Optimal Two-Stage Decoupled Partial PIC Receivers for Multiuser Detection

Yu-Tao Hsieh and Wen-Rong Wu

***Abstract*—Parallel interference cancellation (PIC) is considered a simple yet effective multiuser detector for direct-sequence code-division multiple-access (DS-CDMA) systems. However, system performance may deteriorate due to unreliable interference cancellation in the early stages. Thus, a partial PIC detector in which partial cancellation factors (PCFs) are introduced to control the interference cancellation level has been developed as a remedy. Although PCFs are crucial, complete solutions for their optimal values are not available. In this paper, we consider a two-stage decoupled partial PIC receiver. Using the minimum bit error rate (BER) criterion, we derive a complete set of optimal PCFs. This includes optimal PCFs for periodic and aperiodic spreading codes in additive white Gaussian channels and multipath channels. Simulation results show that our theoretical optimal PCFs agree closely with empirical ones. Our two-stage partial PIC using derived optimal PCFs outperforms not only a two-stage, but also a three-stage full PIC.**

***Index Terms*—Direct-sequence code-division multiple-access (DS-CDMA), multiple-access interference, multiuser detection, parallel interference cancellation.**

I. Introduction

DIRECT-SEQUENCE code-division multiple-access (DS-CDMA) is considered a promising technique in cellular and personal communications. Conventional matched-filter receivers suffer from multiple access interference (MAI) and the near-far effect. A maximum-likelihood multiuser detector was proposed [1] to mitigate these problems. Unfortunately, the computational complexity of this approach grows exponentially with the user number, prohibiting its practical applications. Suboptimum receivers were then considered [2]–[4] to reduce the computational complexity. The decorrelator, being a linear receiver, can effectively eliminate the MAI; however, it may greatly enhance noise [7]. The linear minimum mean square error (LMMSE) detector [7], [8], an improvement to the decorrelator, represents a compromise between interference suppression and noise enhancement. Although these suboptimal approaches are much simpler than the optimal solution, they require matrix inversion operations that are undesirable in practice.

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Y.-T. Hsieh is with the Department of Communication Engineering, National Chiao Tung University, Hsinchu, 300, Taiwan, R.O.C. (e-mail: ythsieh.cm86g@nctu.edu.tw).

W.-R. Wu is with the Department of Communication Engineering, National Chiao Tung University, 830R, Hsinchu, 300, Taiwan, R.O.C. (e-mail: wrwu@faculty.nctu.edu.tw).

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In addition to the aforementioned linear detectors, another category of interest is the subtractive-type interference cancellation method. Cancellation of this type involves only vector operations making it a good candidate for real-world implementation. For a particular desired user, the subtractive-type canceller estimates the interference from other users, regenerates it, and cancels it from the received signal. This cancellation process can be carried out for each interfering user either successively, i.e., successive interference cancellation (SIC) [10], [11], or in parallel, i.e., parallel interference cancellation (PIC) [12]–[15]. SIC estimates and cancels each interference one by one while PIC cancels all interferences simultaneously. SIC can perform better than PIC; however, its computational complexity is higher and processing delays are larger. There are two types of PIC classified according to the tentative decision devices used in each stage: hard-decision PICs (HPIC) [12], [16], [18], and soft-decision PICs (SPIC) [13], [16], [19]. It has been observed that HPICs perform better than SPICs [25].

Conventional PIC receivers permit a full cancellation of MAI. One problem associated with this approach is that the MAI estimate may not be reliable in the earlier canceling stages. This makes the PIC less effective when the number of users is large. The partial PIC detector has been proposed in which partial cancellation factors (PCFs) are introduced to control the interference cancellation level as a remedy [16], [17]. However, theoretical analysis of the partial HPIC is difficult due to the nonlinear decision devices used in the receiver. Consequently, optimal PCFs are usually obtained either by training via the least-mean-square (LMS) adaptive algorithm [20], [28], or theoretical derivation under some simplifying assumptions [21]. It is well known that the LMS algorithm is simple but converges slowly. The approaches in [20], [28] may not be adequate in fast fading environments. On the other hand, the optimal PCFs derived in [21] are only valid when the number of users is small. The work in [27] has derived optimal PCFs for partial SPIC; however, the results only apply to a perfect power control scenario. As mentioned above, HPICs perform better than SPIC's. Thus, one may expect that partial HPICs will also perform better than partial SPIC's. Although this seems intuitively reasonable, it may not be true. Our primary study shows that for a two-stage canceller, an optimal partial SPIC performs similar to an optimal partial HPIC.

A complete partial PIC requires $K(K-1)$ PCFs where K is the number of users and the computational complexity is high. Simplified partial PIC's have been proposed, in which only K PCFs are needed. Two structures are commonly used for simplified partial PICs; we call them the coupled and decoupled structures. In the coupled structure, each user output is related to all K PCFs [20], while in the decoupled structure, each user output is only related to a specific PCF. The partial HPICs mentioned in the preceding paragraph all use the coupled structure. While the partial SPIC described in [17] used a decoupled structure, optimal PCFs were not derived. A complete comparison of these two structures is not available in the literature. Our primary study shows that both structures (employing optimal PCFs) perform similarly. Since optimizing one PCF is much easier than optimizing K PCFs, it is preferable to deal with the decoupled structure.

In this paper, we focus on a two-stage partial SPIC receiver with a decoupled structure. Our motivation for using two-stage processing is that it requires low computational complexity and is particularly suitable for real-world implementation. As indicated in [17] that in higher stage processing, the PCFs will approach unity for stages greater than two. In other words, the PCFs in the second stage will dominate system performance. We first consider the additive white Gaussian noise (AWGN) channel and derive optimal PCFs for systems employing periodic codes. The criterion for optimization is the bit error rate (BER). We then extend the result to systems with aperiodic spreading codes. Finally, we consider optimal PCFs with multipath channels. Simulations show that the performance of our theoretical optimal PCFs is close to that of empirical ones. In addition, the optimal two-stage partial SPIC outperforms not only the two-stage full SPIC, but also the three-stage full SPIC. The remainder of this paper is organized as follows. In Section II, we describe the two-stage full and partial SPIC receiver structures. In Sections III and IV, we derive optimal PCFs with periodic and aperiodic codes, both in AWGN and multipath channels. Simulation results are presented and discussed in Section V. Finally, conclusions are given in Section VI.

II. System Model

Consider a synchronous CDMA system accommodating K users. Let $r(t)$ denote the received signal (for a certain bit interval), $s_k(t)$ the kth user's transmitted signal, and $n(t)$ AGWN. The equivalent baseband received signal can be described as

$$\begin{aligned} r(t) &= \sum_{k=1}^{K} s_k(t) + n(t) \\ &= \sum_{k=1}^{K} A_k b_k a_k(t) + n(t), \quad t \in [0, T] \end{aligned} \tag{1}$$

where A_k and b_k are the kth user's amplitude and data bit, $a_k(t)$ denotes its signature waveform, and T is the bit period. The signature waveform can be expressed as

$$a_k(t) = \sum_{i=0}^{N-1} a_{k,i} \Pi_{T_c}(t - iT_c) \tag{2}$$

where $a_{k,i} \in \{1/\sqrt{N}, -1/\sqrt{N}\}$ is the binary spreading chip sequence for User k, N is the processing gain, Π_{T_c} is a rectangular pulse waveform with support T_c and unit magnitude. Note that T_c is the chip period.

The first stage of a PIC receiver is the conventional matched filter bank. The output can be represented as

$$\begin{aligned} y_k &= \int_0^T r(t) a_k(t) dt \\ &= A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + \eta_k \end{aligned} \tag{3}$$

where ρ_{jk} is a correlation coefficient and η_k is the noise term after despreading. They are defined as

$$\rho_{jk} \triangleq \int_0^T a_j(t) a_k(t) dt \tag{4}$$

and

$$\eta_k \triangleq \int_0^T n(t) a_k(t) dt. \tag{5}$$

It can be seen that the output metric in (3) consists of three parts: the desired signal, MAI, and η_k. The conventional detector makes a decision based on y_k. Thus, MAI is treated as another noise source. When the number of users is large, MAI will seriously degrade the system performance. A PIC, being a multiuser detection scheme, was proposed to alleviate this problem. Let $\hat{r}_k(t)$ be an interference-subtracted signal (for User k) given by

$$\hat{r}_k(t) = r(t) - \sum_{j \neq k} \hat{s}_j(t) \tag{6}$$

where $\hat{s}_j(t)$ is a regenerated signal for User j. For SPIC, this signal is obtained by

$$\hat{s}_j(t) = y_j a_j(t). \tag{7}$$

Thus, the output signal in the second stage is then

$$z_k = \int_0^T \hat{r}_k(t) a_k(t) dt. \tag{8}$$

Finally, the symbol data is detected as $\hat{b}_k = sgn(z_k)$. In principle, the interference cancellation procedure in (6)–(8) can be repeated with multiple stages to obtain better performance. It is apparent from (3) and (7) that the regenerated signal is noisy. Thus, fully cancelling the regenerated interference may not yield best results. One solution to this problem is to partially cancel the interference. This idea is implemented by modifying (6) as

$$\hat{r}_k(t) = r(t) - \sum_{j \neq k} C_{jk}\ \hat{s}_j(t). \tag{9}$$

The constants C_{jk}s are called the partial cancellation factors (PCFs) for User k and their amplitudes should reflect the fidelity of the interference estimate. The structure of a partial SPIC receiver with three users is shown in Fig. 1.

Generally, $K \times (K-1)$ PCFs are needed for a two-stage partial PIC. It is apparent that the computational complexity of the partial PIC is high when the number of users is large [on the

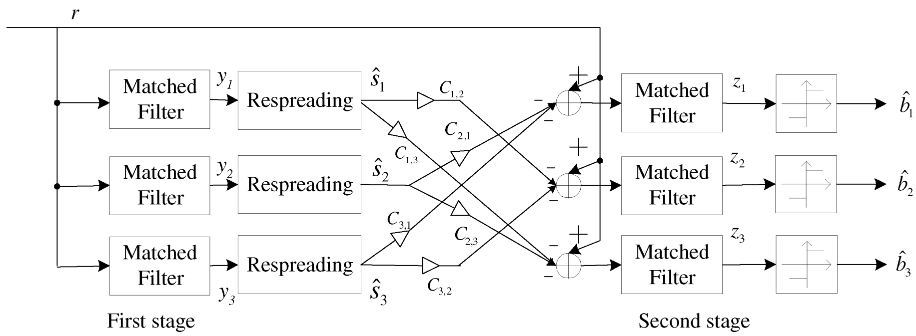

Fig. 1. General partial SPIC receiver structure.

order of $O(K^2)$]. Two simplified structures, whose complexities are on the order of $O(K)$, were investigated in the literature. The first one corresponds to the case in which $C_{jk} = C_j$ [in (9)]. In this case, all regenerated signals are first weighted and then summed. Thus, each regenerated interference signal in (9) has an individual PCF and the signal to be estimated is a function of all PCFs. We call this structure the coupled structure. The other structure is one in which $C_{jk} = C_k$. In this case, all regenerated signals are summed first and then weighted. Thus, there is one PCF for the signal to be estimated. We thus call this structure the decoupled structure. A thorough discussion of both structures is not available in the literature. Optimal PCFs have only been derived for the coupled structure under equal power scenarios [27]. In what follows, we focus on a two-stage partial SPIC receiver with a decoupled structure. Primary simulation results (in Section V) show that both PIC structures with optimal PCFs perform similarly.

III. Optimal PCFs for AWGN Channels

In this section, we derive the optimal PCFs for a two-stage partial SPIC under an AWGN channel. For ease of description, we only give the results associated with synchronous transmission. Periodic and aperiodic spreading codes are both considered.

A. Periodic Code Scenario

Assuming perfect chip synchronization, we first sample the received continuous-time signal in (1) with period T_c. Let $\mathbf{r} = [r(0), r(T_c), \ldots, r((N-1)T_c)]^T$ be the received signal sample vector, $\mathbf{a}_k \triangleq [a_{k,0}, a_{k,1}, \ldots, a_{k,N-1}]^T$ be the kth user's spreading sequence vector, and $\mathbf{n} \triangleq [n(0), n(T_c), \ldots, n((N-1)T_c)]^T$ be the noise sample vector. From (1), we have

$$\mathbf{r} = \sum_k A_k b_k \mathbf{a}_k + \mathbf{n}. \tag{10}$$

Thus, we can obtain the matched filter output as

$$\begin{aligned} y_k &= \mathbf{r}^T \mathbf{a}_k \\ &= A_k b_k + \sum_{j \neq k} A_j b_j \mathbf{a}_j^T \mathbf{a}_k + \mathbf{a}_k^T \mathbf{n}. \end{aligned} \tag{11}$$

Note that $\mathbf{a}_j^T \mathbf{a}_k$ is a discrete version of the correlation term ρ_{jk} shown in (4). Similarly, $\mathbf{a}_k^T \mathbf{n}$ is a discrete version of the noise-related term η_k in (5). For notational simplicity, we still use ρ_{jk} to represent $\mathbf{a}_j^T \mathbf{a}_k$ and η_k to represent $\mathbf{a}_k^T \mathbf{n}$. Thus, (11) can be rewritten as

$$y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + \eta_k. \tag{12}$$

For the second stage of a partial SPIC (with the decoupld structure), the regenerated signal for User k is

$$\hat{\mathbf{r}}_k = \mathbf{r} - C_k \sum_{j \neq k} \hat{\mathbf{s}}_j \tag{13}$$

where $\hat{\mathbf{s}}_j = y_j \mathbf{a}_j$. The second stage output is then

$$\begin{aligned} z_k &= \hat{\mathbf{r}}_k^T \mathbf{a}_k \\ &= y_k - C_k \sum_{j \neq k} y_j \rho_{jk} \\ &= A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + \eta_k \\ &\quad - C_k \sum_{j \neq k} \left(A_j b_j + \sum_{m \neq j} A_m b_m \rho_{mj} + \eta_j \right) \rho_{jk} \\ &= A_k b_k \left(1 - C_k \sum_{j \neq k} \rho_{jk}^2 \right) + \left(\eta_k - C_k \sum_{j \neq k} \eta_j \rho_{jk} \right) \\ &\quad + \sum_{j \neq k} A_j b_j \left(\rho_{jk} - C_k \rho_{jk} - C_k \sum_{m \neq j,k} \rho_{jm} \rho_{mk} \right). \end{aligned} \tag{14}$$

The bit error probability for User k, denoted as $\mathcal{P}(z_k)$, can be written as

$$\begin{aligned} \mathcal{P}(z_k) &= \frac{1}{2} \mathcal{P}(z_k \mid b_k = 1) + \frac{1}{2} \mathcal{P}(z_k \mid b_k = -1) \\ &= \mathcal{P}(z_k \mid b_k = 1). \end{aligned} \tag{15}$$

In (15), we assume that the occurrence probabilities for $b_k = 1$ and $b_k = -1$ are equal, and the error probabilities for $b_k = 1$ and $b_k = -1$ are also equal. As we can see, there are three terms in (14). The first term corresponds to the desired user bit. If we let $b_k = 1$, it is a deterministic value. The second term corresponds to noise interference which is Gaussian distributed. The third term corresponds to the interference from other users and each interference is binomial distributed. Note that correlation coefficients in (14) are small and CDMA systems are usually operated in low signal-to-noise ratio (SNR) environments. The variance of the third term is then much smaller than that of the second term. Thus, we can assume that z_k conditioned on $b_k = 1$ is Gaussian distributed. The error probability is then

$$\mathcal{P}(z_k) = \mathcal{Q}\left\{\sqrt{\frac{\mathcal{M}_k}{\mathcal{V}_k}}\right\} \tag{16}$$

where $Q\{.\}$ is the Q-function and

$$\mathcal{M}_k = \left(E\{z_k \mid b_k = 1\}\right)^2 \tag{17}$$

$$\mathcal{V}_k = E\{z_k^2\} - \mathcal{M}_k. \tag{18}$$

Note that the expectations in (17) and (18) are operated on interfering user bits and noise. Let $E\{\mathbf{n}\mathbf{n}^T\} = \sigma^2\mathbf{I}_N$ and $\gamma_j \triangleq A_j^2/\sigma^2$. Evaluating (17), we obtain

$$\mathcal{M}_k = A_k^2\left(1 - C_k\Lambda_k\right)^2 \tag{19}$$

where

$$\Lambda_k \triangleq \sum_{j\neq k}\rho_{jk}^2. \tag{20}$$

Similarly, we obtain the variance as

$$\mathcal{V}_k = \sigma^2(\Omega_{1,k}C_k^2 - 2\Omega_{2,k}C_k + \Omega_{3,k}) \tag{21}$$

where the coefficients of $\mathcal{V}_k$ are represented by

$$\Omega_{1,k} = \sum_{j\neq k}\gamma_j^2\left(\rho_{jk} + \sum_{m\neq j,k}\rho_{jm}\rho_{mk}\right)^2 + \sum_{j\neq k}\left(\rho_{jk}^2 + \sum_{m\neq j,k}\rho_{jm}\rho_{mk}\rho_{jk}\right) \tag{22}$$

$$\Omega_{2,k} = \sum_{j\neq k}\gamma_j^2\left(\rho_{jk}^2 + \sum_{m\neq j,k}\rho_{jm}\rho_{mk}\rho_{jk}\right) + \sum_{j\neq k}\rho_{jk}^2 \tag{23}$$

$$\Omega_{3,k} = \sum_{j\neq k}\gamma_j^2\rho_{jk}^2 + 1. \tag{24}$$

The optimal PCF for User k can be found as

$$C_{k,\text{opt}} = \arg\max_{C_k}\left\{\frac{\mathcal{M}_k}{\mathcal{V}_k}\right\} = \left\{C_{k,\text{opt}} : \mathcal{V}_k\frac{d\mathcal{M}_k}{dC_k} - \mathcal{M}_k\frac{d\mathcal{V}_k}{dC_k} = 0\right\}. \tag{25}$$

Substituting (19) and (21) into (25) and simplifying the result, we have the following equation:

$$(1 - C_{k,\text{opt}}\,\Lambda_k)\left[C_{k,\text{opt}}\,(\Omega_{1,k} - \Lambda_{1,k}\Omega_{2,k}) + \Lambda_k\Omega_{3,k} - \Omega_{2,k}\right] = 0. \tag{26}$$

We have two possible solutions now. The first solution for the first parenthesis is trivial since it makes the squared mean value $\mathcal{M}_k$ in (19) zero. The optimum PCF is then

$$C_{k,\text{opt}} = \frac{\Omega_{2,k} - \Omega_{3,k}\Lambda_k}{\Omega_{1,k} - \Omega_{2,k}\Lambda_k}. \tag{27}$$

We also derived optimal PCFs for an asynchronous CDMA system. The results are summarized in Appendix I. In what follows, we discuss some special cases to give a better understanding of the optimal PCF characteristics. Let the correlations between any two user spreading codes be equal ($\rho_{jk} = \rho$ for $j \neq k$) and the power control be perfect ($A_k = A$ and $\gamma_k = \gamma$). The optimal PCF can then be expressed as

$$C_{k,\text{opt}} = \frac{\gamma}{1 + \gamma[1 + \rho(K-2)]}. \tag{28}$$

As we can see from (28), the optimal PCF is smaller when ρ or K is larger, because when the correlations between user codes are higher and the number of users is larger, the MAI is larger, and the regenerated signal is unreliable. As a result, the PCF should be smaller. Also, when the user power is larger or the noise is smaller (γ is larger), the optimal PCF is larger. If we assume that the noise is much smaller than the signal power ($\gamma \gg 1$), the optimal PCF can be further simplified to

$$C_{k,\text{opt}} = \frac{1}{1 + \rho(K-2)}. \tag{29}$$

Now the optimal PCF is independent of the transmission signal power. The bit error performance would also be saturated in this interference-limited region. From (28), we can also see that when the noise is large ($\gamma \ll 1$), the optimal PCF tends to be small ($C_k \to 0$). Note that the effect of the processing gain N is reflected in the receiving SNR. If N is larger, the receiving SNR will become smaller.

B. Aperiodic Code Scenario

In commercial CDMA systems, the users' spreading codes are often modulated with another code having a very long period. As far as the received signal is concerned, the spreading code is not periodic. In other words, there will be many possible spreading codes for each user. If we use the result derived above, we then have to calculate the optimum PCFs for each possible code and the computational complexity will become very high. Since the period of the modulating code is usually very long, we can treat the code chips as independent random variables and approximate the correlation coefficient, ρ_{jk}, as a Gaussian random variable. As a result, the expectations in (17) and (18) can be further operated on ρ_{jk}. This greatly simplifies the optimal PCF evaluation. We now rewrite (16) as

$$\mathcal{P}(z_k) = \mathcal{Q}\left\{\sqrt{\frac{E_{\mathcal{L}}\{\mathcal{M}_k^{(l)}\}}{E_{\mathcal{L}}\{\mathcal{V}_k^{(l)}\}}}\right\} \tag{30}$$

where $E_{\mathcal{L}}\{\cdot\}$ denotes the expectation operator over the spreading code set $\mathcal{L}$ and $\mathcal{M}_k^{(l)}$ and $\mathcal{V}_k^{(l)}$ are the expected squared mean and variance of z_k, respectively, given the lth possible code in $\mathcal{L}$. Letting $I_k = \sum_{j\neq k}\gamma_j^2$, considering ρ_{jk} as a Gaussian random variable, and evaluating (17) and (18), we have

$$E_{\mathcal{L}}\{\mathcal{M}_k^{(l)}\} = A_k^2\left(1 - C_kE_{\mathcal{L}}\{\Lambda_k^{(l)}\}\right)^2 \tag{31}$$

where

$$E_{\mathcal{L}}\left\{\Lambda_k^{(l)}\right\} = \frac{K-1}{N} \tag{32}$$

and

$$E_{\mathcal{L}}\{\mathcal{V}_k^{(l)}\}=\sigma^2\left(E_{\mathcal{L}}\{\Omega_{1,k}^{(l)}\}C_k^2-2E_{\mathcal{L}}\{\Omega_{2,k}^{(l)}\}C_k+E_{\mathcal{L}}\{\Omega_{3,k}^{(l)}\}\right) \tag{33}$$

where

$$E_{\mathcal{L}}\left\{\Omega_{1,k}^{(l)}\right\}=I_k\left(\frac{1}{N}+\frac{3(K-2)}{N^2}+\frac{(K-2)(K-3)}{N^3}\right)+\frac{K-1}{N}+\frac{(K-1)(K-2)}{N^2} \tag{34}$$

$$E_{\mathcal{L}}\left\{\Omega_{2,k}^{(l)}\right\}=I_k\left(\frac{1}{N}+\frac{K-2}{N^2}\right)+\frac{K-1}{N} \tag{35}$$

$$E_{\mathcal{L}}\left\{\Omega_{3,k}^{(l)}\right\}=\frac{I_k}{N}+1. \tag{36}$$

In the above expressions, the notation $X^{(l)}$ denotes the X value given the lth possible spreading code in $\mathcal{L}$. Equation (25) can be re-expressed as

$$\begin{aligned} &C_{k,\text{opt}}\\ &=\arg\max_{C_k}\left\{\frac{E_{\mathcal{L}}\{\mathcal{M}_k^{(l)}\}}{E_{\mathcal{L}}\{\mathcal{V}_k^{(l)}\}}\right\}\\ &=\left\{C_{k,\text{opt}}: E_{\mathcal{L}}\{\mathcal{V}_k^{(l)}\}\frac{dE_{\mathcal{L}}\{\mathcal{M}_k^{(l)}\}}{dC_k}-E_{\mathcal{L}}\{\mathcal{M}_k^{(l)}\}\frac{dE_{\mathcal{L}}\{\mathcal{V}_k^{(l)}\}}{dC_k}=0\right\}. \end{aligned} \tag{37}$$

Substituting (31)–(36) into (37) and simplifying the result, we finally obtain

$$C_{k,\text{opt}}=\frac{E_{\mathcal{L}}\{\Omega_{2,k}^{(l)}\}-E_{\mathcal{L}}\{\Omega_{3,k}^{(l)}\}E_{\mathcal{L}}\{\Lambda_k^{(l)}\}}{E_{\mathcal{L}}\{\Omega_{1,k}^{(l)}\}-E_{\mathcal{L}}\{\Omega_{2,k}^{(l)}\}E_{\mathcal{L}}\{\Lambda_k^{(l)}\}}. \tag{38}$$

As we can see, (38) only involves (32) and (34)–(36) and these expressions are easy to work with. We further consider the case in which noise is small ($I_k \gg K$). Eq. (38) can be simplified to

$$C_{k,\text{opt}}=\frac{N}{N+2K-4}. \tag{39}$$

This result is remarkably simple. We only require N and K to calculate optimal PCFs; this will be useful in real-world applications.

IV. Optimal PCFs for Multipath Channels

A. Periodic Code Scenario

Let the transfer function for User k's channel be

$$W_k(z)=\sum_{i=1}^{L}h_{k,i}z^{-\tau_{k,i}}. \tag{40}$$

As we can see from (40), the number of paths is L and the gain and delay for the ith channel path are $h_{k,i}$ and $\tau_{k,i}$, respectively. We use two vectors to represent these parameters: $\mathbf{t}_k=[\tau_{k,1},\tau_{k,2},\ldots,\tau_{k,L}]^T$ and $\mathbf{h}_k=[h_{k,1},h_{k,2},\ldots,h_{k,L}]^T$. Let $\tau_{k,1}\le\tau_{k,2}\le\cdots\le\tau_{k,L}$ and the channel power is normalized ($\sum h_{k,i}^2=1$). Without loss of generality, we may assume that $\tau_{k,1}=0$ for each user and L is the maximum possible number of paths. When a user's path number, say L', is less than L, we can let all the elements in $\tau_{k,i}$ and $h_{k,i}$ be zero for $L'+1\le i\le L$. We may also assume that the maximum delay is much smaller than the processing gain N [32]. Before our formulation, we first define a $(2N-1)\times L$ composite signature matrix $\mathbf{S}_k$ as

$$\mathbf{S}_k\triangleq[\tilde{\mathbf{a}}_{k,1},\tilde{\mathbf{a}}_{k,2},\ldots,\tilde{\mathbf{a}}_{k,L}] \tag{41}$$

where $\tilde{\mathbf{a}}_{k,i}$ is a vector containing ith delayed spreading code for User k. It is defined as

$$\tilde{\mathbf{a}}_{k,i}\triangleq[\overbrace{0\ldots0}^{\tau_{k,i}},\ \mathbf{a}_k^T,\ \overbrace{0,\ldots,0}^{N-\tau_{k,i}-1}]^T. \tag{42}$$

Since a multipath channel is involved, the current received bit signal will be interfered by previous bit signals. As mentioned above, the maximum path delay is much smaller than the processing gain. The interference will not be severe and for simplicity, we may ignore this effect. Let $\mathbf{f}_k=\mathbf{S}_k\mathbf{h}_k$. As that in (10), we can obtain the received signal vector as

$$\mathbf{r}=\sum_k A_k b_k\mathbf{f}_k+\mathbf{n}. \tag{43}$$

To have better results, we use a maximum ratio rake combing scheme in the receiver. Let $\varrho_{jk}=\mathbf{f}_j^T\mathbf{f}_k$, $\varrho_k=\varrho_{kk}$, and $v_k=\mathbf{n}^T\mathbf{f}_k$. The output of the receiver is then

$$\begin{aligned} y_k&=\mathbf{r}^T\mathbf{f}_k\\ &=A_kb_k\mathbf{f}_k^T\mathbf{f}_k+\sum_{j\neq k}A_jb_j\mathbf{f}_j^T\mathbf{f}_k+\mathbf{n}^T\mathbf{f}_k\\ &=A_kb_k\varrho_k+\sum_{j\neq k}A_jb_j\varrho_{jk}+v_k. \end{aligned} \tag{44}$$

This result is similar to that in (12) except that ρ_{jk} is replaced by ϱ_{jk} and η_k is replaced by v_k. For the second stage of a partial SPIC, the regenerated signal is

$$\begin{aligned} \hat{\mathbf{r}}_k&=\mathbf{r}-C_k\sum_{j\neq k}\hat{\mathbf{s}}_j\\ &=\mathbf{r}-C_k\sum_{j\neq k}y_j\mathbf{f}_j. \end{aligned} \tag{45}$$

We then have the output signal for the second stage as

$$\begin{aligned} z_k&=\hat{\mathbf{r}}_k^T\mathbf{f}_k\\ &=A_kb_k\left(\varrho_k-C_k\sum_{j\neq k}\varrho_{j,k}^2\right)+v_k-C_k\sum_{j\neq k}v_j\varrho_{jk}\\ &\quad+\sum_{j\neq k}A_jb_j\left(\varrho_{jk}-C_k\varrho_{jk}-C_k\sum_{m\neq j,k}\varrho_{jm}\varrho_{mk}\right). \end{aligned} \tag{46}$$

As previously, we assume that z_k is Gaussian distributed, the interfering bits and noise are random, and parameters N, K, $\mathbf{t}_k$, $\mathbf{h}_k$, γ_k, and ϱ_{jk} are known beforehand. Thus, the output error probability is expressed as in (16) where the squared mean for z_k, similar to that of (19), is obtained from (17) and (46) as

$$\mathcal{M}_k = A_k^2 \left(\varrho_k - C_k \Gamma_k\right)^2 \tag{47}$$

where

$$\Gamma_k \triangleq \sum_{j \neq k} \varrho_{jk}^2 \tag{48}$$

and the variance is obtained from (18) and (46) as

$$\mathcal{V}_k = \sigma^2 \left(\Xi_{1,k} C_k^2 - 2\Xi_{2,k} C_k + \Xi_{3,k}\right) \tag{49}$$

where

$$\begin{aligned}\Xi_{1,k} = &\sum_{j \neq k} \gamma_j^2 \left(\varrho_{jk}\varrho_j + \sum_{m \neq j,k} \varrho_{jm}\varrho_{mk}\right)^2 \\ &+ \sum_{j \neq k} \left(\varrho_{jk}^2 \varrho_j + \sum_{m \neq j,k} \varrho_{jm}\varrho_{mk}\varrho_{jk}\right)\end{aligned} \tag{50}$$

$$\Xi_{2,k} = \sum_{j \neq k} \gamma_j^2 \left(\varrho_{jk}^2 \varrho_j + \sum_{m \neq j,k} \varrho_{jm}\varrho_{mk}\varrho_{jk}\right) + \sum_{j \neq k} \varrho_{jk}^2 \tag{51}$$

$$\Xi_{3,k} = \sum_{j \neq k} \gamma_j^2 \varrho_{jk}^2 + \varrho_k. \tag{52}$$

The optimal PCF derivation for the multipath channels is similar to that in (25). Substituting (47) and (49) into (25), we then obtain

$$C_{k,\text{opt}} = \frac{\varrho_k \Xi_{2,k} - \Xi_{3,k}\Gamma_k}{\varrho_k \Xi_{1,k} - \Xi_{2,k}\Gamma_k}. \tag{53}$$

B. Aperiodic Code Scenario

If aperiodic codes are utilized, ϱ_{jk}'s can be seen as Gaussian random variables. Using the method in Section III, we can obtain the corresponding optimal PCFs. From (47), we have the expected squared mean as

$$\begin{aligned}E_{\mathcal{L}}\{\mathcal{M}_k^{(l)}\} &= A_k^2 \left(E_{\mathcal{L}}\{\varrho_k^{(l)}\} - C_k E_{\mathcal{L}}\{\Gamma_k^{(l)}\}\right)^2 \\ &= A_k^2 \left(1 - C_k E_{\mathcal{L}}\{\Gamma_k^{(l)}\}\right)^2\end{aligned} \tag{54}$$

and the variance as

$$E_{\mathcal{L}}\{\mathcal{V}_k^{(l)}\} = \sigma^2 \left(E_{\mathcal{L}}\{\Xi_{1,k}^{(l)}\} C_k^2 - 2E_{\mathcal{L}}\{\Xi_{2,k}^{(l)}\} C_k + E_{\mathcal{L}}\{\Xi_{3,k}^{(l)}\}\right). \tag{55}$$

Comparing (54)–(55) with (31)–(33), we see that the optimal PCF here is similar to that in (37). We then have the optimal PCF as

$$C_{k,\text{opt}} = \frac{E_{\mathcal{L}}\{\Xi_{2,k}^{(l)}\} - E_{\mathcal{L}}\{\Xi_{3,k}^{(l)}\} E_{\mathcal{L}}\{\Gamma_k^{(l)}\}}{E_{\mathcal{L}}\{\Xi_{1,k}^{(l)}\} - E_{\mathcal{L}}\{\Xi_{2,k}^{(l)}\} E_{\mathcal{L}}\{\Gamma_k^{(l)}\}}. \tag{56}$$

Unlike that in AWGN channel, the result for the aperiodic code scenario is more difficult to obtain because there are more correlation terms in (48) and (50)–(52) to work with. Before evaluating expectation terms in (56), we define some functions as follows:

$$h_{jk}(p,q) = h_{j,p} h_{k,q} \tag{57}$$

$$\tau_{jk}(p,q) = \tau_{j,p} - \tau_{k,q} \tag{58}$$

$$\zeta_{jk}(p,q) = \tilde{\mathbf{a}}_{j,p}^T \tilde{\mathbf{a}}_{k,q}. \tag{59}$$

Thus, (57)–(59) define some relative figures between the pth channel path of the jth user and the qth channel path of the kth user. The notation $h_{jk}(p,q)$ denotes the path gain product, $\tau_{jk}(p,q)$ the relative path delay, and $\zeta_{jk}(p,q)$ the code correlation with the relative delay $\tau_{jk}(p,q)$. Expanding (50)–(52), we have seven expectation terms to evaluate. For purpose of illustration, we show how to evaluate the first term, $E_{\mathcal{L}}\{\varrho_{jk}^2\}$, here. By definition, we have ϱ_{jk} as

$$\begin{aligned}\varrho_{jk} &= \mathbf{f}_j^T \mathbf{f}_k \\ &= \left(\sum_{p=1}^{L} \tilde{\mathbf{a}}_{j,p} h_{j,p}\right)^T \left(\sum_{q=1}^{L} \tilde{\mathbf{a}}_{k,q} h_{k,q}\right) \\ &= \sum_{p=1}^{L}\sum_{q=1}^{L} h_{j,p} h_{k,q} \tilde{\mathbf{a}}_{j,p}^T \tilde{\mathbf{a}}_{k,q} \\ &= \sum_{p=1}^{L}\sum_{q=1}^{L} h_{jk}(p,q)\zeta_{jk}(p,q).\end{aligned} \tag{60}$$

The expectation of ϱ_{jk} over all possible codes is then obtained as given in (61), shown at the bottom of the page. Let

$$\mathcal{F}_{jk}(p_1,q_1,p_2,q_2) = N^2 E\Big\{\zeta_{jk}(p_1,q_1)\zeta_{jk}(p_2,q_2)\Big\}. \tag{62}$$

The coefficient N^2 in (62) is only a normalization constant. Since the spreading codes are seen as random, only when $\tau_{jk}(p_1,q_1)$ is equal to $\tau_{jk}(p_2,q_2)$ will $\mathcal{F}_{jk}(\cdot)$ be nonzero. Consider a specific set of $\{p_1,q_1,p_2,q_2\}$ such that $\tau_{jk}(p_1,q_1) = \tau_{jk}(p_2,q_2) = \tau$ and $\tau \geq 0$. We then have

$$\begin{aligned}\mathcal{F}_{jk}(p_1,q_1,p_2,q_2) &= N^2 \sum_{w=0}^{N-\tau-1} E\Big\{a_{j,w+\tau}^2 a_{k,w}^2\Big\} \\ &= N - \tau.\end{aligned} \tag{63}$$

$$\begin{aligned}E_{\mathcal{L}}\left\{\varrho_{jk}^2\right\} &= E\Big\{\sum_{p_1=1}^{L}\sum_{q_1=1}^{L}\sum_{p_2=1}^{L}\sum_{q_2=1}^{L} h_{jk}(p_1,q_1)\zeta_{jk}(p_1,q_1)h_{jk}(p_2,q_2)\zeta_{jk}(p_2,q_2)\Big\} \\ &= \sum_{p_1=1}^{L}\sum_{q_1=1}^{L}\sum_{p_2=1}^{L}\sum_{q_2=1}^{L} h_{jk}(p_1,q_1)h_{jk}(p_2,q_2)E\Big\{\zeta_{jk}(p_1,q_1)\zeta_{jk}(p_2,q_2)\Big\}\end{aligned} \tag{61}$$

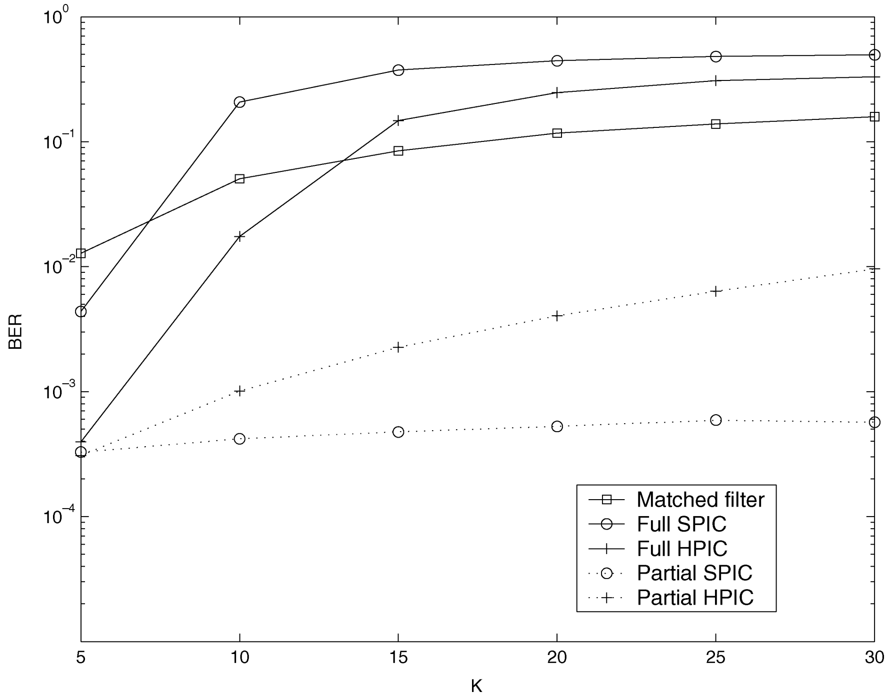

Fig. 2. Performance comparison for HPIC and SPIC ($N = 31$, $p = \frac{1}{\sqrt{N}}$, and $E_b/N_0 = 8$ dB). Optimal PCFs for the partial HPIC were obtained by trial and error and those for the SPIC were obtained from (27).

For $\tau < 0$, we have the same result except that the sign of τ in (63) is plus. We then conclude that the function $\mathcal{F}_{jk}(\cdot)$ in (62) is

$$\mathcal{F}_{jk}(p_1, q_1, p_2, q_2) = \begin{cases} N - |\tau|, & \text{if } \tau_{jk}(p_1, q_1) = \tau_{jk}(p_2, q_2) = \tau \\ 0, & \text{otherwise} \end{cases}. \quad (64)$$

Using (62), (64), and (61), we can evaluate $E_{\mathcal{L}}\{\varrho_{jk}^2\}$ in (50)–(52). The general formulations for the other six expectation terms are summarized in Appendix II.

We now provide a simple example to show the multipath effect on the optimal PCFs. Let $\mathbf{t}_k = [0, D]^T$ and $\mathbf{h}_k = [\alpha, \beta]^T$ for all k's ($\alpha^2 + \beta^2 = 1$). Also, let $\mathcal{G}_a \triangleq (N - D)\alpha^2\beta^2$, and $\mathcal{G}_b \triangleq (N - 2D)\alpha^4\beta^4$. Then

$$E_{\mathcal{L}}\{\Gamma_k^{(l)}\} = E_{\mathcal{L}}\{\Lambda_k^{(l)}\} + \frac{2\mathcal{G}_a(K-1)}{N^2} \quad (65)$$

$$\begin{aligned} E_{\mathcal{L}}\{\Xi_{1,k}^{(l)}\} &= E_{\mathcal{L}}\{\Omega_{1,k}^{(l)}\} + 2\mathcal{G}_a\bigg\{\frac{I_k}{N^4}\Big[N^2 + 10N + 4\mathcal{G}_a \\ &\quad + 2(K-2)(4N + 3K + \mathcal{G}_a + 1)\Big] \\ &\quad + \frac{K-1}{N^3}(N+3K-2)\bigg\} + 4\mathcal{G}_b\Big\{\frac{I_k \cdot K}{N^4} + 6K - 12\Big\} \\ &\quad + \frac{I_k}{N^4}(6N - 10D)\alpha^4\beta^4 \end{aligned} \quad (66)$$

$$E_{\mathcal{L}}\{\Xi_{2,k}^{(l)}\} = E_{\mathcal{L}}\{\Omega_{2,k}^{(l)}\} + 2\mathcal{G}_a\left[\frac{I_k}{N^3}(N+3K-2) + \frac{K-1}{N^2}\right] \quad (67)$$

$$E_{\mathcal{L}}\{\Xi_{3,k}^{(l)}\} = E_{\mathcal{L}}\{\Omega_{3,k}^{(l)}\} + 2\mathcal{G}_a\frac{I_k}{N^2}. \quad (68)$$

Note that the first terms in (65)–(68) are those in (32) and (34)–(36) which correspond to the optimal PCFs in an AWGN channel. Other terms are due to the multipath channel effect. It is evident to see that if $\beta = 0$, $\mathcal{G}_a = \mathcal{G}_b = 0$ and the metrics above are then degenerated to (32) and (34)–(36).

V. Simulation Results

A. *Performance Comparison for Various Partial PIC Structures*

In this section, we provide simulation results to verify the validity of our derived PCFs. Before we do that, we give some comparison results to justify the PIC structure we considered. First, we compare the performance of a partial SPIC and that of a partial HPIC. We used periodic codes of length 31 as spreading codes. Let E_b/N_0 be 8 dB ($\sigma^2 = N_0/2$), and assume a perfect power control scenario. It is straightforward to see that in the perfect power control case, optimal PCFs are equal for the coupled and decoupled structures. Fig. 2 shows the BER performance versus the number of users. Here, optimal PCFs for the partial HPIC were determined empirically (trial and error with a resolution of 0.01). Surprisingly, we found that the optimal partial SPIC outperformed the optimal partial HPIC. This result differs from the result given in [25] where the full SPIC was found to be inferior to the full HPIC.

In the second set of simulations, we compared the performance of the coupled and decoupled structures (using a partial SPIC). As mentioned above, optimal PCFs are equal for both structures under perfect power control. Thus, we compared their performance in an imperfect power control scenario. The optimal PCFs for the coupled structure were determined empirically. Let the number of users be three and the spreading code be aperiodic (of length 31). We assumed that the third user had a fixed E_b/N_0 of 8 dB, and the other two users had variable E_b/N_0 values of $8 - \Delta\gamma$ and $8 - 2\Delta\gamma$ dB, respectively. Fig. 3

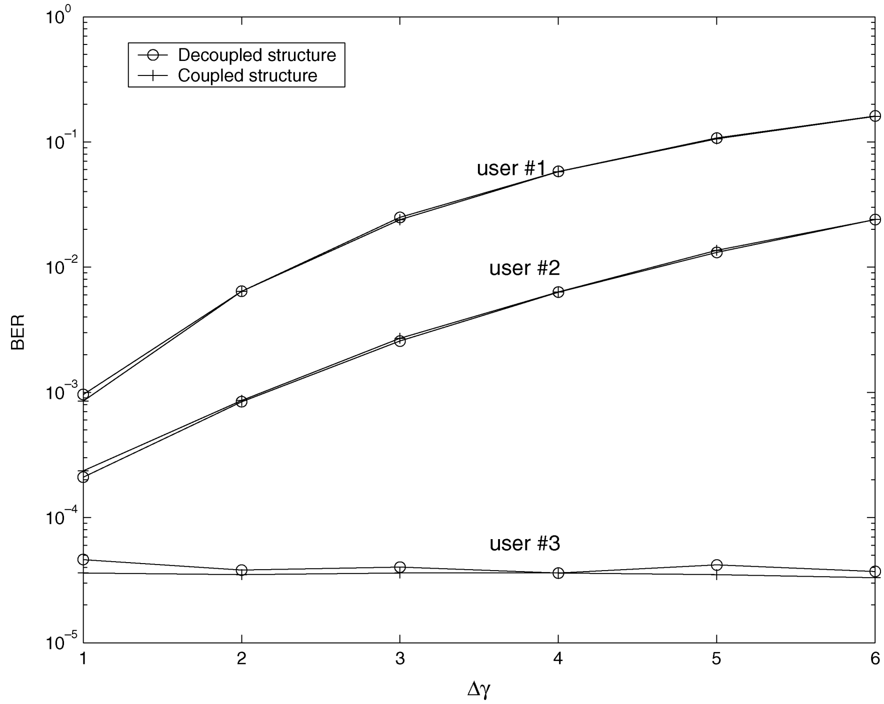

Fig. 3. Performance comparison for the coupled and decoupled structures (three users with $E_b/N_0 = 8 - 2\Delta\gamma$, $8 - \Delta\gamma$, and 8 dB). Optimal PCFs for the coupled structure were obtained by trial and error, and those for the decoupled structure were obtained from (38).

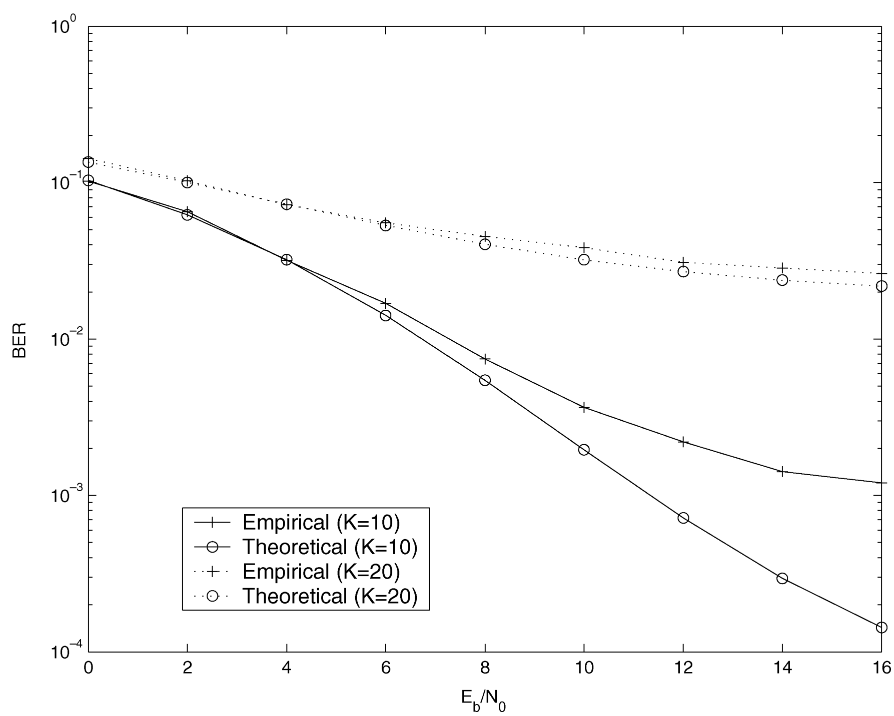

Fig. 4. BER of the partial SPIC detector versus E_b/N_0 (aperiodic AWGN channels and perfect power control).

shows the BER performance versus $\Delta\gamma$ for these two structures. As we can see, both structures performed similarly.

B. Validity of Derived PCFs

In this paragraph, we report simulation results demonstrating the accuracy of our theoretical solutions. A two-stage decoupled partial SPIC was considered. For the simulations conducted, we used Gold codes for periodic code systems and random codes for aperiodic code systems. Fig. 4 gives the empirical and theoretical BERs for the optimal partial SPIC detector (with the aperiodic code scenario). This figure shows the validity of the Gaussian assumption used in our derivation. As we can see, when the number of users was smaller and E_b/N_0 was higher, the Gaussian approximation was less valid. Fig. 5 shows the optimal PCFs in (27) and the empirical optimal PCFs versus the number of users. The channel here was an asynchronous AWGN channel, the spreading codes were periodic, and E_b/N_0

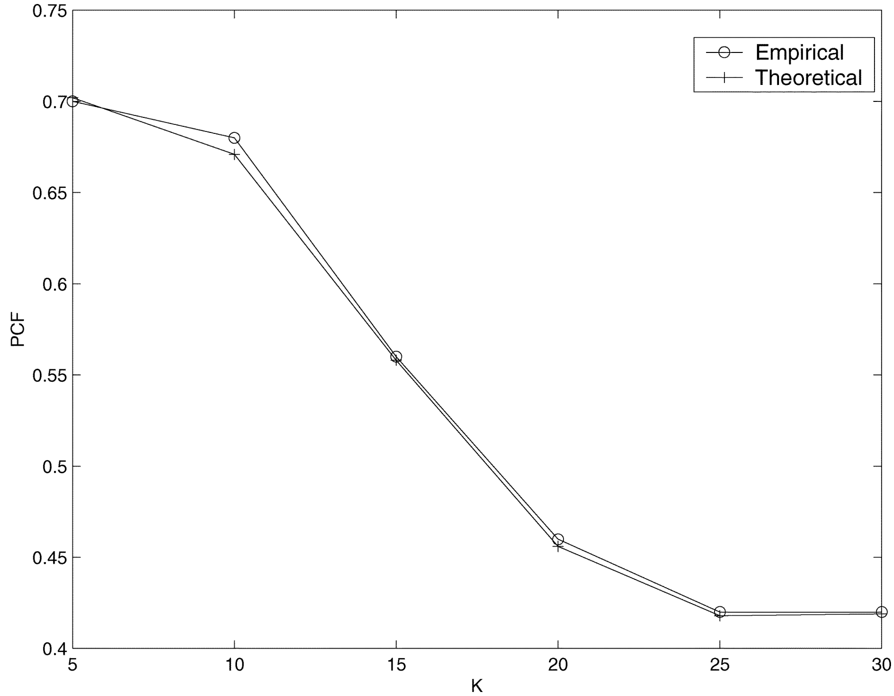

Fig. 5. Optimal PCF versus number of users (Gold codes, asynchronous AWGN channels, $E_b/N_0 = 8$ dB, and perfect power control).

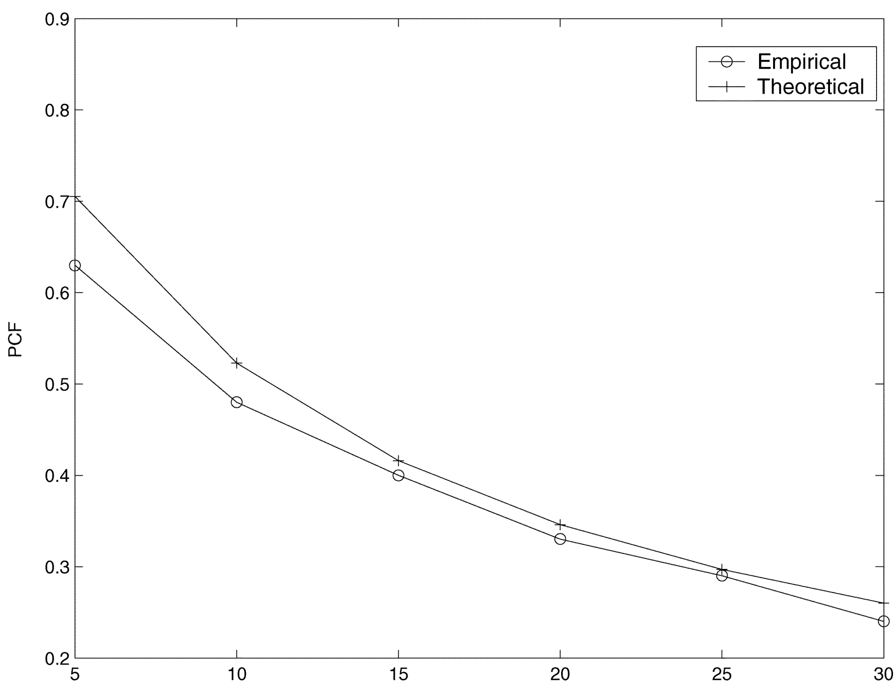

Fig. 6. Optimal PCF versus number of users (aperiodic codes, multipath channels, $E_b/N_0 = 10$ dB, and perfect power control).

was 8 dB for each user. In the figure, it can be seen that the theoretical optimal PCFs were very close to the empirical ones in all cases. We then considered optimal PCFs for a multipath channel. The multipath channel assumed was a two-ray channel with the transfer function $W_k(z) = 0.762 + 0.648z^{-2}$ (for all users). Theoretical optimal PCFs derived in (56) were compared with empirical PCFs and the results are shown in Fig. 6. We can observe that the theoretical results also matched with the empirical ones satisfactorily. Note that when the number of users was smaller, the theoretical values were less accurate. This was because when the user number is small, the Gaussian approximation in (30) is less valid. This was also consistent with the result observed in Fig. 4.

C. BER Performance Comparison

In what follows, we report the BER performance for various SPIC detectors. Fig. 7 gives the performance comparison for an optimal two-stage partial SPIC, a conventional matched-filter receiver, a two-stage full SPIC, and a three-stage full SPIC. The

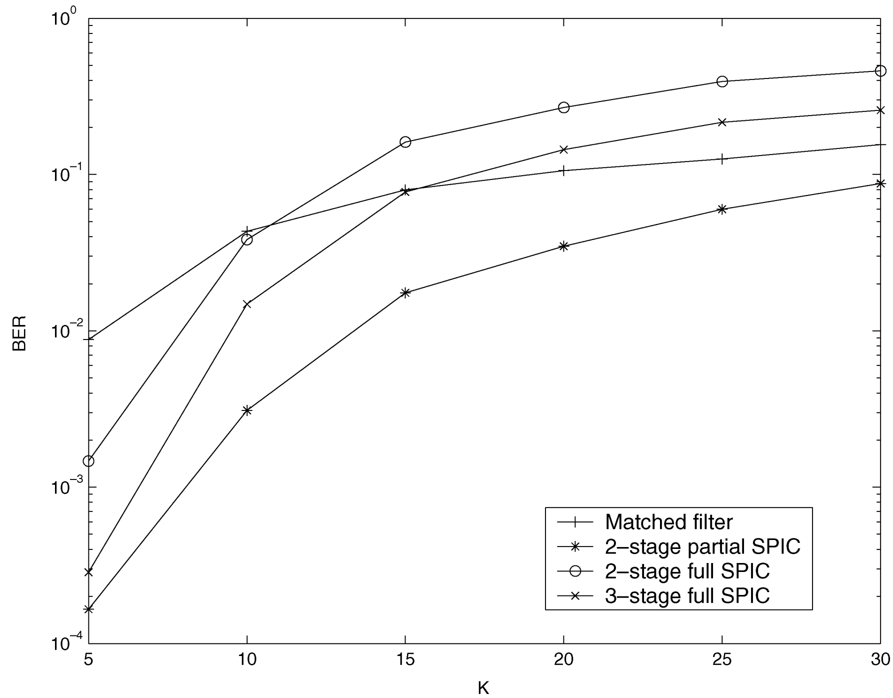

Fig. 7. BER versus number of users (gold codes, asynchronous AWGN channels, $E_b/N_0 = 10$ dB, and perfect power control).

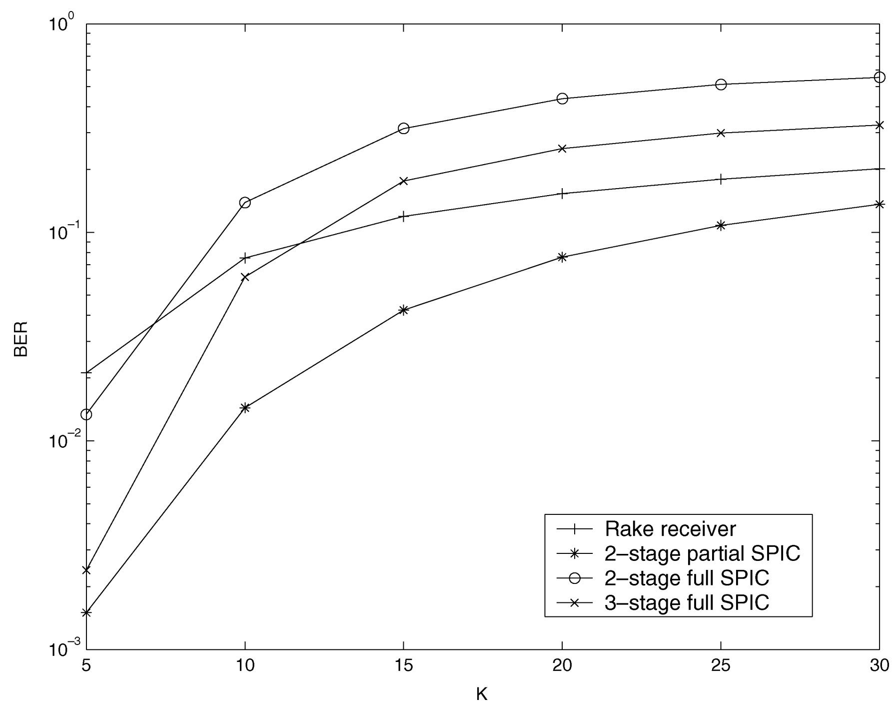

Fig. 8. BER versus number of users (aperiodic spreading codes, multipath channels, $E_b/N_0 = 10$ dB, and perfect power control).

spreading codes were periodic and the channel was an asynchronous AWGN channel. Also, E_b/N_0 was 10 dB and perfect power control was assumed. From the figure, we can see that the optimal two-stage partial SPIC outperformed others in all cases. The two-stage and three-stage SPIC receivers performed even worse than the conventional matched-filter receiver when the number of users was large. The optimal two-stage partial SPIC always performed better than the matched-filter receiver. Finally, Fig. 8 shows the performance comparison for the detectors considered above in the multipath channel. The simulation setup was identical to that in the previous cases except that the spreading code was aperiodic. The PCFs for the optimal two-stage partial SPIC were calculated using (56). As in the AWGN channel, the optimal two-stage partial SPIC outperformed other types of detectors.

D. *Effect of Imperfect Parameter Estimation*

In the optimal PCF formulation, we assumed that the required parameters are perfectly known. In practice, this may not be always possible. Some parameters will have to be estimated for

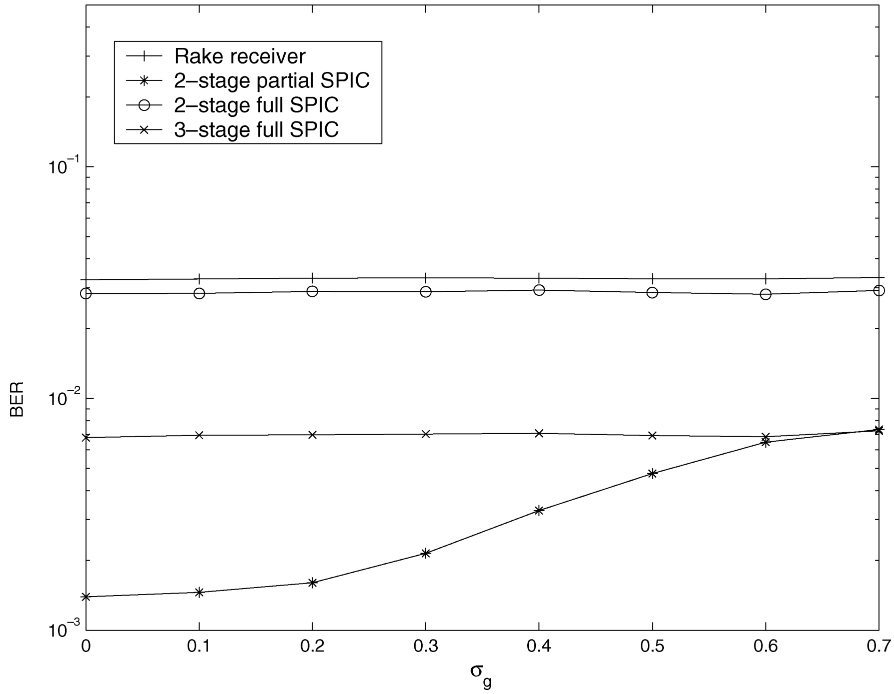

Fig. 9. BER with channel estimation error (aperiodic spreading codes, multipath channels, $K = 6$, $E_b/N_0 = 10$ dB, and perfect power control).

time-varying channels which may introduce errors. The main parameters we need to know are the channel responses and the noise variance. Once the channel responses are known, A_k's, ρ_{jk}'s and γ_k's can be calculated accordingly. We modeled the channel estimation error as follows. Let $g_{k,i} = A_k h_{k,i}$ be the ith path channel of user k, and $g'_{k,i} = g_{k,i} + \Delta g_{k,i}$, where $g'_{k,i}$ was the estimated channel response, $g_{k,i}$ was the actual response, and $\Delta g_{k,i}$ was a Gaussian random variable denoting the estimation error. We first let the noise variance be exactly known and varied the channel estimation error. The performance impact is shown in Fig. 9. The result corresponds to the case in which the user number was six, the spreading code was aperiodic, the channel was the multipath channel, and E_b/N_0 was 10 dB. In the figure, σ_g denotes the standard deviation of $\Delta g_{k,i}$ (same for all k's and i's). Since the matched-filter and the full SPIC receivers do not rely on channel information, the channel estimation error had no influence on their performance. (The BER variations in Fig. 9 were due to the random data used in different runs). As we can see, the partial SPIC performance was not affected until $\sigma_g = 0.3$. Note that the magnitude of the main path was 0.762. Thus, the estimation error was quite large in this case. The second case we considered was noise variance estimation error. The simulation setup was identical to the previous one. We let the channel responses be known and varied noise variance from $0.1 \times \sigma_n^2$ to $10 \times \sigma_n^2$, where σ_n^2 was the actual noise variance. We found that the optimal SPIC performance was almost unaffected. Thus, we conclude that the optimal partial SPIC has good immunity to parameter estimation errors.

VI. Conclusion

In DS-CDMA communication systems, MAI is considered as the main factor in the system performance degradation. Among multiuser detection schemes, the PIC receiver is considered as a simple and effective approach. It has been shown that the performance of the PIC can be further improved if interference is not fully cancelled. The performance of a partial PIC depends totally on the PCFs. Thus, how to determine PCFs optimally is then of great concern. In this paper, we have considered a two-stage decoupled partial SPIC and derived a set of closed-form solutions for optimal PCFs. These PCFs are useful for periodic and aperiodic spreading codes in AWGN channels, and those in multipath channels. Simulation results show that the derived optimal PCFs agree closely with empirical optimal PCFs. The performance of the optimal two-stage partial SPIC outperformed a conventional matched filter detector, a two-stage full SPIC detector, and even a three-stage full SPIC. We have also shown that the derived PCFs are not sensitive to parameter estimation errors. The optimal PCFs for aperiodic spreading code systems in AWGN channel have a simple expression. This will be a great advantage for real world applications since the optimal PCFs can be calculated efficiently online in a time-varying environment. In this paper, we are mainly concerned with BPSK modulation. Note that the same result can be extended to accommodate QAM modulation. In this case, however, we have to take the interference between inphase and quadrature components into account. It turns out that for the inphase or quadrature component of one user, we may treat the number of interfering users as $2K - 1$.

Appendix I
Periodic Code System Optimal PCFs for Asynchronous AWGN Channels

Let $b_{k,i}$ denote the ith bit for the kth user and τ_k the user delay. Then the received signal for asynchronous channels can then be represented by

$$r(t) = \sum_k \sum_i b_{k,i} a_k(t-iT-\tau_k)\Pi_T(t-iT-\tau_k)+n(t). \quad (69)$$

We further define the relative delay between User j and k as $\tau_{jk} = \tau_j - \tau_k$, and the cross-correlation functions are given by

$$\rho_{jk}(\tau_{jk}) = \begin{cases} \int_{T+\tau_{jk}}^{T} a_j(t-\tau_{jk}-T)a_k(t)dt, & \tau_{jk} < 0 \\ \int_{\tau_{jk}}^{T} a_j(t-\tau_{jk})a_k(t)dt, & \tau_{jk} \geq 0 \end{cases} \tag{70}$$

and

$$\hat{\rho}_{jk}(\tau_{jk}) = \begin{cases} \int_{0}^{T+\tau_{jk}} a_j(t-\tau_{jk})a_k(t)dt, & \tau_{jk} < 0 \\ \int_{0}^{\tau_{jk}} a_j(t-\tau_{jk}+T)a_k(t)dt, & \tau_{jk} \geq 0 \end{cases}. \tag{71}$$

For simplicity, we use ρ_{jk} and $\hat{\rho}_{jk}$ instead of $\rho_{jk}(\tau_{jk})$ and $\hat{\rho}_{jk}(\tau_{jk})$ in the sequel. The matched filter output for the kth users' ith bit is obtained as

$$\begin{aligned} y_{k,i} &= \int_{iT+\tau_k}^{(i+1)T+\tau_k} r(t)a_k(t-\tau_k)dt \\ &= A_k b_{k,i} + \sum_{j\neq k} A_j \left(b_{j,i-l_{jk}}\hat{\rho}_{jk} + b_{j,i-l_{jk}+1}\rho_{jk}\right) + \eta_{k,i} \end{aligned} \tag{72}$$

where the delay index and noise term are expressed as l_{jk} and $\eta_{k,i}$. They are defined as

$$l_{jk} \triangleq \begin{cases} 1, & \tau_{jk} \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{73}$$

and

$$\eta_{k,i} = \int_{iT+\tau_k}^{(i+1)T+\tau_k} n(t)a_k(t-\tau_k)dt. \tag{74}$$

The regenerated received signal using partial SPIC is given by

$$\hat{r}_k(t) = r(t) - C_k \sum_{j\neq k}\sum_{i} y_{j,i}a_j(t-iT-\tau_j)\Pi_T(t-iT-\tau_j). \tag{75}$$

Thus, the second stage output is obtained in (76), shown at the bottom of the page. Without loss of generality, we may assume that $\tau_j \geq \tau_k,\ j \neq k$. Then $l_{jk} = 1$ for all j's and the result can be simplified to

$$\begin{aligned} z_{k,i} &= A_k b_{k,i}\left\{1 - C_k \sum_{j\neq k}\left(\hat{\rho}_{jk}^2 + \rho_{jk}^2\right)\right\} \\ &\quad - A_k C_k \sum_{j\neq k}\left(b_{k,i-1} + b_{k,i+1}\right)\hat{\rho}_{jk}\rho_{jk} \\ &\quad - C_k \sum_{j\neq k}\sum_{m\neq j,k}^{\tau_m\leq\tau_j} A_j b_{j,i-2}\hat{\rho}_{jm}\hat{\rho}_{mk} \\ &\quad - C_k \sum_{j\neq k}\sum_{m\neq j,k}^{\tau_m>\tau_j} A_j b_{j,i+1}\rho_{jm}\rho_{mk} \\ &\quad + \sum_{j\neq k} A_j b_{j,i-1}\Phi_{jk} + \sum_{j\neq k} A_j b_{j,i}\Psi_{jk} + \eta_{k,i} \\ &\quad - C_k \sum_{j\neq k}\left(\eta_{j,i-l_{jk}}\hat{\rho}_{jk} + \eta_{j,i-l_{jk}+1}\rho_{jk}\right) \end{aligned} \tag{77}$$

where Φ_{jk} and Ψ_{jk} are defined as

$$\begin{aligned} \Phi_{jk} &= \hat{\rho}_{jk}(1-C_k) - C_k \\ &\quad \times \left\{\sum_{m\neq j,k}^{\tau_m>\tau_j}\hat{\rho}_{jm}\hat{\rho}_{mk} + \sum_{m\neq j,k}^{\tau_m\leq\tau_j}(\rho_{jm}\hat{\rho}_{mk} + \hat{\rho}_{jm}\rho_{mk})\right\} \\ \Psi_{jk} &= \rho_{jk}(1-C_k) - C_k \\ &\quad \times \left\{\sum_{m\neq j,k}^{\tau_m\leq\tau_j}\rho_{jm}\rho_{mk} + \sum_{m\neq j,k}^{\tau_m>\tau_j}(\rho_{jm}\hat{\rho}_{mk} + \hat{\rho}_{jm}\rho_{mk})\right\}. \end{aligned} \tag{78}$$

The squared-mean for $z_{k,i}$ is obtained from (17) and (77) as

$$\mathcal{M}_k = A_k^2\left(1 - C_k\Lambda_k\right)^2 \tag{79}$$

where

$$\Lambda_k \triangleq \sum_{j\neq k}\left(\hat{\rho}_{jk}^2 + \rho_{jk}^2\right). \tag{80}$$

$$\begin{aligned} z_{k,i} &= \int_{iT+\tau_k}^{(i+1)T+\tau_k} \hat{r}(t)a_k(t-\tau_k)dt = y_{k,i} - C_k\sum_{j\neq k}\left(y_{j,i-l_{jk}}\hat{\rho}_{jk} + y_{j,i-l_{jk}+1}\rho_{jk}\right) \\ &= A_k b_{k,i} + \sum_{j\neq k} A_j\left(b_{j,i-l_{jk}}\hat{\rho}_{jk} + b_{j,i-l_{jk}+1}\rho_{jk}\right) \\ &\quad + \eta_{k,i} - C_k\sum_{j\neq k}\left\{A_j\left(b_{j,i-l_{jk}}\hat{\rho}_{jk} + b_{j,i-l_{jk}+1}\rho_{jk}\right)\right. \\ &\qquad + \sum_{m\neq j} A_m\left(b_{m,i-l_{mj}-l_{jk}}\hat{\rho}_{mj}\hat{\rho}_{jk} + b_{m,i-l_{mj}-l_{jk}+1}\rho_{mj}\hat{\rho}_{jk} + b_{m,i-l_{mj}-l_{jk}+1}\hat{\rho}_{mj}\rho_{jk}\right. \\ &\qquad \left.\left. + b_{m,i-l_{mj}-l_{jk}+2}\rho_{mj}\rho_{jk}\right) + \eta_{j,i-l_{jk}}\hat{\rho}_{jk} + \eta_{j,i-l_{jk}+1}\rho_{jk}\right\} \end{aligned} \tag{76}$$

Similarly, the variance can also be obtained as

$$\mathcal{V}_k = \sigma^2\left(\Omega_{1,k}C_k^2 - 2\Omega_{2,k}C_k + \Omega_{3,k}\right) \tag{81}$$

where $\Omega_{i,k}, 1 \le i \le 3$, are defined as

$$\begin{aligned}
&\Omega_{1,k}\\
&=2\gamma_k^2\sum_{j\neq k}\hat{\rho}_{jk}^2\rho_{jk}^2+\sum_{j\neq k}\gamma_j^2\left(\Phi_{jk}^2+\Psi_{jk}^2\right)+\left(\sum_{j\neq k}\sum_{m\neq j,k}^{\tau_m\le\tau_j}\gamma_j^2\hat{\rho}_{jm}\hat{\rho}_{mk}\right)^2\\
&\quad+\left(\sum_{j\neq k}\sum_{m\neq j,k}^{\tau_m>\tau_j}\gamma_j^2\rho_{jm}\rho_{mk}\right)^2+\sum_{j\neq k}\left(\hat{\rho}_{jk}^2+\rho_{jk}^2\right)\\
&\quad+\sum_{j\neq k}\sum_{m\neq j,k}^{\tau_m<\tau_j}2\left(\rho_{jm}\hat{\rho}_{j,k}\hat{\rho}_{m,k}+\rho_{jm}\rho_{jk}\rho_{mk}+\hat{\rho}_{jm}\hat{\rho}_{jk}\rho_{mk}\right)\\
&\quad+\sum_{j\neq k}\sum_{m\neq j,k}^{\tau_m=\tau_j}\left(\rho_{jm}\hat{\rho}_{jk}\hat{\rho}_{mk}+\rho_{jm}\rho_{jk}\rho_{mk}\right)
\end{aligned} \tag{82}$$

$$\begin{aligned}
&\Omega_{2,k}\\
&=\sum_{j\neq k}\gamma_j^2(\hat{\rho}_{jk}\Phi_{jk}+\rho_{jk}\Psi_{jk})+\sum_{j\neq k}\left(\hat{\rho}_{jk}^2+\rho_{jk}^2\right)
\end{aligned} \tag{83}$$

$$\begin{aligned}
&\Omega_{3,k}\\
&=\sum_{j\neq k}\gamma_j^2\left(\hat{\rho}_{jk}^2+\rho_{jk}^2\right)+1.
\end{aligned} \tag{84}$$

Thus, the optimal PCF can be obtained by substituting (80) and (82)–(84) into (27).

Appendix II
Expressions for the Expected Terms in (50)–(51)

Extending the definition in (62), we have

$$\mathcal{F}_{jk}(p_1,q_1,\ldots,p_i,q_i)=N^iE\left\{\zeta_{jk}(p_1,q_1),\ldots,\zeta_{jk}(p_i,q_i)\right\} \tag{85}$$

where i is an integer. To make expression simpler, we let $\mathbf{w}_i = \{p_i, q_i\}$. Equation (85) can then be rewritten as

$$\mathcal{F}_{jk}(\mathbf{w}_1,\ldots,\mathbf{w}_i) = N^iE\left\{\zeta_{jk}(\mathbf{w}_1),\ldots,\zeta_{jk}(\mathbf{w}_i)\right\}. \tag{86}$$

We further omit the subscript in $\mathcal{F}(\cdot)$ and use the following notational substitution:

$$\sum_{\mathbf{w}_i=0}^{L^2} \longrightarrow \sum_{p_i=0}^{L}\sum_{q_i=0}^{L}. \tag{87}$$

In what follows, six expected terms are given without detailed derivation. The first term is (88), found as the first equation at the bottom of the page. The second term is (89), found as the second equation at the bottom of the page. The third term is (90), found as the first equation on the following

$$\begin{aligned}
E_{\mathcal{L}}\{\varrho_{jk}^2\varrho_j\} = \frac{1}{N^3}\sum_{\mathbf{w}_1=1}^{L^2}\sum_{\mathbf{w}_2=1}^{L^2}\sum_{\mathbf{w}_3=1}^{L^2} & h_{jk}(\mathbf{w}_1)h_{jk}(\mathbf{w}_2)h_{jj}(\mathbf{w}_3)\mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3),\\
&\text{IF } \tau_{jk}(\mathbf{w}_1)=\tau_{jk}(\mathbf{w}_2), \tau_{jj}(\mathbf{w}_3)=0,\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3) = N\left(N-\left|\tau_{jk}(\mathbf{w}_1)\right|\right).\\
&\text{ELSE IF } \left|\tau_{jk}(\mathbf{w}_1)-\tau_{jk}(\mathbf{w}_2)\right| = \left|\tau_{jj}(\mathbf{w}_3)\right|,\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3) = N-\max\left\{\left|\tau_{jk}(\mathbf{w}_1)\right|,\left|\tau_{jk}(\mathbf{w}_2),\right|\tau_{jj}(\mathbf{w}_3)\right|\right\}.\\
&\text{ELSE}\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3) = 0\\
&\text{END.}
\end{aligned} \tag{88}$$

$$\begin{aligned}
E_{\mathcal{L}}\{\varrho_{jk}\varrho_{jm}\varrho_{mk}\} = \frac{1}{N^3}\sum_{\mathbf{w}_1=1}^{L^2}\sum_{\mathbf{w}_2=1}^{L^2}\sum_{\mathbf{w}_3=1}^{L^2} & h_{jk}(\mathbf{w}_1)h_{jm}(\mathbf{w}_2)h_{mk}(\mathbf{w}_3)\mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3),\\
&\text{IF } \tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_2)=\tau_{jk}(\mathbf{w}_3),\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3) = N-\max\left\{\left|\tau_{jk}(\mathbf{w}_1)\right|,\left|\tau_{jm}(\mathbf{w}_2)\right|,\left|\tau_{mk}(\mathbf{w}_3)\right|\right\}.\\
&\text{ELSE}\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3) = 0\\
&\text{END.}
\end{aligned} \tag{89}$$

$$
\begin{aligned}
E_{\mathcal{L}}\{\varrho_{jk}^2\varrho_j^2\} = \frac{1}{N^4}\sum_{\mathbf{w}_1=1}^{L^2}\sum_{\mathbf{w}_2=1}^{L^2}\sum_{\mathbf{w}_3=1}^{L^2}\sum_{\mathbf{w}_4=1}^{L^2} & h_{jk}(\mathbf{w}_1)h_{jk}(\mathbf{w}_2)h_{jj}(\mathbf{w}_3)h_{jj}(\mathbf{w}_4)\mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4),\\
& \text{IF } \tau_{jk}(\mathbf{w}_1)=\tau_{jk}(\mathbf{w}_2), \tau_{jj}(\mathbf{w}_3)=\tau_{jj}(\mathbf{w}_4)=0,\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N^2\Big(N-\big|\tau_{jk}(\mathbf{w}_1)\big|\Big).\\
& \text{ELSE IF } \tau_{jk}(\mathbf{w}_1)=\tau_{jk}(\mathbf{w}_2), \tau_{jj}(\mathbf{w}_3)=\tau_{jj}(\mathbf{w}_4),\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = \Big(N-\big|\tau_{jk}(\mathbf{w}_1)\big|\Big)\Big(N-\big|\tau_{jj}(\mathbf{w}_3)\big|\Big).\\
& \text{ELSE IF } \big|\tau_{jk}(\mathbf{w}_1)-\tau_{jk}(\mathbf{w}_2)\big| = \tau_{jj}(\mathbf{w}_3), \tau_{jj}(\mathbf{w}_4)=0,\\
& \quad \text{or}\big|\tau_{jk}(\mathbf{w}_1)-\tau_{jk}(\mathbf{w}_2)\big| = \tau_{jj}(\mathbf{w}_4), \tau_{jj}(\mathbf{w}_3)=0,\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jk}(\mathbf{w}_1)\big|,\big|\tau_{jk}(\mathbf{w}_2),\big|\tau_{jj}(\mathbf{w}_3)\big|,\big|\tau_{jj}(\mathbf{w}_4)\big|\Big\}.\\
& \text{ELSE IF } \big|\tau_{jk}(\mathbf{w}_1)-\tau_{jk}(\mathbf{w}_2)\big| = \big|\tau_{jj}(\mathbf{w}_3)\pm\tau_{jj}(\mathbf{w}_4)\big|, \tau_{jk}(\mathbf{w}_1)\neq 0, \tau_{jk}(\mathbf{w}_2)\neq 0,\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jk}(\mathbf{w}_1)\big|,\big|\tau_{jk}(\mathbf{w}_2),\big|\tau_{jj}(\mathbf{w}_3)\big|,\big|\tau_{jj}(\mathbf{w}_4)\big|,\\
& \quad \big|\tau_{jk}(\mathbf{w}_1)-\tau_{jk}(\mathbf{w}_2)\big|,\big|\tau_{jk}(\mathbf{w}_1)\pm\tau_{jj}(\mathbf{w}_3)\big|,\big|\tau_{jk}(\mathbf{w}_1)\pm\tau_{jj}(\mathbf{w}_4)\big|\Big\}.\\
& \text{ELSE IF } \big|\tau_{jk}(\mathbf{w}_1)-\tau_{jk}(\mathbf{w}_2)\big| = \big|\tau_{jj}(\mathbf{w}_3)\pm\tau_{jj}(\mathbf{w}_4)\big|,\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jk}(\mathbf{w}_1)\big|,\big|\tau_{jk}(\mathbf{w}_2),\big|\tau_{jj}(\mathbf{w}_3)\big|,\big|\tau_{jj}(\mathbf{w}_4)\big|\Big\}.\\
& \text{ELSE}\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = 0\\
& \text{END.}
\end{aligned}
\tag{90}
$$

$$
\begin{aligned}
E_{\mathcal{L}}\{\varrho_{jm}^2\varrho_{mk}^2\} = \frac{1}{N^4}\sum_{\mathbf{w}_1=1}^{L^2}\sum_{\mathbf{w}_2=1}^{L^2}\sum_{\mathbf{w}_3=1}^{L^2}\sum_{\mathbf{w}_4=1}^{L^2} & h_{jm}(\mathbf{w}_1)h_{jm}(\mathbf{w}_2)h_{mk}(\mathbf{w}_3)h_{mk}(\mathbf{w}_4)\mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3\mathbf{w}_4),\\
& \text{IF } \tau_{jm}(\mathbf{w}_1)=\tau_{jm}(\mathbf{w}_2), \tau_{mk}(\mathbf{w}_3)=\tau_{mk}(\mathbf{w}_4),\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3\mathbf{w}_4) = \Big(N-\big|\tau_{jm}(\mathbf{w}_1)\big|\Big)\Big(N-\big|\tau_{mk}(\mathbf{w}_3)\big|\Big).\\
& \text{ELSE IF } \big|\tau_{jm}(\mathbf{w}_1)-\tau_{jm}(\mathbf{w}_2)\big| = \big|\tau_{mk}(\mathbf{w}_3)-\tau_{mk}(\mathbf{w}_4)\big|, \tau_{jm}(\mathbf{w}_1)\neq 0, \tau_{jm}(\mathbf{w}_2)\neq 0,\\
& \quad \tau_{mk}(\mathbf{w}_3)\neq 0, \tau_{mk}(\mathbf{w}_4)\neq 0,\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jm}(\mathbf{w}_1)\big|,\big|\tau_{jm}(\mathbf{w}_2),\big|\tau_{mk}(\mathbf{w}_3)\big|,\big|\tau_{mk}(\mathbf{w}_4)\big|,\\
& \quad \big|\tau_{jm}(\mathbf{w}_1)-\tau_{jm}(\mathbf{w}_2)\big|,\big|\tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_3)\big|,\big|\tau_{jm}(\mathbf{w}_1)+_{mk}(\mathbf{w}_4)\big|\Big\}.\\
& \text{ELSE IF } \big|\tau_{jm}(\mathbf{w}_1)-\tau_{jm}(\mathbf{w}_2)\big| = \big|\tau_{mk}(\mathbf{w}_3)-\tau_{mk}(\mathbf{w}_4)\big|,\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jm}(\mathbf{w}_1)\big|,\big|\tau_{jm}(\mathbf{w}_2),\big|\tau_{mk}(\mathbf{w}_3)\big|,\big|\tau_{mk}(\mathbf{w}_4)\big|\Big\}.\\
& \text{ELSE}\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = 0\\
& \text{END.}
\end{aligned}
\tag{91}
$$

$$
\begin{aligned}
E_{\mathcal{L}}\{\varrho_{jm}\varrho_{mk}\varrho_{jn}\varrho_{nk}\} = \frac{1}{N^4}\sum_{\mathbf{w}_1=1}^{L^2}\sum_{\mathbf{w}_2=1}^{L^2}\sum_{\mathbf{w}_3=1}^{L^2}\sum_{\mathbf{w}_4=1}^{L^2} & h_{jm}(\mathbf{w}_1)h_{mk}(\mathbf{w}_2)h_{jn}(\mathbf{w}_3)h_{nk}(\mathbf{w}_4)\mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4),\\
& \text{IF } \tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_2)=\tau_{jn}(\mathbf{w}_3)+\tau_{nk}(\mathbf{w}_4), \tau_{jm}(\mathbf{w}_1)\neq 0, \tau_{mk}(\mathbf{w}_2)\neq 0, \tau_{jn}(\mathbf{w}_3)\neq 0,\\
& \quad \tau_{nk}(\mathbf{w}_4)\neq 0,\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jm}(\mathbf{w}_1)\big|,\big|\tau_{mk}(\mathbf{w}_2),\big|\tau_{jn}(\mathbf{w}_3)\big|,\big|\tau_{nk}(\mathbf{w}_4)\big|,\\
& \quad \big|\tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_2)\big|,\big|\tau_{jm}(\mathbf{w}_1)-\tau_{jn}(\mathbf{w}_3)\big|\Big\}.\\
& \text{ELSE IF } \tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_2)=\tau_{jn}(\mathbf{w}_3)+\tau_{nk}(\mathbf{w}_4),\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jm}(\mathbf{w}_1)\big|,\big|\tau_{mk}(\mathbf{w}_2),\big|\tau_{jn}(\mathbf{w}_3)\big|,\big|\tau_{nk}(\mathbf{w}_4)\big|\Big\}.\\
& \text{ELSE}\\
& \quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = 0\\
& \text{END.}
\end{aligned}
\tag{92}
$$

$$
\begin{aligned}
E_{\mathcal{L}}\{\varrho_{jm}\varrho_{mk}\varrho_{jk}\varrho_{jj}\} = &\frac{1}{N^4}\sum_{\mathbf{w}_1=1}^{L^2}\sum_{\mathbf{w}_2=1}^{L^2}\sum_{\mathbf{w}_3=1}^{L^2}\sum_{\mathbf{w}_4=1}^{L^2} h_{jm}(\mathbf{w}_1)h_{mk}(\mathbf{w}_2)h_{jk}(\mathbf{w}_3)h_{jj}(\mathbf{w}_4)\mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4),\\
&\text{IF } \tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_2)=\tau_{jk}(\mathbf{w}_3), \tau_{jj}(\mathbf{w}_4)=0,\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N\Big(N-\max\Big\{\big|\tau_{jm}(\mathbf{w}_1)\big|, \big|\tau_{mk}(\mathbf{w}_2), \big|\tau_{jk}(\mathbf{w}_3)\big|\Big\}\Big).\\
&\text{ELSE IF } \big|\tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_2)-\tau_{jk}(\mathbf{w}_3)\big| = \big|\tau_{jj}(\mathbf{w}_4)\big|, \tau_{jm}(\mathbf{w}_1)\neq 0, \tau_{mk}(\mathbf{w}_2)\neq 0, \tau_{jk}(\mathbf{w}_3)\neq 0,\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jm}(\mathbf{w}_1)\big|, \big|\tau_{mk}(\mathbf{w}_2), \big|\tau_{jk}(\mathbf{w}_3)\big|, \big|\tau_{jj}(\mathbf{w}_4)\big|\\
&\quad \big|\tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_2)\big|, \big|\tau_{jm}(\mathbf{w}_1)\pm\tau_{jj}(\mathbf{w}_4)\big|\Big\}.\\
&\text{ELSE IF } \big|\tau_{jm}(\mathbf{w}_1)+\tau_{mk}(\mathbf{w}_2)-\tau_{jk}(\mathbf{w}_3)\big| = \big|\tau_{jj}(\mathbf{w}_4)\big|,\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = N-\max\Big\{\big|\tau_{jm}(\mathbf{w}_1)\big|, \big|\tau_{mk}(\mathbf{w}_2), \big|\tau_{jk}(\mathbf{w}_3)\big|, \big|\tau_{jj}(\mathbf{w}_4)\big|\Big\}.\\
&\text{ELSE}\\
&\quad \mathcal{F}(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) = 0\\
&\text{END.}
\end{aligned}
\tag{93}
$$

of the page. The fourth term is (91), found as the second equation on the previous page. The fifth term is (92), found as the third equation on the previous page. Finally, the sixth term is (93), found at the top of the page.

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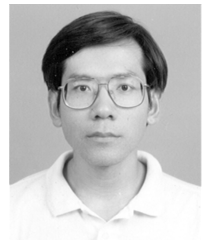
Yu-Tao Hsieh received the B.S. and M.S. degrees in communication engineering from National Chiao Tung University, Hsinchu, Taiwan, R.O.C., in 1995 and 1997, respectively, where he is currently working towards the Ph.D. degree in the Department of Communication Engineering.

His research interests include multiuser detection, wireless communication, and adaptive signal processing.



Wen-Rong Wu received the B.S. degree in mechanical engineering from Tatung Institute of Technology, Taiwan, in 1980, and the M.S. degrees in mechanical and electrical engineering, and the Ph.D. degree in electrical engineering from State University of New York, Buffalo in 1985, 1986, and 1989, respectively.

Since August 1989, he has been a faculty member in the Department of Communication Engineering, National Chiao Tung University, Taiwan. His research interests include statistical signal processing and digital communications.