ELSEVIER

15 October 1997

Optics Communications 142 (1997) 229–233

OPTICS
COMMUNICATIONS

Spontaneous emission factor versus cavity volume in low dimensional photonic micro-cavities

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Received 27 November 1996; revised 22 May 1997; accepted 29 May 1997

Abstract

Micro-ring and micro-disk lasers are formed at the same time by transferring the epitaxial layers of 1.55 μm wavelength InGaAs/InGaAsP quantum wells on a glass substrate. The spontaneous emission factor β in such micro-rings of 5 μm diameter and $0.2 \times 0.5\ \mu m^2$ cross-section is 0.16, which is four times that in micro-disk lasers of the same diameter. The result confirms for the first time that the micro-ring being a lower dimensional photonic system has a larger β. Study of β versus the cavity size indicates that the observed spontaneous emission factor is approximately the reciprocal of the normalized cavity volume, when normalization is done to the cubic of the wavelength in the medium. © 1997 Elsevier Science B.V.

PACS: 42.60.Da; 42.50.Lc

Dielectric confinement in wavelength-scale microstructures can reduce the dimensionality of the density states of light. The vacuum field fluctuations being inversely proportional to the square root of the mode volume can also be greatly enhanced when the dimension of resonators approaches the optical wavelength. These effects have led to interesting cavity quantum electrodynamic phenomena of atomic systems [1], of semiconductor systems [2], and of novel photonic bandgap structures [3]. Along this line of thinking, semiconductor micro-disk [4,5] and micro-ring lasers [6] were also recently demonstrated. Micro-disk and micro-ring cavities can be regarded as photonic quantum wells and wires respectively due to their tightly one- or two-dimensionally confined optical mode area. In these photonic low dimensional systems, a large portion of the spontaneously emitted energy is channeled into the cavity mode instead of the radiation modes to yield a high spontaneous emission factor β. Thus these lasers are useful for ultra low threshold laser applications [7]. Chin, Chu and Ho [8], and Chu and Ho [9] have modeled the spontaneous emission in micro-disk and micro-ring cavities respectively. It can be concluded that a micro-ring, being a lower dimensional confined photonic system, has a larger β value than a micro-disk. However, in previous reports of experimental investigation, micro-disk and micro-ring lasers have never been studied in parallel and the direct determination of their β values from the measured data has not been done systematically. In this Letter we report our experimental investigation of semiconductor micro-disk and micro-ring lasers. We developed an epitaxial transfer method to allow the rings and disks to form on a glass substrate simultaneously. One of our goals is the direct comparison of the β values for both systems. By keeping the experimental parameters and conditions the same, except the width of the ring, we confirm for the first time that indeed the micro-ring has a larger β than the micro-disk. We also studied the dependence of β versus the cavity volume. The result indicates that the β value is approximately the reciprocal of the normalized cavity volume. The normalization is done to the cubic of the wavelength in the medium λ/n, which is the free

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PII S0030-4018(97)00294-0

space wavelength divided by the refractive index n of the medium that forms the cavity.

The waveguide structure of our micro-disk or micro-ring contains three layers of 1.55 μm wavelength InGaAs quantum wells, sandwiched between four InGaAsP barriers. The InGaAs wells and the central two InGaAsP barriers are 100 Å thick. The thicknesses of the top and bottom InGaAsP barriers are 750 Å. The metal organic vapor phase deposition method was used to grow the quantum wells on the InP substrate with a total thickness of 0.2 μm. In order to form the optically confined structure, the high index epitaxial layer needs to be surrounded by a low index environment. Photo-lithography technique and wet chemical etching in $HCl:H_2O_2:HBr:H_2O$ were first applied to form the ring and disk patterns on the InP substrate. The wafer was attached epitaxy-side down to the sapphire substrate by ultra-violet cured epoxy of index 1.5. Then, the InP substrate was removed by selective chemical etching in HCl. The rings and disks are now surrounded by a low index environment and become photonic confined micro-cavities. The 0.2 μm waveguide thickness is about 0.33 λ/n_{eff}. An effective index n_{eff} of 2.5 is used in our estimation [4,5]. According to the calculation by Ho et al. [7] made for the planar waveguide structure, the vertical dipole emission from these structures will be strongly suppressed when the thickness falls between 0.2 to 0.4 λ/n_{eff}. For a 5 μm diameter micro-disk, the active material volume and the cavity volume are only 0.58 and 3.9 μm^3, respectively. The volume of the micro-disk is about 28 times the cubic of the wavelength in the medium. The cavity volume for micro-rings is reduced, when comparing to the same diameter micro-disk, by a factor $2w/r$, where w is the width and r is the radius. For a 5 μm diameter ring with 0.5 μm width, the normalized cavity volume is 10. At this width, the ring waveguide supports only one mode along the radial direction. It is often conjectured that the spontaneous emission factor in these strongly optical confined structures is roughly the reciprocal of the normalized cavity volume. Thus, the above mentioned micro-disk and micro-ring are expected to have a high value of β.

In the experiment, we use a 632.8 nm helium neon laser as the pump source. The pump was normal incident to the plane of the rings and its intensity was scanned by an acoustic optical modulator. The light emission was collected at an angle about 80° from surface normal of the substrate. According to McCall et al. [4], the emission of the micro-disk lasers is peaked at $\theta = 90°$, taking the surface normal being parallel to $\theta = 0°$ in a spherical coordinate. The width of the emission pattern along the θ direction is about $2/\sqrt{m}$, where the azimuthal angular mode number m is defined as $2\pi r n_{\mathrm{eff}}/\lambda$. A similar spatial emission distribution is expected for the micro-ring because the light transmission of the two structures is due to the same photon tunneling mechanism [4,9]. When the disk or ring diameter is 5 μm, m is 25 and the emission angle is 23°. Therefore our optical arrangement allows direct collection of lasing mode emission. We employed also a spatial and spectral filter in the measurement of the laser output versus the pump input curve. This is because the derivation of the spontaneous emission factor β relies on a single mode rate equation analysis. Spatial filtering was facilitated by an aperture and spectral filtering was via a spectrometer by adjusting the width of the slits to the cavity mode width. Before measuring the lasing versus pumping curve by the spectral filter, emission spectra for pumping near and above the threshold have been measured. From these data, we can estimate the proper spectral window to cover the effect of linewidth change and line center shift and to exclude unwanted spontaneous light. In our case, a spectral window of 10 Å is used to insure that the measurement is done for a single mode both above and below threshold pumping.

Fig. 1 plots a typical lasing output as a function of the pumping power for a 5 μm diameter and 0.5 μm wide micro-ring at 77 K and cw condition. The threshold behavior is not evident, indicating the spontaneous emission factor is very large. The emission spectra at different pumping levels are shown in Fig. 2. Unlike the power output versus pumping characteristics, optical spectra show clearly the onset of single mode lasing. We have confirmed that the lasing linewidth just below the threshold is about 1.0 nm and is reduced to 0.3 nm at 4 times the threshold value. Figs. 3 and 4 are the corresponding data of Figs. 1 and 2 for a 5 μm diameter micro-disk, located within several mm from the ring in the same substrate. The emission spectra show a similar scenario when the pump increases. But the disk exhibits a much sharper threshold behavior in the light output versus pumping curve, indicating the disk has smaller β. The exact wavelength of the single mode emission peak varies from device to device

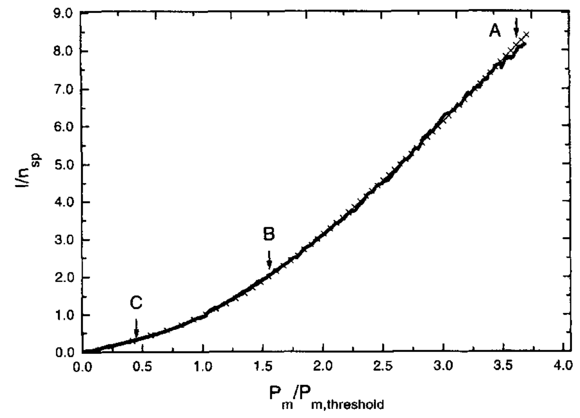

Fig. 1. Typical lasing output versus pump power characteristics for a micro-ring of 5 μm diameter and 0.5 μm wide ring. The vertical axis is the cavity photon number in units of n_{sp}. The horizontal axis is plotted in units of the threshold pumping value. Dots are data and the curve with ×-marks is from fitting, where β equal 0.16 is used.

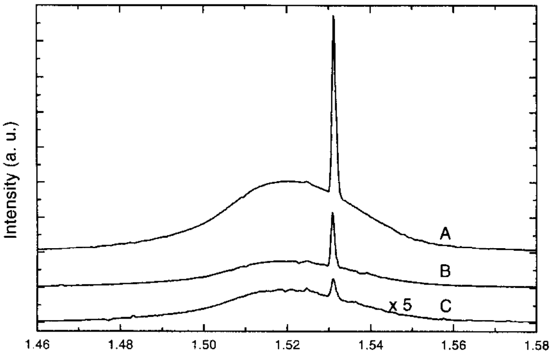

Fig. 2. Emission spectra of the micro-ring shown in Fig. 1 at 3.5 (A), 1.6 (B) and 0.5 (C) times the threshold pumping, respectively.

and is seen to cover the entire spectral bandwidth of the bulk spontaneous emission spectrum. The variation is due to the size variation. Specifically one percent difference in diameter will give rise to 15 nm shift in wavelength. From the measured linewidth just below the threshold (about 1.0 nm for both systems), we estimate that the Q factors for both micro-disk and micro-ring cavities with a diameter of 5 μm are about the same value, $Q \approx \lambda/\Delta\lambda \approx 1500$. This value is a little larger than the values reported in the literature (Q below 1000). Currently we are not sure what causes the difference. However, the fact that both ring and disk systems have a similar Q value indicates that the interior etched interface does not reduce the Q factor significantly.

In deducing β, we analyzed the lasing output versus the pumping characteristics based on the single mode rate equations [10],

$$\mathrm{d}I/\mathrm{d}t = -\gamma I + GI + R, \tag{1}$$

$$\mathrm{d}N/\mathrm{d}t = P - N\gamma_{\mathrm{sp}} - GI, \tag{2}$$

where I is the total photon number in the cavity, N is the total number of carriers, P is the pumping rate of the carriers, γ is the cavity photon decay rate, γ_{sp} is the spontaneous decay rate of carriers, $R = \beta N\gamma_{\mathrm{sp}}$ is the spontaneous emission rate of the lasing mode and $G = R/n_{\mathrm{sp}}$ is the stimulated emission rate. Here n_{sp} is the inversion factor. The photon number I is related to the measured lasing power P_{L} by $I = f_{\mathrm{L}} P_{\mathrm{L}}$, where the constant factor f_{L} accounts for finite collection efficiency and cavity transmission rate. The carrier pumping rate is related to the pumping power P_{m} by $P = f_{\mathrm{m}} P_{\mathrm{m}}$, where the constant factor f_{m} accounts for the actual absorption and other trivial conversion parameters. From the above equations, one can show that the relation between the measured pumping power P_{m} and the measured lasing power P_{L} is given by

$$P_{\mathrm{m}} = (c_1/\beta)\, P_{\mathrm{L}}(1 + \beta c_2 P_{\mathrm{L}})/(1 + c_2 P_{\mathrm{L}}), \tag{3}$$

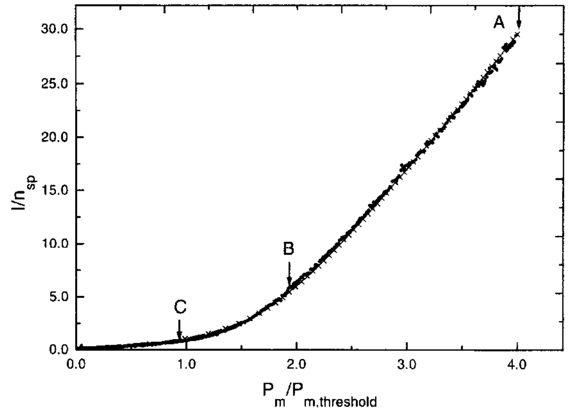

Fig. 3. Typical lasing power versus pump power characteristics for a micro-disk of 5 μm diameter. Dots are data and the curve with ×-marks is from fitting.

where $c_1 = \gamma f_{\mathrm{L}}/f_{\mathrm{m}}$, $c_2 = f_{\mathrm{L}}/n_{\mathrm{sp}}$. One can see that although in general we have many unknown parameters, it is very fortunate that there are only three independent degrees of freedom in the P_{L}–P_{m} relation and β is one of them. Because of this, we can use Eq. (3) to fit the measured P_{L}–P_{m} curve and determine the value of β unambiguously.

Theoretical P_{L}–P_{m} curves for the ring and disk are superimposed in Figs. 1 and 3. The vertical axes in Figs. 1 and 3 are actually the photon number in units of n_{sp}, which is simply $c_2 P_{\mathrm{L}}$. The advantage of this vertical scale is in the determination of the lasing threshold defined here as when the net stimulation emission rate equals the spontaneous emission rate. From Einstein's relation, this happens when $I/n_{\mathrm{sp}} = 1$. Excellent fitting is found in Figs. 1 and 3, where the pumping levels are up to four times the threshold value. β is deduced to be 0.16 and 0.04 for ring and disk, respectively. The ratio of β in these two structures is four. The observed β values are to be compared

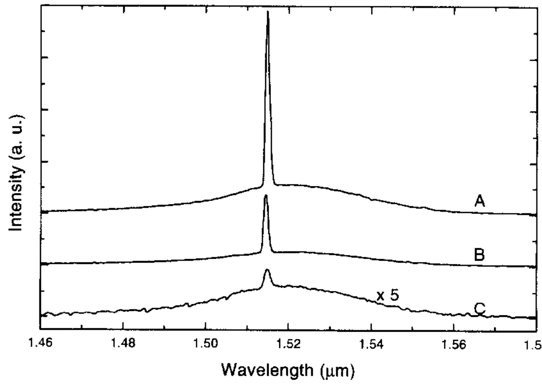

Fig. 4. Emission spectra of the micro-disk shown in Fig. 3, at 4.0 (A), 1.9 (B) and 0.9 (C) times the threshold pumping, respectively.

with the reciprocal of the normalized cavity volume, 0.1 and 0.035.

In the above modeling we have assumed a constant n_{sp}. In semiconductors, n_{sp} is expressed as $n/(n-n_{tr})$, where n_{tr} is the transparent carrier density. It is negative when n is smaller than n_{tr}, and approaches 1 when the system is highly inverted. In our case, the photon number at the transparent carrier density is defined as $\beta V_{gain,total} n_{tr} \gamma_{sp}/\gamma$ and its value is less than one. This justifies the use of a constant value for n_{sp} in the analysis. In other words, the threshold carrier density is larger than n_{tr}. That the photon number at the transparent carrier density being less than one implies lasing without inversion [11] does not occur. Excellent fitting quality further confides the validity of the assumption. With the inclusion of n_{tr} in the analysis and the use of a non-constant n_{sp}, the analysis may yield a slightly larger value of β than we have obtained [7,12].

As has been mentioned, spectral filtering has been used to insure the measurement is done for a single mode both above and below threshold. However, a small amount of undesired spontaneous emissions within the spectral window still can enter the measurement and cause some uncertainties. The net power of these undesired spontaneous emissions is expected to be linearly proportional to the pumping power and thus will result in an overestimate of the β value. Quantitatively, we have confirmed that if 10% of the measured power at the threshold comes from such undesired contributions, then the β value is roughly overestimated by 10%. In reality it is difficult to accurately decide the ratio of these undesired spontaneous emissions. However, since we have reduced the spectral window to the allowed minimum value and all the measurements are performed and analyzed in the same way, we expect such undesired spontaneous emissions will not affect the main conclusions presented in this Letter.

Measurements have also been made for other rings with the same diameter but larger width. Excellent fitting quality was obtained for all the measurements. Fig. 5 summarizes the β values deduced from 12 devices of 5 μm diameter, plotted as a function of the width w. It is evident that β is inversely proportional to the width (and thus volume), indicated by the $1/w$ solid curve.

As mentioned, β is found to be about the reciprocal of the normalized cavity volume. We now give a physical account for this observation. The total spontaneous emission rate into the lasing mode can be expressed as

$$\gamma_c = \frac{\hbar\omega}{2\epsilon V_{mode}} \frac{2\pi p^2}{\hbar^2} \rho_c(\omega) \frac{\Delta\nu_c}{\Delta\nu_{sp}} \frac{V_{gain,eff}}{V_{gain,total}}$$

$$= \frac{2\pi}{\hbar} \frac{p^2\omega}{2\epsilon V_{cavity}} \rho_c(\omega) \frac{\Delta\nu_c}{\Delta\nu_{sp}}. \qquad (4)$$

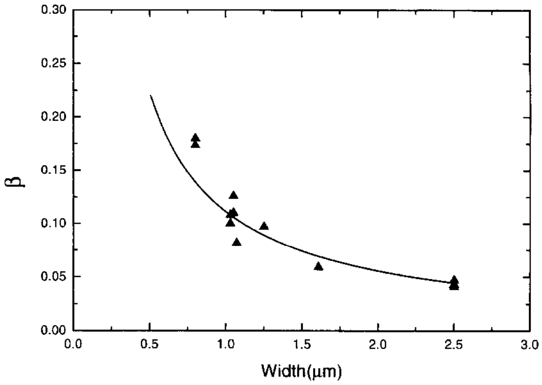

Fig. 5. Plot of the spontaneous emission factor versus the width for 5 μm diameter micro-rings. Note that when the width equals 2.5 μm, the ring becomes a disk.

The first term in the middle expression stands for the square of the vacuum field and is proportional to the inverse of the mode volume of the lasing mode. p is the dipole moment and $\rho_c = 2\pi r/(2\pi)\mathrm{d}k/\mathrm{d}\omega$ is the density of states for the 1-d ring. The spectral factor $\Delta\nu_c/\Delta\nu_{sp}$ is needed to account for the overlap factor of the spontaneous emission linewidth $\Delta\nu_{sp}$ and the cavity mode linewidth $\Delta\nu_c$. Because the interior portion of the gain medium which is not overlapped with the lasing mode cannot emit photons into the lasing mode either, the net spontaneous emission rate into the lasing mode is also reduced by a filling factor $V_{gain,eff}/V_{gain,total}$, under the assumption that the spontaneous emission lifetime of carriers in the microdisk and micro-ring is approximately the same as the that in the bulk [8,9]. Here $V_{gain,eff}$ is the effective gain volume that can interact with the lasing mode and $V_{gain,total}$ is the total gain volume. The key points here are that: (i) carrier diffusion lengths can be the order of microns so that gain is shared in the whole gain medium; and (ii) the beta value determined from the measured *L–I* curve is not the beta value for carriers at a particular position but the averaged beta value for all carriers in the whole gain medium. After combining all the factors, γ_c is found to be proportional to the inverse of the cavity volume instead of the mode volume, as is already shown in Eq. (4). A simple estimate of β can be obtained by taking the ratio of γ_c with the bulk spontaneous emission rate $\gamma_b = n\omega^3 p^2/(3\pi\hbar\epsilon_0 c^3)$. After some algebra, we get

$$\beta = \frac{3m}{8\pi} \frac{\Delta\nu_c}{\Delta\nu_{sp}} \frac{(\lambda/n)^3}{V_{cavity}}. \qquad (5)$$

The pre-factor $3m\Delta\nu_c/8\pi\Delta\nu_{sp}$ is 0.44 in our experiment. The agreement is good in view of the simplicity of the argument.

In conclusion, we have succeeded in demonstrating the operation of micro-ring and micro-disk lasers formed at the same time by transferring the epitaxial layers of 1.5 μm wavelength InGaAs/InGaAsP quantum well structure on a glass substrate. We confirm for the first time that the micro-ring being a lower dimension photonic system has a larger β than the micro-disk of the same diameter. We also find that the β value in strongly confined optical systems is scaled with the reciprocal of the cavity volume to the cubic of the mode wavelength with the scaling factor of order unity.

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