Optimum Beamformers for Monopulse Angle Estimation Using Overlapping Subarrays

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***Abstract*— We here derive the optimum sum and difference beamformers for monopulse target localization using a linear array. The beamformers are constructed, treating as superelements two overlapping subarrays. Removing the common factor associated with the superelement pattern from the angle error function leads to a closed-form target angle estimator independent of any adaptive nulling performed. Performance analysis of the angle estimator is conducted, and a procedure is developed to construct the beamformers, which achieve the minimum estimation variance under Gaussian noise. It is shown that the optimum angle estimator using the maximum overlapping subarrays is efficient for a moderately high signal–noise ratio (SNR) and a small off-boresight angle. The proposed method can be easily modified to incorporate interference cancellation.**

I. Introduction

THE classical sum-and-difference monopulse tracker represents a type of angle estimator that works in beam pattern domain. With the sum and difference patterns known *a priori*, the angle of a target may be determined via the ratio of the difference data to the sum data. In phased array systems, the sum and difference beams can be formed independently by two sets of complex weights applied at the array elements. With the flexibility of independent beamforming, optimum patterns can be synthesized under various criteria, such as minimum power [1], uniformly low sidelobes [2], [3], and maximum likelihood (ML) [4]. These optimum beamformers are constructed in an attempt to achieve some degree of improvement in angle estimation by enhancing the sidelobe suppression capability of the array.

In the above-described open-loop monopulse scheme, the sum and difference patterns must be computed or measured beforehand. In case the analytic expressions of the patterns are not available, a look-up table may be used as a mapping from the difference-to-sum ratio to the target angle. However, the table look-up method is of limited use for an array performing adaptive nulling. In that case, the sum and difference patterns vary with time. It is thus desired to obtain an angle estimator independent of any nulling that must be performed. Vu [5] described a method of beamforming to achieve this. In his method, two identical nonoverlapping subarrays of a uniform, linear array (ULA), tapered with the same set of weights, are used as two superelements possessing the same pattern. As a result, the sum and difference patterns synthesized with the superelements share a common factor. Dividing out this factor from the angle error function yields a simple, exact solution for the angle estimate. A similar idea was used earlier in the development of the three-aperture method for low-angle radar tracking [6]. A trade-off in Vu's two-aperture method is that the number of interferers that can be suppressed via simultaneous nulling is reduced to slightly less than one-half the number of elements.

As an extension to the two-aperture method, a class of beamforming schemes working with two identical overlapping subarrays of a ULA is developed. Closed-form angle estimators are derived accordingly. To assess the performance of the proposed method, the first- and second-order statistics of the angle estimator are derived, and a procedure is presented for constructing the optimum beamformers that achieve the minimum estimation variance under Gaussian noise. It is shown analytically that the optimum estimator using the maximum overlapping subarrays attains the Cramer–Rao lower bound (CRLB) for a moderately high SNR and a small off-boresight angle. On the other hand, the optimum two-aperture estimator exhibits better accuracy for a low SNR and a small off-boresight angle, or for a moderately high SNR and a large off-boresight angle. The proposed method can be easily modified to incorporate simultaneous nulling via either supervised or unsupervised techniques when the environment is contaminated with strong external interference.

II. Problem Formulation

Consider the scenario of a single far-field target illuminated by a uniform, linear array (ULA) radar consisting of M identical elements separated by a half wavelength. Due to the planewave assumption, the array data, in complex envelopes, received at a certain sampling instant can be put in the following $M \times 1$ vector form:

$$\mathbf{x} = \xi \mathbf{a}_M(u_t) + \mathbf{n} \tag{1}$$

where $u_t = sin(\theta_t)$, with θ_t being the angle of the target with respect to the broadside of the array, as shown in Fig. 1. The scalar ξ represents the target echo received at some reference point of the array. The $M \times 1$ vector $\mathbf{a}_M(u_t)$ is the direction vector accounting for the phase variation across the array. Finally, the $M \times 1$ vector $\mathbf{n}$ is composed of the additive noise (including both internal and external ones) present at the M elements. It is assumed that these noise components are jointly Gaussian distributed. Setting the reference point at the array center, we have (2), as shown at the bottom of the next page, with $K = M$ and $u = u_t$ for (1). The superscript

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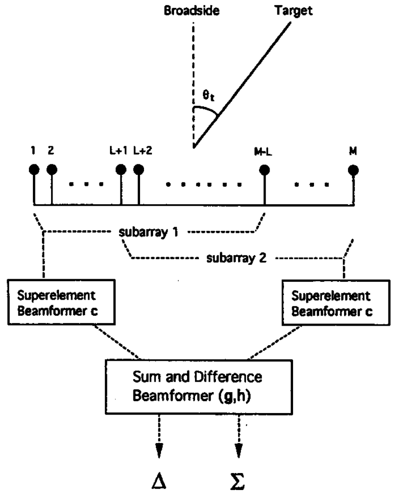

Fig. 1. Geometry of the array.

T denotes the transpose. We observe that the direction vector exhibits conjugate symmetry. The sum and difference data for monopulse estimation are formed by applying two sets of weights on $\mathbf{x}$:

$$\Sigma = \mathbf{s}^H \mathbf{x}; \qquad \Delta = \mathbf{d}^H \mathbf{x} \tag{3}$$

where $\mathbf{s}$ and $\mathbf{d}$ are the $M \times 1$ sum and difference weight vectors, respectively, and the superscript H denotes the complex conjugate transpose.

In open-loop monopulse localization, the target angle estimate $\hat{u}_t$ is determined via

$$\rho \overset{\triangle}{=} \frac{\Delta}{\Sigma} = \frac{d(u)}{s(u)}\Big|_{u=\hat{u}_t} \overset{\triangle}{=} \Phi(u)\Big|_{u=\hat{u}_t} \tag{4}$$

where $s(u) = \mathbf{s}^H \mathbf{a}_M(u)$ and $d(u) = \mathbf{d}^H \mathbf{a}_M(u)$ represent the sum and difference patterns, respectively. $\Phi(u)$ is referred to as the angle error function and represents the true ρ value under no noise. Equation (4) can be solved by searching over the spatial passband of the sum beam for a best match between both sides. If $\Phi(u)$ is known *a priori*, then a calibration curve relating ρ and $\hat{u}_t$ can be used. The former approach is time consuming for a large array, whereas the latter is of limited use in the case of adaptive nulling, for $\Phi(u)$ varies with time. Motivated by the work of Vu [5] and Cantrell *et al.* [6], we here propose a method of beamforming based on two overlapping subarrays that yields a closed-form angle estimate, regardless of any nulling performed. We first investigate the distinctive structures of the sum and difference weight vectors obtained with this scheme.

III. Structure of Beamformers and Angle Estimation

For convenience, assume that M is even. Consider decomposing an M-element ULA into two sub-ULA's of size $M - L$, as shown in Fig. 1. Suppose that a beamformer is attached to each of the subarrays with the same $(M - L) \times 1$ weight vector $\mathbf{c}$. These tapered subarrays may be viewed as *superelements* having the same pattern $c(u) = \mathbf{c}^H \mathbf{a}_{M-L}(u)$. The sum and difference beams are then formed with weight vectors $\mathbf{g} = [g_1, g_2]^T$ and $\mathbf{h} = [h_1, h_2]^T$, respectively, treating these superelements as two elements separated by $L/2$ wavelengths. Invoking the principle of arrays, the sum and difference patterns can be factorized as

$$s(u) = c(u)\Big(g_1^* e^{-j\pi L u/2} + g_2^* e^{j\pi L u/2}\Big)$$

$$d(u) = c(u)\Big(h_1^* e^{-j\pi L u/2} + h_2^* e^{j\pi L u/2}\Big). \tag{5}$$

It is noteworthy that $s(u)$ and $d(u)$ share a common factor $c(u)$. For a specific boresight angle u_o, we choose

$$g_1 = e^{-j\pi L u_o/2} = g_2^*$$

$$h_1 = -je^{-j\pi L u_o/2} = h_2^* \tag{6}$$

to make $\mathbf{g}$ and $\mathbf{h}$ conjugate symmetric. Substituting (6) into (5) and taking the ratio yields

$$\Phi(u) = \tan\left\{\frac{\pi L(u - u_o)}{2}\right\}. \tag{7}$$

A. Structure of Weight Vectors

It is easily verified that the subarray-based weight vectors exhibit the following factorizations:

$$\mathbf{s} = \mathbf{G}\mathbf{c} \quad ; \quad \mathbf{d} = \mathbf{H}\mathbf{c} \tag{8}$$

where

$$\mathbf{G} = \begin{bmatrix} z_o^* & & & \mathbf{O} \\ \mathbf{0}_{L-1} & z_o^* & & \\ z_o & \mathbf{0}_{L-1} & \ddots & \\ & z_o & \ddots & z_o^* \\ & & \ddots & \mathbf{0}_{L-1} \\ \mathbf{O} & & & z_o \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} -jz_o^* & & & \mathbf{O} \\ \mathbf{0}_{L-1} & -jz_o^* & & \\ z_o & \mathbf{0}_{L-1} & \ddots & \\ & jz_o & \ddots & z_o^* \\ & & \ddots & \mathbf{0}_{L-1} \\ \mathbf{O} & & & jz_o \end{bmatrix} \tag{9}$$

are $M \times (M - L)$ banded Toeplitz matrices, with $z_o = e^{j\pi L u_o/2}$, and $\mathbf{0}_K$ the $K \times 1$ zero vector.

Exploiting the similarity in structure between $\mathbf{G}$ and $\mathbf{H}$ reveals that $\mathbf{s}$ and $\mathbf{d}$ share a set of $2L$ common weights in the following fashion:

$$\mathbf{s} = \begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_2 \end{bmatrix}; \qquad \mathbf{d} = \begin{bmatrix} -j\mathbf{r}_1 \\ \vdots \\ j\mathbf{r}_2 \end{bmatrix} \tag{10}$$

$$\mathbf{a}_K(u) = \left[e^{-j\pi\left(\frac{K-1}{2}\right)u}, e^{-j\pi\left(\frac{K-3}{2}\right)u}, \ldots, e^{-j\pi\left(\frac{K-3}{2}\right)u}, e^{-j\pi\left(\frac{K-1}{2}\right)u}\right]^T \tag{2}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are $L \times 1$ vectors. This represents a saving of $2L$ independent complex weights in the implementation of the beamformers.

B. Closed-Form Angle Estimator and Performance Analysis

Substituting (7) in (4) and taking the real part of both sides yields

$$\hat{u}_t = u_o + \frac{2}{\pi L}\tan^{-1}\{\mathrm{Re}\{\rho\}\}. \tag{11}$$

For a reasonably accurate angle estimate, ρ should be nearly a real scalar. This requires $|\xi \mathbf{s}^H \mathbf{a}_M(u_t)| \gg |\mathbf{s}^H \mathbf{n}|$ and $|\xi \mathbf{s}^H \mathbf{a}_M(u_t)| \gg |\mathbf{d}^H \mathbf{n}|$. In other words, the SNR at the array elements and the sum beamformer SNR gain must be moderately high.

Under Gaussian noise assumption and the conditions stated above, the mean and mean square of the estimation error $\delta u_t = \hat{u}_t - u_t$ are given approximately by [7]

$$E\{\delta u_t\} \approx -\frac{\bar{\rho}}{\pi L(1+\bar{\rho}^2)^2}\frac{\mathbf{b}^H \mathbf{R}_{nn}\mathbf{b}}{|\xi|^2|\mathbf{s}^H\mathbf{a}_M(u_t)|^2} \tag{12}$$

and

$$E\{(\delta u_t)^2\} \approx \frac{2}{\pi^2 L^2(1+\bar{\rho}^2)^2}\frac{\mathbf{b}^H \mathbf{R}_{nn}\mathbf{b}}{|\xi|^2|\mathbf{s}^H\mathbf{a}_M(u_t)|^2} + \frac{3\bar{\rho}^2}{\pi^2 L^2(1+\bar{\rho}^2)^4}\frac{|\mathbf{b}^H \mathbf{R}_{nn}\mathbf{b}|^2}{|\xi|^4|\mathbf{s}^H\mathbf{a}_M(u_t)|^4} \tag{13}$$

where $\bar{\rho} = \Phi(u_t)$, $\mathbf{b} = \mathbf{d} - \bar{\rho}\mathbf{s}$, and $\mathbf{R}_{nn} = E\{\mathbf{n}\mathbf{n}^H\}$ is the noise correlation matrix. The estimator is less biased for a small $\bar{\rho}$, or a small off-boresight angle $|u_t - u_o|$.

IV. Optimum Quiescent Beamformers

In the absence of any external interference, it is adequate to model the noise as spatially white, i.e., the components of $\mathbf{n}$ are zero-mean, uncorrelated complex Gaussian random variables with the same variance σ_n^2. In this case, $E\{\delta u_t\}$ and $E\{(\delta u_t)^2\}$ are obtained by substituting $\mathbf{R}_{nn} = \sigma_n^2 \mathbf{I}_M$ in (12) and (13), respectively, where $\mathbf{I}_M$ denotes the $M \times M$ identity matrix. Our goal here is to determine the sum and difference weight vectors that minimize $E\{(\delta u_t)^2\}$.

A. Minimum Variance and Efficiency of the Estimator

In constructing the optimum beamformers, it is assumed that the off-boresight angle is small, such that $E\{\delta u_t\} \approx 0$ and

$$E\{(\delta u_t)^2\} \approx \mathrm{Var}\ \{\hat{u}_t\} \approx \frac{2\sigma_n^2}{\pi^2 L^2 |\xi|^2}\frac{\mathbf{d}^H\mathbf{d}}{|\mathbf{s}^H\mathbf{a}_M(u_o)|^2} = \frac{2\sigma_n^2}{\pi^2 L^2 |\xi|^2}\frac{\mathbf{c}^H\mathbf{H}^H\mathbf{H}\mathbf{c}}{|\mathbf{c}^H\mathbf{G}^H\mathbf{a}_M(u_o)|^2} \tag{14}$$

where we have used the approximations $\bar{\rho} \approx 0$, $\mathbf{b} \approx \mathbf{d}$ and $\mathbf{a}_M(u_t) \approx \mathbf{a}_M(u_o)$.

Minimizing (14) with respect to $\mathbf{c}$ is a linearly constrained minimum variance (LCMV) problem [8] whose solution is given by

$$\mathbf{c} = \lambda\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{G}^H\mathbf{a}_M(u_o) \tag{15}$$

where λ is a normalizing scalar. Substituting (15) back into (14) yields the expression of the minimum variance:

$$\mathrm{Var}\ \{\hat{u}_t\}_{\min} \approx \frac{2\sigma_n^2}{\pi^2 L^2|\xi|^2}\frac{1}{\mathbf{a}_M^H(u_o)\mathbf{G}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{G}^H\mathbf{a}_M(u_o)}. \tag{16}$$

Some algebraic manipulations [7] show that $\mathrm{Var}\{\hat{u}_t\}_{\min}$ is independent of u_o, and in particular

$$\mathrm{Var}\{\hat{u}_t\}_{\min} \approx \frac{6\sigma_n^2}{\pi^2|\xi|^2 M(M^2-1)} \qquad \text{for } L = 1,$$

$$\mathrm{Var}\{\hat{u}_t\}_{\min} \approx \frac{8\sigma_n^2}{\pi^2|\xi|^2 M^3} \qquad \text{for } L = \frac{M}{2}. \tag{17}$$

Note that $L = 1$ corresponds to the case of maximum overlapping subarrays, and $L = M/2$ corresponds to the two-aperture method. For a large M, the latter is approximately 33% larger than the former. The values of $\mathrm{Var}\{\hat{u}_t\}_{\min}$ as a function of L are depicted in Fig. 2 for $M = 10, 20, 40$ and 80. For all cases, $\mathrm{Var}\{\hat{u}_t\}_{\min}$ appears to be an increasing function of L, though it is not monotonic. With a certain degree of confidence, we can say that $\mathrm{Var}\{\hat{u}_t\}_{\min}$ are bounded between by the two expressions in (17).

Interestingly, it is found that the minimum variance achieved with $L = 1$ is identical to the CRLB for $\hat{u}_t$ given $\mathbf{x}$ as data [9], [10]. Brennan [9] showed that under the high SNR, large M, and small pointing error assumptions, the CRLB can be nearly achieved with judiciously designed amplitude comparison and phase comparison monopulse estimators. On the other hand, the ML angle estimator [4] was shown to be asymptotically efficient in that the estimation variance approaches the CRLB as the SNR or M increases [10]. Similar results hold for the proposed estimator. As stated in Section III-B, the analysis was based on the assumption of moderately high SNR and sum beamformer SNR gain. Since the SNR gain is proportional to M, and inversely proportional to the pointing error, the analysis results will be more accurate as the SNR and M increase, and as the pointing error decreases. This in turn implies that the optimum angle estimator for $L = 1$ becomes efficient under the same three assumptions. The minimum variance achieved with the two-aperture method coincides with that of the phase comparison estimator using the uniform weighting [9]. As will be shown shortly, the optimum weights for the two-aperture method are indeed uniform. Numerical results demonstrate that the two-aperture estimator performs relatively well with a low SNR or a large off-boresight angle. This is the case for which the assumptions made in Section III-B do not hold.

B. Optimum Quiescent Weight Vectors

Substituting (9) and (15) into (8), along with some algebraic manipulations [7] yields the closed-form expressions of $\mathbf{s}_o$ and $\mathbf{d}_o$, the optimum quiescent weight vectors associated with $u_o = 0$ (the weight vectors for $u_o \neq 0$ are obtained via progressive phase shifting)

$$\mathbf{s}_o(k) = \mathbf{s}_o(M-k+1) = (M+1)(2k-1) - 2k^2,$$
$$\mathbf{d}_o(k) = -\mathbf{d}_o(M-k+1) = -j(M-2k+1),$$
$$k = 1, \ldots, \tfrac{M}{2} \tag{18}$$

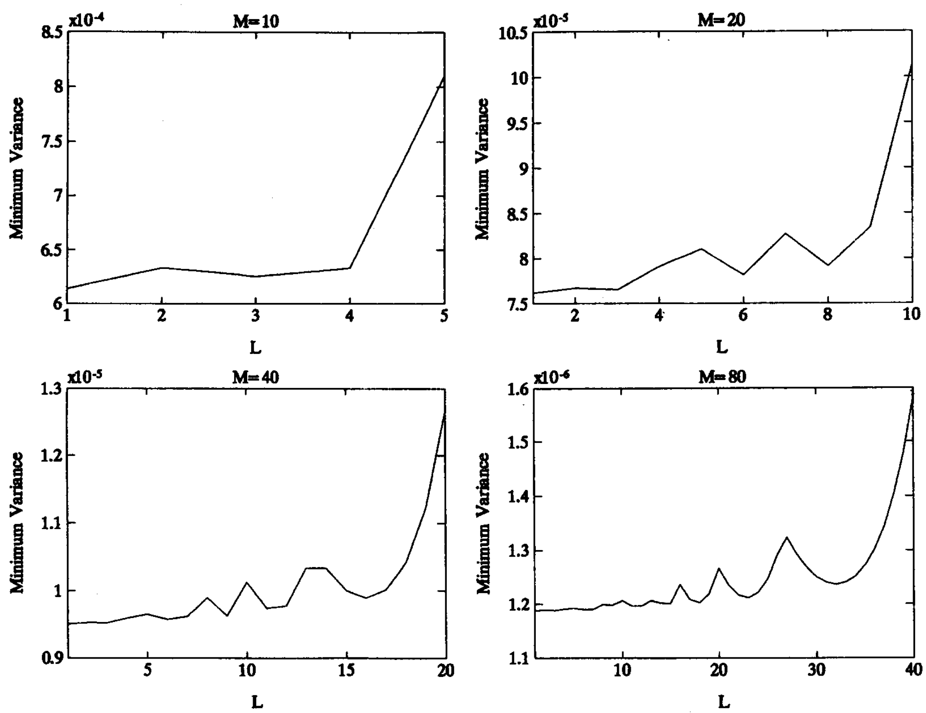

Fig. 2. Values of the minimum variance achieved with the optimum quiescent estimator for $M = 10, 20, 40$, and 80. The SNR was fixed at 0 dB.

for $L = 1$, and

$$\mathbf{s}_o(k) = \mathbf{s}_o(M - k + 1) = 1,$$

$$\mathbf{d}_o(k) = -\mathbf{d}_o(M - k + 1) = -j, \qquad k = 1, \ldots, \tfrac{M}{2} \quad (19)$$

for $L = M/2$, where $\mathbf{v}(k)$ denotes the kth component of the vector $\mathbf{v}$. It is noteworthy that the optimum difference weights for $L = 1$ exhibit the linear odd symmetry. Brennan [9] showed that the linear odd symmetric weighting yields the minimum angle estimation variance attaining the CRLB for the phase comparison monopulse estimator. Davis *et al.* [4] showed that the difference weights associated with the ML monopulse estimator under spatially white Gaussian noise exhibit the linear odd symmetry as well. In fact, the linear odd symmetric weighting yields the maximum normalized boresight slope for the difference pattern associated with a ULA with one-half wavelength spacing [11]. The optimum weights for the two-aperture method are uniformly distributed as expected. This is the best weighting method if the same set of weights are used for both the sum and difference channels.

V. Optimum Beamformers Incorporating Interference Cancellation

In the presence of strong external interferers, it is necessary to perform simultaneous nulling for the sum and difference beamformers. Simultaneous nulling can be performed in either the supervised or unsupervised way. In the supervised case, the interfering directions are first estimated via some kinds of off-line direction finding algorithms during the passive mode of the radar. The sum and difference beams are then formed accordingly to put hard nulls in these directions [5]. In the unsupervised case, the interfering signals are suppressed via some kinds of adaptive beamforming techniques, such as those based on the minimum power criterion [1] and the ML criterion [4]. We now derive the optimum beamformers for both cases.

A. Supervised Case

Let $\hat{u}_i, \quad i = 1, \ldots, J$, be the J estimated interfering directions. The execution of simultaneous nulling requires that a null be synthesized in each of the interfering directions for the common pattern $c(u)$. Denote as $\mathbf{e} = [e_1, e_2, \ldots, e_{J+1}]^T$ the weight vector associated with the constrained pattern factor $e(u) = \mathbf{e}^H \mathbf{a}_{J+1}(u)$ of $c(u)$ with J nulls at $\hat{u}_i$, $i = 1, \ldots, J$. Decomposing $\mathbf{c}$ in accordance with (8) gives

$$\mathbf{c} = \mathbf{E}\mathbf{f} \qquad (20)$$

where

$$\mathbf{E} = \begin{bmatrix} e_1 & & & \mathbf{O} \\ e_2 & e_1 & & \\ \vdots & e_2 & \ddots & \\ e_{J+1} & \vdots & \ddots & e_1 \\ & e_{J+1} & & e_2 \\ & & \ddots & \vdots \\ \mathbf{O} & & & e_{J+1} \end{bmatrix} \qquad (21)$$

is an $(M-L)\times(M-L-J)$ banded, Toeplitz matrices. The $(M-L-J)\times 1$ vector $\mathbf{f}$ corresponds to the unconstrained pattern factor $f(u)=\mathbf{f}^H\mathbf{a}_{M-L-J}(u)$ of $c(u)$. Thus $\mathbf{c}$ may be viewed as composed of a fixed part $\mathbf{E}$ and a free part $\mathbf{f}$. Assume that $\hat{u}_i, \quad i=1,\ldots,J$, are fairly accurate, such that the noise parts in the beamformer outputs are essentially free of any interfering signals and can be assumed to be spatially white. In this case, the optimization problem described previously remains unchanged in structure, but it is modified into that of minimizing

$$\mathrm{Var}\{\hat{u}_t\}\approx\frac{2\sigma_n^2}{\pi^2L^2|\xi|^2}\frac{\mathbf{f}^H\mathbf{E}^H\mathbf{H}^H\mathbf{H}\mathbf{E}\mathbf{f}}{|\mathbf{f}^H\mathbf{E}^H\mathbf{G}^H\mathbf{a}_M(u_o)|^2} \quad (22)$$

with respect to $\mathbf{f}$. Solving for $\mathbf{f}$ and using (20), we obtain the optimum $\mathbf{c}$ vector

$$\mathbf{c}=\lambda\mathbf{E}\left(\mathbf{E}^H\mathbf{H}^H\mathbf{H}\mathbf{E}\right)^{-1}\mathbf{E}^H\mathbf{G}^H\mathbf{a}_M(u_o) \quad (23)$$

Substituting (23) into (22) yields the minimum variance achieved with simultaneous nulling

$$\mathrm{Var}\{\hat{u}_t\}_{\min}\approx\frac{2\sigma_n^2}{\pi^2L^2|\xi|^2}\times\frac{1}{\mathbf{a}_M^H(u_o)\mathbf{G}\mathbf{E}\left(\mathbf{E}^H\mathbf{H}^H\mathbf{H}\mathbf{E}\right)^{-1}\mathbf{E}^H\mathbf{G}^H\mathbf{a}_M(u_o)} \quad (24)$$

B. Unsupervised Case

In this case, the original noise correlation matrix $\mathbf{R}_{nn}$ is used in (12) and (13). In practice, $\mathbf{R}_{nn}$ can be estimated by the sample data correlation matrix formed with a set of independent sample vectors collected during the passive mode of the radar. Following the procedure described in Section IV-A, we obtain the optimum weight vectors and minimum variance by replacing $\mathbf{H}^H\mathbf{H}$ with $\mathbf{H}^H\mathbf{R}_{nn}\mathbf{H}$ in (14)–(16). In order to minimize the estimation variance, the optimum beamformers will attempt to suppress the interfering power contained in $\mathbf{R}_{nn}$ by putting a deep null in each of the interfering directions. This is similar in principle to the minimum variance adaptive array schemes [8]. In particular, the proposed method so modified may be regarded as a variation of the Duvall's adaptive array [12].

VI. Simulations Results

Computer simulations were conducted to ascertain the performance of the proposed angle estimator. The ULA employed was composed of 20 identical elements spaced by one-half wavelength. The target angle was $u_t=0$. In the results, all sample statistics were obtained based on 500 independent runs, and the SNR in dB was defined at element level as $10\log_{10}(|\xi|^2/\sigma_n^2)$. For brevity, we show only the results associated with $L=1$, 5, and 10.

The first set of simulations examines the performance of the optimum quiescent angle estimator under spatially white Gaussian noise. The optimum beamformers constructed in Section IV-A were used for all cases, and the corresponding patterns are plotted in Fig. 3 for $L=5$. Note that the sum and difference patterns exhibit 14 common nulls corresponding to

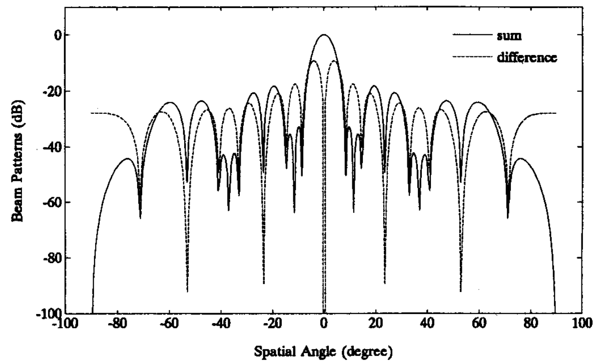

Fig. 3. Sum and difference patterns associated with the optimum quiescent beamformers. $M=20$, $L=5$, $u_o=0$.

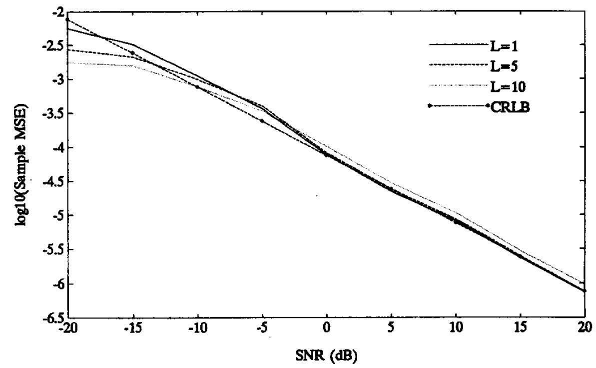

Fig. 4. Sample mean square error of the optimum quiescent angle estimator for several SNR values. $M=20$, $u_o=u_t=0$.

$c(u)$. Fig. 4 shows the sample mean square error (MSE) of $\hat{u}_t$ for several SNR values. The corresponding CRLB was also plotted for reference. As expected, the angle estimates were more accurate as the SNR was increased. The sample MSE decreased as L was decreased, attaining the CRLB for $L=1$ with SNR >-5 dB. This confirms the analysis results for the optimum estimator. At low SNR, the sample MSE behaved oppositely as a decreasing function of L, being lower than the CRLB for the extreme case of SNR <-10 dB. An evaluation of the corresponding sample means in this case indicates that the estimator was biased.

The second set of simulations examines the effect of pointing error on the optimum quiescent estimator. In this case, the boresight angle of the optimum beamformers was varied from $-5°$ to $5°$, and the SNR was fixed at 10 dB. Note that the 3-dB beamwidth of the array is approximately 2.9°. Figs. 5 and 6 show the sample MSE of $\hat{u}_t$ and the corresponding theoretical values from analysis, respectively. We observe that the simulation results are consistent with the analysis results for a broad range of boresight angles. For a small off-boresight angle, the estimator performed better with a small L. For a large off-boresight angle, the opposite was true. In particular, the sample MSE achieved with the two-aperture estimator

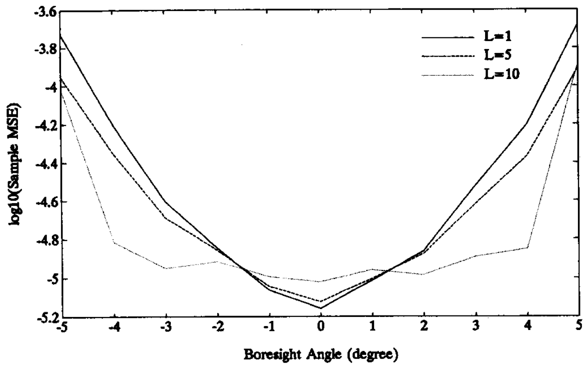

Fig. 5. Sample mean square error of the optimum quiescent angle estimator for several off-boresight angles. $M = 20$. The SNR was fixed at 10 dB.

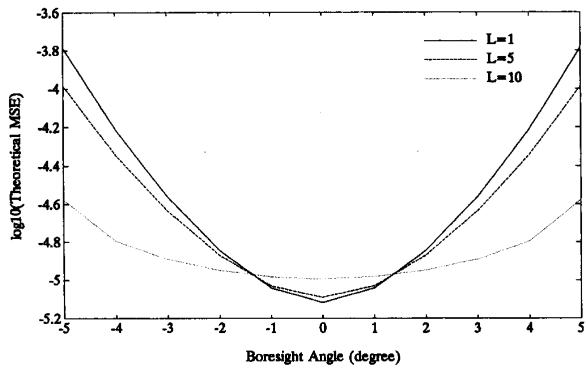

Fig. 6. Theoretical mean square error of the optimum quiescent angle estimator for several off-boresight angles. $M = 20$. The SNR was fixed at 10 dB.

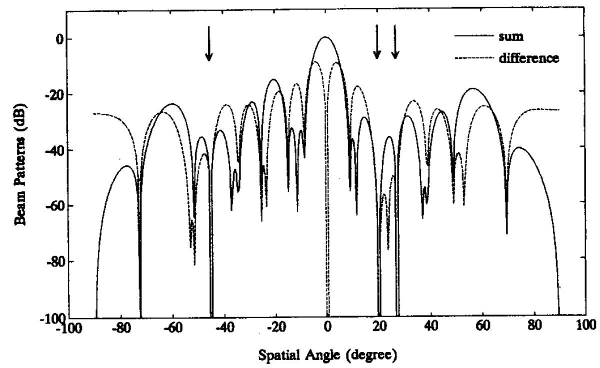

Fig. 7. Sum and difference patterns associated with the optimum beamformers incorporating supervised interference cancellation. $M = 20$, $L = 5$, $u_o = 0$. The interfering directions are $-45°$, $20°$, and $27°$.

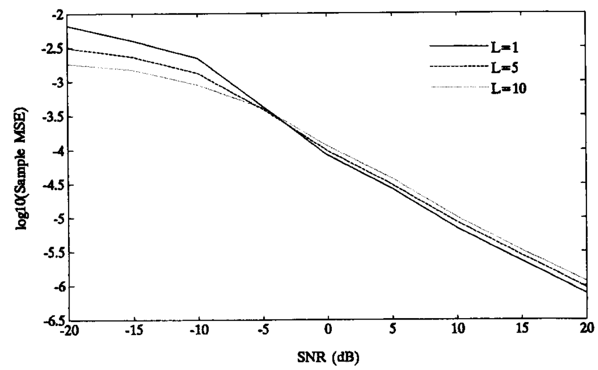

Fig. 8. Sample mean square error of the optimum angle estimator incorporating supervised interference cancellation for several SNR values. $M = 20$, $u_o = u_t = 0$. The interfering directions are $-45°$, $20°$, and $27°$.

was significantly smaller than the other two over the range $2° \leq |u_t - u_o| \leq 4°$.

Summarizing at this point, it is found that the optimum estimator improves as L is decreased under good estimation conditions, while the opposite is true under poor estimation conditions. This serves as a criterion for choosing the appropriate subarray size in constructing the beamformers. For example, the two-aperture estimator may be used in the preliminary stage, in which a coarse angle estimate is obtained. The subarray size is then increased as the boresight angle approaches the target angle. When the radar nearly boresights the target, it should switch to the estimator using the maximum overlapping subarrays to exploit its efficiency.

The third set of simulations examines the performance of the optimum angle estimator incorporating interference cancellation. Three Gaussian jammers were assumed at $\theta_1 = -45°$, $\theta_2 = 20°$, and $\theta_3 = 27°$, with power levels equal to 20 dB, 10 dB and 10 dB, respectively, relative to the echo power. First, the optimum supervised beamformers constructed in Section V-A were used, and the corresponding patterns are plotted in Fig. 7 for $L = 5$. We assumed that the estimated interfering directions were correct. Note that although the sidelobes were significantly modified within the interfered regions, the general pattern shapes were not changed much compared to the quiescent ones. This confirms the efficacy of the minimum variance objective function in preserving the desired pattern shapes. Fig. 8 shows the sample MSE of $\hat{u}_t$ for several SNR values. The results are observed to be similar in trend to those shown in Fig. 4. Second, the optimum unsupervised beamformers described in Section V-B was used. For evaluation purposes, the true noise correlation matrix was used. The resulting patterns obtained with SNR = 10 dB are plotted in Fig. 9. They look quite similar to those shown in Fig. 7, except that the nulls at the interfering directions are less defined. As in the supervised case, we found that the sample MSE's shown in Fig. 10 are comparable to those obtained with the quiescent estimator. These results demonstrate that the minimum variance property was well retained in the process of interference cancellation.

VII. Conclusion

A simple method of monopulse target localization using a ULA has been proposed. A closed-form angle estimator was obtained with judiciously chosen beamformers constructed

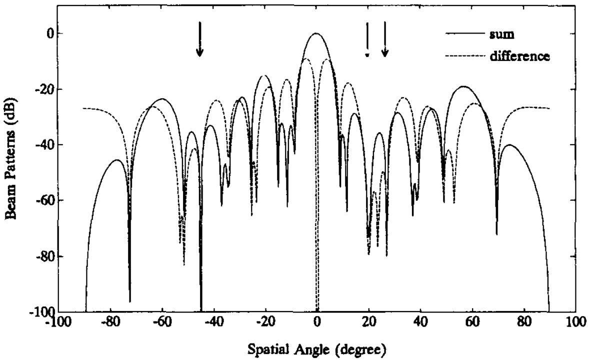

Fig. 9. Sum and difference patterns associated with the optimum beamformers incorporating unsupervised interference cancellation. $M = 20$, $L = 5$, $u_o = 0$. SNR = 10 dB. The interfering directions are $-45°$, $20°$, and $27°$.

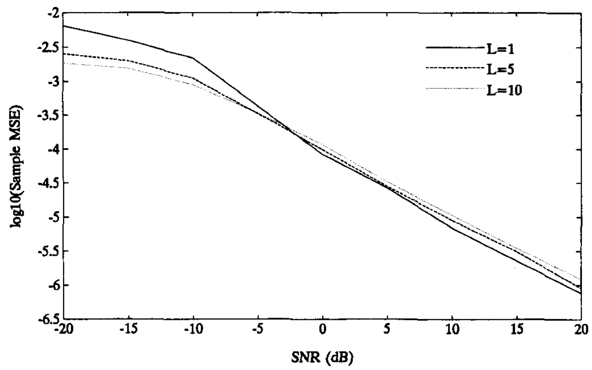

Fig. 10. Sample mean square error of the optimum angle estimator incorporating unsupervised interference cancellation for several SNR values. $M = 20$, $u_o = u_t = 0$. The interfering directions are $-45°$, $20°$, and $27°$.

with two overlapping subarrays. The optimum beamformers that achieve the minimum variance of the estimator were derived for the Gaussian noise case. Supervised/unsupervised simultaneous nulling was incorporated to retain the performance of the angle estimators under strong external interference. Analysis results and computer simulations confirm that the optimum estimator using the maximum overlapping subarrays exhibits the following merits: 1) It attains the CRLB with a moderately high SNR and a small off-boresight angle, and 2) it provides the largest degree of freedom for adaptive nulling. The two-aperture method, on the other hand, exhibits robustness under poor estimation conditions in that it produces the smallest mean square error 1) with a low SNR and a small off-boresight angle, and 2) with a moderately high SNR and a large off-boresight angle. The results presented in the paper may be used as criteria for choosing the right subarray size in target localization.

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