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Analytical design of symmetrical Kerr-lens mode-locking laser cavities

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Analytical expressions for the spot size and the Kerr-lens mode-locking (KLM) strength can be represented as explicit functions of the position in the cavity, the laser power, and the stability parameter for a four-mirror figure-Z laser resonator. The results indicate that the KLM strength achieves its maximum value at the edge of the stability range. Self-amplitude modulation and group-velocity dispersion compensation can be established by a prism pair. Simultaneously obtaining a large pumping efficiency and KLM is possible for this cavity.

1. INTRODUCTION

Recently the optical Kerr effect has been used for mode locking solid-state lasers to generate ultrashort laser pulses. Kerr-lens mode locking (KLM), simulating fast saturable absorber action, can achieve sub-20-fs pulses from Ti:sapphire lasers.[1,2] By maximizing self-amplitude modulation and minimizing intracavity third-order dispersion, Curley *et al.* observed the generation of 12-fs pulses by placing a hard aperture in the short nondispersive arm of the asymmetrical cavity.[2] Several approaches were developed to analyze KLM lasers.[3–6] An analysis[3] of the power-dependent change of focal position and beam waist in a thin laser medium to the first order of the average intracavity power also showed that the mode-locked operation can be obtained only by placement of an aperture in the short resonator arm, and the hard-aperture KLM laser operates most efficiently near the limits of the stable range.[3] A relatively symmetrical figure-Z-shaped cavity was used by another group[1] to generate 17-fs pulses without using any aperture but using some clipping of the beam by the apexes of the prisms in the longer resonator arm to induce self-mode locking.

Only with the approximation that the change of refractive index that is due to the optical Kerr effect is much smaller than its unperturbed refractive index[3,4] (i.e., the intracavity power is much less than the critical power of self-trapping) and use of the averaged beam waist throughout the Kerr medium can the transfer matrix be found and the q parameter be analytically solved for the asymmetrical figure-Z cavity. However, since precise beam-spot variation throughout the Kerr medium cannot be calculated with the averaged-beam-waist and low-intracavity-power approximations, the optimization of coupling between the pump and the cavity beams cannot be achieved. Another group proposed an alternative transfer matrix for the Kerr medium, in which the concept of Gaussian beam propagation over a negative distance was introduced.[5] Although this method can be applied to more general conditions than that of Refs. 3 and 4, one first has to calculate the continuous-wave (low-power) cavity-beam waist and the spot size at the center of the Kerr medium to obtain the transfer matrix of the Kerr medium for mode-locking (high-power) operation. By use of the paraxial approximation and introduction of the renormalized q parameter in a nonlinear medium,[7] which satisfies the free-space propagation equation, a problem involving self-focusing can be analyzed as free-space propagation. A cavity consisting of a plane mirror backed against a Kerr medium, a converging lens, and another end flat mirror can be analytically described in this manner.[7]

We present a fully analytic approach, based on a renormalized q parameter[7] that is transformed by the *ABCD* law as the beam propagates in the cavity, to study a four-mirror figure-Z laser cavity with a Kerr medium placed between the curved-mirror pairs. With the self-similar point of the q parameter located at one of the exit planes of the Kerr medium, the derivation is drastically simplified. Various optical properties, such as power-dependent beam-spot sizes, stable conditions for KLM, KLM strength, and optimal mode locking are analytically investigated.

2. GENERAL FORMULATION

To develop an analytic theory of KLM in a resonator, we use the q representation of a Gaussian beam of amplitude A_0 and beam radius w, and we approximate the self-phase shift $\Delta\Phi$ in the Kerr medium of length dz and nonlinear refractive index n_2 by a parabola:

$$\Delta\Phi = \frac{2\pi}{\lambda} n_2 {A_0}^2 \exp(-2r^2/w^2)\mathrm{d}z \approx \frac{2\pi}{\lambda} n_2 {A_0}^2 \left(1 - \frac{2r^2}{w^2}\right)\mathrm{d}z .$$

Haus *et al.*[7] showed that the propagation equation of $1/q$ satisfies

$$-\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{q}\right) = \frac{1}{q^2} + K\,\mathrm{Im}^2\left(\frac{1}{q}\right),$$

and the Kerr parameter K is defined as

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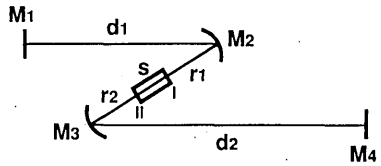

Fig. 1. Four-mirror figure-Z laser cavity with a Kerr medium of length s placed between curved mirrors. M_1 and M_4 are flat mirrors. M_2 and M_3 are curved mirrors with focal length f.

$$K = \frac{8P}{\pi}\left(\frac{\pi}{\lambda}\right)^2 n_0 n_2 ,$$

in which P is the cavity power and n_0 is the linear refractive index. After renormalizing the q parameter as

$$\frac{1}{q'} = \mathrm{Re}\left[\frac{1}{q}\right] + j\ \mathrm{Im}\left[\frac{1}{q}\right]\sqrt{1-K} ,$$

they find that

$$-\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{q'}\right) = \frac{1}{q'^2} ,$$

which is identical to the free-space propagation equation of $1/q$. Therefore the problem involving self-focusing can be analyzed as free-space propagation.

For precise fitting of experimental data[8] an adjustable correction factor α can be contained in Kerr parameter K. The definition of K is modified as $K = P/P_{\mathrm{cr}}$, with $P_{\mathrm{cr}} = \alpha\lambda^2/(8\pi n_0 n_2)$, where α is the correction factor and P_{cr} is the critical power of self-trapping. Since the above formula of critical power contains an adjustable correction factor, the results presented here can be applied even for high beam power ($P \approx P_{\mathrm{cr}}$).

The laser under consideration is a four-mirror figure-Z cavity laser as shown in Fig. 1. The cavity consists of two flat end mirrors (M_1 and M_4) and a pair of curved mirrors (M_2 and M_3) with the same focal length f. A laser rod (also a Kerr medium with nonlinear index n_2) of length s and index of refraction n_0 is placed between the curved mirrors. The distance between M_1 and M_2 (M_3 and M_4) is d_1 (d_2), and the distance between M_2 (M_3) and end face (could be Brewster-angle cut) I (II) of the Kerr medium is r_1 (r_2). The q parameter at M_1 is $q_1 = jy_1 = j\pi w_1{}^2/\lambda$, where λ is the free-space wavelength and w_1 the Gaussian beam radius at the output couplers at M_1.

Let the $ABCD$ matrix propagating from M_1 via M_2 to end face I be

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix},$$

and the round-trip matrix from end face II via M_3 and M_4 back to end face II be

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}.$$

Using the renormalized q-parameter concept to transform the q_1 parameter through the Kerr medium and the requirement of self-consistence at end face II, one obtains a quartic equation of $y_1{}^2$, i.e.,

$$a_4(y_1{}^2)^4 + a_3(y_1{}^2)^3 + a_2(y_1{}^2)^2 + a_1(y_1{}^2) + a_0 = 0 ,$$

where the coefficients a_4, a_3, a_2, a_1, and a_0 are functions of the Kerr medium length and the matrix elements of the two $ABCD$ matrices. Since a quartic equation has analytic solutions, the KLM laser cavity can, in principle, be described by an analytic approach.

3. RESULTS AND DISCUSSION

To simplify our discussion, we consider the case of a symmetrical laser cavity with $r_1 = r_2 = r$ and $d_1 = d_2 = d$. Since q_1, the q parameter at M_1, must be equal to q_4 at M_4 because of cavity symmetry, with $q_1 = q_4 = jy_1 = j\pi w_1{}^2/\lambda$, the q parameter at I, q_{I}, can be related to q_1 by the linear transfer matrix. Furthermore, the renormalized q parameter at I, q_{I}', is calculated as

$$\frac{1}{q_{\mathrm{I}}'} = \mathrm{Re}\left(\frac{1}{q_{\mathrm{I}}}\right) + j\ \mathrm{Im}\left(\frac{1}{q_{\mathrm{I}}'}\right)\sqrt{1-K}$$
$$= \frac{-y_1{}^2(f-r) + (f-d)(rf+df-rd) - jy_1 f^2\sqrt{1-K}}{y_1{}^2(f-r)^2 + (rf+df-rd)^2} . \quad (1)$$

One can obtain q_{II} at output II by transforming the q_{I}' parameter at the input into q_{II}' at output II by the relation $q_{\mathrm{II}}' = q_{\mathrm{I}}' + s/n_0 = q_{\mathrm{I}}' + L$, where $L = s/n_0$ is the optical path and n_0 the effective index of refraction.

Because of the symmetry property the imaginary parts (beam waists) of both $1/q_{\mathrm{I}}$ and $1/q_{\mathrm{II}}$ are equal, and their real parts (curvatures) have opposite signs. By relating $1/q_{\mathrm{II}} = -\mathrm{Re}(1/q_{\mathrm{I}}) + j\ \mathrm{Im}(1/q_{\mathrm{I}})$, one obtains an equation involving only $y_1{}^2$ and simply a guadratic equation of $y_1{}^2$, i.e.,

$$a(y_1{}^2)^2 + b(y_1{}^2) + c = 0 , \quad (2)$$

with

$$a = (f-r)^2(L+2r-2f) ,$$
$$b = 2(f-r)(rf+df-rd)[(L+2r)(d-f) - 2df + f^2] + Lf^4(1-K) ,$$
$$c = (d-f)(rf+df-rd)^2[(L+2r)(d-f) - 2df]$$
$$= (d-f)^2(rf+df-rd)^2\left(L + 2r - \frac{2df}{d-f}\right).$$

Before solving Eq. (2), let us consider the asymptotic limit of low nonlinear index or low laser power, i.e., $K \approx 0$. Since the spot size is always a positive real number, the only possible solution to $y_1{}^2$ for $K = 0$ is $y_{10}{}^2 = (d-f)^2[2df/(d-f) - (L+2r)]/(L+2r-2f)$. For $d > f$, which is generally true for mode-locked lasers, the stable condition of this cavity must satisfy $2f < (L + 2r) < 2df/(d-f)$ or $0 < \delta < [2df/(d-f)] - 2f$, where $L + 2r = s/n_0 + 2r$ is the effective separation of curved mirrors and $\delta = L + 2r - 2f$ is defined as the stability parameter. The stable condition also ensures that $a > 0$ and $c < 0$. It is trivial to solve the general solution to Eq. (2) as $y_1{}^2 = [(b^2 - 4ac)^{1/2} - b]/2a$. We obtain that the stable conditions are not only $ac < 0$ but $b^2 - 4ac > 0$. The former condition is equivalent to the condition for $K = 0$, which regulates the lower bound of the stable condition to be $\delta > 0$, and the latter condition determines the upper limit of the stability parameter.

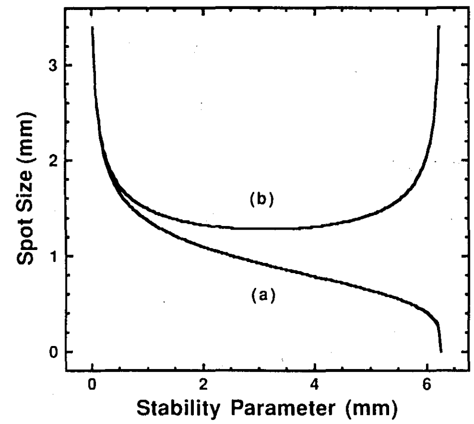

Fig. 2. Calculated spot size: curve (a) at M_1 and curve (b) at M_2 as functions of the stability parameter under cw operation.

After knowing y_1, we can calculate the beam radius at M_1 and the radii anywhere inside the cavity by transformation by the *ABCD* law. For example, the spot size $y(z)$ at a distance z from M_1 is presented as $y(z) = y_1[1 + (z/y_1)^2]$.

Consider that the symmetric Ti:sapphire laser has $d = 85$ cm, $f = 5$ cm, and a 9-mm-long Ti:sapphire rod as the Kerr medium ($n_0 = 1.76$ and $n_2 = 3 \times 10^{-20}$ m^2 W^{-1}, with correction factor $\alpha \approx 5.35$ and $P_{cr} \approx 2.6$ MW as in Ref. 8). In the no KLM ($K = 0$) limit, the spot size at the output coupler decreases monotonically as the separation of the curved mirrors increases, as is shown in Fig. 2, curve (a) in terms of the stability parameter. The spot size goes to infinity when $\delta = 0$ and to zero when $\delta = 6.25$ mm. However, the spot size in front of the curved mirror goes to infinity when the separation is adjusted to the edges of the stable region [see Fig. 2, curve (b)]. The output spot size is ~1.0 mm for cw operation at the center of the stable region ($\delta \approx 3$ mm).

Because the efficiency of self-amplitude modulation that is due to the optical Kerr effect is determined by the rate of change of the spot size as the laser power increases, we define the KLM strength[4] as

$$F = -\frac{1}{w}\left.\frac{\mathrm{d}w}{\mathrm{d}P}\right|_{K=0} = -\frac{8\pi n_0 n_2}{\alpha\lambda^2}\,\frac{1}{4y^2}\left.\frac{\mathrm{d}y^2}{\mathrm{d}K}\right|_{K=0} \cdot \quad (3)$$

The larger the F value, the larger the KLM efficiency is. By deriving $y_1{}^2$ with respect to power P (or K), we obtain $\mathrm{d}y_1{}^2/\mathrm{d}K = (Lf^4 y_1{}^2)/(b^2 - 4ac)^{1/2}$, which is always greater than zero. The spot size at the output coupler is a monotonically increasing function of laser power, and thus KLM is not attainable by insertion of an aperture at the output coupler for this type of laser. This result contradicts that for an asymmetrical cavity design,[2,4] where the hard aperture was inserted at the output coupler.

Since $\mathrm{d}y^2/\mathrm{d}K = (\mathrm{d}y^2/\mathrm{d}y_1{}^2)\,(\mathrm{d}y_1{}^2/\mathrm{d}K)$, where y is the function of y_1 with a Gaussian beam transformation, the F value can be calculated throughout the laser cavity. Figure 3 shows that the KLM strength F is a function of the position z and the stability parameter δ. By setting $\delta = 6$ mm, we can see that the KLM strength changes rather smoothly from approximately $z = 30$ cm to $z = 90$ cm [see Fig. 3, curve (a)]. Figure 3, curve (b) shows that the KLM strength at $z = 80$ cm increases as the stability parameter increases. To yield positive F for KLM by insertion of an aperture at $z = 80$ cm, the stability parameter range is $3 < \delta < 6.25$ mm, which is more restrictive than that of cw operation. The KLM strength reaches a maximum of 10^{-7} W^{-1} at the edge of the stable range, where $\delta = 6.25$ mm. Since the KLM strength achieves its maximum value at end face I, one can insert a hard aperture close to end face I to obtain KLM action. Instead of inserting an aperture, we can also use the apexes of the dispersion prisms to clip the optical beam. For instance, we can place a prism as close to M_2 as possible (say, its apex is located at $z = 80$ cm), and locate the other at a distance that is adjusted to compensate for the second-order group-velocity dispersion, e.g., 51 cm for Ref. 1. Each apex clips the opposite side of the optical beam. The behavior of this clipping mechanism is equivalent to that of a vertical slit, and KLM action is obtained.

Since the renormalized q parameter is valid with intracavity power $P < P_{cr}$, Fig. 4, curve (a) shows that the beam spot size at $z = 80$ cm decreases monotonically and reaches a minimum of 1.32 mm at the cavity power near $0.83P_{cr}$. To show how the spot size at the center of the Kerr medium is affected by increased cavity power, Fig. 4, curve (b) depicts the beam spot size first mildly changing from 19 to 16 μm as the cavity power varies from 0 to $0.73P_{cr}$, then drastically increasing as a result of self-trapping. Besides the small change ($\approx$17%) of the spot

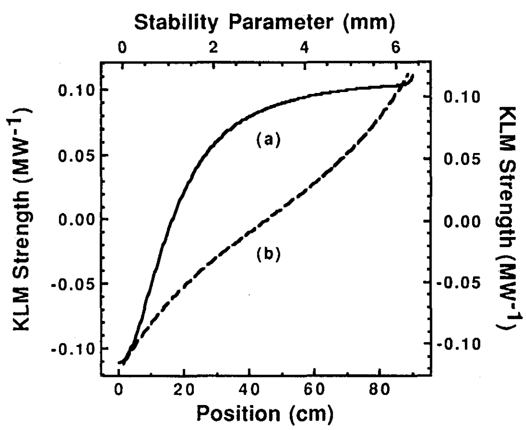

Fig. 3. Curve (a) KLM strength versus position measured from M_1 at $\delta = 6$ mm (left vertical scale); curve (b) KLM strength versus stability parameter calculated at $z = 80$ cm (right vertical scale).

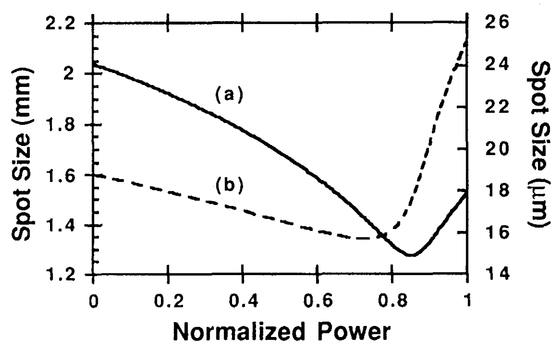

Fig. 4. Calculated power-dependent spot size at 80 cm for M_1 [curve (a) left vertical scale] and the center of Kerr medium [curve (b) right vertical scale].

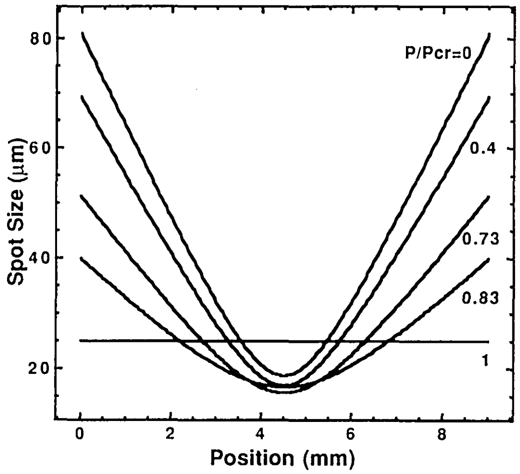

Fig. 5. Calculated spot size versus position inside the Kerr medium for various cavity powers P.

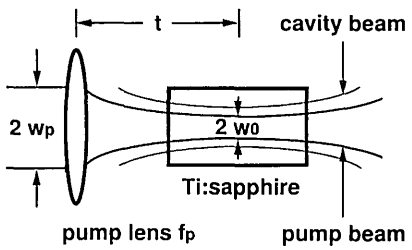

Fig. 6. Pumping design to favor KLM operation. The pump beam must be focused to match the cavity mode.

size and because of the symmetry of the cavity, the beam waist is located at the center of the gain material. Therefore the overlap of the pump and the cavity beams can be maintained to favor mode locking, and hence the laser efficiency may be better than that of an asymmetric cavity[2,4] as long as the cavity power does not go beyond $0.73P_{cr}$.

4. PUMPING CONSIDERATIONS

In addition to the hard-aperturing effect caused by insertion of an aperture into the laser cavity, a soft-aperturing effect caused by coupling between the pump-laser beam and the cavity laser beam also occurs in the end-pumping KLM lasers. While hard aperturing discriminates between the round-trip losses for KLM and cw operations, the soft aperturing then discriminates between the pumping efficiencies and thus the gains for KLM and cw operations. Simultaneously minimizing insertion loss and maximizing coupling efficiency for a high-power mode will then optimize the mode-locking operation. Therefore, for efficient KLM action, the soft-aperturing effect must be considered.

To allow us to consider the soft-aperturing effect, beam-spot sizes inside the Kerr medium are calculated and plotted in Fig. 5 for various cavity powers. We observe that beam-spot size for higher-power modes are always smaller than lower-power modes as long as $P < 0.73P_{cr}$. When cavity power is larger than $0.73P_{cr}$, the spot size near the center of the Kerr medium begins to increase. Finally, at $P = P_{cr}$, the beam propagates in a guided mode without variations of spot size (self-trapping).

Since the threshold pump power is lower and the coupling efficiency is higher with smaller pump and cavity beam-spot sizes,[9,10] one can make the spot size of the pump beam smaller than that of the cavity beam in the Kerr medium by properly adjusting the location and focal length of pump lens. For $P < 0.73P_{cr}$, the spot size of the KLM mode is always smaller than the cw mode in the Kerr medium; thus soft aperturing always favors mode-locking operation. Precise calculation will involve the maximization of the coupling factor F_0 introduced in Refs. 9 and 10, where

$$F_0 = \int r(x,y,z)s(x,y,z)\mathrm{d}v\,,$$

in which $r(x,y,z)$ and $s(x,y,z)$ are the pump- and the cavity-beam distribution functions, respectively, and the integration must be carried out all over the Kerr medium and can only be done by our approach and the renormalized-q-parameter method.

Once the spot size of the pump-beam waist is determined, we can proceed to calculate the location and focal length of the the pump lens. Let the focal length of the pump lens be f_p; the optical length between the beam waist and pump lens is t, as shown in Fig. 6. The pump lens has to transform the incident pump beam of $2w_p$ (assuming a plane wave) down to $2w_0$. Using the *ABCD* law, we have

$$t = [y_0(y_p - y_0)]^{1/2}\,,$$

$$f_p = (y_0{}^2 + t^2)/t\,,$$

where $y_0 = n\pi w_0{}^2/\lambda$ and $y_p = m\pi w_p{}^2/\lambda$. Here t must be larger than the optical length between the beam waist and the curved cavity mirror; i.e., the pump lens must be located outside the curved-mirror pair. For example, let $w_0 = 8\ \mu\mathrm{m}$; if we use an Ar^+ laser as the pump source, $\lambda = 514.5$ nm and $w_p = 1.1$ mm (Coherent Innova 310), then $t \approx f_p = 7.13$ cm.

5. CONCLUSION

In this work we have introduced a new analytic approach based on the aberrationless theory of self-focusing to study a four-mirror figure-Z laser cavity with a Kerr medium placed between the curved-mirror pairs. By proper choice of self-similar points of the q parameter and introducing the renormalized q parameter in the Kerr medium, with this method we reduce the solution of a nonlinear cavity to a quartic algebraic equation of y^2. In the symmetrical case the quartic equation is further reduced to a quadratic equation. Analytical expressions of the spot size and KLM strength can be represented as explicit functions of any position, through the cavity, laser power, and stability parameter for a symmetric figure-Z laser resonator. Both hard-aperturing and soft-aperturing effects are discussed. We conclude that in this cavity (1) the range of stability parameters for KLM operation is more restrictive than that of cw operation; (2) KLM strength achieves maximum value at the edge of stability range; (3) self-amplitude modulation and group-velocity dispersion compensation can be established by a prism pair; (4) simultaneously obtaining a large efficiency of pumping and KLM by self-aperturing is possible for this cavity; (5) higher pumping efficiency and lower insertion loss can be simultaneously obtained to favor KLM operation.

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