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ORIGINAL ARTICLE

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Capability measure for asymmetric tolerance non-normal processes applied to speaker driver manufacturing

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Abstract Process capability indices, $C_p(u, v)$, including C_p, C_{pk}, C_{pm}, and C_{pmk}, have been proposed in the manufacturing industry to provide numerical measures on process potential and performance for normal processes. Earlier studies considered a class of flexible capability indices, called $C_{Np}(u, v)$, for processes with non-normal distributions where the tolerances are symmetric. In this paper we consider an extension of $C_{Np}(u, v)$, called $C''_{Np}(u, v)$, to handle non-normal processes with asymmetric tolerances. The extension takes into account the important property of the asymmetric loss function, which is shown to be more sensitive to process shift and more accurate than $C_{Np}(u, v)$ in measuring process capability, hence provides better manufacturing quality assurance. Comparisons between $C_{Np}(u, v)$ and the extension $C''_{Np}(u, v)$ are provided. We propose a sample percentile estimator, and apply the bootstrap method to find the lower confidence bound for testing manufacturing capability. We also develop an integrated S-PLUS program to calculate the percentile estimator and the corresponding lower confidence bound. As an illustration, the proposed approach is applied to capability testing of home-theater speaker systems.

Keywords Asymmetric tolerances · Bootstrap method · Lower confidence bound · Non-normal processes · Percentile estimator

1 Introduction

Process capability indices $C_p(u, v)$, which include the two basic indices C_p and C_{pk} [1], and the two more advanced indices, C_{pm} and C_{pmk} [2, 3] as special cases, have been proposed in the manufacturing industry to provide numerical measures on process potential and process performance. The superstructure indices $C_p(u, v)$ are defined as the following [4]:

$$C_p(u, v) = \frac{d - u\,|\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}, \tag{1}$$

where μ is the process mean, σ is the process standard deviation, $d = (\mathrm{USL} - \mathrm{LSL})/2$ is half of the length of the specification interval, $m = (\mathrm{USL} + \mathrm{LSL})/2$ is the midpoint between the upper and the lower specification limits, T is the target value, and $u, v \geqq 0$. By setting u and v equal to 0 or 1, we obtain $C_p(0, 0) = C_p,\ C_p(1, 0) = C_{pk},\ C_p(0, 1) = C_{pm},\ C_p(1, 1) = C_{pmk}$. Those four indices have been investigated extensively by Kane [1], Choi and Owen [5], Chan et al. [2], Pearn et al. [3, 6], and Kotz et al. [7].

Applications of those indices include the manufacturing of semiconductor products [8] head/gimbals assembly for memory storage system [9], jet-turbine engine components [10], flip-chips and chip-on-board [11], rubber edge [12], wood products [13], aluminum electrolytic capacitors [14], audio-speaker drivers [15], and Pulux surround [16]. Other applications of those indices include performance measures on processes with tool-wear problem [17], production process monitoring [18], and many others.

Flexible capability indices $C_{Np}(u, v)$

The indices $C_p(u, v)$ are appropriate for normal and near-normal processes, but have been shown to be inappropriate for non-normal processes. Pearn and Chen [14] considered the generalization of $C_p(u, v)$ defined in the following, called $C_{Np}(u, v)$, which can be applied to processes with arbitrary distributions:

$$C_{Np}(u, v) = \frac{d - u\,|M - m|}{3\sqrt{\left[\frac{P_{99.865} - P_{0.135}}{6}\right]^2 + v(M - T)^2}}. \tag{2}$$

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In developing the generalization $C_{Np}(u, v)$, Pearn and Chen [14] replaced the process mean μ by the process median, M (a more robust measure for process central tendency), and the process standard deviation σ by $(P_{99.865} - P_{0.135})/6$ calculated from the distribution percentiles in the definition of the original indices $C_p(u, v)$. If the process follows the normal distribution, then clearly $C_{Np}(u, v)$ reduces to the basic indices $C_p(u, v)$. Pearn and Chen [14] investigated the generalization $C_{Np}(u, v)$, and considered a sample percentile method to calculate $C_{Np}(u, v)$. But, their investigation was restricted to processes with symmetric tolerances, and therefore may not be applied to processes with asymmetric tolerances.

2 The subwoofer speaker system

In this section we present a case on investigating the free air resonance, F_o, of the speaker driver used in home-theater subwoofer speaker systems. The case we investigate is taken from a speaker driver supplier in Taiwan, which manufactures various types of speaker drivers including 3-inch tweeters, 3-inch and 4-inch full-ranges, 5-inch mid-ranges, 6.5-inch woofers, 8-inch, 10-inch, 12-inch, 15-inch, and 18-inch subwoofers. A standard woofer or subwoofer driver, depicted in Fig. 1, consists of the following components: edge, cone, dust cap, spider (also called a damper), voice coil, lead wire, frame, magnet, front plate, and back plate (also called a T-york). The edge (on the top) and the spider (on the bottom) are glued onto the frame to hold the cone during the piston movements, and the dust cap is glued onto the center top of the cone, to cover the voice coil, which decouples the noise from the musical signals.

One characteristic that critically determines the bass performance, musical image, clarity and cleanness of the sound, transparence, and compliance (excursion movement) of the midrange, full range, woofer, and subwoofer driver units, is the free-air resonance, known as F_o. Some key factors determining the $F_o$values include the hardness, thickness, and weight of the damper, the hardness, thickness, and weight of the edge and cone. Some typical ranges of F_o are 60–15 000 Hz for the full ranges, 500–5000 Hz for the mid-ranges, 1000–18 000 Hz for hard-dome tweeters, and 1500–20 000 Hz for soft-dome tweeters. A typical subwoofer system with a front reflex tube, as depicted in Fig. 2, plays an important role in most home-theater applications.

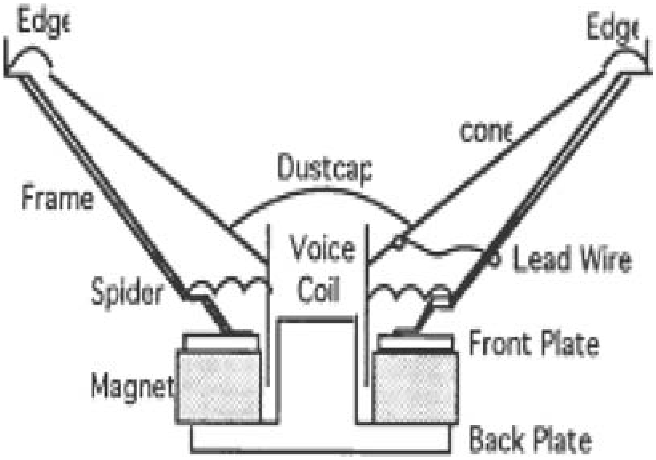

Fig. 1. A subwoofer driver

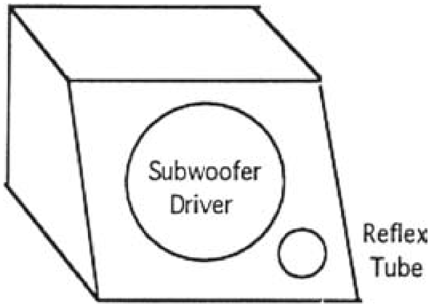

Fig. 2. A subwoofer system

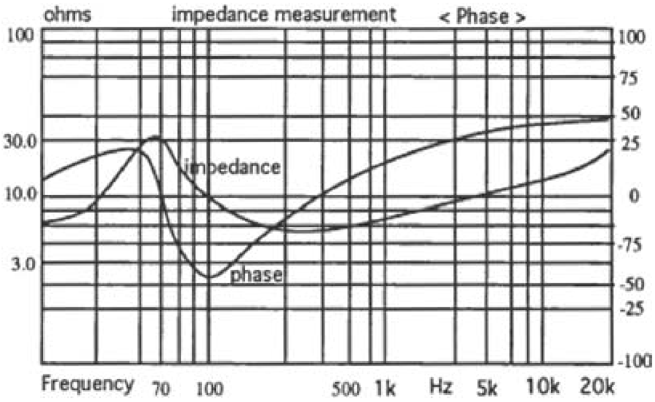

Fig. 3. LMS drawing for impedance and phase curves

A standard home-theater system contains two main speaker systems (normally with tweeters, midranges, and woofers) for the front channels, one speaker (normally with full ranges, or tweeters and midranges) for the center channel, one subwoofer system, and two speakers for the (rear) background channels. One particular home-theater application we investigated uses a subwoofer system with a 10-inch subwoofer driver. The upper and lower specification limits, the USL and LSL, for this subwoofer driver are set at 35 Hz and 20 Hz, respectively, and the target value is set at $T = 30$ Hz. The LMS (Learning management system) is the computer software commonly used in speaker driver manufacturing industry for measuring the impedance, phase curve and other characteristics of the drivers (shown in Fig. 3). This is the case where the manufacturing tolerance is asymmetric.

3 Capability measure for asymmetric tolerances

For asymmetric tolerances, the work of several researchers including Kane [1], and Kushler and Hurley [19], simply shift the specification limits to make them symmetric around the target value T. These methods obviously can severely understate or overstate process capability and thus reflect process performance inaccurately. To overcome the problem, Vännman [20] considered an alternative method by adding a new term, $|\mu - T|$, in the numerator of the capability definitions, which has been defined as:

$$C_{pa}(u, v) = \frac{d - |\mu - m| - u\,|\mu - T|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}, \tag{3}$$

where u and v are non-negative parameters. The index $C_{pa}(u, v)$ has the advantage of having its maximum when $\mu = T$, and it decreases as μ shifts away from T in either direction.

Recently, Pearn and Chen [15] developed a new method to generalize C_{pk} for asymmetric tolerance. Chen et al. [21], and Pearn et al. [22] have applied the method to indices C_{pm}, and C_{pmk}, respectively, for normal processes with asymmetric tolerance. Pearn et al. [23] also applied the method to Clements' formula for calculating capability indices for non-normal processes with asymmetric tolerance. The method takes into account the important property of the asymmetric loss function, which is shown to be superior to the other existing methods, and is an appropriate method for asymmetric tolerances. We note, however, that Clements' formula requires the underlying process distribution to be of the Pearson family type, and the computation requires checking extensive percentile tables of the Pearson family distributions. In the following, we apply the method shown in [15] to extend the flexible indices $C_{Np}(u, v)$, which we refer to as $C''_{Np}(u, v)$, to cover arbitrary distributions with asymmetric tolerances. The extension $C''_{Np}(u, v)$ can be expressed as:

$$C''_{Np}(u, v) = \frac{d^* - uA^*}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + vA^2}}, \tag{4}$$

where $A = \max\{d(M - T)/d_u, d(T - M)/d_l\}$, $A^* = \max\{d^*(M - T)/d_u,\ d^*(T - M)/d_l\}$, $d_u = \mathrm{USL} - T$, $d_l = T - \mathrm{LSL}$, $d^* = \min\{d_u, d_l\}$. We note that if $T = m$ (tolerance is symmetric), then $A = A^* = |M - T|$, and the extension $C''_{Np}(u, v)$ reduces to the original index $C_{Np}(u, v)$. By setting u and v equal to 0 or 1 we obtain $C''_{Np}(0, 0) = C''_{Np}$, $C''_{Np}(1, 0) = C''_{Npk}$, $C''_{Np}(0, 1) = C''_{Npm}$, and $C''_{Np}(1, 1) = C''_{Npmk}$, which can be expressed as the following:

$$\begin{aligned}
C''_{Np} &= \frac{d^*}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2}},\\
C''_{Npk} &= \frac{d^* - A^*}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2}},\\
C''_{Npm} &= \frac{d^*}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + A^2}},\\
C''_{Npmk} &= \frac{d^* - A^*}{3\sqrt{\left(\frac{P_{99.865} - P_{0.135}}{6}\right)^2 + A^2}}.
\end{aligned} \tag{5}$$

The merit of the extension is that it takes into consideration the asymmetric loss function that other existing methods have not implemented. For processes with asymmetric tolerances, the corresponding loss function is also asymmetric to the target value T. Consider the popular quadratic loss function defined as $L(x) = [(T - x)/(T - \mathrm{LSL})]^2$, for $\mathrm{LSL} < x \le T$, $L(x) = [(x - T)/(\mathrm{USL} - T)]^2$, for $T \le x < \mathrm{USL}$, and $L(x) = 1$, otherwise. Then, for $x_1 = (T + \mathrm{LSL})/2$, and $x_2 = (T + \mathrm{USL})/2$, the corresponding loss can be calculated as $L(x_1) = L(x_2) = 1/4$. Obviously, x_1 and x_2 have the same departure ratio $k = (T - x_1)/d_l = (x_2 - T)/d_u = 1/2$ (equal departure relative to the tolerance). Thus, a desired property for a capability index with asymmetric tolerances is that the capability measures for shifted processes with equal departure ratios are the same. While we do not employ the quadratic loss function, the extension penalizes processes with equal departure ratios equally.

Justification of the extension

To justify that the extension indeed possesses the important property of the asymmetric loss function, we consider the following example with asymmetric tolerance (LSL, T, USL) = (100, 120, 130) and fixed process variations $P_{99.865} - P_{0.135} = 0.9d$, $P_{99.865} - M = 0.55d$, and $M - P_{0.135} = 0.35d$, where $100 \leqq M \leqq 130$. Table 1 displays the capability measures of those processes using $C''_{Np}(u, v)$. The proposed extensions, $C''_{Np}(u, v)$,

Table 1. $C''_{Np}(u, v)$ for processes A, B, satisfying $(M_A - T)/d_u = (T - M_B)/d_\ell$

M	C''_{Np}	C''_{Npk}	C''_{Npm}	C''_{Npmk}	M	C''_{Np}	C''_{Npk}	C''_{Npm}	C''_{Npmk}
100	1.482	0.000	0.220	0.000	110	1.482	0.741	0.426	0.070
130	1.482	0.000	0.220	0.000	125	1.482	0.741	0.426	0.070
102	1.482	0.148	0.244	0.005	112	1.482	0.889	0.520	0.117
129	1.482	0.148	0.244	0.005	124	1.482	0.889	0.520	0.117
104	1.482	0.296	0.273	0.013	114	1.482	1.037	0.663	0.198
128	1.482	0.296	0.273	0.013	123	1.482	1.037	0.663	0.198
106	1.482	0.444	0.310	0.024	116	1.482	1.185	0.889	0.331
127	1.482	0.444	0.310	0.024	122	1.482	1.185	0.889	0.331
108	1.482	0.593	0.359	0.042	118	1.482	1.333	1.233	0.516
126	1.482	0.593	0.359	0.042	121	1.482	1.333	1.233	0.516

obtain their maximal values at the target value $T = 120$. It is easy to verify that when $M = T$, we obtain that $A = A^* = 0$, and the equations defined in Eq. 4 are indeed maximized. Table 1 displays the index values of the extension $C''_{Np}(u, v)$ obtained for processes with equal departure ratios satisfying the property $(M_A - T)/d_u = (T - M_B)/d_l)$. For example, consider processes A and B with $M_A = 124$ and $M_B = 112$. It is easy to verify that $(M_A - T)/d_u = (124 - 120)/10 = 2/5$, and $(T - M_B)/d_l = (120 - 112)/20 = 2/5$, thus satisfying $(M_A - T)/d_u = (T - M_B)/d_l$. Checking Table 1 the extensions give same values for A and B.

4 Performance comparisons

In the following, we show that the proposed extension $C''_{Np}(u, v)$ outperforms the original index $C_{Np}(u, v)$ in detecting process shifting. We consider the following example with an on-target process A, and three shifted processes A_1, A_2 and A_3, where the manufacturing tolerance (LSL, T, USL) is set to (100, 120, 130). Table 2 displays the characteristics of the four processes A, A_1, A_2, A_3 the index values of C_{Np}, C_{Npk}, C_{Npm}, C_{Npmk}, and the extensions C''_{Np}, C''_{Npk}, C''_{Npm}, and C''_{Npmk}.

We note that the index C_{Npm} detects process shifts of A_1, A_2 and A_3. But, C_{Npm} fails to differentiate the high quality process A_2 (with nearly 100% process yield) from the low quality process A_3 (with only 50% process yield), as $C_{Npm} = 0.140$ for both A_2, and A_3. Therefore, we consider C_{Npm} inaccurate. On the other hand, the extensions detect the shifts of A_1, A_2, A_3, and differentiate process A_2 from process A_3 by giving larger values to A_2 and smaller values to A_3 (except for C''_{Np} which never takes into account the process median and the target value hence provides no sensitivity to process departure at all). We also note that for the extensions C''_{Npk}, C''_{Npm}, and C''_{Npmk}, the on-target process A receives larger index values than the other three off-target processes A_1, A_2, and A_3. But, for the original indices C_{Npk} the off-target process A_1 receives larger index values than the on target process A, and the capability measure is considered inaccurate. Since the generalization is proposed to handle non-normal processes with asymmetric tolerances, it has to deal with a large number (theoretically an infinite number) of arbitrary shapes of distributions, it can only reflect process quality approximately (often conservatively), and may not be very accurate.

Table 2. Characteristics of processes A, A_1, A_2, A_3, the corresponding index values of the original indices and extensions

	A	A_1	A_2	A_3
M	120	119	110	130
$P_{99.865}$	130	129	120	140
$P_{0.135}$	115	114	105	125
C_{Np}	2.000	2.000	2.000	2.000
C_{Npk}	1.333	1.467	1.333	0.000
C_{Npm}	2.000	1.765	0.140	0.140
C_{Npmk}	1.333	1.294	0.093	0.000
C''_{Np}	1.333	1.333	1.333	1.333
C''_{Npk}	1.333	1.267	0.667	0.000
C''_{Npm}	1.333	1.240	0.157	0.043
C''_{Npmk}	1.333	1.178	0.078	0.000

5 Percentile estimator of $C''_{Np}(u, v)$

Pearn and Chen [14] considered a sample percentile estimator to calculate the index $C_{Np}(u, v)$. The estimator essentially applies the sample percentile along with interpolation for calculating the sample percentiles, $P_{99.865}$, $P_{0.135}$, and the median M. The estimator can be expressed as the following:

$$\hat{C}_{Np}(u, v) = \frac{d - u\left|\hat{M} - m\right|}{3\sqrt{\left(\frac{\hat{P}_{99.865} - \hat{P}_{0.135}}{6}\right)^2 + v\left(\hat{M} - T\right)^2}},$$

$$\hat{P}_{99.865} = X_{([R_1])} + \{R_1 - [R_1]\} \times \left\{X_{([R_1]+1)} - X_{([R_1])}\right\},$$

$$\hat{P}_{0.135} = X_{([R_2])} + \{R_2 - [R_2]\} \times \left\{X_{([R_2]+1)} - X_{([R_2])}\right\},$$

$$\hat{M} = X_{([R_3])} + \{R_3 - [R_3]\} \times \left\{X_{([R_3]+1)} - X_{([R_3])}\right\},$$

$$R_1 = \left(\frac{99.865n + 0.135}{100}\right), R_2 = \left(\frac{0.135n + 99.865}{100}\right),$$

$$R_3 = \left(\frac{n+1}{2}\right). \tag{6}$$

In this setting, the notation $[R]$ is defined as the greatest integer less than or equal to the number R, and $x_{(i)}$ is defined as the ith order statistic. The sample percentile estimator for the extension $C''_{Np}(u, v)$, therefore, can be easily obtained as the following. We note that the percentile formula developed in the following requires no assumption on the underlying process distributions to be of the Pearson types, and the computation does not require any tables. Hence, the sample percentile method is more general and convenient to use than Clements' formula:

$$\hat{C}''_{Np}(u, v) = \frac{d^* - u\hat{A}^*}{3\sqrt{\left(\frac{\hat{P}_{99.865} - \hat{P}_{0.135}}{6}\right)^2 + v\hat{A}^2}},$$

$$\hat{A}^* = \max\left\{\frac{d^*(\hat{M} - T)}{d_u}, \frac{d^*(T - \hat{M})}{d_l}\right\},$$

$$\hat{A} = \max\left\{\frac{d(\hat{M} - T)}{d_u}, \frac{d(T - \hat{M})}{d_l}\right\}. \tag{7}$$

An S-PLUS computer program for calculating the percentile formula was developed and is listed in the Appendix. The S-PLUS program reads the sample data as an input, then outputs with the estimated values of the indices. Since the percentile method involves a complicated function of linear combinations of the order statistics and, given that the underlying process distribution is unknown, the problem of finding an exact distribution is analytically intractable. Approximation approaches or bootstrap resampling methods (an effective simulation technique for non-normal distributions) may be applied to establish the lower confidence bound for capability testing purposes.

6 Distribution plot of the percentile estimator

In the following, we apply a simulation approach to investigate the distribution of the sample percentile estimator. We set the asymmetric specification limits to (LSL, T, USL) = (8, 18, 23), and the underlying process distribution to:

1. Normal distribution N(17,1), with probability density function $f(x) = (\sqrt{2\pi})^{-1} e^{-(x-17)^2/2}$, for $-\infty < x < \infty$,
2. Uniform distribution U(14, 20), with probability density function $f(x) = 1/6$, for $14 < x < 20$,
3. Weibull distribution W(2, 2) shifted for 17 units, with probability density function $f(x) = 4xe^{-2x^2}$, for $x > 0$,
4. Gamma distribution G(289, 17), with probability density function $f(x) = 17^{289} x^{289-1} e^{-17x} / \Gamma(289)$, for $x > 0$,
5. Beta distribution B(17, 1), with probability density function $f(x) = [\Gamma(18)/\Gamma(17)]\, x^{16}$, for $0 < x < 1$,
6. Lognormal distribution LN(0.5, 1) shifted for 17 units, with probability density function of LN(0.5, 1) as $f(x) = (\sqrt{2\pi})^{-1} x^{-1} e^{-(\ln x - 0.5)^2/2}$, for $x > 0$,
7. Chi-square distribution with degrees of freedom two shifted for 17 units, where the probability density function of χ_2^2 is $f(x) = e^{-x/2}/2$, for $x > 0$, and
8. t distribution, shifted 18 units and with eight degrees of freedom, where the probability density function of t_8 is, $f(x) = [\Gamma(9/2)/\Gamma(4)]\,(\sqrt{8\pi})^{-1}(1 + x^2/8)^{-9/2}$ for $-\infty < x < \infty$. For each distribution, we randomly generate $N = 15,000$ samples of sizes $n = 50, 100, 250, 500, 1000$, and then calculate the estimated capability index C''_{Npmk}.

Figures 4–11 plot the distribution of C''_{Npmk} for the eight process distributions, normal distribution N(17, 1), uniform distribution U(14, 20), Weibull distribution W(2, 2) shifted for 17 units, gamma distribution G(289, 17), beta distribution B(17, 1),

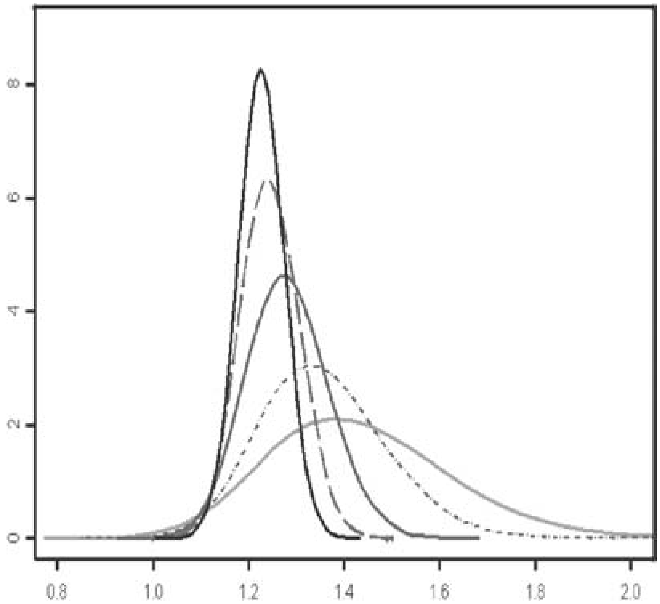

Fig. 4. Distribution plot of C''_{Npmk} for normal distribution N(17,1), with $n = 50, 100, 250, 500, 1000$ (bottom to top)

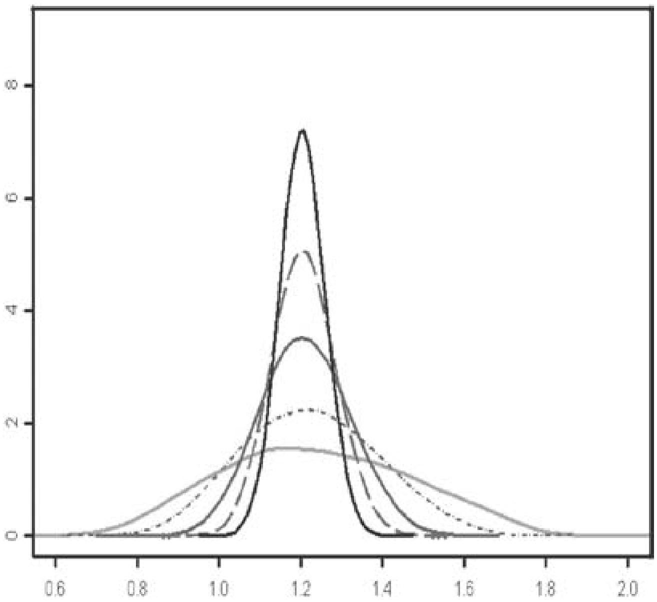

Fig. 5. Distribution plot of C''_{Npmk} for uniform distribution U(14,20), with $n = 50, 100, 250, 500, 1000$ (bottom to top)

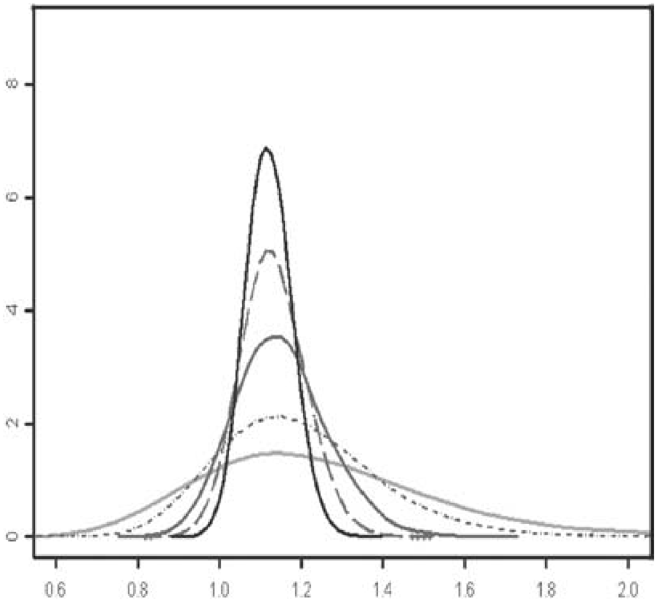

Fig. 6. Distribution plot of C''_{Npmk} for Weibull distribution W(2,2), with $n = 50, 100, 250, 500, 1000$ (bottom to top)

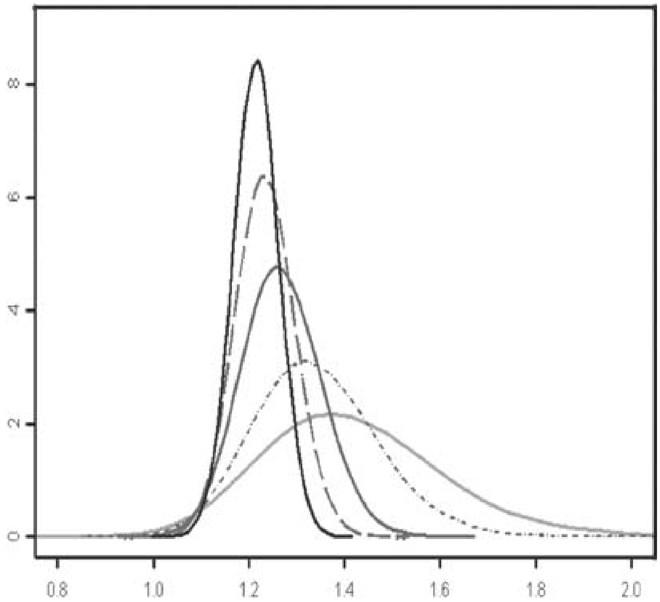

Fig. 7. Distribution plot of C''_{Npmk} for gamma distribution G(289,17), with $n = 50$, 100, 250, 500, 1000 (bottom to top)

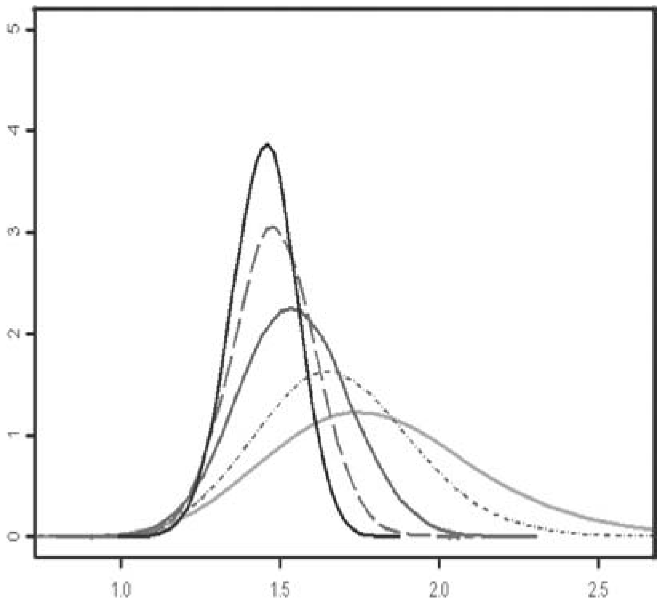

Fig. 8. Distribution plot of C''_{Npmk} for beta distribution B(17,1), with $n = 50$, 100, 250, 500, 1000 (bottom to top)

lognormal distribution LN(0.5, 1) shifted for 17 units, chi-square distribution with degrees of freedom two shifted for 17 units, and t_8 distribution with degrees of freedom eight shifted for 18 units, respectively. For moderate and large sample size n, the distributions of the estimated capability index all appear to be normal. Therefore, for processes where large sample data may be collected (product items may be inspected by automatic controlled machines), normal approximations may be used for capability testing.

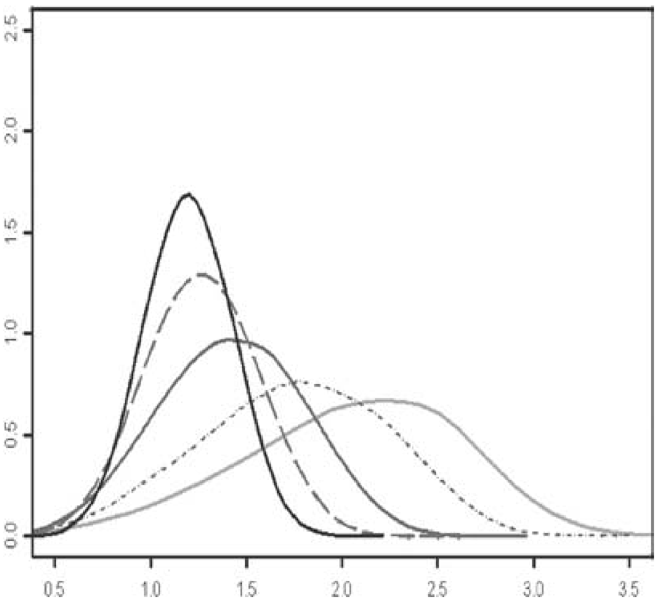

Fig. 9. Distribution plot of C''_{Npmk} for lognormal distribution LN(0.5,1), $n =$ 50, 100, 250, 500, 1000 (bottom to top)

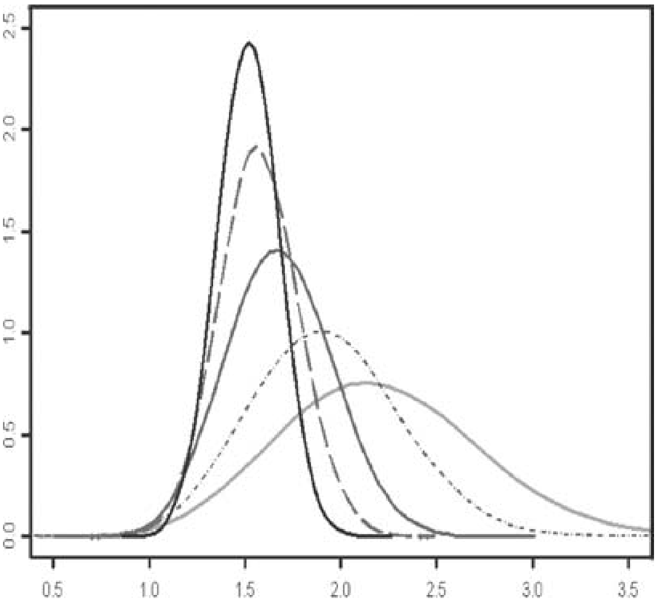

Fig. 10. Distribution plot of C''_{Npmk} for chi-square distribution with $df = 2$, $n = 50$, 100, 250, 500, 1000 (bottom to top)

7 Bootstrap for manufacturing capability testing

In statistical analysis, the researcher is usually interested in obtaining not only a point estimate of a statistic but also an estimate of the variation of this point estimate, and a confidence interval for the true value of the parameter. For example, a researcher may calculate not only a sample mean but also the standard error of the mean and a confidence interval for the mean. Tra-

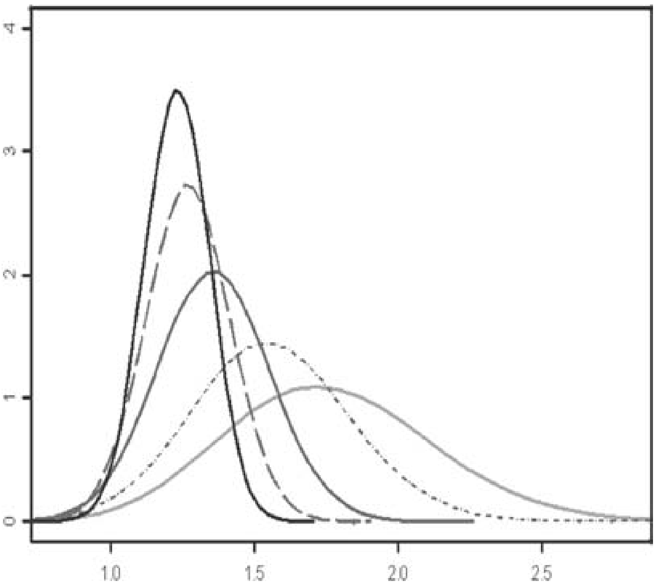

Fig. 11. Distribution plot of C''_{Npmk} for t distribution with $df = 8$, and $n =$ 50, 100, 250, 500, 1000 (bottom to top)

ditionally, researchers have relied on the central limit theorem and normal approximations to obtain standard errors and confidence intervals. These techniques are valid only if the statistic, or some known transformation of it, is asymptotically normally distributed. Hence, if the normality assumption does not hold, then the traditional methods should not be used to obtain confidence intervals. A major motivation for the traditional reliance on normal-theory methods has been computational tractability. Now, with the availability of modern computing power, researchers need no longer rely on asymptotic theory to estimate the distribution of a statistic. Instead, they may use resampling methods, which return inferential results for either normal or non-normal distributions. This is particularly useful in our case, for small or moderate size of non-normal data.

Bootstrapping, as presented by Efron [24, 25] is a data based estimation method for statistical inference that is effective, nonparametric but also computationally intensive. In this method, B new samples, each of the same size as the observed data, are drawn *with replacement* from the available sample. The statistic of interest is then calculated for each new set of resampled data, yielding a bootstrap distribution for the statistic. The fundamental assumption of bootstrapping is that the observed data are representative of the underlying population. By resampling observations from the observed data, the process of sampling observations from the population is mimicked. The merit, in its simplest form, is that the nonparametric bootstrap does not rely on any distributional assumptions about the underlying population. Efron and Tibshirani [26] developed three types of bootstrap confidence interval. Those include the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), and the biased corrected percentile bootstrap confidence interval (BCPB). Franklin and Wasserman [27] investigated the lower confidence bounds for the capability indices, C_p, C_{pk} and C_{pm} using the three bootstrap methods. Some simulations were conducted and a comparison was made among the results from the three bootstrap methods against the known values of the capability indices. Their simulation results indicate that, for normal processes, the bootstrap confidence limits perform equally well. And for non-normal processes the SB method performed significantly better than other methods. Furthermore, the proportion of coverage of the SB method limits and confidence intervals are closer to the desired value, whatever the underlying process distribution, and the lengths of the SB intervals are much shorter than others. Therefore, the SB method is more useful in determining the value of the process index. In the following, we will describe how to construct the bootstrap lower confidence bound by means of the standard bootstrap method.

The bootstrap lower confidence bound

In our application, $B = 10000$ bootstrap resamples (each of the same size as the available data) are drawn randomly from the original sample. A $100(1-\alpha)\%$ bootstrap lower confidence limit of the SB method for $C''_{Np}(u, v)$ is constructed. If the calculated bootstrap lower confidence limit is found to be smaller than the specified index value, we would judge the process as incapable. Quality improvement activities will be initiated. Otherwise, the process is considered to be capable. From the 10000 bootstrap estimates, the sample average can be calculated as:

$$\overline{\hat{C}''_{Np}}(u, v) = \frac{1}{10000}\sum_{i=1}^{10000}\hat{C}''_{Np(i)}(u, v),$$

and the sample standard deviation of the $\hat{C}''_{Np}(u, v)$ can be obtained as:

$$S_{\hat{C}''_{Np}(u,v)} = \sqrt{\frac{1}{10000-1}\sum_{i=1}^{10000}\left(\hat{C}''_{Np(i)}(u, v) - \overline{\hat{C}''_{Np(i)}}(u, v)\right)^2}.$$

where $\hat{C}''_{Np(i)}(u, v)$ is the ith bootstrap estimates and by setting $u, v = 0, 1$, we obtain $\hat{C}''_{Np}(0, 0) = \hat{C}''_{Np}$, $\hat{C}''_{Np}(1, 0) = \hat{C}''_{Npk}$, $\hat{C}''_{Np}(0, 1) = \hat{C}''_{Npm}$, and $\hat{C}''_{Np}(1, 1) = \hat{C}''_{Npmk}$. Thus, the $100(1-\alpha)\%$ SB lower confidence bound (LCB) for $C''{}_{Np}(u, v)$ can be constructed as:

$$\mathrm{LCB} = \overline{\hat{C}''_{Np}}(u, v) - Z_\alpha \times S_{\hat{C}''_{Np}(u,v)}. \tag{8}$$

In order to make use of the methodology more convenient and accelerate the computation time, an integrated S-PLUS computer program has been developed (see Appendix). The practitioners only need to input the manufacturing specifications, USL, LSL, target value T, a specified quality level of $C''_{Np}(u, v)$ and a collected sample data of size n. The estimated value $\hat{C}''_{Np}(u, v)$ and the SB lower confidence bound of $C''_{Np}(u, v)$ may be easily obtained. Thus, whether or not the process is capable may be determined.

Table 3. The 100 sample observations

28	28	26	32	28	27	29	25	25	28	28	26	27
31	27	26	32	29	27	26	26	26	30	25	27	29
27	31	30	30	27	31	27	25	30	28	26	27	31
27	34	27	28	32	33	25	29	28	28	29	29	25
29	29	30	31	28	28	30	28	26	28	28	25	29
28	29	31	28	28	27	30	27	25	30	25	29	26
26	27	33	29	26	31	32	27	26	29	26	28	30
26	29	28	29	25	30	27	28	32				

The collected data (a total of 100 observations) are displayed in Table 3. Proceeding with the calculations by running the integrated S-PLUS program with 95% of confidence, we obtain the values of the sample percentile estimators and the corresponding bootstrap lower confidence bound (LCB) as:

$$\hat{C}''_{Np} = 1.35, \quad \text{with} \quad \text{LCB} = 1.25,$$
$$\hat{C}''_{Npk} = 1.20, \quad \text{with} \quad \text{LCB} = 1.10,$$
$$\hat{C}''_{Npm} = 1.18, \quad \text{with} \quad \text{LCB} = 1.08,$$
$$\hat{C}''_{Npmk} = 1.05, \quad \text{with} \quad \text{LCB} = 0.94.$$

We note that the estimated index values for all the four extensions are greater than 1.00. In fact, all 100 observations fall within the specification interval (LSL, USL) with one observation (34) fairly close to the upper specification limit (35). Checking the corresponding lower confidence bounds, 1.25, 1.10, 1.08, and 0.94, we may only conclude that the process is marginally capable, with 95% of confidence.

8 Concluding remarks

Process capability indices are practical and powerful tools for measuring process performance. Many quality engineers and statisticians have proposed methodologies for assessing product/process quality but limited their proposals to the inspection of data that are normally distributed where the manufacturing tolerances are symmetric. In this paper, we considered an extension of the existing capability index, called $C''_{Np}(u, v)$, to handle non-normal processes with asymmetric tolerances. The extension takes into account the important property of the asymmetric loss function, which is shown to be more sensitive to process shift and more accurate than the existing ones in measuring process capability, and hence provides better quality assurance. We also presented a percentile estimator to calculate the asymmetric process capability with non-normal data.

Since the proposed percentile method involves a complicated function of linear combinations on the order statistics and, given that the underlying process distribution is unknown, the problem of finding an exact distribution is analytically intractable. We applied the nonparametric bootstrap method to establish the lower confidence bound for capability testing purposes. We also developed an integrated S-PLUS computer program to calculate the percentile estimator and the corresponding lower confidence bound. The practitioners only need to input the manufacturing specifications, USL, LSL, target value T, a specified quality level of $C''_{Np}(u, v)$ and the collected sample data. The estimated value $\hat{C}''_{Np}(u, v)$ and the SB lower confidence bound of $C''_{Np}(u, v)$ may be applied to determine whether the process is capable or not.

Appendix

Integrated S-PLUS computer program

```
#---------------------------------------#
# Input the specification limits, USL, LSL,
# and the target value T
#---------------------------------------#
USL<-35
LSL<-20
Target<-29

#---------------------------------------#
# Store the input of the original sample data
# of size n =100
#---------------------------------------#
data0<-c( 28,28,26,32,28,27,29,25,25,28,
28,26,27,31,27,26,32, 29,27,26,26,26,30,25,
27,29,27,31,30,30,27, 31,27,25, 30,28,26,27,
31,27,34,27,28,32,33,25,29,28, 28,29,29,25,
29,29,30,31,28,28,30,28,26,28,28,25,29,28,
29,31, 28,28,27,30,27,25,30,25,29,26,26,27,
33,29, 26,31,32, 27,26,29,26,28,30,26,29,28,
29,25,30,27,28, 32)

#---------------------------------------#
# The function to calculate the estimated
# C''_Np (u, v) based on the given data
#---------------------------------------#
CNp.hat<-function(u,v,data){
  data.order<-sort(c(data))
  n<-length(data)
  R1<-(99.865*n+0.135)/100
  R2<-(0.135*n+99.865)/100
  R3<-(n+1)/2
P99.865hat<-data.order[floor(R1)]
   +(R1-floor(R1))*(data.order[floor(R1+1)]
   -data.order[floor(R1)])
  P0.135hat<-data.order[floor(R2)]
   +(R2-floor(R2))*(data.order[floor(R2+1)]
   -data.order[floor(R2)])
```

```
  Mhat<-data.order[floor(R3)]
   +(R3-floor(R3))*(data.order[floor(R3+1)]
    -data.order[floor(R3)])
  du<-USL-Target
  dl<-Target-LSL
  d<-(USL-LSL)/2
  dstar<-min(du,dl)
  Ahat.star<-max(dstar*(Mhat-Target)
    /du,dstar*(Target-Mhat)/dl)
  Ahat<-max(d*(Mhat-Target)/du,
    d*(Target-Mhat)/dl)
CNp.return<-(dstar-u*Ahat.star)
  /(3*sqrt(((P99.865hat-P0.135hat)/6)^2
  +v*Ahat^2))
}

#-----------------------------------------#
# Calculate the estimated C''_Np (u, v)
# based on the original data
#-----------------------------------------#
CNp.est<-CNp.hat(0,0,data0)
CNpk.est<-CNp.hat(1,0,data0)
CNpm.est<-CNp.hat(0,1,data0)
  CNpmk.est<-CNp.hat(1,1,data0)

#-----------------------------------------#
# Generate 10000 bootstrap resamples from
# the original sample of n = 100
#-----------------------------------------#
  m<-10000
CNp.boot<-rep(0,m)
CNpk.boot<-rep(0,m)
CNpm.boot<-rep(0,m)
  CNpmk.boot<-rep(0,m)
for (i in 1:m){
    dataB<-sample(data0,100,replace=T)
    CNp.boot[i]<-CNp.hat(0,0,dataB)
  CNpk.boot[i]<-CNp.hat(1,0,dataB)
    CNpm.boot[i]<-CNp.hat(0,1,dataB)
    CNpmk.boot[i]<-CNp.hat(1,1,dataB)
}

#-----------------------------------------#
# Calculate the lower confidence bound based
# on the bootstrap resampling
#-----------------------------------------#
CNp.boot.95lowerbound<-mean(CNp.boot)
  -qnorm(0.95)*var(CNp.boot)^0.5
CNpk.boot.95lowerbound<-mean(CNpk.boot)
  -qnorm(0.95)*var(CNpk.boot)^0.5
CNpm.boot.95lowerbound<-mean(CNpm.boot)
  -qnorm(0.95)*var(CNpm.boot)^0.5
CNpmk.boot.95lowerbound<-mean(CNpmk.boot)
  -qnorm(0.95)*var(CNpmk.boot)^0.5

#-----------------------------------------#
```

The output of sample program based on the above settings:

1. The estimated $C''_{Np}(u, v)$ based on the original data are:
   ```
   > CNp.est = 1.353432
   > CNpk.est = 1.20305
   > CNpm.est = 1.178897
   > CNpmk.est = 1.047908
   ```

2. The bootstrap lower confidence bound of $C''_{Np}(u, v)$ are:
   ```
   > CNp.boot.95lowerbound = 1.250352
   > CNpk.boot.95lowerbound = 1.104946
   > CNpm.boot.95lowerbound = 1.084890
   > CNpmk.boot.95lowerbound = 0.9366828
   ```

References

1. Kane VE (1986) Process capability indices. J Qual Technol 18(1): 41–52
2. Chan LK, Cheng SW, Spiring FA (1988) A new measure of process capability C_{pm}. J Qual Technol 20(3):162–175
3. Pearn WL, Kotz S, Johnson NL (1992) Distributional and inferential properties of process capability indices. J Qual Technol 24(4): 216–231
4. Vännman K (1995) A unified approach to capability indices. Stat Sin 5:805–820
5. Choi BC, Owen DB (1989) A study of a new capability index. Commun Stat Theory Methods 19(4):1231–1245
6. Pearn WL, Lin GH, Chen KS (1998) Distributional and inferential properties of the process accuracy and process precision indices. Commun Stat Theory Methods 27(4):985–1000
7. Kotz S, Pearn WL, Johnson NL (1993) Some process capability indices are more reliable than one might think. J R Stat Soc C 42(1):55–62
8. Hoskins J, Stuart B, Taylor J (1988) A Motorola commitment: a six sigma mandate. The Motorola guide to statistical process control for continuous improvement towards six-sigma quality
9. Rado LG (1989) Enhance product development by using capability indexes. Qual Progress 22(4):38–41
10. Hubele NF, Montgomery DC, Chih WH (1991) An application of statistical process control in jet-turbine engine component manufacturing. Qual Eng 4(2):197–210
11. Noguera J, Nielsen T (1992) Implementing six sigma for interconnect technology. ASQC Quality Congress Transactions, Nashville, TN, pp 538–544
12. Pearn WL, Kotz S (1994) Application of Clements' method for calculating second and third generation process capability indices for non-normal Pearsonian populations. Qual Eng 7(1):139–145
13. Lyth DM, Rabiej RJ (1995) Critical variables in wood manufacturing's process capability: species, structure, and moisture content. Qual Eng 8(2):275–281
14. Pearn WL, Chen KS (1997) Capability indices for non-normal distributions with an application in electrolytic capacitor manufacturing. Microelectron Reliab 37(12):1853–1858
15. Pearn WL, Chen KS (1998) New generalizations of the process capability index C_{pk}. J Appl Stat 25(6):801–810
16. Pearn WL, Chang CS (1998) An implementation of the precision index for contaminated processes. Qual Eng 11(1):101–110
17. Spiring FA (1989) An application of C_{pm} to the tool-wear problem. ASQC Quality Congress Transactions, Toronto, ON, pp 123–128
18. McCoy PF (1991) Using performance indexes to monitor production processes. Qual Progress February:49–55
19. Kushler RH, Hurley P (1992) Confidence bounds for capability indices. J Qual Technol 24(4):188–195
20. Vännman K (1997) A general class of capability indices in the case of asymmetric tolerances. Commun Stat Theory Methods 26(8):2049–2072

21. Chen KS, Pearn WL, Lin PC (1999) A new generalization of the capability index C_{pm} for asymmetric tolerances. Int J Reliab Qual Safety Eng 6(4):383–398
22. Pearn WL, Chen KS, Lin PC (1999) On the generalizations of the capability index C_{pmk} for asymmetric tolerances. Far East J Theor Stat 3(1):47–66
23. Pearn WL, Chen KS, Lin GH (1999) A generalization of Clements‘ method for non-normal Pearsonian processes with asymmetric tolerances. Int J Qual Reliab Manage 16(5):507–521
24. Efron B (1979) Bootstrap methods: another look at the Jackknife. Ann Stat 7:1–26
25. Efron B (1982) The Jackknife, the bootstrap and other resampling plans. Society for Industrial and Applied Mathematics, Philadelphia, PA
26. Efron B, Tibshirani RJ (1986) Bootstrap methods for standard errors, confidence interval, and other measures of statistical accuracy. Stat Sci 1:54–77
27. Franklin LA, Wasserman GS (1992) Bootstrap lower confidence limits for capability indices. J Qual Technol 24(4):196–210