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Stability of an Umbilical Director Field of Nematic Liquid Crystal in a Closed Conical Cavity

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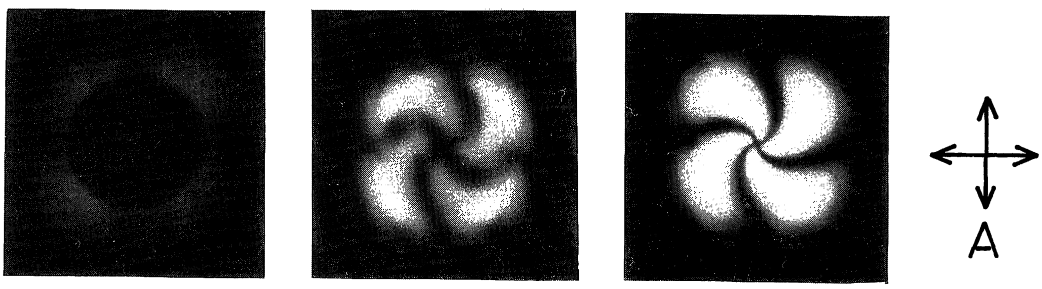
This letter studies the stability of a single umbilical director field of nematic liquid crystal (NLC) in a closed cavity with external electric field. To form the single umbilic structure, we filled NLC with negative dielectric anisotropy in a closed conical cavity and made molecules align perpendicular to the cavity wall. Electric field was applied along the symmetry axis of the cavity. Using the continuum theory, we show that an umbilical director field will transform to a director field of a hyperbolic singular point with disclination ring instead of a Poincaré structure at a critical voltage.

KEYWORDS: nematic liquid crystal, stability, director field, closed cavity, umbilic, Poincaré, disclination, continuum theory

Since polymer-dispersed liquid crystal (PDLC) can be employed as a material for displays and light shutters, the characteristics of nematic droplets dispersed in the polymer are currently eliciting much interest. Especially, the stability of confined nematic liquid crystal (NLC) has become the object of active research. The director field in a closed cavity is determined by the geometry of the cavity, boundary conditions, elastic forces of liquid crystal, and external field. Because the droplets in PDLC often possess spherical geometry, the nematic director field in a spherical cavity has been investigated thoroughly.[1–10] For a perpendicular boundary condition, the director fields found in the spherical cavity are the radial hedgehog and the axial structure with or without disclination ring (depending on the surface-anchoring strength). Other interesting studies deal with the closed cylinder of liquid crystal (CCLC).[11–15] Except for the nematic director field in a spherical cavity, the hyperbolic hedgehog and disclination ring were found in CCLC under a perpendicular boundary condition. After an external field is applied to the cavity, the above director fields can transform between themselves, leading to the production of twisted structures.[9,15] We note that the umbilical director field, which occurs in a closed cavity, has not been reported so far. In fact, in a perpendicularly aligned NLC thin film with negative dielectric anisotropy, the umbilic structure[16] was found when an electric field beyond the threshold of Freedericksz transition was applied parallel to the directors. Based on the topology, however, pairs of different kinds of umbilic structures must exist in the NLC film. Moreover, it was predicted that[17] higher electric field might transform the nonsingular umbilic structure to the Poincare structure which contains a disclination line of strength +1 ending with an unlike pair of 1/2 singular points. We are the first to report on the *single umbilical* director field structure of NLC in a *closed cavity*. The initiated single umbilic structure is not induced by the electric field, but by the geometry of the cavity and the boundary condition. We demonstrate that if the electric field is applied in a closed conical cavity, the umbilical director field will transform to a hyperbolic hedgehog with a disclination ring structure instead of a Poincaré structure at a critical voltage.

In our experiment, the sample was prepared using two glass plates coated with indium-tin-oxide (ITO) for transparent electrodes and a film of silicon oxide (SiO) with conical holes sandwiched between ITO. The height of the closed cavity is 4.7 μm. The radii of the upper base and lower base are 19.61 and 35.95 μm, respectively. To make the NLC have perpendicular boundary condition in the cavity, we coated the walls with N,N-dimethyl-N-octadecyl-3-aminopropyltrimethoxysilyl-chloride (DMOAP). The NLC used was ZLI-14627 whose dielectric anisotropy $\Delta\varepsilon=-3.52$. The sinusoidal electric field with 1 kHz was applied along the symmetry axis of the closed conical cavity. Then we placed the sample in a Leitz polarizing microscope to observe the optical patterns from a top view of the conical cell. Figures 1(a)–1(c) show the textures at different applied voltages. Since the behavior of the tex-



(a) (b) (c)

Fig. 1. The optical patterns of NLC in a closed conical cavity viewed between crossed polarizers: (a) $V_0=0$ V, (b) $V_0=1.77$ V, and (c) $V_0=4.0$ V. $R_U=19.61$ μm, $R_L=35.95$ μm, and $D=4.7$ μm.

tures was not changed when we rotated the sample between the crossed polarizers, we concluded that the director fields of the NLC were cylindrically symmetrical. Furthermore, we observed that the twisted textures could be a right-handed twist or a left-handed one at the same voltage, indicating that the right-handed and left-handed twists of the director configuration are degenerate in the closed cavity. It is also found that in the lower voltage regime, the width of the dark brushes is almost the same from the center to the edge, and as voltage increases, the width of dark brushes near the center becomes smaller and more twisted. The optical patterns reveal that the projecting configuration of the director field on the plane perpendicular to the symmetry axis of the cavity is a twisted radial type with a singularity at the center of the plane.

Because it is too difficult to identify the NLC director field by visual observation of the optical pattern from the side view of the conical cavity, we use theoretical studies to explain those optical patterns. From the elastic continuum theory, the total free energy of a confined liquid crystal is expressed as

$$F=\frac{1}{2}\int_{\text{vol}}[K_{11}(\nabla\cdot\boldsymbol{n})^2+K_{22}(\boldsymbol{n}\cdot\nabla\times\boldsymbol{n})^2+K_{33}(\boldsymbol{n}\times\nabla\times\boldsymbol{n})^2 -\varepsilon_{\perp}\varepsilon_0|\boldsymbol{E}|^2-\Delta\varepsilon\varepsilon_0(\boldsymbol{n}\cdot\boldsymbol{E})^2]\,\mathrm{d}v \tag{1}$$

where K_{11}, K_{22}, and K_{33} are the splay, twist, and bend bulk elastic constants, respectively. $\varepsilon_{\perp}$, $\Delta\varepsilon$ and ε_0 are the dielectric constants perpendicular to the director, the dielectric anisotropy, and the permittivity of free space, respectively. For the twisted textures observed in experiment, the nematic director $\boldsymbol{n}$ is specified in cylindrical coordinates by $\boldsymbol{n}(r, z)=\boldsymbol{i}_r\sin\theta(r, z)\cos\phi(r, z)+\boldsymbol{i}_{\varphi}\sin\theta(r, z)\sin\phi(r, z)+\boldsymbol{i}_z\cos\theta(r, z)$, where θ is the angle between $\boldsymbol{n}$ and the symmetry axis $\boldsymbol{i}_z$ and ϕ is the angle between $\boldsymbol{i}_r$ and the projection of $\boldsymbol{n}$ on $\boldsymbol{i}_r-\boldsymbol{i}_{\varphi}$ plane. $\boldsymbol{E}$ is the electric field. Here we have assumed a strong anchoring condition on the boundary and neglected the elastic energy induced by saddle-splay elastic constant K_{24}. Minimizing the total free energy by the variational method, we can obtain the Euler-Lagrange equation as follows:[15]

$$\Theta\left(r, \theta, \phi, \frac{\partial\theta}{\partial r}, \frac{\partial\theta}{\partial z}, \frac{\partial\phi}{\partial r}, \frac{\partial\phi}{\partial z}, \frac{\partial^2\theta}{\partial r^2}, \frac{\partial^2\theta}{\partial z^2}, E_r, E_z\right)=0 \tag{2}$$

$$\Phi\left(r, \theta, \phi, \frac{\partial\theta}{\partial r}, \frac{\partial\theta}{\partial z}, \frac{\partial\phi}{\partial r}, \frac{\partial\phi}{\partial z}, \frac{\partial^2\theta}{\partial r^2}, \frac{\partial^2\theta}{\partial z^2}, E_r, E_z\right)=0. \tag{3}$$

Functions Θ and Φ contain too many terms to express in detail. The electric field $\boldsymbol{E}(r, z)$ of an applied voltage V_0 is obtained by following equations:

$$\nabla\cdot[\boldsymbol{\varepsilon}(r, z)\varepsilon_0\nabla V(r, z)]=0 \tag{4}$$

$$\boldsymbol{E}(r, z)=-\nabla V(r, z) \tag{5}$$

where $\boldsymbol{\varepsilon}(r, z)$ is the dielectric constant tensor and V is the electric potential. We have set θ, ϕ, ε, $\boldsymbol{E}$, and V have cylindrical symmetry in eqs. (2)–(5), i.e., they are only functions of r and z.

We use the relaxation method to solve three kinds of director fields achievable in a closed conical cavity whose height is D, radius of upper base $R_U=D$, and radius of lower base $R_L=2D$. These director fields have the umbilic structure, hyperbolic singular point structure, and Poincaré structure. Figure 2 shows the dependence of free energy F of these structures on applied voltage V_0 in a typical closed conical cavity. The twisted structures are obtained by setting initial condition $\phi(r, z)\neq 0$. Here, we have added the isotropic core energy of disclination ring with radius D to the hyperbolic singular point structure and the isotropic core energy of a segment of disclination line with strength $+1$ to the Poincaré structure. The directors on the walls are always perpendicular to the walls. The constants for NLC used are $K_{11}=6\times10^{-12}$ N, $K_{22}=4\times10^{-12}$ N, $K_{33}=13.5\times10^{-12}$ N, $\varepsilon_{\perp}=7.35$, $\Delta\varepsilon=-3.52$, and $\varepsilon_0=8.85\times10^{-12}$ F/m. The twisted umbilical director field is stable until the applied voltage exceeds a critical voltage $V_t=4.9$ V. As the applied voltage is greater than V_t, the director field of twisted hyperbolic hedgehog with disclination ring structure is stable. The stable director fields for different voltage regimes are shown in Figs. 3(a) and 3(b). In order to compare the theoretical results with experimental

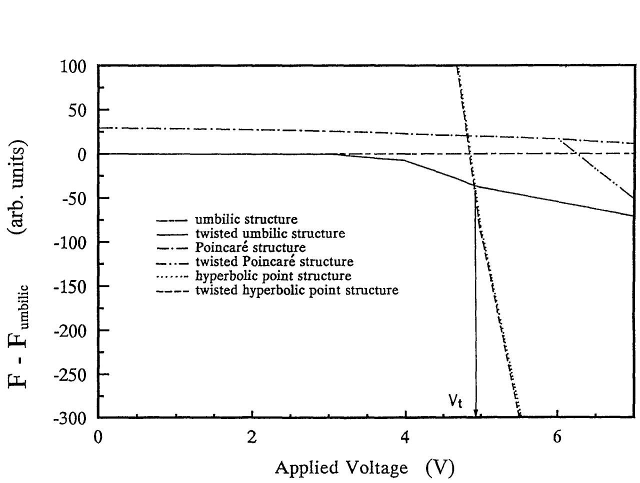


Fig. 2. Dependence of relative free energy on applied voltage. V_t is the critical voltage at which the twisted umbilic structure transforms to the twisted hyperbolic singular point structure with disclination ring.

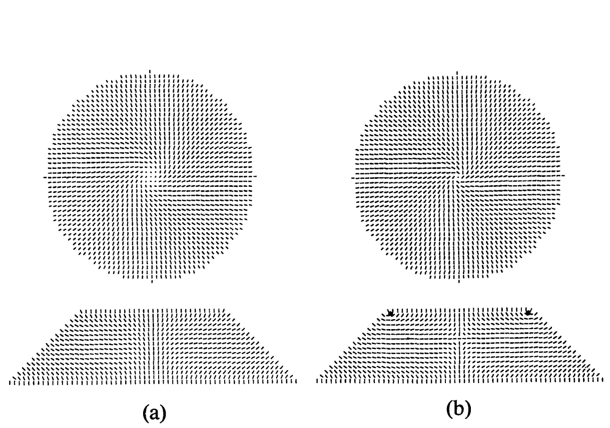


Fig. 3. The stable NLC director field in closed conical cavity. (a) $V_0=4$ V and twisted umbilic structure is stable. (b) $V_0=5$ V and hyperbolic singular point structure with disclination ring near the upper corners of the base is stable. ● is added at the upper corners to indicate the existence of disclination ring.

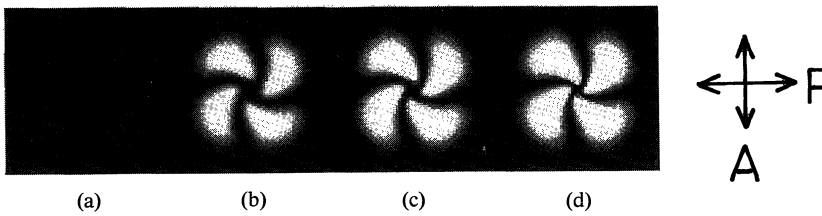


Fig. 4. The simulated optical patterns of NLC in a closed conical cavity between crossed polarizers: (a) $V_0=0$ V, (b) $V_0=4$ V, (c) $V_0=4.8$ V, and (d) $V_0=5$ V. $R_U=D$ and $R_L=2D$.

ones, we simulate the optical patterns of stable director fields in the closed conical cavity by the Jones matrix method.[14] Figures 4(a)–4(d) demonstrate the simulated optical patterns at different applied voltages, in which Figs. 4(a)–4(c) represent the stable twisted umbilical director fields, and Fig. 4(d) represents the stable twisted hyperbolic singular point structure.

From the simulated optical patterns of umbilic structures, we find that when the applied voltage is low, the dark brush has a width almost equivalent to that observed in Fig. 1(b). The twisted pattern is more apparent as the applied voltage increases, which is also demonstrated in Fig. 1. This implies that even though the directors perpendicularly anchor on the boundaries, the stable director field in the cavity will be twisted. We note that the distribution of the ϕ component of $\boldsymbol{n}$ will induce not only twist deformation but also splay and bend deformations. The small value of K_{22} is the main factor that makes the total free energy of the twisted director field less than that of an untwisted one. As the applied voltage is increased, angle ϕ increases to induce more twist deformation to compensate for the splay and bend deformations. This is why the dark brushes of optical patterns become more twisted at higher voltage.

On the other hand, in the high voltage regime, the directors prefer to align with the $\boldsymbol{i}_r-\boldsymbol{i}_\varphi$ plane rather than to be parallel to the $\boldsymbol{i}_z$ axis, causing shrinkage of the radius of the nonsingular core and an increase of the energy density near the core. It is shown by numerical results that if the applied voltage is greater than V_t, the nonsingular core breaks down to a hyperbolic singular point with a disclination ring near the corner of the upper base (see Fig. 3(b)). This also demonstrates that a hyperbolic singular point with disclination ring structure can be more stable than a Poincaré structure in the high voltage regime. The drastic change of director field around the $\boldsymbol{i}_z$ axis is not easily observed in the experiment, because the optical patterns of umbilic and hyperbolic point structures observed from a direction parallel to the symmetry axis of the conical cavity are almost the same (see Fig. 4). We need a more efficient method to overcome these problems.

In conclusion, based on our experiments as well as the continuum theory, we have demonstrated that a single umbilical director field of NLC can be stabilized in a closed conical cavity and the electric field applied along the symmetry axis of the cavity will make the director field more twisted. We also show that a transformation from the umbilical director field to a director field of hyperbolic hedgehog with disclination ring structure will occur in the closed conical cavity as the applied voltage exceeds a critical value V_t obtained theoretically. It is a subject of future study to investigate the dependency of the stability of the umbilical director field on the elastic constants of NLC and the geometry of the closed conical cavity. Furthermore, an experimental method to confirm the existence of hyperbolic hedgehog with disclination ring structure is required.

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