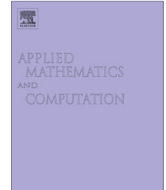




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Diversified portfolios with different entropy measures

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ABSTRACT

One of the major issues for Markowitz mean–variance model is the errors in estimations cause “corner solutions” and low diversity in the portfolio. In this paper, we compare the mean–variance efficiency, realized portfolio values, and diversity of the models incorporating different entropy measures by applying multiple criteria method. Differing from previous studies, we evaluate twenty-three portfolio over-time rebalancing strategies with considering short-sales and various transaction costs in asset diversification. Using the data of the most liquid stocks in Taiwan's market, our finding shows that the models with Yager's entropy yield higher performance because they respond to the change in market by reallocating assets more effectively than those with Shannon's entropy and with the minimax disparity model. Furthermore, including entropy in models enhances diversity of the portfolios and makes asset allocation more feasible than the models without incorporating entropy.

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1. Introduction

The Markowitz [1] mean–variance (MV) model has been widely applied both in academia and in the real-world for portfolio selection. Numerous studies have analyzed the effectiveness of diversification in investment strategies based on its risk–return trade-off relation [2]. However, such optimal portfolios are found problematic to be exercised in practice due to the error in estimation of the moments of asset returns. The major issues for asset management are (1) the MV portfolios are often extremely concentrated in a limited number of assets, and (2) the out-of-sample performances generated are poor. The estimation errors may bias the optimal portfolio weights and may cause those portfolios to generate “corner solutions” that involve infeasible asset allocations [3]. There has been extensive research on reducing statistical errors in the mean and variance–covariance matrixes due to the bias caused by the use of unpredicted information [4,5]. Therefore the statistics generated by the entropies provide additional information in forming optimal portfolios, particularly to increase asset diversification by reducing errors in estimating associated parameters. The low diversity of the MV portfolio may result in loss while some of the invested assets experience unexpected gains [4,6–8]. In this paper, we evaluate the performance of the portfolio selections incorporating different entropy measures by applying multiple criteria method. To improve the feasibility of models, we consider the impact of short-sale constraints and transaction costs on portfolios.

Portfolio diversification implies that the idiosyncratic risk can be decreased to zero as the assets included in the investment increase [9]. Therefore, the purpose of a diversified portfolio is to invest in as many mean–variance efficient assets as

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possible. One of the major issues of realization of a conventional portfolio selection model is low diversity or the corner solutions. Chiou et al. [10] suggest that setting the upper bound of weight reduces the uncertainty but sacrifice the performance of diversified portfolio. DeMiguel et al. [8] show that a naïve equally-weighted portfolio demonstrates higher out-of-sample diversification performance than the optimal MV model. In our study, we apply various entropy measures, including Shannon's, Yager's entropy, minimax disparity model, as theoretical foundation to model portfolio rebalancing and further to compare the naïve portfolio suggested by DeMiguel et al. [8].

Trading costs play a crucial role in constructing realistic strategies that can be applied in the financial industry [6,11–15]. In our study, we incorporate real brokerage fee and taxes and apply multiple criteria decision making (MCDM) to maximize the portfolio realized return. The technical difficulty for computing realization portfolio values with considering trading costs in this study is higher than previous literature using return and mean–variance efficiency. This paper contributes to literature by applying advanced methodology to evaluate effectiveness of various optimal portfolios in the real world.

The entropy serves an alternative measure of uncertainty in information theory, econometrics, and finance [16]. Among the measures, the Jaynes [17] selection criterion is referred as the maximum entropy criterion [18], which is a rule to assign numerical values to probability in circumstances that certain partial information is available [19]. Assuming the given moments represent known information, the maximum entropy (ME) principle chooses the one having maximum entropy or equivalently the most uncertain distribution [20].

Entropy is useful to model a least biased distribution from the partial information represented by certain moment restrictions [21]. Shannon [16] proposes a non-linear model to estimate entropy. Yager [22] later applies the maximum entropy principle. Wang and Parkan [23] proposed a linear entropy model, the minimax disparity model, which minimizes the maximum difference between each pair of weights. Lutgens and Schotman [24] use a mini-max portfolio strategy to analyze the decisions and performance of a robust decision maker. Philippatos and Wilson [25] first apply entropy as a measurement of the uncertainty in portfolio selection. They suggest that entropy is more general than variance as a measure of risk, because it is free from reliance on the assumption of symmetric probability distributions and can be computed from non-metric data [26]. Jiang et al. [18] present a maximum entropy portfolio model for large scale portfolio problems. Though the above models are free of any assumption regarding the distribution, they are static and fail to include rebalancing process.

The weights of portfolio obtained through the maximum entropy (ME) approach are in the form of “probabilities,” therefore the weights are non-negative. However, allowing short-sale gives investors flexibility in managing their portfolios, particularly during the bearish market or for managing hedge funds [4]. To our best knowledge, previous studies related to entropy do not consider portfolio short-sale in models. White [27] points out that short-sale can be used to lower investment risks and to improve a portfolio's risk-return trade-off [28]. Investors also can use short-sale to engage portfolio arbitrage [29]. In our study, we compare the realized portfolio values of twenty-three models and their change in portfolio weights [30].

To generate realistic results, we rebalance the optimal portfolios that (1) allow short selling assets, (2) apply various entropy measures, and (3) consider various transaction costs. Our finding shows that the models with Yager's entropy yield higher portfolio value than those with Shannon's entropy and those with the minimax disparity model. This is because the models of Yager's entropy respond the change in market by reallocating assets in the portfolio more effectively. In addition, including entropy in models enhance diversity of the portfolios. Given the fact that our portfolios with entropy measures are less subject to the variation in sample, these portfolios are robust [24].

The remainder of this paper is organized as follows. Section 2 introduces the entropy models and portfolio selection models. In Section 3, we present portfolio rebalancing method. Section 4 reports the numerical results. The conclusions are presented in Section 5.

2. Entropy models and portfolio selection models

We first introduce Shannon's entropy, Yager's entropy and the mini-max disparity model in this section. We then present portfolio selection models.

2.1. Shannon's entropy

Shannon's entropy [16] is first developed to solve communication problems and later is applied in finance to measure the amount of information given by observing the market. Simonelli [31] points out that Shannon's entropy is more useful in constructing a portfolio than using variance or other deviation measures. Using Shannon's entropy in portfolio selection can diversify the allocation on various assets, while meeting the requirement of investors.

The following is Shannon's entropy:

$$H = -\sum_{i=1}^n w_i \ln w_i, \quad \sum_{i=1}^n w_i = 1, \quad i = 1, 2, \dots, n, \quad (1)$$

w_i the weight of security i (the probability of outcome i); n the number of invested securities (the number of states).

H has the maximum value, while $w_i = \frac{1}{n}$; the larger the H , the more information is gained by the observations. The other extreme case occurs when $w_i = 1$ for one i , and $=0$ for the rest, then $H = 0$. Therefore, Shannon's entropy provides a measure

of disorder in a system or expected information in a probability distribution. In addition, Shannon’s entropy measure can be used to evaluate the degree of diversification of portfolios.

Ke and Zhang [7] used Shannon’s entropy to modify the mean–variance model and construct a diversified model as follows:

$$\text{Min } w^T \Omega w + \sum_{i=1}^n w_i \ln w_i, \tag{2}$$

$$\text{s.t. } \sum_{i=1}^n r_i w_i = E \quad i = 1, 2, \dots, n, \tag{3}$$

$$\sum_{i=1}^n w_i = 1, \tag{4}$$

$$w : (w_1, w_2, \dots, w_n)^T,$$

r_i the average return of asset i ; Ω the covariance matrix; and E the default expected return of the portfolio.

By considering entropy and minimizing the objective, the model prevents the portfolio concentrate on a limited amount of assets. Jiang et al. [18] combine Shannon’s entropy and the mean–variance model to contribute to the portfolio choice of investors. Zheng et al. [32] use Shannon’s entropy to replace the variance in the portfolio selection problem:

$$\text{Max } - \sum_{i=1}^n w_i \ln w_i, \tag{5}$$

$$\text{s.t. } \sum_{i=1}^n r_i w_i = E, \tag{6}$$

$$\sum_{i=1}^n \sum_{t=1}^m w_i (r_{it} - \bar{r}_i)^2 \leq \text{Var}, \tag{7}$$

$$\sum_{i=1}^n w_i = 1, \tag{8}$$

r_{it} the return of asset i at time t ; and Var variance assigned by investors.

Using the Lagrangian function ϕ , where λ , γ_1 and γ_2 are constants; w_i can be obtained by the method proposed by Jiang et al. [18]:

$$\phi = - \sum_{i=1}^n w_i \ln w_i + \gamma_1 \left(\sum_{i=1}^n r_i w_i - E \right) + \gamma_2 \left(\sum_{i=1}^n \sum_{t=1}^m w_i (r_{it} - \bar{r}_i)^2 - \text{Var} \right) + \lambda \left(\sum_{i=1}^n w_i - 1 \right), \tag{9}$$

$$w_i = \frac{e^{[\gamma_1 \bar{r}_i + \gamma_2 \sum_{t=1}^m (r_{it} - \bar{r}_i)^2]}}{\sum_{i=1}^n e^{[\gamma_1 \bar{r}_i + \gamma_2 \sum_{t=1}^m (r_{it} - \bar{r}_i)^2]}}, \quad i = 1, 2, \dots, n. \tag{10}$$

2.2. Yager’s entropy

Yager’s entropy [22] aims to minimize the distance between the weight of the invested asset and $1/n$ in terms of the portfolio selection. The budget allocates on securities more equally while Yager’s entropy is larger. Therefore, Yager’s entropy has the minimum value if $w_i \approx 1/n$. The following is the definition of Yager’s entropy:

$$Q(w) = - \left(\sum_{i=1}^n \left| w_i - \frac{1}{n} \right|^z \right)^{1/z}, \tag{11}$$

where z is a constant and $z \geq 1$. While $z = 1$, Yager’s entropy can be transferred into a linear type [33]. The following is Yager’s entropy as $z = 1$:

$$Q(w) = - \sum_{i=1}^n \left| w_i - \frac{1}{n} \right|, \tag{12}$$

Yager’s entropy can be reformulated as the following optimization problem:

$$\text{Min } \sum_{i=1}^n (e_i^+ + e_i^-), \quad (13)$$

$$\text{s.t. } w_i - \frac{1}{n} - e_i^+ + e_i^- = 0, e_i^+ \geq 0, e_i^- \geq 0, \quad (14)$$

$$\sum_{i=1}^n w_i = 1 \quad i = 1, 2, \dots, n, \quad (15)$$

$$w_i \in [0, 1],$$

where e_i^+ and e_i^- are the upside deviation and downside deviation between w_i and $1/n$ respectively. When $z \rightarrow \infty$ then

$$Q(w) = \text{Max}_i [w_i] - \frac{1}{n}. \quad (16)$$

Yager's entropy allocates the budget onto assets equally by applying the maximum entropy principle.

2.3. Minimax disparity model

The minimax disparity model [23] is to minimize the maximum distance between weights. The following is the minimax disparity model:

$$\text{Min } \{ \text{Max}_{i \in \{1, \dots, n-1\}} |w_i - w_{i+1}| \} \quad (17)$$

$$\text{s.t. } \text{orness}(w) = \alpha = \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i, \quad 0 \leq \alpha \leq 1, \quad (18)$$

$$\sum_{i=1}^n w_i = 1, \quad (19)$$

$$0 \leq w_i \leq 1, \quad i = 1, \dots, n,$$

where α is a given value from the decision maker, and orness is a component with a value between $[0, 1]$. Orness is used to explain the aggregation between weights [34,35]. For this study, we do not consider the aggregation between each weight. Specifically:

$$\text{Min } \{ \text{Max}_{i \in \{1, \dots, n-1\}} |w_i - w_{i+1}| \}, \quad (20)$$

$$\text{s.t. } \sum_{i=1}^n w_i = 1, \quad (21)$$

$$0 \leq w_i \leq 1, \quad i = 1, \dots, n.$$

The model can be transformed as a linear model:

$$\text{Min } \delta, \quad (22)$$

$$\text{s.t. } \sum_{i=1}^n w_i = 1, \quad (23)$$

$$w_i - w_{i+1} - \delta \leq 0, \quad i = 1, \dots, n-1, \quad (24)$$

$$w_i - w_{i+1} + \delta \geq 0, \quad i = 1, \dots, n-1, \quad (25)$$

$$w_i \geq 0, \quad i = 1, \dots, n,$$

where δ is the maximum distance between any two weights. Because the above is linearized, it can be solved directly by linear programming. Given $w_i \geq 0$ for any asset i in the above three entropy measures, these models cannot be applied directly to the scenarios when short selling is allowed.

2.4. Mean–variance model

The mean–variance model [1] aims to determine the composition for a portfolio of n securities, which minimizes risks, while achieving a given level of expected returns, as follows:

$$\text{Min } \sigma_p = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} w_i w_j, \tag{26}$$

$$\text{s.t. } \sum_{i=1}^n r_i w_i \geq \mu, \tag{27}$$

$$\sum_{i=1}^n w_i = 1, \tag{28}$$

n the number of available securities; w_i the investment portion in security i , for $i = 1, \dots, n$; r_i the return on security i ; μ the portfolio expected return; σ_i^2 the variance of the return of security i ; and σ_{ij} the covariance between the returns of securities i and j .

The first constraint expresses the requirements of a portfolio return, while the second is the budget constraint. If $w_i \geq 0$, for $i = 1, 2, \dots, n$, short selling is prohibited. In our study, short selling is allowed. Therefore, in the proposed models, w_i has an unrestricted sign.

According to Yu and Lee [36], the mean–variance model can be regarded as a multiple objective problem. To enhance the feasibility in portfolio selection, they consider transaction costs, short selling, skewness and kurtosis as criteria in determining portfolio rebalancing. Their results show that skewness and short selling are important criteria and the models with higher moments or those that adopt short selling strategies perform better. We apply their mean, variance, transaction cost and short selling (MVTS) model to rebalance portfolio as a foundation for our work on entropy.

2.5. The Mean–Variance–Short Selling–Transaction Cost (MVST) model [36]

There are four objectives in this model: maximization of the portfolio expected return, minimization of the portfolio variance, minimization of the short selling proportion of the portfolio, and minimization of the transaction costs in the MVST model. To incorporate short selling, w_i is an unrestricted sign, which is decomposed into $w_i^+ - w_i^-$ as follows:

$$\text{Max } \sum_{i=1}^n r_i (w_i^+ - w_i^-), \tag{29}$$

$$\text{Min } \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} (w_i^+ - w_i^-) (w_j^+ - w_j^-), \tag{30}$$

$$\text{Min } \sum_{i=1}^n w_i^-, \tag{31}$$

$$\text{Min } \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \tag{32}$$

$$\text{s.t. } \sum_{i=1}^n (w_i^+ + k w_i^- + p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-) = 1, \tag{33}$$

$$w_i^+ = w_{i,0}^+ + l_i^+ - l_i^-, \tag{34}$$

$$w_i^- = w_{i,0}^- + s_i^+ - s_i^-, \tag{35}$$

$$0.05 u_i \leq w_i^+ \leq 0.2 u_i, \tag{36}$$

$$0.05 v_i \leq w_i^- \leq 0.2 v_i, \tag{37}$$

$$u_i + v_i = y_i, \tag{38}$$

for $i = 1, 2, \dots, n$,

$w_{i,0}^+$ the proportion of security i bought by investors prior to portfolio rebalancing; $w_{i,0}^-$ the proportion of security i sold short by investors prior to portfolio rebalancing; w_i^+ the total proportion of security i desired by the investors upon portfolio rebalancing; and w_i^- the total proportion of security i sold short by investors upon portfolio rebalancing. For each rebalancing, l_i^+ the proportion of security i bought by investors; l_i^- the proportion of security i sold by investors; s_i^+ the proportion of security i sold short by investors; s_i^- the proportion of security i repurchased by investors; u_i the binary variable that indicates whether security i is selected for long; v_i the binary variable that indicates whether security i is selected for short selling; k the initial margin requirement for short selling; and p_j for $j = 1, 2, 3,$ and 4 : the transaction costs of buying, selling, selling short, and repurchasing, respectively.

k and p_j are given constants. Eq. (33) defines the budget allocated to buying, short selling, and transaction costs. Eq. (34) shows the current long position after rebalancing. Eq. (35) represents the current short selling position after rebalancing, while Eqs. (36), (37) state the upper and lower bounds of the total positions of each security in buying and short selling, respectively. The conventional portfolio model needs to set the upper bound of each weight, for example, such as Eq. (36) and (37), to avoid the corner solutions, i.e., allocating portfolio in a limited number of assets. Chiou et al. [10] found that setting weight upper bound would reduce the performance of the portfolio. However, the concept of entropy is employed to achieve a diversified portfolio instead of setting the upper bound of the corresponding weight. Therefore, both constraints can be eliminated since the entropy measure is taken into consideration. Eq. (38) shows the number of invested securities for both buying and short selling.

MVST model is introduced to construct the diversified portfolio in the study. To see the impact of the entropy measure, the transaction cost is incorporated into the portfolio return to become a single objective instead of two individual objectives. To our best knowledge, most entropy measures cannot be applied to portfolio rebalancing when short selling is allowed. The contribution of this paper is to introduce the portfolio rebalancing models with various entropy measures allowing short selling.

3. Portfolio rebalancing method

In this study, we classify the models as various groups according to two conditions. Table 1 shows that the first criterion is whether short selling is allowed and the second is what objectives are considered in model. We compare the portfolio performance of twenty-three portfolio models that incorporate different entropy under various conditions. Among them, Model V is the conventional mean–variance model which minimizes portfolio variance and serves as the benchmark in the short-selling prohibited single-objective models. Model V_S is the benchmark in the short selling-allowed single-objective models. For the two and three-objective models, we use Model MV and MV_S as benchmarks to evaluate the portfolio performance after incorporate different entropy under different conditions.

For the group of short-selling allowed models, MV_S is the modified conventional mean–variance model with two objectives. We further consider the other models with short-selling, such as portfolio return considering transaction cost and Shannon’s entropy (MS_S), portfolio return considering transaction cost and Yager’s entropy (MY_S), and portfolio return considering transaction cost and minimax disparity (MD_S). Short-selling is not set as an objective in these models.

3.1. Model MS_S

We exemplify Model MS_S to give readers an idea about the structure of our analyses. Model MS_S consists of two objectives, as shown in Eqs. (39) and (40), namely, the maximization of Shannon’s entropy and the maximization of portfolio

Table 1
The criteria of each model.

	Short selling prohibited		Short selling allowed	
	Model	Criteria of objective(s)	Model	Criteria of objective(s)
Single objective	V	Portfolio variance	V_S	Portfolio variance
	S	Shannon’s entropy	S_S	Shannon’s entropy
	Y	Yager’s entropy	Y_S	Yager’s entropy
	D	Minimax disparity	D_S	Minimax disparity
Two objectives	MV	Portfolio return–transaction cost Portfolio variance	MV_S	Portfolio return–transaction cost Portfolio variance
	MS	Portfolio return–transaction cost Shannon’s entropy	MS_S	Portfolio return–transaction cost Shannon’s entropy
	MY	Portfolio return–transaction cost Yager’s entropy	MY_S	Portfolio return–transaction cost Yager’s entropy
	MD	Portfolio return–transaction cost Minimax disparity model	MD_S	Portfolio return–transaction cost Minimax disparity model
	MVS	Portfolio return–transaction cost Portfolio variance Shannon’s entropy	MVS_S	Portfolio return–transaction cost Portfolio variance Shannon’s entropy
Three objectives	MVY	Portfolio return–transaction cost Portfolio variance Yager’s entropy	MVY_S	Portfolio return–transaction cost Portfolio variance Yager’s entropy
	MVD	Portfolio return–transaction cost Portfolio variance Minimax disparity model	MVD_S	Portfolio return–transaction cost Portfolio variance Minimax disparity model

return considering transaction costs. In order to consider short selling, w_i is set unrestricted sign, which is decomposed into $w_i^+ - w_i^-$. The details of Model MS_S are as follows:

$$\text{Max } \sum_{i=1}^n r_i(w_i^+ - w_i^-) - \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \tag{39}$$

$$\text{Max } \sum_{i=1}^n w_i^+ \ln w_i^+ + \sum_{i=1}^n w_i^- \ln w_i^-, \tag{40}$$

$$\text{s.t. } \sum_{i=1}^n (w_i^+ + k w_i^- + p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-) = 1, \tag{41}$$

$$w_i^+ = w_{i,0}^+ + l_i^+ - l_i^-, \tag{42}$$

$$w_i^- = w_{i,0}^- + s_i^+ - s_i^-, \tag{43}$$

$$0 \leq w_i^+ \leq 1 v_i, \tag{44}$$

$$0 \leq w_i^- \leq 1 u_i, \tag{45}$$

$$v_i + u_i \leq 1, \tag{46}$$

$$w_i^+ \geq 10^{-9}, \tag{47}$$

$$w_i^- \geq 10^{-9}, \tag{48}$$

$$v_i, u_i \in (0, 1), \tag{49}$$

$$w_i^+, w_i^-, w_{i,0}^+, w_{i,0}^-, l_i^+, l_i^-, s_i^+, s_i^- \geq 0, \text{ for } i = 1, 2, \dots, n. \tag{50}$$

The objective function $\text{Max } \sum_{i=1}^n w_i^+ \ln w_i^+ + \sum_{i=1}^n w_i^- \ln w_i^-$ of entropy decomposes investing into buying and short selling; therefore the entropy is calculated separately. Eqs. (47) and (48) define that all weights should be positive by taking natural logarithm. No upper bounds are needed in this model since the principle of entropy has been already taken into consideration in this model.

The above multiple criteria model can be solved by fuzzy multiple objective programming [37,38] and can be transformed from multiple objectives to a single objective model. Fuzzy multiple objective programming is based on the concept of the fuzzy set, which uses a minimization operator to calculate the membership function value of the aspiration level, λ . The program forces each goal to achieve its aspiration level and then provides a trade-off among the conflicting objectives or criteria. The ideal and anti-ideal solutions must be obtained in advance. By employing fuzzy multiple objective programming, Model MS_S can be reformulated as a single objective model, which allows the solution to be generated easily, as follows:

$$\text{Max } \lambda, \tag{51}$$

$$\text{s.t. } \lambda \leq \frac{(r^* - r_l)}{(r_g - r_l)}, \tag{52}$$

$$\lambda \leq \frac{(\omega^* - \omega_l)}{(\omega_g - \omega_l)}, \tag{53}$$

$$r^* = \sum_{i=1}^n r_i(w_i^+ - w_i^-) - \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \tag{54}$$

$$\omega^* = \sum_{i=1}^n w_i^+ \ln w_i^+ + \sum_{i=1}^n w_i^- \ln w_i^-, \tag{55}$$

Eqs. (41)–(50),

r^* the return of the portfolio minus transaction cost; r_l the anti-ideal return of the portfolio minus transaction cost; r_g the ideal return of the portfolio minus transaction cost; ω^* Shannon's entropy; ω_l the anti-ideal of Shannon's entropy; ω_g the ideal of Shannon's entropy.

Eqs. (52)–(55) altogether achieve the goals of maximizing Shannon's entropy and maximizing of portfolio returns. Both ideal and anti-ideal values of the criteria are required from investors or from historical data. In our case, the ideal and

anti-ideal solutions for portfolio returns are pre-specified by the best and the worst values in history, taken from the n securities at each rebalancing. The ideal solution for the variance of a portfolio is 0, and the anti-ideal default value is the variance with the worst value according to the historical data. The ideal and anti-ideal solution of Shannon's entropy are $\frac{1}{n} \sum_{i=1}^n \ln(\frac{1}{n}) + \frac{1}{n} \sum_{i=1}^n \ln(\frac{1}{n})$ and 0, respectively. The transactions costs for purchasing stock (the percentage in terms of the transaction value) are p_1 , selling stock p_2 , short-selling stock p_3 , and repurchasing short-sold stock p_4 . The rate of initial margin requirements for short selling, k , depends on the market condition. Since the sample size affects the ideal and anti-ideal solution for each entropy measure, sample selection is critical in this model.

Models MY_S and MD_S are constructed according to the similar ideas. We further consider portfolio variance and model three-objective portfolio. For our study, the performances of single-objective models are compared with Model V (or V_S) and the performances of two and three-objective models are compared with Model MV (or MV_S).

3.2. Model MY_S

Model MY_S consists of two objectives, as shown in Eqs. (56) and (57), namely, the minimization of Yager's entropy and the maximization of portfolio return minus transaction costs. The major concept of Yager's entropy regarding deviation in portfolio weights is shown in Eq. (58). The model is as following:

$$\text{Max } \sum_{i=1}^n r_i (w_i^+ - w_i^-) - \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \quad (56)$$

$$\text{Min } \sum_{i=1}^n (e_i^+ + e_i^-), \quad (57)$$

$$\text{s.t. } (w_i^+ + w_i^-) - \frac{1}{n} - e_i^+ + e_i^- = 0, e_i^+ \geq 0, e_i^- \geq 0. \quad (58)$$

Eqs. (41)–(46), (49), and (50),

where e_i^+ and e_i^- are the upside deviation and downside deviation between i -th security's weight to $1/n$. After incorporate Eqs. (57) and (58), MY_S aims to allocate the same budget to assets. For our study, Yager's entropy is used to allocate the budget to assets instead of to deal with variance that is linearized in the model. This is also the difference between MV_S and MY_S.

3.3. Model MD_S

Model MD_S consists of two objectives, as shown in Eqs. (59) and (60), namely, the maximization of portfolio return minus transaction costs and the minimization of minimax disparity value. Here the concept of minimax disparity method is used as linear entropy. Eqs. (61) and (62) are to incorporate the minimax disparity method that minimizes the distance among weights. The formulation is as following:

$$\text{Max } \sum_{i=1}^n r_i (w_i^+ - w_i^-) - \sum_{i=1}^n (p_1 l_i^+ + p_2 l_i^- + p_3 s_i^+ + p_4 s_i^-), \quad (59)$$

$$\text{Min } \delta, \quad (60)$$

$$\text{s.t. } (w_i^+ - w_{i+1}^+) + (w_i^- - w_{i+1}^-) - \delta \leq 0, i = 1, \dots, n-1, \quad (61)$$

$$(w_i^+ - w_{i+1}^+) + (w_i^- - w_{i+1}^-) + \delta \geq 0, i = 1, \dots, n-1, \quad (62)$$

Eqs. (41)–(46), (49), and (50).

Eqs. (61) and (62) mean the difference among weights would be limit within δ and mean the maximum distance among weights. Specifically, Eqs. (60)–(62) altogether are to minimize the maximum distance among weights.

4. Numerical results

The 1380 daily returns of the stocks in Taiwan 50 index and Mid-Cap 100 index from the Taiwan Stock Exchange (TSE) during a period of November 1, 2006 and June 19, 2012 are collected from the database of the Taiwan Economic Journal (<http://www.tej.com.tw/>). We exclude the stocks of missing data during the sample period and use the data of 136 stocks to demonstrate the numerical example. The market value of these stocks represents more than 80% of the overall capitalization in Taiwan. Therefore our results should be free of illiquidity issue and should be useful to portfolio management. In addition, the conclusion of the profitability of the models should be robust since the sample includes the period of financial

Table 2
Summary statistics of *ex ante* Sharpe ratio of portfolio models.

		Sharpe ratio			
		Mean	Max	Min	% of Negative
<i>Short selling prohibited</i>					
Single objective	1/n	0.099	0.651	-0.230	28.36
	V	0.114	0.761	-0.468	32.84
	S	0.072	0.606	-0.246	32.84
	Y	0.071	0.611	-0.247	34.33
	D	0.071	0.604	-0.247	34.33
Two objective	MV	0.343	0.868	-0.094	4.48
	MS	0.237	0.727	-0.152	8.96
	MY	0.185	0.573	-0.192	14.93
	MD	0.281	0.683	-0.115	2.99
Three objective	MVS	0.264	0.741	-0.170	8.96
	MVY	0.197	0.592	-0.204	13.43
	MVD	0.339	0.857	-0.104	4.48
<i>Short selling allowed</i>					
Single objective	V_S	0.165	0.859	0.191	22.39
	S_S	0.003	0.406	-0.486	47.76
	Y_S	0.075	0.579	-0.245	32.84
	D_S	-0.071	0.247	-0.604	65.67
Two objective	MV_S	0.513	0.900	0.250	0.00
	MS_S	0.330	0.730	0.031	0.00
	MY_S	0.265	0.604	-0.005	1.49
	MD_S	0.331	0.709	0.061	0.00
Three objective	MVS_S	0.380	0.738	0.083	0.00
	MVY_S	0.290	0.607	0.064	0.00
	MVD_S	0.453	0.898	0.106	0.00

crisis. To compare the results of the portfolio models, we also present the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) as a proxy for the overall market portfolio.

We form and rebalance the optimal portfolios by using the models presented in Section 3. The portfolios are first formed by using the data in the first 60 transaction days and are rebalanced every 20-trading-day by using a rolling window of the previous 60 daily data. The initial investments are assumed to be \$1 million. Investors can follow the strategies in our paper by proportionally increasing their portfolios. According to the rules in Taiwan's stock markets, the percentage of transaction costs (including brokerage fee and taxes) of buying (p_1), selling (p_2), selling short (p_3), and repurchasing (p_4) are 0.1425% (the brokerage fee charged over the total trading value for buying), 0.4425% (including the 0.1425% brokerage fee charged over the total trading value and the Security Transaction Tax for selling 0.3%), 0.5225% (including the 0.1425% brokerage fee charged over the total trading value and the Security Trading Tax for short-selling 0.38%), and 0.1425% (the brokerage fee charged over the total trading value for repurchasing the short-sale position), respectively. The rate of initial margin requirements for short selling, k is 0.9.¹ To compare the performance of naïve diversification, we also include a portfolio of $1/n$ weight on each of assets [8]. The models are run on a Genuine Intel CPU T2400 1.83 GHz and a 1 GB RAM notebook computer, with Lingo 11.0 software [39]. For our study, we rebalance each of portfolios 67 times over the sample period and compute their return, volatility, trading costs, and the proportion of short selling to compare their performance.

Table 2 presents the over-time statistics of *ex ante* Sharpe ratio for each model, including mean, maximum, and minimum of Sharpe ratio of each model over the sample period. The percentage of the negative Sharpe ratio over the period is reported. For $1/n$ portfolio and single-objective models, they in general yield low mean–variance efficiency. On the other hand, the risk-adjusted performance for the short-sale allowed portfolios is higher than their corresponding short-sale prohibited portfolios. The portfolios with two or three objectives and with allowing short-selling seem yield higher in-the-sample effectiveness in managing portfolio. This might attribute to the portfolio flexibility brought by short selling in asset management. The addition of portfolio objectives and entropies increases the mean–variance efficiency.

The above results suggest that it is not necessary for the models with fewer objectives generate higher *ex ante* mean–variance efficiency. Instead, the two and three-objective models seem to demonstrate higher Sharpe ratio than the single-objective model. This is particularly significant for those portfolios allow asset short-selling. The multiple-objective models with allowing short-sale also tend to generate positive Sharpe ratio over the sample period.

¹ One may obtain the trading cost and taxes from the website of Taiwan Stock Exchange Corporation (TWSE) http://www.twse.com.tw/en/products/trading_rules/costs.php.

Table 3

The summary statistics ex post performance for each model.

	Model	Mean of realized return	Standard deviation of realized return	Ex post sharpe ratio	Average market value (\$)	Ending market value (\$)	Average number of invested assets	Average transaction cost (\$)	t-value of difference in market value
<i>Short selling prohibited</i>									
Single objective	1/n	-0.0186	0.2684	-0.0694	1,285,021	1,610,120	136	21	-
	V	-0.1385	0.1800	-0.7693	671,438	432,574	14	13,424	-
	S	0.1357	0.2923	0.4644	1,335,669	1,640,038	136	236	13.65***
	Y	0.1326	0.2925	0.4535	1,316,514	1,612,717	136	501	13.47***
	D	-0.1185	0.2847	-0.4162	677,980	421,331	136	13,800	0.20
Two objectives	MV	0.1121	0.3051	0.3675	1,152,539	1,418,066	14	1502	-
	MS	0.1059	0.3550	0.2984	1,065,768	1,256,630	136	905	-1.75*
	MY	0.1329	0.3469	0.3832	1,188,280	1,474,488	114	796	0.63
	MD	0.0844	0.4146	0.2036	872,800	1,001,369	7	1,285	-6.09**
Three objectives	MVS	0.1020	0.2975	0.3428	1,115,514	1,356,280	134	988	-0.79
	MVY	0.1191	0.2943	0.4046	1,196,307	1,489,306	111	796	0.84
	MVD	0.1110	0.3045	0.3643	1,094,640	1,411,244	14	1451	-1.15
<i>Short selling allowed</i>									
Single objective	V_S	-0.2863	0.0688	-4.1638	485,713	208,833	81	8708	-
	S_S	-0.0715	0.1058	-0.6759	753,727	660,820	136	4991	8.99***
	Y_S	0.0631	0.2674	0.2361	989,114	1,156,123	136	4092	13.09***
	D_S	-0.1829	0.3264	-0.5602	522,751	277,243	114	1903	0.89
Two objectives	MV_S	0.1797	0.2911	0.6172	1,373,868	2,061,413	15	2348	-
	MS_S	0.1485	0.3372	0.4404	1,464,589	1,619,451	111	1646	1.08
	MY_S	0.1765	0.3060	0.5767	1,596,083	1,983,857	102	1373	2.36**
	MD_S	0.2362	0.4230	0.5585	1,965,464	2,180,131	13	3539	4.82***
Three objectives	MVS_S	0.1465	0.2674	0.5479	1,503,702	1,802,904	136	1803	1.59
	MVY_S	0.1580	0.2542	0.6217	1,585,368	1,952,982	120	1433	2.36**
	MVD_S	0.2045	0.2899	0.7056	1,548,374	2,370,631	23	2393	1.87*

*, **, and *** represent p-value <0.1, 0.05, and 0.001 respectively.

4.1. Realized portfolio performance

In the real world, what interest investors *more* is the profitability of executing these asset strategies. We further discuss their *ex post* economic benefits in portfolio management. Table 3 shows the comparison of realization performance between the diversification portfolio models with various entropy measures and multiple objectives. To evaluate the benefit of the entropy models, our study includes various portfolio objectives, such as return, and variance respectively with short-selling allowed or prohibited. For two or more objectives, the solution is generated from multiple objective programming. We also generate the market value of the portfolio that is generated at the end of the each rebalancing.

The models that allow portfolio short selling and consider multiple objectives, on average, yield higher *ex post* performance than the others in most cases. The *ex post* risk-adjusted return for 1/*n* strategy and single-objective models, especially those with short-sale allowed, is generally low. For multiple-objective models, the portfolio flexibility brought short-sale seems enhance the mean–variance efficiency. For instance, among the models based on the minimax disparity, the Sharpe ratio of MVD_S is 0.706 while MD_S and D_S are 0.559 and –0.560, respectively.

Similar results can be found from the portfolio market values. For the single-objective short-sale-prohibited models, all models except Model D yield higher realization market value than the benchmark model (V). On the other hand, for the multiple objective models, the models including Yager's entropy (MY and MVY) outperform the benchmark MV model. When short selling is allowed, the conclusion for the single-objective models is similar but the market values are lower than the corresponding short-sale-prohibited models. For the two objectives models and three objectives models, they generate superior average market value to the benchmark (MV_S). However, for the two and three objective models, the short-selling allowed portfolios, in general, yield higher market value than the corresponding short-selling prohibited portfolios. Taken together, the models that consider multiple objectives and allow short selling demonstrate more significant *ex ante* diversification benefits than those do not incorporate both of them. Due to the lack of predictability of parameters, the portfolios can be *ex ante* profitable though they suffer *ex post* loss, and *vice versa*. This deviation of result between the realized performance and *ex ante* return is obvious for the portfolios without including entropies.

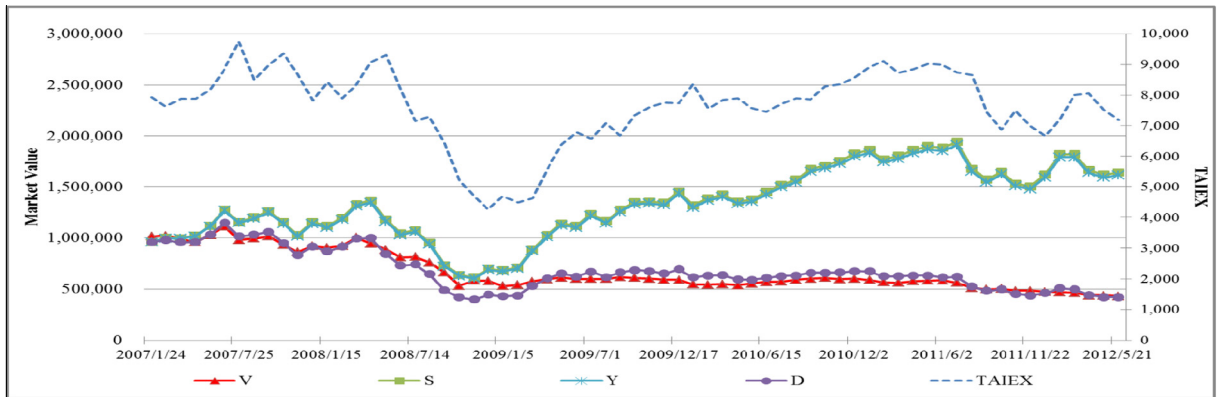
Table 3 also shows that a high realization market value is not necessarily caused by low transaction costs of the portfolio models. When short selling is allowed, Model D_S has the lowest transaction costs comparing to the other models with single objective but yield a low market value. For the models with two objectives, Model MY_S has the lowest transaction costs but has low realized market value. The low transaction costs of these models are attributed to their low rebalancing frequencies but do not necessarily lead to outperformance in profitability.

Table 3 reports the summary *t*-value statistics of testing the differences of market value between the Model V (the benchmark for the single objective models) or the Model MV (the benchmark for the two- and three-objective models) and the other short-sale prohibited portfolio models. The null hypothesis is the tested portfolio model differs from the *ex post* market value of benchmark (V or MV) portfolio in terms of realized portfolio value. The *p*-value shows that the realization values of the entropy portfolios are higher than that generated by the benchmarks, though their outperformance is not always statistically significant. Similarly, among the short-selling allowed portfolios, Model V_S is used to serve as the benchmark for the single objective models and the Model MV_S for the benchmark for the two- and three-objective models. The *t*-values of all for the single objective models are statistically significant except Model D_S. For the two- and three-objective models, there is statically significant outperformance in market value for the short-selling allowed models, such as the Models MY_S, MD_S, MVY, S, and MVD_S, comparing to the benchmark Model MV_S.

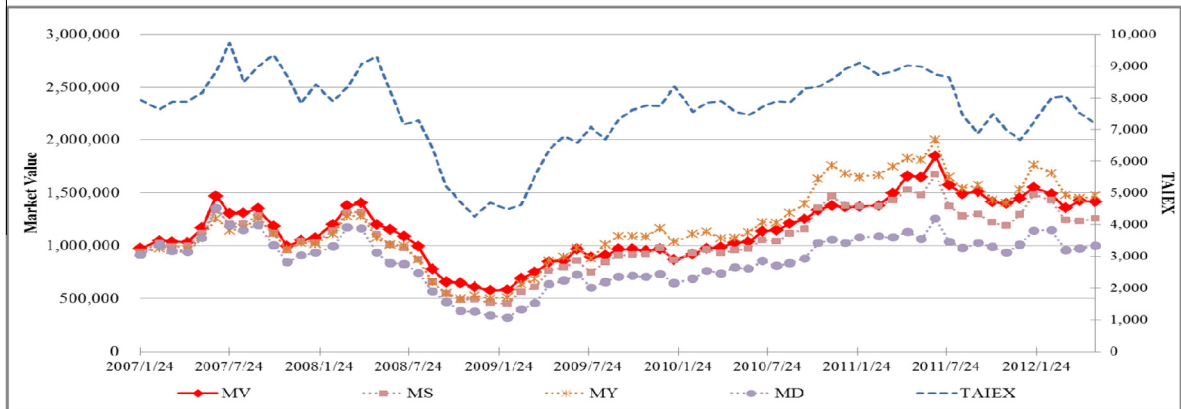
Our results also show that the models considering short selling and incorporating multiple objectives yield higher realization market values than others in the most time. When short selling is allowed, the models incorporating Yager's entropy outperform the other models. Though the models with minimax disparity demonstrate higher market value than the model with Yager's entropy for two-objective model, the market value of the minimax disparity measure is more volatile intertemporally.

Fig. 1 show the time-variation of the portfolio values of the short-sale prohibited models. We also demonstrate the overall market index as the reference in each graph. The market experienced a significant downturn in 2008 and 2009. The market values of the models which consider entropy in general generate positive return though the overall market yields loss during the sample period. Comparing with the benchmark portfolios (Model V for the single-objective portfolios and Model MV for the multiple-objective portfolios), the market values of the models incorporating Yager's entropy (i.e., MY and MVY) are higher than the other models in the same group. The performance of the multiple-objective models seem to be enhanced by incorporating Yager's entropy.

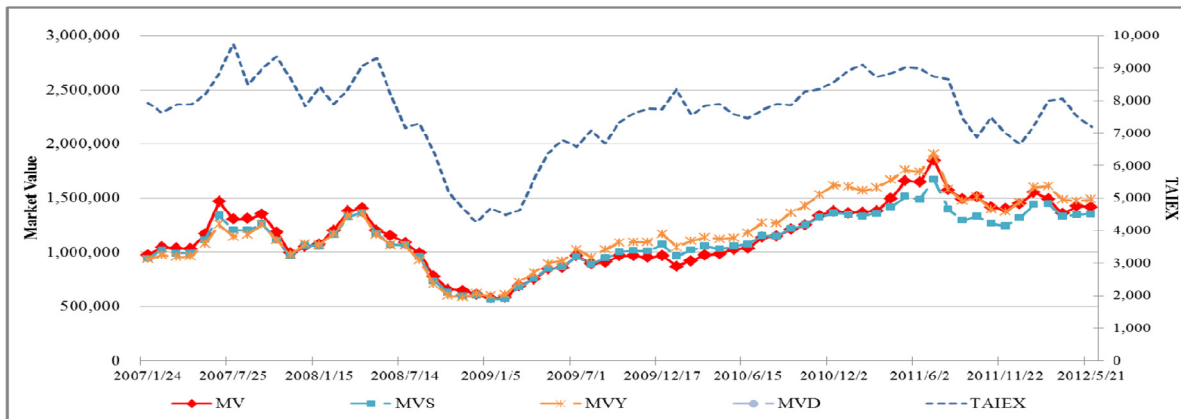
Fig. 2 shows the time-variation of portfolio value of short-sale allowed models. Among them, Models MD_S, MY_S and MVY_S demonstrate higher market value than others. Furthermore, the difference between V_S and the other models that incorporate entropy become more significant after early 2009. Intertemporally, the performance of Yager's entropy is more stable than other entropy measures. The minimax disparity model, on the other hand, demonstrates superior performance to the other models when short-selling is allowed. Therefore, the performance of the short-sale allowed models with multiple objectives can be improved by incorporating entropy in portfolio modeling.



A: Single-objective portfolios



B: Two-objective portfolios

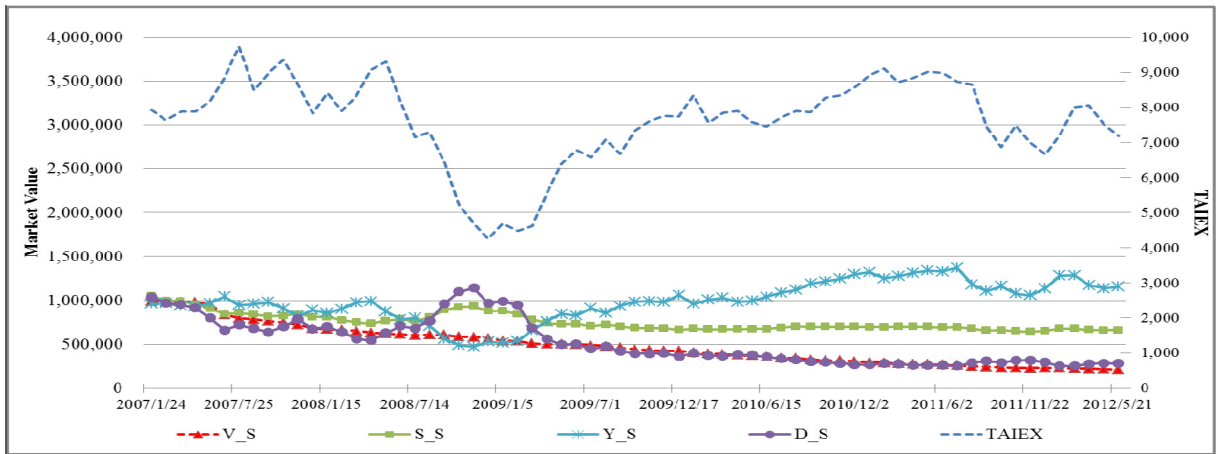


C: Three-objective portfolios

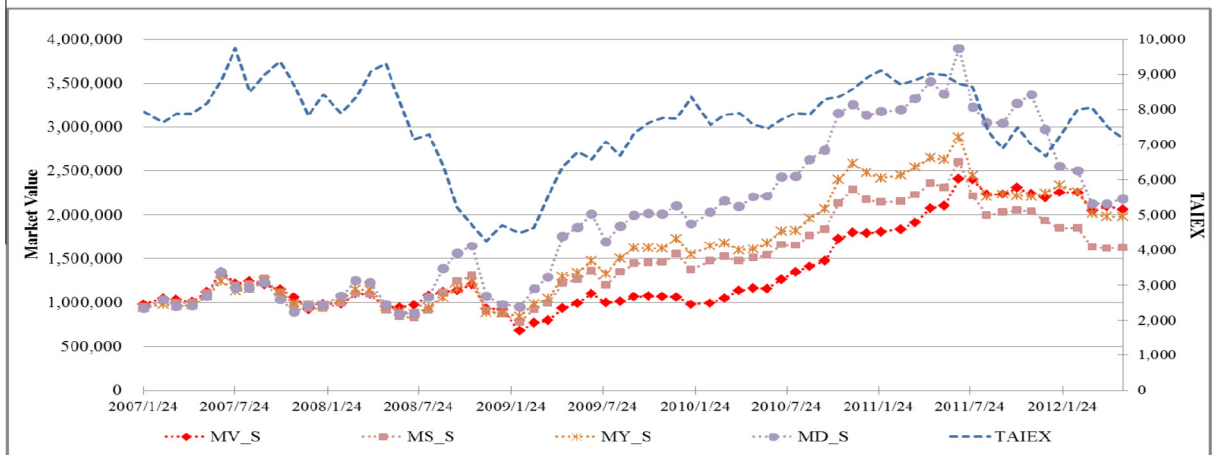
Fig. 1. Market values of short-selling prohibited portfolios. The market values of short-selling prohibited portfolios and Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) are presented. The initial investment is assumed \$1 million for each portfolio.

4.2. Portfolio diversity and flexibility of adjustment

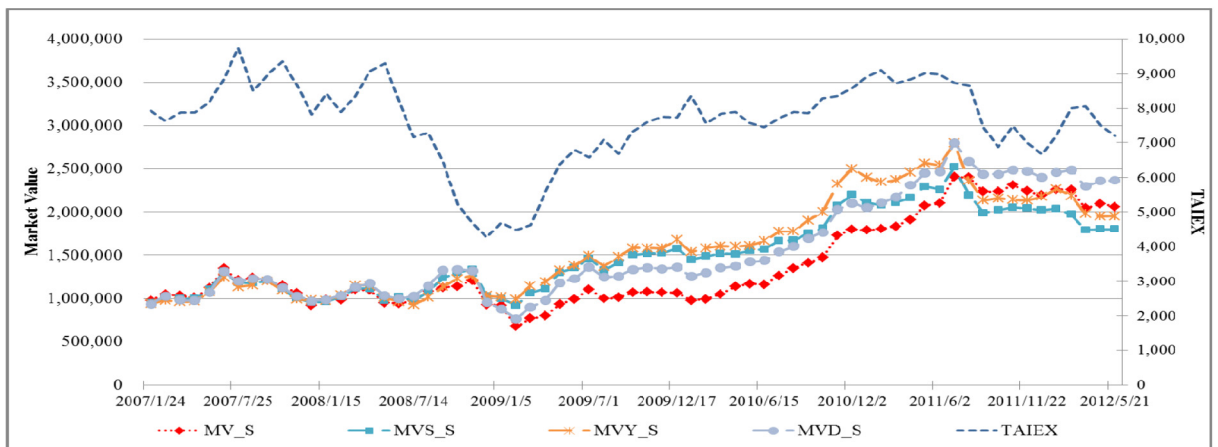
Table 4 presents the over-time statistics of the weight of short-sale position, the number of assets, and Herfindahl-Hirschman index (HHI) of portfolio weights for different models. For the models that short-sales are prohibited, the negative weight, of course, is zero. In general, among short-sales allowed models, the single-objective portfolios tend to have higher negative weight, except D_S model, than the models of two or three objectives. For our benchmark models, such as V, MV,



A: Single-objective portfolios



B: Two-objective portfolios



C: Three-objective portfolios

Fig. 2. Market values of short-selling allowed portfolios. The market values of short-selling allowed portfolios and Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) over the sample period are presented. The initial investment is assumed \$1 million for each portfolio.

Table 4
Concentration of asset and portfolio weight.

	Short selling proportion	Number of assets			Herfindahl–Hirschman index		
		Mean	Max	Min	Mean	Max	Min
1/n	0.0000	136.0	136	136	0.0095	0.0131	0.0073
V	0.0000	14.0	29	7	0.1959	0.5019	0.0771
S	0.0000	136.0	136	136	0.0074	0.0074	0.0073
Y	0.0000	136.0	136	136	0.0073	0.0074	0.0073
D	0.0000	136.0	136	136	0.0071	0.0073	0.0069
MV	0.0000	13.6	22	5	0.1403	0.2758	0.0694
MS	0.0000	136.0	136	136	0.0595	0.1228	0.0374
MY	0.0000	114.4	126	97	0.0580	0.0995	0.0274
MD	0.0000	7.5	11	4	0.1551	0.2479	0.1092
MVS	0.0000	134.3	136	100	0.0472	0.0861	0.0264
MVY	0.0000	111.4	122	94	0.0391	0.088	0.0142
MVD	0.0000	13.9	20	7	0.1182	0.2027	0.0717
V_S	0.5510	81.2	136	19	0.0528	0.1345	0.0111
S_S	0.5854	135.9	136	132	0.0108	0.0174	0.0082
Y_S	0.0382	136.0	136	136	0.0073	0.0074	0.0073
D_S	0.9291	114.0	114	114	0.0076	0.0076	0.0073
MV_S	0.2843	15.5	24	8	0.1410	0.2517	0.0686
MS_S	0.1809	111.1	136	5	0.0494	0.1202	0.0208
MY_S	0.1610	102.2	128	5	0.0448	0.0984	0.0165
MD_S	0.1779	13.0	43	7	0.1127	0.2236	0.0369
MVS_S	0.2242	135.8	136	129	0.0505	0.2032	0.0231
MVY_S	0.1741	120.3	134	102	0.0422	0.0983	0.014
MVD_S	0.2166	23.2	38	9	0.0826	0.1518	0.0334

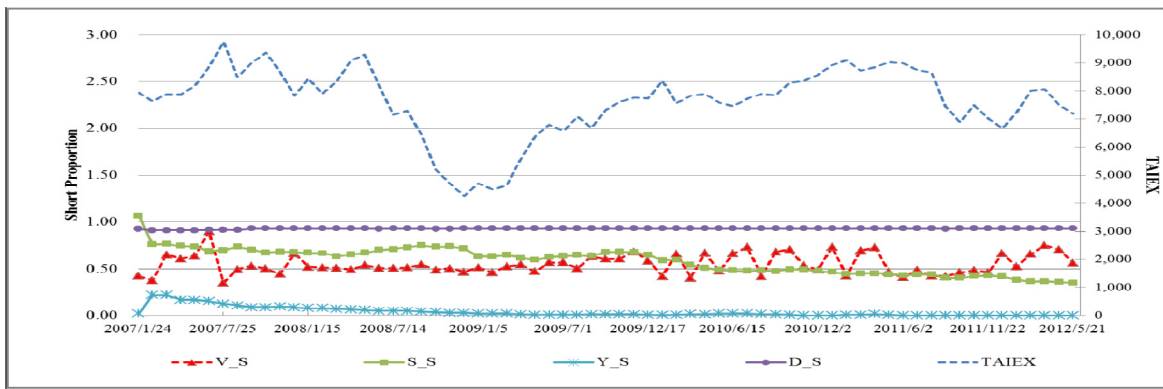
V_S, and MV_S, they tend to generate portfolios that concentrate on smaller number of assets and are of lower diversity than other models.

The performance of a portfolio model is affected by its sensitivity and extent of adjustment of asset holding to the market movement. The change between a long proportion and short proportion in a portfolio can be viewed as the model's response to a change from bull market to bear market or from bear market to bull market. That is, if the bull (bear) market is about to turn into a bear (bull) market, some long (short) position of the portfolio would be shifted as short selling (buying) proportions speedily.

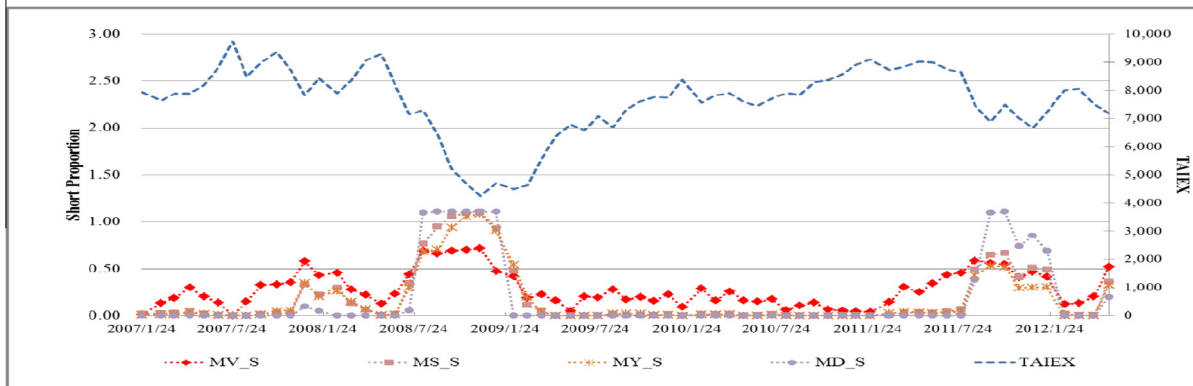
Fig. 3 shows the short selling weight of various portfolio models. The models that incorporate entropy seem capture the change in market better than Model V_S. The weights in the portfolios that incorporate entropy adjust rapidly as the market condition changes. In addition, these models had higher short selling position than V_S during the bear market, e.g., in 2008. The models incorporating entropy outperform the V_S because these portfolio models are sensitive to the change in the market.

In sum, the portfolio values of the models incorporating entropy are the higher than their corresponding benchmarks. Interestingly, these entropy portfolios also respond to movement of the overall market speedily when short selling is allowed. Specifically, the short-selling weight of the models with entropy decrease (increase) rapidly when the market turns to bullish (bearish). The models incorporating entropy also have higher short selling position than benchmark models do during the market downturn. It seems that the entropy models, particularly those allowing short selling, respond to the change in the market effectively.

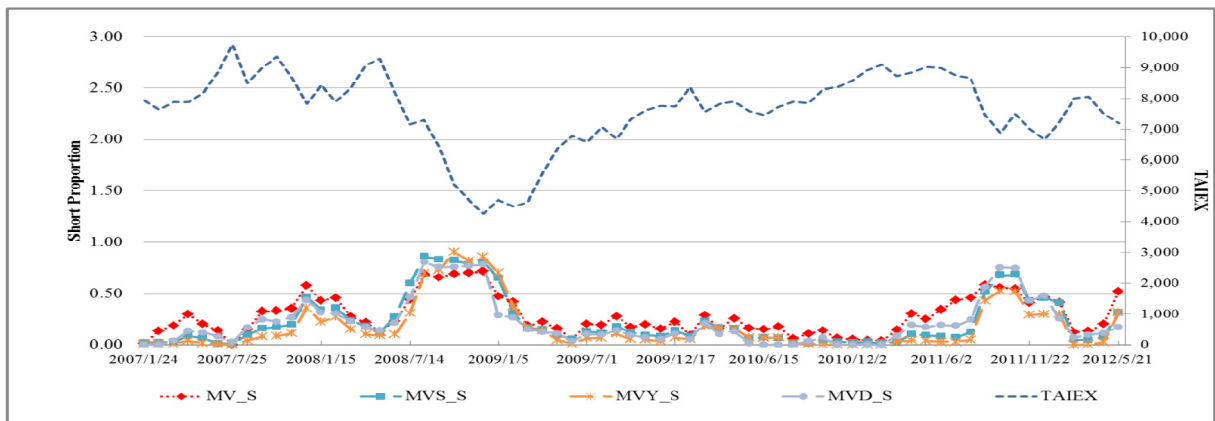
The results of numerical example suggest that asset managers need to consider the following issues when they exercise these portfolio models. First, the Shannon's entropy measure is non-linear in portfolio selection since its weights take natural logarithm. This can result in yielding small positive weights for some asset in the portfolio. Second, the models with minimax disparity may outperform the model with Yager's entropy under certain scenarios. In Table 3, the ranking of average market value shows MD_S > MY_S > MS_S > MV_S but MY > MV > MS > MD and MVY > MVS > MVD. From the average market value perspective, using minimax disparity measure may cause higher fluctuation in market value than other models. Third, the linearization of Yager's entropy can directly obtain a diversified portfolio because of the linear feature of Yager's entropy. It is clear to see that the models with Yager's entropy can effectively cope with the market change especially under the bear market. Hence, the Yager's entropy measure is suggested.



A: Single-objective models



B: Two-objective models



C: Three-objective models

Fig. 3. Weights of short selling in portfolios and market trend. The overall weight of short sales for short-selling allowed portfolios and Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) over the sample period are presented.

4.3. Comparing 1/n portfolio

The effectiveness of portfolio models to improve the out-of-sample performance is still questionable. DeMiguel et al. [8] use a naïve portfolio, or 1/n strategy, as a benchmark and find that the gain from portfolio optimization in risk portfolio models is offset by estimation error. We study the difference of realization benefits in managing portfolio between using entropy measures and adapting the naïve strategy. The null hypothesis is the tested portfolio model (V, MV, etc.) differs from the 1/n strategy in terms of realization portfolio value.

Table 5 presents the summary statistics of the comparisons of market value between the value of 1/n portfolio and those generated by the other models. The negative (positive) *t*-value represents that the portfolio value of the 1/n strategy is higher

Table 5

Difference of market value between each portfolio and 1/n strategy portfolio.

	Model	t-Statistics
<i>Short selling prohibited</i>		
Single objective	V	-13.32***
	S	0.87
	Y	0.55
	D	-13.12***
Two objectives	MV	-2.49**
	MS	-4.15***
	MY	-1.63
	MD	-8.36***
Three objectives	MVS	-3.39***
	MVY	-1.61
	MVD	-3.58***
<i>Short selling allowed</i>		
Single objective	V_S	-16.58***
	S_S	-12.79***
	Y_S	-6.15***
	D_S	-15.00***
Two objectives	MV_S	1.22
	MS_S	2.56**
	MY_S	3.79***
	MD_S	5.99***
Three objectives	MVS_S	3.26***
	MVY_S	3.91***
	MVD_S	3.26***

p-Value <0.1, 0.05, and 0.001 are indicated as *, **, and *** respectively

(lower) than that of the risk model portfolio. When short selling is prohibited, the 1/n strategy significantly outperforms the majority of the portfolio models with entropies. However, when short selling is allowed, the model with entropies and multiple objectives dominates 1/n strategy significantly in terms of realization market value. The finding suggests that greater portfolio flexibility brought by short-sale grant portfolios with entropy measures higher out-of-sample performance. The above finding also supports the conclusion of DeMiguel et al. [8] that no model consistently demonstrates higher performance than the naïve portfolio.

5. Conclusions

In this paper, we evaluate the performance of the portfolio models that are used to rebalance with allowing or prohibiting short selling, considering transaction cost of the portfolio, minimizing portfolio risk, and applying entropy in modeling asset allocation. The existence of transaction costs and the estimation error may affect the ability to reflect mean–variance in portfolio optimization. To enhance feasibility in the real world, we directly incorporate real transaction costs in the optimization procedure. The introduction of entropy in the multi-objective optimization will help to avoid under-diversified portfolios (due to corner solutions) and to reduce the impact of the estimation risk.

The numerical test using the most liquid 136 stocks in the Taiwan Stock Exchange shows that the models with Yager's entropy yield the higher economic value of diversification than other portfolio models, such as Shannon's entropy, the mini-max disparity model, minimal-variance model, and mean–variance model. This is because the portfolio models constructed by Yager's entropy respond the change in market by reallocating assets effectively. In addition, including entropy in models also enhance diversity of the portfolios.

In practice, fund managers need to deal with over-diversification occurs in the model with Shannon or Yager entropy. To control the number of the invested assets, the upper bound of the invested assets, $\sum_{i=1}^n v_i + u_i$, can be set in the corresponding models.

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