l}\right)=0
\end{aligned}
\tag{20f}
$$

Once these six constants (b_1, b_2, …, b_6) are known, one can evaluate H_{21} and H_{22} at any location and time. These constants are presented in Appendix 2.

2.3 Solutions for SGD

The SGDs from the upper and the lower aquifers, Q_U and Q_L, can be calculated, respectively, using following two equations (Li and Jiao 2003a, b; Chuang et al. 2012)

$$
Q_U=T_{11}\int_t^{t+P}\max\left[0,\left.\frac{\partial H_{11}}{\partial x}\right|_{x=0}\right]dt \tag{21a}
$$

$$
Q_L=T_{21}\int_t^{t+P}\max\left[0,\left.\frac{\partial H_{21}}{\partial x}\right|_{x=0}\right]dt \tag{21b}
$$

where P is the tidal period defined as $2\pi/\omega$

2.4 Special cases

Jeng et al. (2002) presented an analytical solution to describe the groundwater fluctuation in a homogeneous coastal aquifer system. The present solution for groundwater fluctuation can be shown to reduce to Jeng et al.'s solution (2002) if considering $l\rightarrow\infty$ or assuming $T_{11}=T_{12}=T_1$, $S_{11}=S_{12}=S_1$, $T_{21}=T_{22}=T_2$, $S_{21}=S_{22}=S_2$. Therefore, Jeng et al.'s solution (2002) may be considered as a special case of the present solution. Moreover, the present solution with $T_{11}\rightarrow\infty$ can give the same predictions as Chuang and Yeh's solution (2011) if there are only two horizontal regions in their leaky aquifer system and the thickness of the outlet capping approaches zero.

3 Results and Discussion

In this section, a hypothetical example is given to illustrate and investigate the effect of inhomogeneity on the tide-induced groundwater fluctuation. To address the effect of inhomogeneity, hydraulic diffusivities are herein defined for the unconfined as $D_{11}=T_{11}/S_{11}$ and $D_{12}=T_{12}/S_{12}$ and the confined aquifers as $D_{21}=T_{21}/S_{21}$ and $D_{22}=T_{22}/S_{22}$. Also, dimensionless diffusivities are introduced as $D_u=D_{11}/D_{12}$ and $D_b=D_{21}/D_{22}$ for the unconfined and confined aquifers, respectively, and dimensionless leakage is introduced as $L_m=L_1/L_2$ for the aquitard. Furthermore, phase lag is specified as

$\Phi_{11}=\tan^{-1}(\mathrm{Im}(h_{11}(x))/\mathrm{Re}(h_{11}(x))$ and $\Phi_{12}=\tan^{-1}(\mathrm{Im}(h_{12}(x))/\mathrm{Re}(h_{12}(x))$ for the unconfined aquifer and $\Phi_{21}=\tan^{-1}(\mathrm{Im}(h_{21}(x))/\mathrm{Re}(h_{21}(x))$ and $\Phi_{22}=\tan^{-1}(\mathrm{Im}(h_{22}(x))/\mathrm{Re}(h_{22}(x))$ for the confined aquifer. The contour for the normalized amplitude and phase lag of head fluctuations in both confined and unconfined aquifers are plotted to explore the joint effect of hydraulic parameters. Typical values of the hydraulic parameters used in Jeng et al. (2002) for coastal aquifers are adopted and shown in Table 1. The amplitude of the head fluctuation is calculated at x=50 m and x=150 m in both confined and unconfined aquifers. Normalized amplitudes of head fluctuation is defined as $\overline{H}$ (=H/A) in both unconfined aquifers and confined aquifers. Note that $\overline{H}_{11}$, $\overline{H}_{21}$, Φ_{11}, Φ_{21}, and Q_U are denoted as solid lines and $\overline{H}_{12}$ and $\overline{H}_{22}$ Φ_{12}, Φ_{22}, and Q_L dashed lines in Figs. 2–6. To compare the results with previous works two other dimensionless parameters $T=T_{11}/T_{21}$ and $T'=T_{11}/T_{21}$ are introduced.

3.1 Joint Effect of Dimensionless Leakage and Upper Aquifer Diffusivity on Head Fluctuation

Fig. 2a and b show the normalized amplitude of head fluctuation in unconfined and confined aquifers, respectively, for both dimensionless hydraulic diffusivity (D_u) and dimensionless leakages (L_m) ranging from 0.5 to 5 when dimensionless diffusivity (D_b) is equal to one. As shown in Fig. 2a, both $\overline{H}_{11}$ and $\overline{H}_{12}$ increase significantly with D_u for a constant L_m and increase slightly with L_m for a constant D_u. Fig. 2b displays that both $\overline{H}_{21}$ and $\overline{H}_{22}$ increase with D_u for a constant L_m and decrease as L_m increases when D_u is kept constant. The figures also demonstrate that the effect of L_m on the normalized amplitude of head fluctuation in the unconfined and confined aquifers is minor when D_u is small. On the other hand, the effect of D_u on $\overline{H}_{21}$ and $\overline{H}_{22}$ is

Table 1 Values of hydraulic parameters used in the hypothetical example

Parameter	value
Amplitude of tide A	0.65 m
Mean sea level h_{MSL}	0
Specific yield of unconfined aquifer S_{11}	0.3
Specific yield of unconfined aquifer S_{12}	0.3
Storativity of confined aquifer S_{21}	0.001
Storativity of confined aquifer S_{22}	0.001
Transmissivity of unconfined aquifer T_{11}	2,000 m^2/day or varying
Transmissivity of unconfined aquifer T_{12}	2,000 m^2/day or varying
Transmissivity of confined aquifer T_{21}	2,000 m^2/day or varying
Transmissivity of confined aquifer T_{21}	2,000 m^2/day or varying
Dimensionless transmissivity of left aquifers $T=T_{11}/T_{21}$	0.5~5
Dimensionless transmissivity of right aquifers $T'=T_{11}/T_{21}$	0.5~5
Leakage of aquitard L_1	1/day or Varying
Leakage of aquitard L_2	1/day or Varying
Dimensionless leakage $L_m=L_1/L_2$	0.25~10
Dimensionless diffusivity of upper aquifers $D_u=D_{11}/D_{12}$	0.5~10
Dimensionless diffusivity of lower aquifers $D_b=D_{21}/D_{22}$	0.5~10
Distance between coastline and inhomogeneity boundary l	100 m

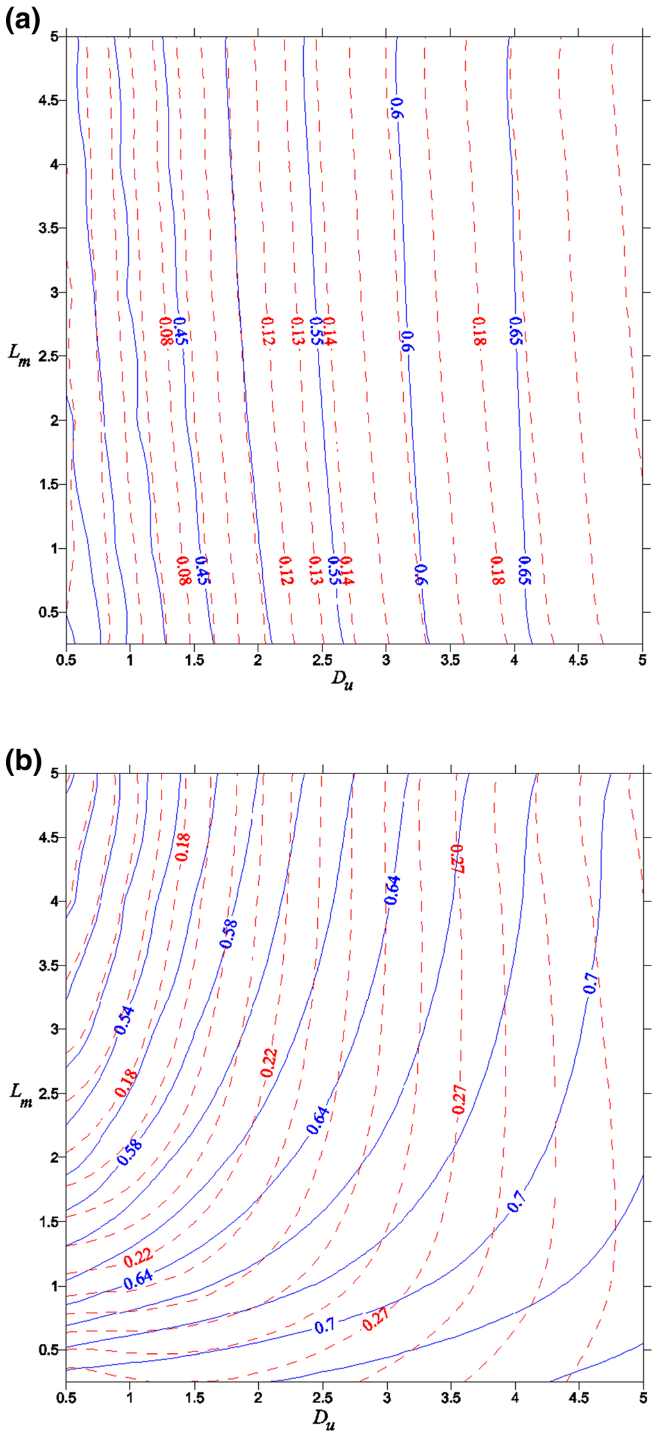

Fig. 2 Normalized amplitude of head fluctuation in **a** unconfined aquifer (*solid lines*) and **b** confined aquifer (*dashed lines*) for various values of dimensionless hydraulic diffusivity and dimensionless leakage

very significant when L_m is large. It is interesting to note that the normalized amplitude of head fluctuation is more sensitive to the change of L_m in confined aquifers than that in unconfined aquifers when D_u is small.

Figure 3a and b shows the phase lag of head fluctuation in the unconfined and confined aquifers, respectively, for different values of dimensionless diffusivity (D_u) and dimensionless leakages when D_b=1. As depicted in Fig. 3a, Φ_{11} decreases significantly as D_u increases, and Φ_{12} increases to reach its maximum and then decreases as D_u increases for L_m ranging from 0.5 to 5. Figure 3a also indicates that Φ_{11} decreases slightly as L_m increases for a constant D_u, and Φ_{12} increases significantly with L_m when D_u is small and increases slightly with L_m when D_u is large. Figure 3b shows that both Φ_{21} and Φ_{22} increase with D_u when L_m<2 and increases and then slightly decreases as D_u increases when L_m>3. Figure 3b also displays Φ_{21} and Φ_{22} increase significantly with L_m for a constant D_u when 0.5<D_u<5. The solid lines in Fig. 3a and b indicate that Φ_{11} is sensitive to the change of D_u, but Φ_{21} is sensitive to the change of L_m.

3.2 Joint Effect of Dimensionless Diffusivities on Head Fluctuation

Figure 4a and b show the normalized amplitude of head fluctuation in the unconfined and confined aquifers, respectively, when the dimensionless hydraulic diffusivity for both upper and lower aquifers ranging from 0.5 to 5 and dimensionless leakages is equal to one. Figure 4a displays that both $\overline{H}_{11}$ and $\overline{H}_{12}$ increase with D_b and D_u. This figure also demonstrates that the effect of the dimensionless diffusivity on $\overline{H}_{11}$ and $\overline{H}_{22}$ is larger when D_b=1 than when D_b=5. Figure 4b shows that both $\overline{H}_{21}$ and $\overline{H}_{22}$ increase with D_b when 0.5<D_u<5; however, the effect of D_u on $\overline{H}_{21}$ and $\overline{H}_{22}$ is larger when D_b=1 than when D_b=5. Figure 5a and b display the phase lags of head fluctuations in the unconfined and confined aquifers, respectively, for dimensionless hydraulic diffusivities of upper and lower aquifers in the range 0.5 to 5. As shown in Fig. 5a, Φ_{11} decreases slightly as D_b increases, but Φ_{12} decreases significantly as D_b increases when 0.5<D_u<5. Furthermore, Φ_{11} increases with $D_u$when 0.5<D_b<5. On the other hand, Φ_{12} increases slightly and then decreases as D_u increases when D_b is greater than one and decreases as D_u increases when D_b is less than one. Figure 5b shows that both Φ_{21} and Φ_{22} decrease significantly as D_b increases when 0.5<D_u<5. The solid lines in Figs. 5a and b indicate that Φ_{11} is more sensitive to the change of D_u, but Φ_{21} is more sensitive to the change of D_b.

3.3 Joint Effect of Dimensionless Leakage and Diffusivity on SGD

Figure 6 shows the SGDs in confined aquifer Q_L denoted by the solid line and in unconfined aquifer Q_U represented by the dashed line when D_b=1 and both D_u and L_m range from 0.5 to 5. The figure displays that Q_L is near 9 m/day when L_m=5 and D_u is near 0.5, but Q_L is close to 3.5 m/day when L_m=0.5 and D_u=5. The figure shows that the SGDs decreases significantly with increasing D_u when 0.5<L_m<5. In addition, the effect of D_u on the change of Q_L is large as L_m is large, and this influence is very significant when L_m=5. The figure also demonstrates that the SGDs from the lower aquifer increase with L_m when 0.5<D_u<5. The figure indicates that Q_U is near 14 m/day for D_u=0.5 but close to 46 m/day for D_u=5 when L_m=0.5.

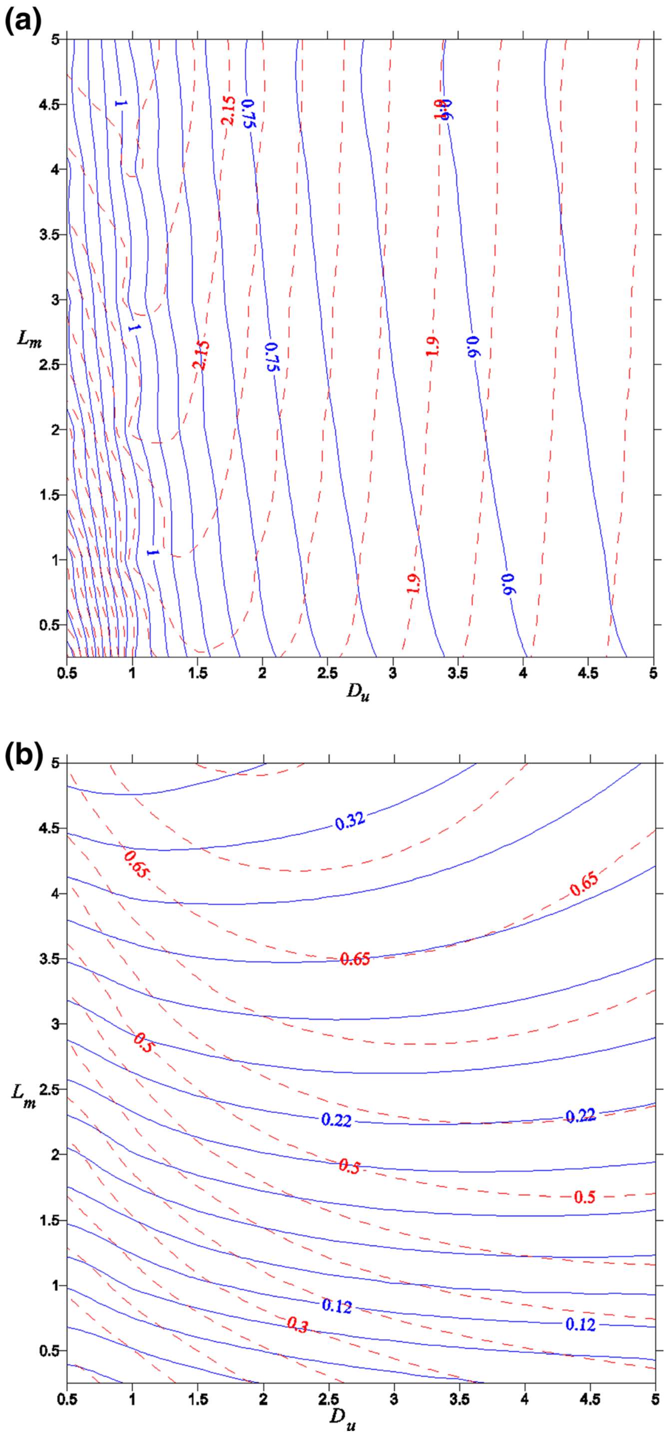

Fig. 3 Phase lag of head fluctuation in **a** unconfined aquifer (*solid lines*) and **b** confined aquifer (*dashed lines*) for various values of dimensionless hydraulic diffusivity and dimensionless leakage

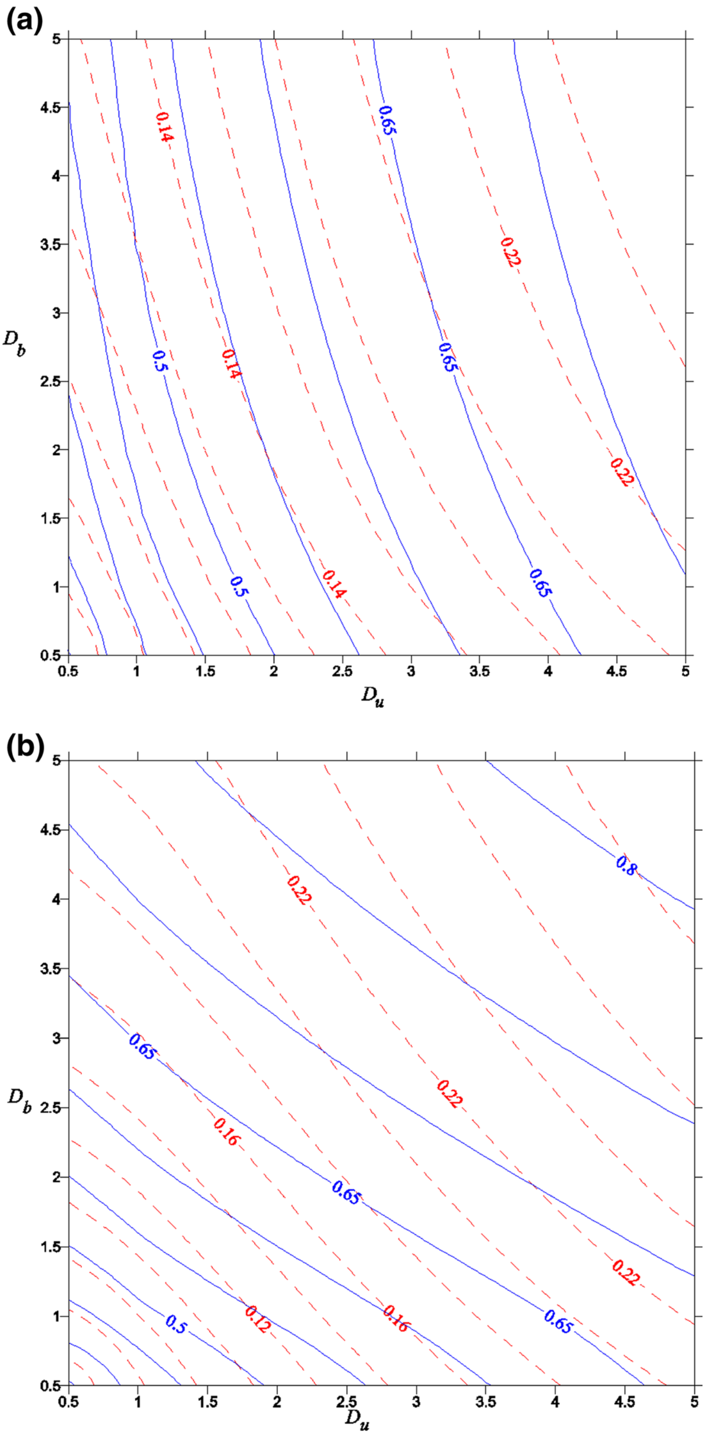

Fig. 4 Normalized amplitude of head fluctuation in **a** unconfined aquifer (*solid lines*) and **b** confined aquifer (*dashed lines*) for various values of dimensionless hydraulic diffusivity of upper and lower aquifers

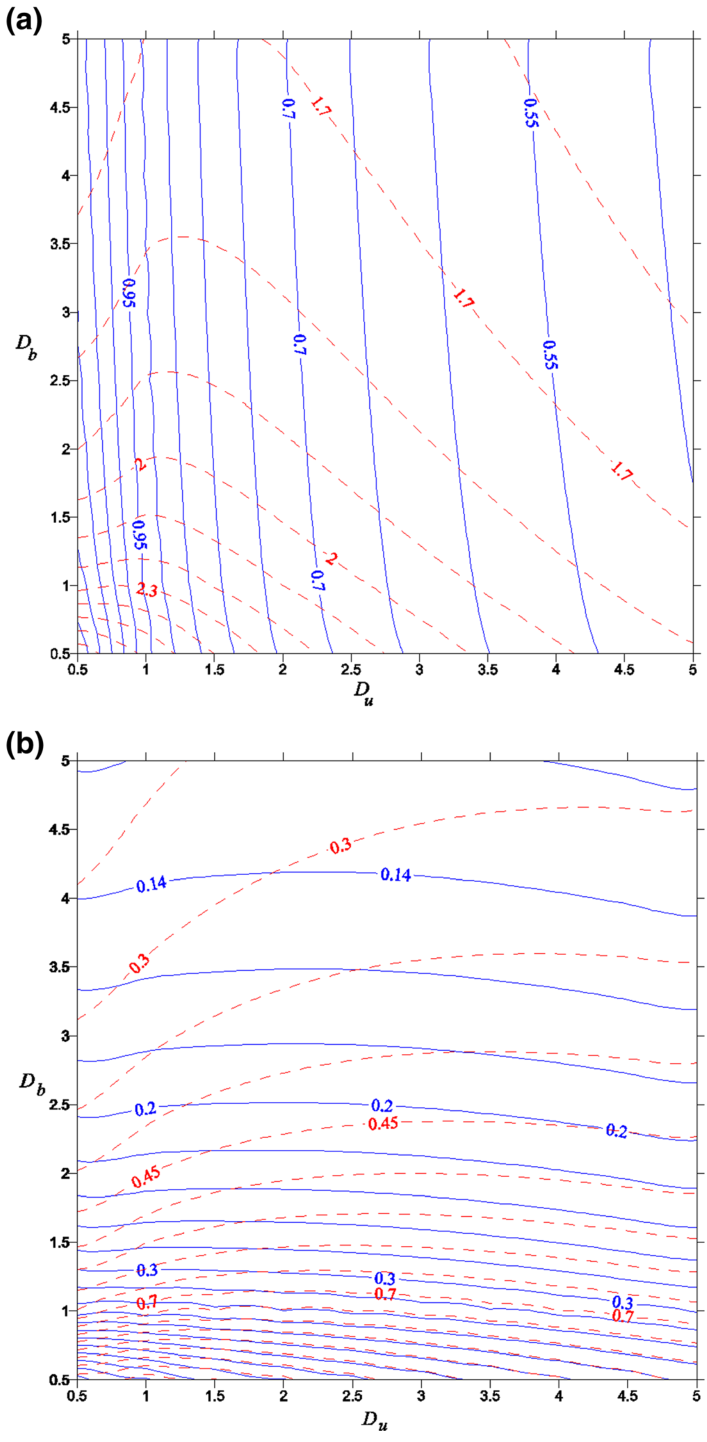

Fig. 5 Phase lag of head fluctuation in **a** unconfined aquifer (*solid lines*) and **b** confined aquifer (*dashed lines*) for various values of dimensionless hydraulic diffusivity of upper and lower aquifers

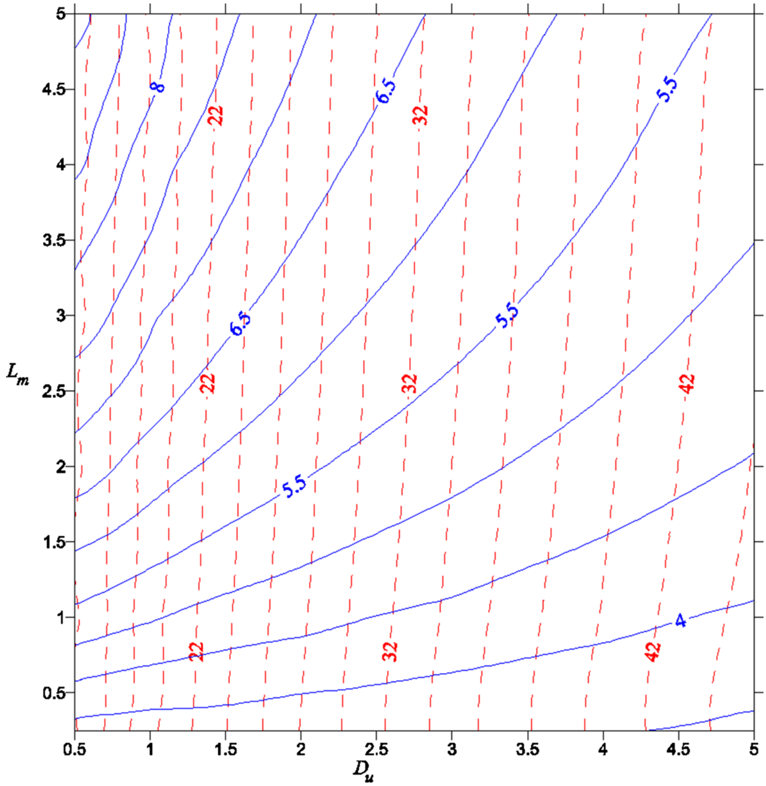

Fig. 6 Submarine groundwater discharge in confined aquifer (*dashed lines*), Q_L, and in unconfined aquifer (*solid lines*), Q_U, with various values of dimensionless leakage and dimensionless diffusivity of the upper aquifers

Furthermore, the SGDs increases significantly with increasing D_u when $0.5<L_m<5$. The figure also demonstrates that the effect of L_m on the SGDs is not significant when D_u is small and the SGDs slightly decreases as L_m increases for large D_u.

3.4 Comparison with Special Cases

As stated earlier, when the hydraulic properties of the left aquifer system equal those of the right aquifer system, the present solution will yield the same results with those of Jeng et al. (2002). The aquifer parameters in the present solution are chosen to be $T_{11}=T_{12}$, $T_{21}=T_{22}$, $S_{11}=S_{12}$, $S_{21}=S_{22}$, and $L_1=L_2$ to compare the results predicted by Jeng et al. (2002). Figure 7a and b show the temporal hydraulic head distributions in unconfined and confined aquifers, respectively, at $x=50$ m for the present and Jeng et al. (2002) solutions. As seen in the figures both solutions give the same results.

To compare the results of Chuang and Yeh (2011) solution, a large value is considered for the transmissivity of the left unconfined aquifer (T_{11}) in the present solution, and the effect of outlet capping is ignored (i.e., by setting its thickness as zero). The comparison between the hydraulic heads at $x=150$ m ($x=50$ in Chuang and Yeh 2011) of two works is presented in Fig. 8a and b for unconfined and confined aquifer, respectively. The figures indicate that the results of both two solutions are identical.

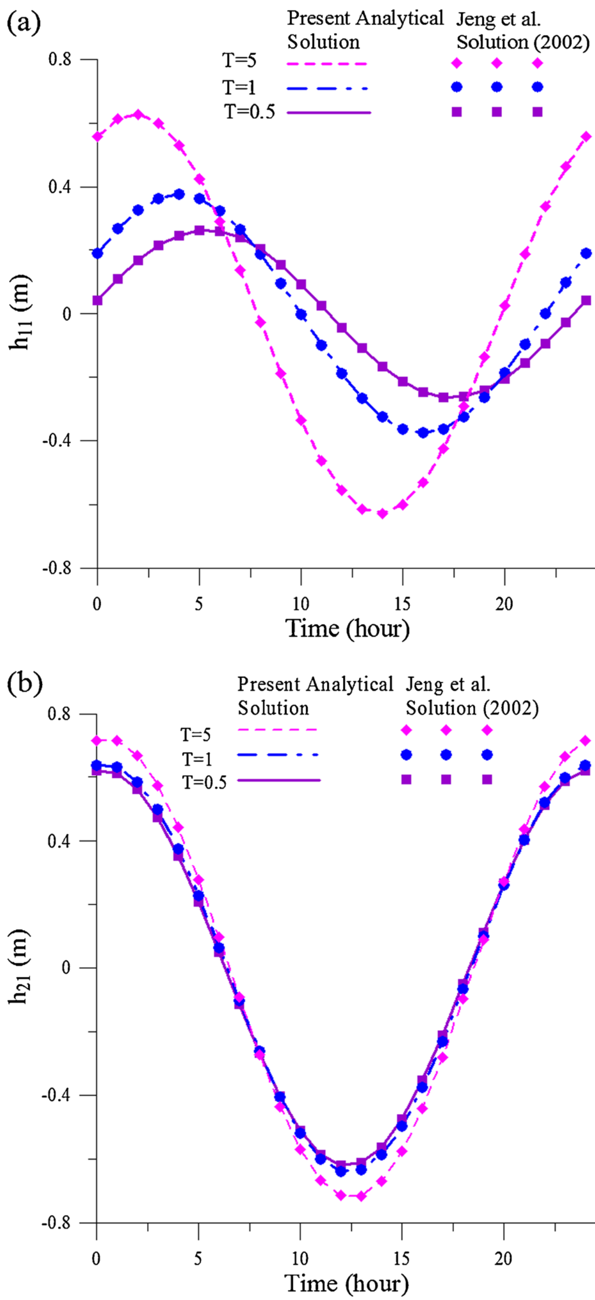

Fig. 7 Groundwater head fluctuation versus time at x=50 m in **a** unconfined aquifer and **b** confined aquifer

4 Concluding Remarks

This paper investigates tide-induced groundwater fluctuation in an inhomogeneous leaky aquifer system, comprising an unconfined aquifer on the top, a confined aquifer at the bottom,

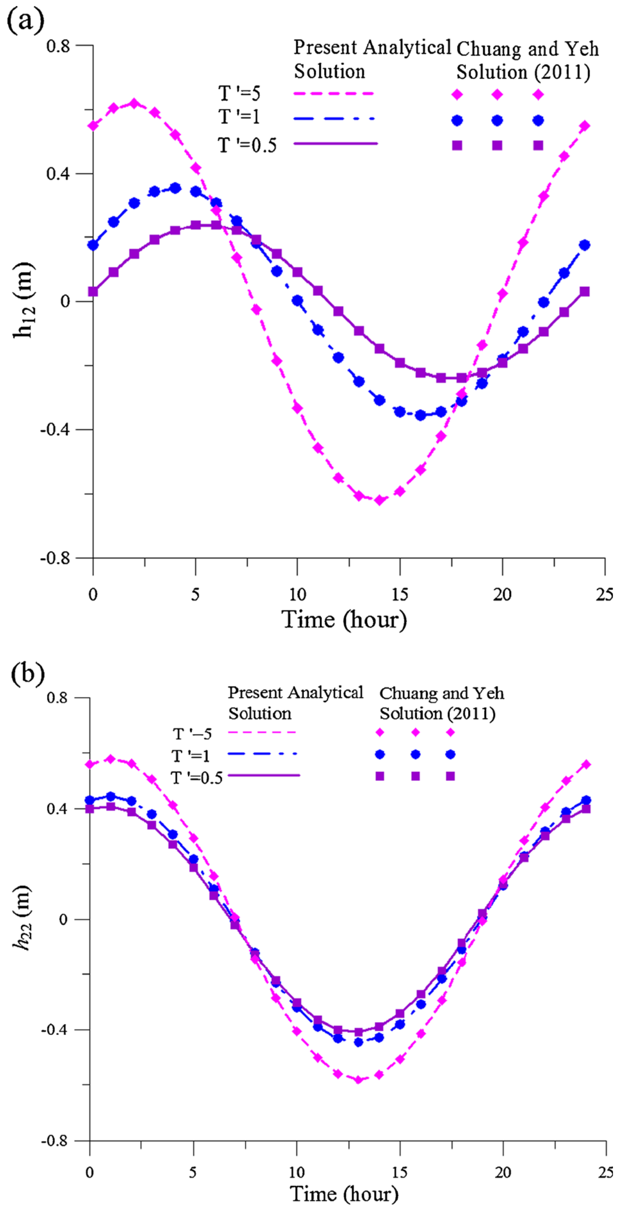

Fig. 8 Groundwater head fluctuation versus time at x=150 m in **a** unconfined aquifer and **b** confined aquifer

and an aquitard between them. The upper left aquifer is a developed land through reclamation. The inhomogeneous boundary occurs at a certain distance from the coast in all layers. An analytical solution is developed to evaluate the head fluctuations in unconfined and confined aquifers. Furthermore the SGDs in the unconfined and confined aquifers are also evaluated. Joint effect of the parameters of the leaky aquifer system on the head fluctuation and SGD is discussed in detail. The results predicted from the present head solution show that the head fluctuations in the unconfined and confined aquifers are dependent on the aquitard leakages. In addition, the head fluctuations in both unconfined and confined aquifers increase with dimensionless hydraulic diffusivity. The SGD from the lower aquifer decreases significantly with increasing dimensionless diffusivity of the upper aquifer and increases with dimensionless leakage. The SGD from the upper aquifer increases significantly with the dimensionless diffusivity of the upper aquifer and the dimensionless leakage does not have significant effect on the SGD.

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Appendixes

Appendix 1

The unknown coefficients a_1 to a_6 in Eqs. (17a) to (17f) can be expressed as:

$$a_1 = \frac{\alpha_1\alpha_2 - \alpha_3\alpha_4}{\alpha_5\alpha_4 - \alpha_6\alpha_1} \tag{A.1}$$

$$a_2 = \frac{\alpha_6\alpha_3 - \alpha_5\alpha_2}{\alpha_5\alpha_4 - \alpha_6\alpha_1} \tag{A.2}$$

$$a_3 = -\frac{1}{\alpha_7}(\alpha_8 a_1 + \alpha_9 a_2 + \alpha_{10}) \tag{A.3}$$

$$a_4 = A - a_1 - a_2 - a_3 \tag{A.4}$$

$$a_5 = -\frac{1}{\alpha_{11}}(\alpha_{12}a_1 + \alpha_{13}a_2 + \alpha_{14}a_3 + \alpha_{15}a_4) \tag{A.5}$$

$$a_6 = -\frac{1}{\alpha_{16}}(\alpha_{17}a_1 + \alpha_{18}a_2 + \alpha_{19}a_3 + \alpha_{20}a_4 + \alpha_{21}a_5) \tag{A.6}$$

Where

$$\alpha_1 = -\frac{\alpha_{13}\alpha_{30} + \alpha_{15}\alpha_{31} - \alpha_{15}\alpha_{30} - \alpha_{12}\alpha_{31} - \alpha_{33}\alpha_{13} + \alpha_{33}\alpha_{12}}{(\alpha_{12} - \alpha_{13})(\alpha_{30} - \alpha_{31})} \tag{A.7}$$

$$\alpha_2 = -\frac{A(-2\alpha_{17}\alpha_{31} + \alpha_{17}\alpha_{30} + \alpha_{31}\alpha_{16} - \alpha_{17} + \alpha_{16})}{(\alpha_{16} - \alpha_{17})(\alpha_{30} - \alpha_{31})} \tag{A.8}$$

$$\alpha_3 = -\frac{A(-2\alpha_{13}\alpha_{31} + \alpha_{13}\alpha_{30} + \alpha_{31}\alpha_{12} - \alpha_{13} + \alpha_{12})}{(\alpha_{12}-\alpha_{13})(\alpha_{30}-\alpha_{31})} \tag{A.9}$$

$$\alpha_4 = -\frac{\alpha_{17}\alpha_{30} + \alpha_{19}\alpha_{31} - \alpha_{19}\alpha_{30} - \alpha_{31}\alpha_{16} - \alpha_{33}\alpha_{17} - \alpha_{33}\alpha_{16}}{(\alpha_{12}-\alpha_{13})(\alpha_{30}-\alpha_{31})} \tag{A.10}$$

$$\alpha_5 = -\frac{\alpha_{17}\alpha_{30} + \alpha_{18}\alpha_{31} - \alpha_{18}\alpha_{30} - \alpha_{31}\alpha_{16} - \alpha_{32}\alpha_{17} - \alpha_{33}\alpha_{16}}{(\alpha_{16}-\alpha_{17})(\alpha_{30}-\alpha_{31})} \tag{A.11}$$

$$\alpha_6 = -\frac{\alpha_{13}\alpha_{30} + \alpha_{14}\alpha_{31} - \alpha_{14}\alpha_{30} - \alpha_{31}\alpha_{12} - \alpha_{32}\alpha_{13} - \alpha_{32}\alpha_{16}}{(\alpha_{12}-\alpha_{13})(\alpha_{30}-\alpha_{31})} \tag{A.12}$$

$$\alpha_7 = \frac{1}{\lambda_6}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)} \tag{A.13}$$

$$\alpha_8 = -\frac{1}{T_{12}\lambda_6}(T_{11}\lambda_1-T_{12}\lambda_6)e^{l(\lambda_1-\lambda_6)} \tag{A.14}$$

$$\alpha_9 = -\frac{1}{T_{12}\lambda_6}(T_{11}\lambda_2-T_{12}\lambda_6)e^{l(\lambda_2-\lambda_6)} \tag{A.15}$$

$$\alpha_{10} = -\frac{1}{T_{12}\lambda_6}(T_{11}\lambda_3-T_{12}\lambda_6)e^{l(\lambda_3-\lambda_6)} \tag{A.16}$$

$$\alpha_{11} = -\frac{1}{T_{12}\lambda_6}(T_{11}\lambda_4-T_{12}\lambda_6)e^{l(\lambda_4-\lambda_6)} \tag{A.17}$$

$$\alpha_{12} = \frac{\alpha_{22}\lambda_5 T_{12}e^{l(\lambda_5-\lambda_6)} - \alpha_{22}T_{12}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \alpha_{24}T_{11}\lambda_1 e^{l(\lambda_3-\lambda_6)} - \alpha_{24}T_{12}\lambda_6 e^{l(\lambda_3-\lambda_6)}}{\alpha_{24}T_{12}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{A.18}$$

$$\alpha_{13} = \frac{\alpha_{23}\lambda_5 T_{12}e^{l(\lambda_5-\lambda_6)} - \alpha_{23}T_{12}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \alpha_{24}T_{11}\lambda_1 e^{l(\lambda_4-\lambda_6)} - \alpha_{24}T_{12}\lambda_6 e^{l(\lambda_4-\lambda_6)}}{\alpha_{24}T_{12}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{A.19}$$

$$\alpha_{14} = \frac{\alpha_{20}\lambda_5 T_{12}e^{l(\lambda_5-\lambda_6)} - \alpha_{20}T_{12}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \alpha_{24}T_{11}\lambda_1 e^{l(\lambda_1-\lambda_6)} - \alpha_{24}T_{12}\lambda_6 e^{l(\lambda_1-\lambda_6)}}{\alpha_{24}T_{12}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{A.20}$$

$$\alpha_{15} = \frac{\alpha_{21}\lambda_5 T_{12} e^{l(\lambda_5-\lambda_6)} - \alpha_{21} T_{12}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \alpha_{24} T_{11}\lambda_1 e^{l(\lambda_2-\lambda_6)} - \alpha_{24} T_{12}\lambda_6 e^{l(\lambda_2-\lambda_6)}}{\alpha_{24} T_{12}(\lambda_5-\lambda_6) e^{l(\lambda_5-\lambda_6)}} \quad (A.21)$$

$$\alpha_{16} = \frac{\alpha_{27}\lambda_5 T_{12} e^{l(\lambda_5-\lambda_6)} - \alpha_{27} T_{12}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \alpha_{29} T_{11}\lambda_1 e^{l(\lambda_3-\lambda_6)} - \alpha_{29} T_{12}\lambda_6 e^{l(\lambda_3-\lambda_6)}}{\alpha_{29} T_{12}(\lambda_5-\lambda_6) e^{l(\lambda_5-\lambda_6)}} \quad (A.22)$$

$$\alpha_{17} = \frac{\alpha_{28}\lambda_5 T_{12} e^{l(\lambda_5-\lambda_6)} - \alpha_{28} T_{12}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \alpha_{24} T_{11}\lambda_1 e^{l(\lambda_4-\lambda_6)} - \alpha_{24} T_{12}\lambda_6 e^{l(\lambda_4-\lambda_6)}}{\alpha_{24} T_{12}(\lambda_5-\lambda_6) e^{l(\lambda_5-\lambda_6)}} \quad (A.23)$$

$$\alpha_{18} = \frac{\alpha_{25}\lambda_5 T_{12} e^{l(\lambda_5-\lambda_6)} - \alpha_{25} T_{12}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \alpha_{29} T_{11}\lambda_1 e^{l(\lambda_1-\lambda_6)} - \alpha_{29} T_{12}\lambda_6 e^{l(\lambda_1-\lambda_6)}}{\alpha_{29} T_{12}(\lambda_5-\lambda_6) e^{-l(\lambda_5-\lambda_6)}} \quad (A.24)$$

$$\alpha_{19} = \frac{\alpha_{26}\lambda_5 T_{12} e^{l(\lambda_5-\lambda_6)} - \alpha_{26} T_{12}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \alpha_{29} T_{11}\lambda_1 e^{l(\lambda_2-\lambda_6)} - \alpha_{29} T_{12}\lambda_6 e^{l(\lambda_2-\lambda_6)}}{\alpha_{29} T_{12}(\lambda_5-\lambda_6) e^{l(\lambda_5-\lambda_6)}} \quad (A.25)$$

$$\alpha_{20} = \frac{L_2 T_{11}{\lambda_1}^2 e^{l\lambda_1} + L_2 i S_{11}\omega - L_1 T_{12}{\lambda_6}^2 e^{l\lambda_1} - L_1 i S_{12}\omega e^{l(\lambda_1-\lambda_6)}}{L_1\left(L_2 e^{l\lambda_6} - T_{12}{\lambda_6}^2 e^{l\lambda_6} - i S_{12}\omega\right)} \quad (A.26)$$

$$\alpha_{21} = \frac{L_2 T_{11}{\lambda_2}^2 e^{l\lambda_2} + L_2 i S_{11}\omega - L_1 T_{12}{\lambda_6}^2 e^{l\lambda_2} - L_1 i S_{12}\omega e^{l(\lambda_2-\lambda_6)}}{L_1\left(L_2 e^{l\lambda_6} - T_{12}{\lambda_6}^2 e^{l\lambda_6} - i S_{12}\omega\right)} \quad (A.27)$$

$$\alpha_{22} = \frac{L_2 T_{11}{\lambda_3}^2 e^{l\lambda_3} + L_2 i S_{11}\omega - L_1 T_{12}{\lambda_6}^2 e^{l\lambda_3} - L_1 i S_{12}\omega e^{l(\lambda_3-\lambda_6)}}{L_1\left(L_2 e^{l\lambda_6} - T_{12}{\lambda_6}^2 e^{l\lambda_6} - i S_{12}\omega\right)} \quad (A.28)$$

$$\alpha_{23} = \frac{L_2 T_{11}{\lambda_4}^2 e^{l\lambda_4} + L_2 i S_{11}\omega - L_1 T_{12}{\lambda_6}^2 e^{l\lambda_4} - L_1 i S_{12}\omega e^{l(\lambda_4-\lambda_6)}}{L_1\left(L_2 e^{l\lambda_6} - T_{12}{\lambda_6}^2 e^{l\lambda_6} - i S_{12}\omega\right)} \quad (A.29)$$

$$\alpha_{24} = \frac{-T_{12}{\lambda_5}^2 e^{l\lambda_5} - i S_{12}\omega + T_{12}{\lambda_6}^2 e^{l\lambda_5} + i S_{12}\omega e^{l(\lambda_5-\lambda_6)}}{L_2 e^{l\lambda_6} - T_{12}{\lambda_6}^2 e^{l\lambda_6} - i S_{12}\omega} \quad (A.30)$$

$$\alpha_{25} = \frac{T_{21}{\lambda_1}^3 L_2 T_{11} - T_{21}\lambda_1 L_2 L_1 + T_{21}\lambda_1 L_2 S_{11}\omega i + L_1 T_{22}\lambda_6 L_2 - L_1 T_{22}{\lambda_6}^3 T_{12} - L_1 T_{22}\lambda_6 S_{12}\omega i}{L_1 T_{22}\lambda_6\left(L_2 - {\lambda_6}^2 T_{12} - S_{12}\omega i\right) e^{l(\lambda_6-\lambda_1)}} \quad (A.31)$$

$$\alpha_{26} = \frac{T_{21}{\lambda_2}^3 L_2 T_{11} - T_{21}\lambda_2 L_2 L_1 + T_{21}\lambda_2 L_2 S_{11}\omega i + L_1 T_{22}\lambda_6 L_2 - L_1 T_{22}{\lambda_6}^3 T_{12} - L_1 T_{22}\lambda_6 S_{12}\omega i}{L_1 T_{22}\lambda_6\left(L_2 - {\lambda_6}^2 T_{12} - S_{12}\omega i\right)e^{l(\lambda_6-\lambda_2)}} \tag{A.32}$$

$$\alpha_{27} = \frac{T_{21}{\lambda_3}^3 L_2 T_{11} - T_{21}\lambda_3 L_2 L_1 + T_{21}\lambda_3 L_2 S_{11}\omega i + L_1 T_{22}\lambda_6 L_2 - L_1 T_{22}{\lambda_6}^3 T_{12} - L_1 T_{22}\lambda_6 S_{12}\omega i}{L_1 T_{22}\lambda_6\left(L_2 - {\lambda_6}^2 T_{12} - S_{12}\omega i\right)e^{l(\lambda_6-\lambda_2)}} \tag{A.33}$$

$$\alpha_{28} = \frac{T_{21}{\lambda_4}^3 L_2 T_{11} - T_{21}\lambda_4 L_2 L_1 + T_{21}\lambda_4 L_2 S_{11}\omega i + L_1 T_{22}\lambda_6 L_2 - L_1 T_{22}{\lambda_6}^3 T_{12} - L_1 T_{22}\lambda_6 S_{12}\omega i}{L_1 T_{22}\lambda_6\left(L_2 - {\lambda_6}^2 T_{12} - S_{12}\omega i\right)e^{l(\lambda_6-\lambda_4)}} \tag{A.34}$$

$$\alpha_{29} = \frac{e^{l(\lambda_5-\lambda_6)}\left(\lambda_5 L_2 - T_{12}{\lambda_5}^3 - \lambda_5 S_{12}\omega i - L_2\lambda_6 + T_{12}{\lambda_6}^3 + \lambda_6 S_{12}\omega i\right)}{\lambda_6\left(L_2 - {\lambda_6}^2 T_{12} - S_{12}\omega i\right)} \tag{A.35}$$

$$\alpha_{30} = 1 - \frac{T_{11}{\lambda_3}^2 + iS_{11}\omega}{L_1} \tag{A.36}$$

$$\alpha_{31} = 1 - \frac{T_{11}{\lambda_4}^2 + iS_{11}\omega}{L_1} \tag{A.37}$$

$$\alpha_{32} = 1 - \frac{T_{11}{\lambda_1}^2 + iS_{11}\omega}{L_1} \tag{A.38}$$

$$\alpha_{33} = 1 - \frac{T_{11}{\lambda_2}^2 + iS_{11}\omega}{L_1} \tag{A.39}$$

Appendix 2

The unknown coefficients b_1 to b_6 in Eqs. (20a) to (20f) can be denoted as:

$$b_1 = \frac{\beta_1\beta_2 - \beta_3\beta_4}{\beta_5\beta_4 - \beta_6\beta_1} \tag{B.1}$$

$$b_2 = \frac{\beta_6\beta_3 - \beta_5\beta_2}{\beta_5\beta_4 - \beta_6\beta_1} \tag{B.2}$$

$$b_3 = -\frac{1}{\beta_7}(\beta_8 b_1 + \beta_9 b_2 + \beta_{10}) \tag{B.3}$$

$$b_4 = A - b_1 - b_2 - b_3 \tag{B.4}$$

$$b_5 = -\frac{1}{\beta_{11}}(\beta_{12} b_1 + \beta_{13} b_2 + \beta_{14} b_3 + \beta_{15} b_4) \tag{B.5}$$

$$b_6 = -\frac{1}{\beta_{16}}(\beta_{17} b_1 + \beta_{18} b_2 + \beta_{19} b_3 + \beta_{20} b_4 + \beta_{21} b_5) \tag{B.6}$$

where

$$\beta_1 = -\frac{\beta_{13}\beta_{30} + \beta_{15}\beta_{31} - \beta_{15}\beta_{30} - \beta_{12}\beta_{31} - \beta_{33}\beta_{13} + \beta_{33}\beta_{12}}{(\beta_{12}-\beta_{13})(\beta_{30}-\beta_{31})} \tag{B.7}$$

$$\beta_2 = -\frac{A(-2\beta_{17}\beta_{31} + \beta_{17}\beta_{30} + \beta_{31}\beta_{16} - \beta_{17} + \beta_{16})}{(\beta_{16}-\beta_{17})(\beta_{30}-\beta_{31})} \tag{B.8}$$

$$\beta_3 = -\frac{A(-2\beta_{13}\beta_{31} + \beta_{13}\beta_{30} + \beta_{31}\beta_{12} - \beta_{13} + \beta_{12})}{(\beta_{12}-\beta_{13})(\beta_{30}-\beta_{31})} \tag{B.9}$$

$$\beta_4 = -\frac{\beta_{17}\beta_{30} + \beta_{19}\beta_{31} - \beta_{19}\beta_{30} - \beta_{31}\beta_{16} - \beta_{33}\beta_{17} - \beta_{33}\beta_{16}}{(\beta_{12}-\beta_{13})(\beta_{30}-\beta_{31})} \tag{B.10}$$

$$\beta_5 = -\frac{\beta_{17}\beta_{30} + \beta_{18}\beta_{31} - \beta_{18}\beta_{30} - \beta_{31}\beta_{16} - \beta_{32}\beta_{17} - \beta_{33}\beta_{16}}{(\beta_{16}-\beta_{17})(\beta_{30}-\beta_{31})} \tag{B.11}$$

$$\beta_6 = -\frac{\beta_{13}\beta_{30} + \beta_{14}\beta_{31} - \beta_{14}\beta_{30} - \beta_{31}\beta_{12} - \beta_{32}\beta_{13} - \beta_{32}\beta_{16}}{(\beta_{12}-\beta_{13})(\beta_{30}-\beta_{31})} \tag{B.12}$$

$$\beta_7 = \frac{1}{\lambda_6}(\lambda_5 - \lambda_6)e^{l(\lambda_5-\lambda_6)} \tag{B.13}$$

$$\beta_8 = -\frac{1}{T_{22}\lambda_6}(T_{21}\lambda_1 - T_{22}\lambda_6)e^{l(\lambda_1-\lambda_6)} \tag{B.14}$$

$$\beta_9 = -\frac{1}{T_{22}\lambda_6}(T_{21}\lambda_2 - T_{22}\lambda_6)e^{l(\lambda_2-\lambda_6)} \tag{B.15}$$

$$\beta_{10} = -\frac{1}{T_{22}\lambda_6}(T_{21}\lambda_3 - T_{22}\lambda_6)e^{l(\lambda_3-\lambda_6)} \tag{B.16}$$

$$\beta_{11} = -\frac{1}{T_{22}\lambda_6}(T_{21}\lambda_4 - T_{22}\lambda_6)e^{l(\lambda_4-\lambda_6)} \tag{B.17}$$

$$\beta_{12} = \frac{\beta_{22}\lambda_5 T_{22}e^{l(\lambda_5-\lambda_6)} - \beta_{22}T_{22}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \beta_{24}T_{21}\lambda_1 e^{l(\lambda_3-\lambda_6)} - \beta_{24}T_{22}\lambda_6 e^{l(\lambda_3-\lambda_6)}}{\beta_{24}T_{22}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{B.18}$$

$$\beta_{13} = \frac{\beta_{23}\lambda_5 T_{22}e^{l(\lambda_5-\lambda_6)} - \beta_{23}T_{22}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \beta_{24}T_{21}\lambda_1 e^{l(\lambda_4-\lambda_6)} - \beta_{24}T_{22}\lambda_6 e^{l(\lambda_4-\lambda_6)}}{\beta_{24}T_{22}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{B.19}$$

$$\beta_{14} = \frac{\beta_{20}\lambda_5 T_{22}e^{l(\lambda_5-\lambda_6)} - \beta_{20}T_{22}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \beta_{24}T_{21}\lambda_1 e^{l(\lambda_1-\lambda_6)} - \beta_{24}T_{22}\lambda_6 e^{l(\lambda_1-\lambda_6)}}{\beta_{24}T_{22}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{B.20}$$

$$\beta_{15} = \frac{\beta_{21}\lambda_5 T_{22}e^{l(\lambda_5-\lambda_6)} - \beta_{21}T_{22}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \beta_{24}T_{21}\lambda_1 e^{l(\lambda_2-\lambda_6)} - \beta_{24}T_{22}\lambda_6 e^{l(\lambda_2-\lambda_6)}}{\beta_{24}T_{22}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{B.21}$$

$$\beta_{16} = \frac{\beta_{27}\lambda_5 T_{22}e^{l(\lambda_5-\lambda_6)} - \beta_{27}T_{22}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \beta_{29}T_{21}\lambda_1 e^{l(\lambda_3-\lambda_6)} - \beta_{29}T_{22}\lambda_6 e^{l(\lambda_3-\lambda_6)}}{\beta_{29}T_{22}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{B.22}$$

$$\beta_{17} = \frac{\beta_{28}\lambda_5 T_{22}e^{l(\lambda_5-\lambda_6)} - \beta_{28}T_{22}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \beta_{24}T_{21}\lambda_1 e^{l(\lambda_4-\lambda_6)} - \beta_{24}T_{22}\lambda_6 e^{l(\lambda_4-\lambda_6)}}{\beta_{24}T_{22}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{B.23}$$

$$\beta_{18} = \frac{\beta_{25}\lambda_5 T_{22}e^{l(\lambda_5-\lambda_6)} - \beta_{25}T_{22}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \beta_{29}T_{21}\lambda_1 e^{l(\lambda_1-\lambda_6)} - \beta_{29}T_{22}\lambda_6 e^{l(\lambda_1-\lambda_6)}}{\beta_{29}T_{22}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{B.24}$$

$$\beta_{19} = \frac{\beta_{26}\lambda_5 T_{22}e^{l(\lambda_5-\lambda_6)} - \beta_{26}T_{22}\lambda_6 e^{l(\lambda_5-\lambda_6)} + \beta_{29}T_{21}\lambda_1 e^{l(\lambda_2-\lambda_6)} - \beta_{29}T_{22}\lambda_6 e^{l(\lambda_2-\lambda_6)}}{\beta_{29}T_{22}(\lambda_5-\lambda_6)e^{l(\lambda_5-\lambda_6)}} \tag{B.25}$$

$$\beta_{20} = \frac{L_2 T_{21}{\lambda_1}^2 e^{l\lambda_1} + L_2 iS_{21}\omega - L_1 T_{22}{\lambda_6}^2 e^{l\lambda_1} - L_1 iS_{22}\omega e^{l(\lambda_1-\lambda_6)}}{L_1\left(L_2 e^{l\lambda_6} - T_{22}{\lambda_6}^2 e^{l\lambda_6} - iS_{22}\omega\right)} \tag{B.26}$$

$$\beta_{21} = \frac{L_2 T_{21} \lambda_2{}^2 e^{l\lambda_2} + L_2 i S_{21}\omega - L_1 T_{22} \lambda_6{}^2 e^{l\lambda_2} - L_1 i S_{22} \omega e^{l(\lambda_2 - \lambda_6)}}{L_1 \left(L_2 e^{l\lambda_6} - T_{22} \lambda_6{}^2 e^{l\lambda_6} - i S_{22} \omega\right)} \tag{B.27}$$

$$\beta_{22} = \frac{L_2 T_{21} \lambda_3{}^2 e^{l\lambda_3} + L_2 i S_{21}\omega - L_1 T_{22} \lambda_6{}^2 e^{l\lambda_3} - L_1 i S_{22} \omega e^{l(\lambda_3 - \lambda_6)}}{L_1 \left(L_2 e^{l\lambda_6} - T_{22} \lambda_6{}^2 e^{l\lambda_6} - i S_{22} \omega\right)} \tag{B.28}$$

$$\beta_{23} = \frac{L_2 T_{21} \lambda_4{}^2 e^{l\lambda_4} + L_2 i S_{21}\omega - L_1 T_{22} \lambda_6{}^2 e^{l\lambda_4} - L_1 i S_{22} \omega e^{l(\lambda_4 - \lambda_6)}}{L_1 \left(L_2 e^{l\lambda_6} - T_{22} \lambda_6{}^2 e^{l\lambda_6} - i S_{22} \omega\right)} \tag{B.29}$$

$$\beta_{24} = \frac{-T_{22} \lambda_5{}^2 e^{l\lambda_5} - i S_{22}\omega + T_{22} \lambda_6{}^2 e^{l\lambda_5} + i S_{22} \omega e^{l(\lambda_5 - \lambda_6)}}{L_2 e^{l\lambda_6} - T_{22} \lambda_6{}^2 e^{l\lambda_6} - i S_{22} \omega} \tag{B.30}$$

$$\beta_{25} = \frac{T_{11}\lambda_1{}^3 L_2 T_{21} - T_{11}\lambda_1 L_2 L_1 + T_{11}\lambda_1 L_2 S_{21}\omega i + L_1 T_{12}\lambda_6 L_2 - L_1 T_{12}\lambda_6{}^3 T_{22} - L_1 T_{12}\lambda_6 S_{22}\omega i}{L_1 T_{12}\lambda_6 \left(L_2 - \lambda_6{}^2 T_{22} - S_{22}\omega i\right) e^{l(\lambda_6 - \lambda_1)}} \tag{B.31}$$

$$\beta_{26} = \frac{T_{11}\lambda_2{}^3 L_2 T_{21} - T_{11}\lambda_2 L_2 L_1 + T_{11}\lambda_2 L_2 S_{21}\omega i + L_1 T_{12}\lambda_6 L_2 - L_1 T_{12}\lambda_6{}^3 T_{22} - L_1 T_{12}\lambda_6 S_{22}\omega i}{L_1 T_{12}\lambda_6 \left(L_2 - \lambda_6{}^2 T_{22} - S_{22}\omega i\right) e^{l(\lambda_6 - \lambda_2)}} \tag{B.32}$$

$$\beta_{27} = \frac{T_{11}\lambda_3{}^3 L_2 T_{21} - T_{11}\lambda_3 L_2 L_1 + T_{11}\lambda_3 L_2 S_{21}\omega i + L_1 T_{12}\lambda_6 L_2 - L_1 T_{12}\lambda_6{}^3 T_{22} - L_1 T_{12}\lambda_6 S_{22}\omega i}{L_1 T_{12}\lambda_6 \left(L_2 - \lambda_6{}^2 T_{22} - S_{22}\omega i\right) e^{l(\lambda_6 - \lambda_3)}} \tag{B.33}$$

$$\beta_{28} = \frac{T_{11}\lambda_4{}^3 L_2 T_{21} - T_{11}\lambda_4 L_2 L_1 + T_{11}\lambda_4 L_2 S_{21}\omega i + L_1 T_{12}\lambda_6 L_2 - L_1 T_{12}\lambda_6{}^3 T_{12} - L_1 T_{12}\lambda_6 S_{22}\omega i}{L_1 T_{12}\lambda_6 \left(L_2 - \lambda_6{}^2 T_{22} - S_{22}\omega i\right) e^{l(\lambda_6 - \lambda_4)}} \tag{B.34}$$

$$\beta_{29} = \frac{e^{l(\lambda_5 - \lambda_6)} \left(\lambda_5 L_2 - T_{22}\lambda_5{}^3 - \lambda_5 S_{22}\omega i - L_2 \lambda_6 + T_{22}\lambda_6{}^3 + \lambda_6 S_{22}\omega i\right)}{\lambda_6 \left(L_2 - \lambda_6{}^2 T_{22} - S_{22}\omega i\right)} \tag{B.35}$$

$$\beta_{30} = 1 - \frac{T_{21}\lambda_3{}^2 + i S_{21}\omega}{L_1} \tag{B.36}$$

$$\beta_{31} = 1 - \frac{T_{21}\lambda_4^{\,2} + iS_{21}\omega}{L_1} \tag{B.37}$$

$$\beta_{32} = 1 - \frac{T_{21}\lambda_1^{\,2} + iS_{21}\omega}{L_1} \tag{B.38}$$

$$\beta_{33} = 1 - \frac{T_{21}\lambda_2^{\,2} + iS_{21}\omega}{L_1} \tag{B.39}$$

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