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ORIGINAL ARTICLE

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Measuring manufacturing capability for couplers and wavelength division multiplexers

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Abstract The couplers and wavelength division multiplexers have been widely used in high-speed, high-volume image data transmission systems to provide sufficient bandwidth and smaller channel spacing for greater throughput. In this paper, we develop a method for measuring the manufacturing capability of a process making couplers and wavelength division multiplexers. The proposed method measures the process capability of reproducing product items meeting the manufacturing specifications where multiple product quality characteristics are involved, including the polarization dependent loss, and the insertion loss, which are critical in fiber-optic transmission quality.

Keywords Bootstrap methods · Couplers · Insertion loss · Multiple characteristics · NCPPM · Polarization dependent loss · Process capability indices · Wavelength division multiplexers · Yield analysis

1 Introduction

The Internet, which in the 1990s spread primarily in North America, has undergone explosive growth over the world in recent years. What were originally only local networks have now expanded to a worldwide scale, and the data sent and received has gone from text only to include moving images, requiring quantum jumps in capacity. In wide-area networks fiber optics has become indispensable in the instantaneous transmission of large volumes of data. The tremendous need for higher data transmission rates has always driven the development of new optical components to the limits of existing technology.

The rapid development of optical and photonic technologies for a variety of applications has resulted in a similarly rapid need for all optical systems, and thus the need for passive optical components. The number of stations or nodes on an all optical fiber data bus is limited by the total allowable system loss. Therefore, fiber optic networks are the present and future choice for high-speed, high-volume data transmission medium. The growth in demand for greater data throughput requires greater bandwidth and smaller channel spacing. This has created a need for more effective utilization of fiber-optic networks. A way of accomplishing this has been with various couplers and wavelength division multiplexing technologies, to split light from one fiber optic route into two fiber optic routes, and a number of signals of differing wavelength can be carried on a single fiber with minimum loss, respectively.

In this paper, we develop a method for measuring manufacturing capability of a process making couplers and wavelength division multiplexers. The proposed method measures the capability of reproducing products meeting the manufacturing specifications where multiple product quality characteristics are involved, including the polarization dependent loss, and the insertion loss, which are critical in fiber-optic transmission quality.

2 Couplers, splitters, and WDM

Couplers, splitters and wavelength division multiplexers (WDM) are important passive components to optic fiber communications. These devices divide route, or combine multiple optical signals, and are highly desirable in fiber optic network maintenance, administration and testing.

2.1 Couplers

Fiber optic couplers either split optical signals into multiple paths or combine multiple signals on one path. Optical signals are more complex than electrical signals, making optical couplers trickier to design than their electrical counterparts. Like electrical currents, a flow of signal carriers, in this case pho-

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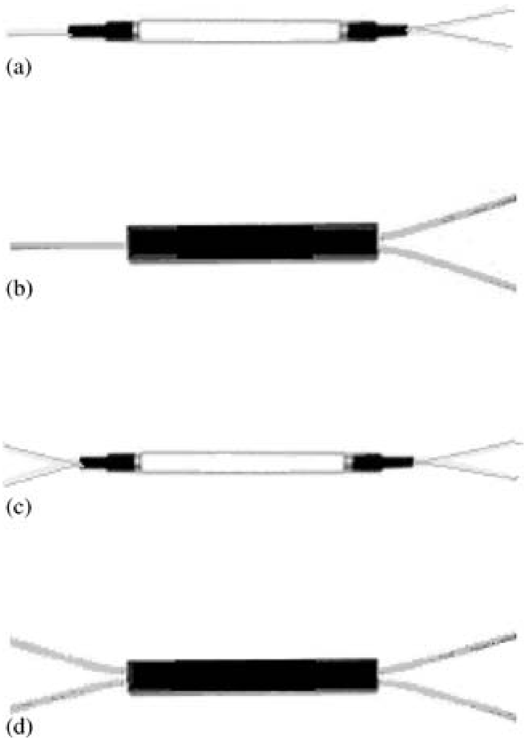

Fig. 1. a 1 × 2, 900 micron, medium duty coupler **b** 1 × 2, 3 mm jacketed, heavy duty coupler **c** 2 × 2, 900 micron, medium duty coupler **d** 2 × 2, 3 mm jacketed, heavy duty coupler

tons, comprise the optical signal. An optical signal does not flow through the receiver to the ground. But, a detector absorbs the signal flow at the receiver. Multiple receivers, connected in a series, would receive no signal past the first receiver, which would absorb the entire signal. Thus, multiple parallel optical output ports must divide the signal between the ports, reducing its magnitude. The number of input and output ports, expressed as an $N \times M$ configuration, characterizes a coupler. The letter N represents the number of input fibers, and M represents the number of output fibers. Fused couplers can be made in any configuration, but they commonly use multiples of two (1 × 2, 2 × 2, 2 × 4, 4 × 4, etc.). Figures 1a,b and 1c,d depict the simple 1 × 2 and 2 × 2 devices, respectively.

2.2 Splitters

The simplest couplers are the fiber optic splitters. These devices possess at least three ports but may have more than 32 for more complex devices. Figure 2 illustrates a simple 3-port device, also called a tee coupler. It can be thought of as a directional coupler. One fiber is called the common fiber, while the other two fibers may be called input or output ports. The coupler manufacturer determines the ratio of the distribution of light between the two output legs. Popular splitting ratios include 50–50%, 10–90%, 5–95% and 1–99%; however, almost any custom value can be achieved. (These values are sometimes specified in dB values.) For example, using a 10–90% splitter with a 50 μW light source, the outputs would equal 5 μW and 45 μW. However, excess loss hinders that performance. All couplers and splitters share this parameter. Excess loss assures that the total output is never as high as the input. Loss figures range from 0.05 dB to 2 dB for different coupler types. An interesting, and unexpected, property of splitters is that they are symmetrical. For example, if the same coupler injected 50 μW into the 10% output leg, only 5 μW would reach the common port.

2.3 Wavelength division multiplexers

The most popular type of fused fiber coupler is the wavelength division multiplexers (WDM) with single mode fiber. An interferometric action forms the WDM within the fused mixing region. Like an interferometer, this causes a sinusoidal response as the length increases. WDMs operate at two specific wavelengths. Adjusting the minimum of the sinusoid to correspond to the first wavelength of interest and the maximum of the sinusoid to correspond to the second wavelength of interest forms a WDM. The fiber optic industry first deployed single wavelength transmission links. As requirements changed, the industry responded with wavelength division multiplexing, which sends two distinct signals per fiber, doubling transmission capacity. Similar to a simple splitter, WDMs typically have a common leg and a number of input or output legs. Unlike the splitter, however, they have very little insertion loss. They do have the same range of excess loss. Two important considerations in a WDM device are crosstalk and channel separation. Crosstalk, also called directivity, refers to separation of demultiplexed channels. Each channel should appear only at its intended port.

The crosstalk specification expresses how well a coupler maintains this port-to-port separation. Channel separation describes a coupler's ability to distinguish wavelengths. In most couplers, the wavelengths must be widely separated allowing light to travel in either direction without the penalty found in splitters. WDMs allow multiple independent data streams to be sent over one fiber. The most common WDM system uses two wavelengths, although four or more wavelength systems are available. Figure 3 illustrates two WDMs permitting two streams of data to be carried on a single fiber. The type of data does not

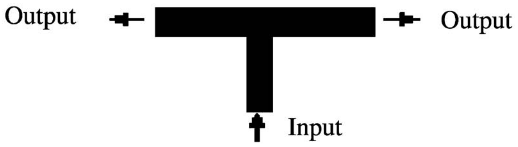

Fig. 2. A typical tee coupler

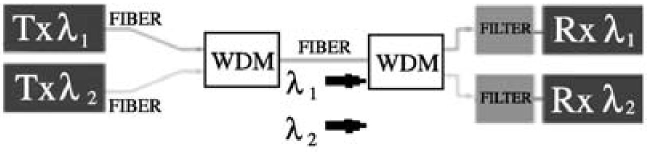

Fig. 3. A WDM application

matter. Both signals could be video signals or higher speed data signals at 2.488 Gb/s. The configuration shown is unidirectional. Bidirectional configurations are also available.

3 Optical characteristics of passive components

For a particular model of the single mode couplers and WDMs, the specifications of characteristics are presented in Tables 1 and 2, respectively. Which is taken form a optical communication manufacturing factory located at Science-based Industrial Park in Hsinchu, Taiwan, devoted to the optical fiber component module products, such as Collimator, Isolator, Coupler, DWDM, CWDM, and EDFA, etc. Many passive components transform one input signal containing many channels into the corresponding number of output signals containing only one channel, or vice versa. It is desirable to characterize all output channels in parallel, in one measurement. Further, polarization dependent loss (PDL) and insertion loss (IL) are the most critical quality characteristics of these passive components used in telecommunications networks. In the following, polarization dependent loss and insertion loss, and their effects in fiber-optic transmission links are described.

3.1 Polarization dependent loss

Polarization dependent loss (PDL) is a measure of the peak-to-peak difference in transmission of an optical component or system with respect to all possible states of polarization. Components in fiber-optic telecommunication networks must be insensitive to changes in the polarization state of the signal, a requirement that is particularly challenging for surface-relief diffraction gratings since those with metallic coatings tend to polarize the light incident on them. It is the ratio of the maximum and the minimum transmission of an optical device with respect to all polarization states. The definition of polarization dependent loss is defined in the following and which is expressed in decibels (dB),

$$\mathrm{PDL}_{\mathrm{dB}} = 10 \times \log_{10}\left(\frac{P_{Max}}{P_{Min}}\right), \tag{1}$$

where P_{Max} and P_{Min} are the maximum and minimum in transmission for light with various states of polarization, respectively. Figure 4 shows the effect of applying all possible states of polarization to an optical component. The polarization of the constant and fully polarized input signal is varied. As the polarization of the incident light varies, the output signal shows a corresponding change in power.

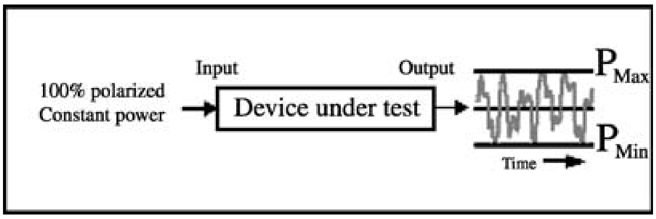

Fig. 4. PDL test of passive optical components

3.2 Insertion loss

Insertion loss (IL) is the measure of reduction in signal magnitude caused by inserting a component, such as a connector, into a previously connected optical path. This measurement allows for analysis of the impact of inserting a single optical component into a system, sometimes called "calculating a loss budget." Insertion loss is measured in decibels (dB) and defined as follows:

$$\text{Insertion Loss}_{\mathrm{dB}} = -10 \times \log_{10}\left(\frac{P_1}{P_0}\right), \tag{2}$$

where P_0 is the initial measured power and P_1 is the measured power after the assembly under test is introduced. The following Table 3 shows typical dB losses represented as percentage loss $((P_0 - P_1)/P_0)$.

In addition, more commonly called attenuation, insertion loss is the loss of signal power between two points. Items that lead to signal loss are excessive cable length, temperature, humidity, and

Table 1. Specifications for single mode coupler (50–50% coupling ratio)

Parameter	Specifications
Operating wavelength	1310 or 1550 nm
Insertion loss	≦ 3.40 dB
Polarization dependent loss	≦ 0.10 dB
Typical excess loss	≦ 0.07 dB
Directivity	≧ 50 dB
Thermal stability	≦ 0.25 dB
Operating temperature	−40 °C to +85 °C
Storage temperature	−50 °C to +85 °C

Table 2. Specifications for WDM

Parameter	Premium	A Grade
Operating wavelength	1310/1550 nm	
Insertion loss	≦ 0.70 dB	≦ 1.00 dB
Polarization dependent loss	≦ 0.08 dB	≦ 0.15 dB
Isolation	≧ 35 dB	≧ 32 dB
Wavelength bandwidth	15 nm	
Typical excess loss	≦ 0.07 dB	≦ 0.07 dB
Directivity	≧ 60 dB	
Thermal stability	≦ 0.25 dB	
Operating temperature	−40 °C to +85 °C	
Storage temperature	−50 °C to +85 °C	

Table 3. Typical dB losses

Loss (dB)	Loss (%)
0.1	2.3
0.2	4.5
0.3	6.7
0.4	8.8
0.5	10.9
0.6	12.9

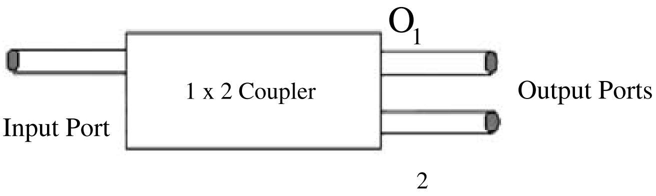

Fig. 5. The 1 × 2 coupler with output powers O_1 and O_2

Table 4. Insertion losses for single mode couplers with coupling ratio form 50–50% to 1–99%

Coupling ratio (%)	Insertion loss (dB)
50/50	3.4/3.4
40/60	4.4/2.5
30/70	5.6/1.8
20/80	7.4/1.1
10/90	10.8/0.6
5/95	14.6/0.4
1/99	21.5/0.2

excess return loss. All devices (such as splitters, amps, etc.) that you add to a cable line have insertion loss. It depends on the coupling ratio and any imperfections in the coupler. Figure 5 shows the 1 × 2 coupler with output powers O_1 and O_2. Coupling ratio (or Split ratio) is the ratio of optical power from one output port to the total output power expressed as a percent, defined as below and denoted by C_R. For a 1 × 2 coupler or WDM with output powers O_1 and O_2, and O_i representing both output powers,

$$C_R(\%) = \frac{O_i}{O_1 + O_2} \times 100\% \,. \tag{3}$$

However, different spilt ratio will correspond to different insertion loss. Table 4 shows the typical insertion losses for modern single-mode couplers with coupling ratio form 50–50% to 1–99%. For example, most common spilt ratio available is 50%, i.e., $O_1 = O_2$. And such a 50–50- coupling ratio a reasonable insertion loss is smaller than 3.4 dB. Furthermore, insertion loss limits always depend on the customer's application. Generally, an insertion loss of 0.50 dB per connection is acceptable. Fiber pulse has imposed a stricter 0.30 dB max on all connector terminations. Doing this involves a high level of control of material selection and processes. The diamond connector range which fiber pulse terminate has a maximum insertion loss of 0.10 dB.

4 Manufacturing capability of couplers and WDMs

The development and testing of new optical components has become more challenging and complex. For example, higher data transmission rates (10 Gbit/sec or 40 Gbit/sec) require shorter pulse duration. In the frequency domain, this results in a broader spectrum. High transmission quality requires broader spectral areas of low polarization dependent loss, to avoid attenuation variations for different spectral components. In addition, due to the rapid growth in the fiber-optic technology market, manufacturers must ramp up production volumes by increasing manufacturing capacity, and by shortening test time while not compromising test accuracy.

4.1 Process capability indices

When focusing on the capability of a process, there are mainly two characteristics of importance, the process location in relation to its target value and the process spread. The closer the process output is to the target value and the smaller the process spread, the more capable is the process. During the last decade, numerous process capability indices (PCIs), including C_p, C_{PU}, C_{PL}, and C_{pk} (see Kane [1], Chan, Cheng and Spiring [2], Pearn, Kotz, and Johnson [3]), have been proposed in the manufacturing industry to provide numerical measures on process performance, which are effective tools for quality/reliability assurance. The larger the value of a process capability index, the more capable is the process. The C_p and C_{pk} indices are appropriate measures for normal processes with two-sided manufacturing specifications, C_{PU} and C_{PL} have been designed particularly for processes with one-sided manufacturing specifications. These indices are defined in the following:

$$C_p = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma} \,,$$

$$C_{PU} = \frac{\mathrm{USL} - \mu}{3\sigma} \,, \quad C_{PL} = \frac{\mu - \mathrm{LSL}}{3\sigma} \,,$$

$$C_{pk} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sigma}, \frac{\mu - \mathrm{LSL}}{3\sigma}\right\} .$$

Process capability index is a function of process parameters and manufacturing specifications, which measures the capability of reproducing products meeting the specifications. For normally distributed processes with one-sided specification limit USL, or LSL, the process yield is the following, where Z follows the standard normal distribution $N(0, 1)$:

$$P(X < \mathrm{USL}) = P\left(\frac{X - \mu}{\sigma} < \frac{\mathrm{USL} - \mu}{\sigma}\right) = \Phi(3C_{PU}) \,, \tag{4}$$

$$P(X > \mathrm{LSL}) = P\left(\frac{X - \mu}{\sigma} > \frac{\mathrm{LSL} - \mu}{\sigma}\right) = \Phi(3C_{PL}) . \tag{5}$$

For convenience of presentation, we let C_I denote either C_{PU} or C_{PL}. Therefore, the corresponding nonconforming units in parts per million (NCPPM) for a well-controlled normal process can be calculated as: $\mathrm{NCPPM} = 10^6 \times [1 - \Phi(3C_I)]$. Consequently, the production yield for usual existing processes should target no more than 88 PPM, noting that $\mathrm{NCPPM} \leqslant 100$ PPM is the common standard used in most microelectronic industries for products with one-sided specification. The production yield for newly set up processes on safety, strength, or with critical parameters, however, should target no more than 0.8 PPM, a more stringent requirement set for possible mean shift or variation change.

4.2 Methodology development

Capability measure for processes with single characteristic has been investigated extensively. But, capability measure for processes with multiple characteristics is comparatively neglected. For processes with multiple characteristics, a simple measure by taking the minimum of the measure of each single characteristic has been considered. Bothe [4] considered a ν-characteristic process with ν yield measures $P_1, P_2, \ldots, P_n$, and suggested that he overall process yield is measured as $P = \min\{P_1, P_2, \ldots, P_\nu\}$. We note that this approach does not reflect the real situation accurately. Suppose the process has five characteristics ($\nu = 5$), with equal characteristic yield measures $P_1 = P_2 = P_3 = P_4 = P_5 = 99.865\%$, then the overall process yield is calculated as $P = \min\{P_1, P_2, P_3, P_4, P_5\} = 99.865\%$ (or 1350 NCPPM). For mutually independent characteristics, the actual overall process yield should be calculated as $P = P_1 \times P_2 \times \ldots \times P_5 = 99.3269\%$ (or 6731 NCPPM), which is significantly less than the calculated one. From the definition of one-sided yield index in Eq. 4, the process yield index can be rewritten as:

$$C_{PU} = \frac{1}{3}\Phi^{-1}\left\{\Phi\left(\frac{\mathrm{USL}-\mu}{\sigma}\right)\right\}, \tag{6}$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal distribution $N(0, 1)$, and Φ^{-1} is the inverse function of Φ. For process with multiple quality characteristics, we propose the following overall capability index, referred to as C_{PU}^T:

$$C_{PU}^T = \frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{\nu}\Phi\left(3C_{PUj}\right)\right\}, \tag{7}$$

where C_{PUj} denotes the C_{PU} value of the jth characteristic for $j = 1, 2, \ldots, \nu$, and ν is the number of characteristics. The index, C_{PU}^T, may be viewed as a generalization of the single characteristic yield index, C_{PU}. Given $C_{PU}^T = c$, we have

$$\left\{\prod_{j=1}^{\nu}\Phi\left(3C_{PUj}\right)\right\} = \Phi(3c).$$

A one-to-one correspondence relationship between the index C_{PU}^T and the overall process yield P can be established as:

$$P = \prod_{j=1}^{\nu} P_j = \prod_{j=1}^{\nu}\Phi\left(3C_{PUj}\right) = \Phi(3C_{PU}^T). \tag{8}$$

Hence, the new index C_{PU}^T provides an exact measure on the overall process yield. For example, if $C_{PU}^T = 1.00$, then the entire process yield would be exactly 99.865%. Table 5 displays various commonly used capability requirement and the corresponding overall process yield associated with NCPPM. For process with ν characteristics, if the requirement for the overall process capability is $C_{PU}^T \geqslant c_o$, a sufficient condition (which is minimal) for the requirement to each single characteristic can be obtained by the followings. Let c' be the minimum C_{PU} required for each single characteristic, then

$$\frac{1}{3}\Phi^{-1}\left\{\prod_{\mathrm{j}=1}^{\nu}\Phi\left(3\,C_{PUj}\right)\right\} \geqslant \frac{1}{3}\Phi^{-1}\left\{\prod_{\mathrm{j}=1}^{\nu}\Phi\left(3\,c'\right)\right\}.$$

Table 5. The corresponding process yield and NCPPM for various C_{PU}^T values

C_{PU}^T	Process yield	NCPPM
1.00	0.9986501020	1350
1.25	0.9999115827	88
1.33	0.9999669634	33
1.45	0.9999931931	6.81
1.50	0.9999966023	3.40
1.60	0.9999992067	0.793
1.67	0.9999997278	0.272
2.00	0.9999999990	0.001

Hence, if

$$C_{PU}^T = \frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{\nu}\Phi\left(3C_{PUj}\right)\right\} \geqslant c_0,$$

then we have

$$c' \geqslant \frac{1}{3}\Phi^{-1}\left(\sqrt[\nu]{\Phi(3c_0)}\right).$$

Thus, the overall process capability requirement is $C_{PU}^T \geqslant c_o$ would be satisfied, if the capability of jth characteristic satisfies $C_{PUj} \geqslant c_L$ for all $j = 1, 2, \ldots, \nu$, where the lower bound c_L on each C_{PUj} can be calculated, respectively, as

$$C_L = \frac{1}{3}\Phi^{-1}\left(\sqrt[\nu]{\Phi(3c_0)}\right).$$

Table 6 displays the lower bound c_L of C_{PUj}, if the requirement of the overall process capability C_{PU}^T are 1.00, 1.25, 1.45 and 1.60 for $\nu = 1(1)15$ characteristics. For example, if c_0 is set to be 1.25 with $\nu = 5$, i.e., the overall process yield is set to be no less than 0.9986501. The overall capability requirement $C_{PU}^T \geqslant 1.00$ would be satisfied, if each single characteristic yield is no less than $(0.9986501)^{1/5} = 0.9997299$ (equivalent to

Table 6. Lower bound of various capability levels for multiple characteristics

c'	c_L			
	1.000	1.250	1.450	1.600
1	1.000	1.250	1.450	1.600
2	1.068	1.307	1.500	1.646
3	1.107	1.339	1.528	1.672
4	1.133	1.361	1.548	1.690
5	1.153	1.379	1.564	1.704
6	1.170	1.392	1.576	1.716
7	1.183	1.404	1.586	1.725
8	1.195	1.414	1.595	1.734
9	1.205	1.423	1.603	1.741
10	1.214	1.431	1.610	1.747
11	1.222	1.438	1.617	1.753
12	1.230	1.444	1.622	1.759
13	1.236	1.450	1.628	1.763
14	1.243	1.455	1.632	1.768
15	1.248	1.460	1.637	1.772

270 NCPPM), and the capability for all the five characteristics is the following, for $j = 1, 2, \dots, 5$

$$C_{PUj} = \frac{1}{3}\Phi^{-1}\left(\sqrt[5]{\Phi(3)}\right) = 1.153321.$$

Similarly, if the characteristics of the product is the smaller-the-better, then the C^T_{PL} can be constructed in the same way.

5 The bootstrap confidence bound

In practice, sample data must be collected in order to calculate these indices since the process mean μ and standard deviation σ usually unknown. Current practices of measuring manufacturing capability by only evaluating the point estimates of the capability indices have been criticized since it ignores sampling errors. Therefore, the decisions made by concluding the capability measures from the sample estimates are unreliable.

Efron [5, 6] introduced a nonparametric, computational intensive but effective estimation method called the "Bootstrap," which is a data based simulation technique for statistical inference. One can use the nonparametric bootstrap method to estimate the sampling distribution of a statistic, while assuming only that the sample is a representative of the population from which it is drawn, and that the observations are independent and identically distributed. The merit of the nonparametric bootstrap approach is that it does not rely on any assumptions regarding the underlying distribution. Rather than using distribution frequency tables to compute approximate p probability values, the bootstrap method generates a unique sampling distribution based on the actual sample rather than the analytic methods.

Efron and Tibshirani [7] developed three types of bootstrap confidence interval, including the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), and the biased corrected percentile bootstrap confidence interval (BCPB). Franklin and Wasserman [8] investigated the lower confidence bounds for the capability indices, C_p, C_{pk} and C_{pm} using these three bootstrap methods. Some simulations were conducted, and a comparison was made among the three bootstrap methods based on the parametric estimates. The simulation results indicate that for normal processes the bootstrap confidence limits perform equally well (see Chou, Owen and Borego [9], Bissell [10], and Boyles [11]). And for non-normal processes the bootstrap estimates performed significantly better than other methods. We performed extensive computational experiments and apply the three bootstrap methods to find the lower confidence bounds of the overall yield measure C^T_{PU}. The results showed that PB method significantly outperformed SB and BCPB methods.

Bootstrap sampling is equivalent to sampling (with replacement) from the empirical probability distribution function. Efron and Tibshirani [7] indicated that a rough minimum of 1000 bootstrap resamples is usually sufficient to compute reasonably accurate confidence interval estimates. We apply the percentile bootstrap (PB) method to the overall process yield measure C^T_{PU} (or C^T_{PL}) to obtain the confidence bounds. In order to obtain more reliable results, $B = 10,000$ bootstrap resamples are taken and these 10,000 bootstrap estimates of C^T_{PU} are calculated and ordered in ascending order. The notations $\hat{C}^T_{PU}$ and $\hat{C}^{T*}_{PU}(i)$ will be used to denote the estimator of an overall yield index and the associated ordered bootstrap estimates. For instance, $\hat{C}^{T*}_{PU}$ (1)is the smallest of the 10,000 bootstrap estimates of C^T_{PU}.

For each single characteristic, the C_{PUj} values can be estimated by their natural estimators $\hat{C}_{PUj} = (USL_j - \bar{x}_j)/s_j$, $j = 1, 2, \dots, \nu$, where $\bar{x}_j$ and s_j are the sample mean and the sample standard deviation of the jth characteristic, respectively. Thus, the bootstrap estimates of C^T_{PU} are defined as:

$$\hat{C}^T_{PU} = \frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{\nu}\Phi\left(3\hat{C}_{PUj}\right)\right\}. \tag{9}$$

From the ordered collection of $\hat{C}^{T*}_{PU}(i)$, the α percentage and the $(1-\alpha)$ percentage points are used to obtain $(1-2\alpha)\%$ PB confidence interval for C^T_{PU} is $[\hat{C}^{T*}_{PU}(\alpha B), \hat{C}^{T*}_{PU}((1-\alpha)B)]$. While a lower $(1-\alpha)\%$ confidence bound can be constructed by using only the lower limit

$$LCB = \hat{C}^{T*}_{PU}(\alpha B). \tag{10}$$

That is, for a 95% LCB for C^T_{PU} based on the PB method with B = 10,000 would be obtained as $\hat{C}^{T*}_{PU}(500)$. This approach makes it feasible for the engineers to perform capability testing using the calculated $\hat{C}^T_{PU}$.

6 Manufacturing capability calculations and control

For efficient capability control and analysis, some recent research suggested various graphical methods to guide the directions for process improvement [12]. One of those graphical methods is called process capability plot, which conveys critical information regarding process spread and process centering. Process capability plots have been proposed and discussed by Gabel [13], Boyles [11, 14], and Deleryd and Vannman [12]. Gable [13] refers to a process capability plot as the process performance chart, which is simply a plot of the spread of a process versus its mean. From the yield index defined early, we adapt the concept of process capability plots but focus on the process parameters (d^*, σ^*)

$$C_{PU} = \frac{\mathrm{USL} - \mu}{3\sigma} = \frac{d^*}{\sigma^*}. \tag{11}$$

Using a process capability plot, we are able to control the closeness to the target and process spread in a more efficient way than using process capability index alone. If the exact values of μ and σ are known, then the (d^*, σ^*)-plot can easily be applied. If the corresponding (d^*, σ^*) value is inside the capability region, then the process is defined to be capable. Otherwise, the process is defined as incapable. In practice, we never know the true

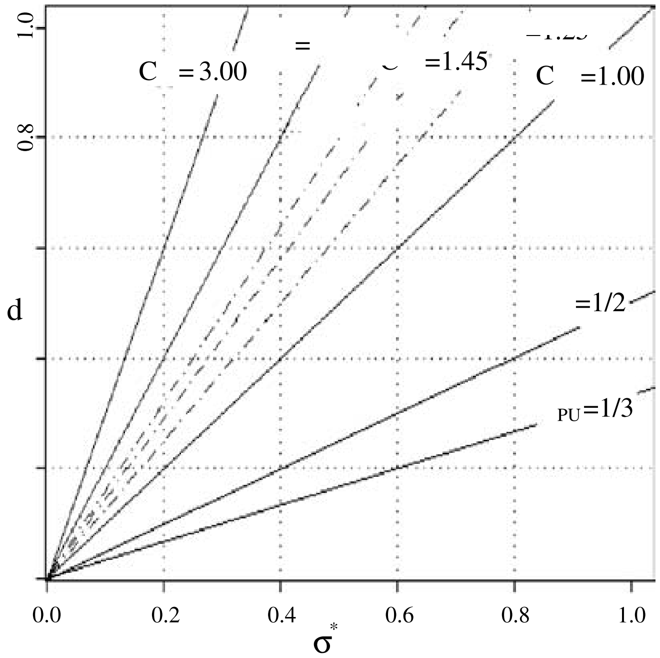

Fig. 6. (d^*, σ^*) plot control chart

values of d^* and σ^*. Hence those parameters must be estimated, and the sampling errors must also be considered.

Chou and Owen [15] showed that under normality assumption the estimator $\hat{C}_{PU}$ and $\hat{C}_{PL}$ are distributed as $(3\sqrt{n})^{-1}t_{n-1}(\delta)$, where $t_{n-1}(\delta)$ is distributed as the noncentral t distribution with $n-1$ degrees of freedom and noncentrality parameter $\delta = 3\sqrt{n}C_{PU}$ and $\delta = 3\sqrt{n}C_{PL}$, respectively. A $100(1-\alpha)\%$ lower confidence bound L_C for C_{PU} satisfies $\Pr(C_{PU} \geqslant L_C) = 1-\alpha$. It can be written as:

$$\Pr\left(\frac{\mathrm{USL}-\mu}{3\sigma} \geqslant L_C\right) = \Pr\left(t_{n-1}(\delta_1) \leqslant t_1\right) = 1-\alpha, \tag{12}$$

where $t_1 = 3\hat{C}_{PU}\sqrt{n}$ and $\delta_1 = 3\sqrt{n}L_C$. Thus, we can obtain the lower confidence bound (LCB) by solving the cumulative distribution function (CDF) of noncentral t distribution with $n-1$ degrees of freedom and noncentrality parameter $\delta_1 = 3\sqrt{n}L_C$. Therefore, to obtain more reliable estimations, we apply this procedure to construct the lower confidence bound of the yield indices C_{PU} for each characteristic. Based on the lower confidence bound, we could plot in the (d^*, σ^*)-plot chart to check if the process output is off target or that the process spread is too large.

Using σ^* as the x-axis, d^* as the y-axis, processes with multiple characteristics having different characteristic specification limits can be plotted simultaneously on a single chart. We call this control chart the multi-characteristic process capability analysis (MCPCA) chart for one-sided specifications. In Fig. 6, processes with same ratios of d^* and σ^*, will be plotted on the same line with the same slope. The larger the slope of the line is, the lower the NCPPM of the process is. For the particular 1 × 2 single mode couplers, an insertion loss of 0.34 dB per connection and the polarization dependent loss is no more than 0.1 dB is acceptable. WDMs have imposed a stricter 0.70 dB maximum of insertion loss and polarization dependent loss must be smaller than 0.08 dB on all connector terminations.

Table 7. Single mode couplers data of 100 measurements for PDL and IL

Couplers IL (dB)				Couplers PDL (dB)			
3.082	3.156	3.162	3.115	0.083	0.083	0.083	0.082
3.133	3.136	3.191	3.120	0.085	0.087	0.083	0.081
3.115	3.201	3.102	3.165	0.074	0.085	0.078	0.082
3.247	3.161	3.126	3.175	0.077	0.074	0.081	0.074
3.162	3.169	3.201	3.196	0.086	0.081	0.079	0.076
3.165	3.192	3.268	3.137	0.078	0.081	0.079	0.081
3.197	3.223	3.121	3.221	0.083	0.084	0.085	0.080
3.023	3.107	3.163	3.210	0.074	0.081	0.079	0.084
3.104	3.223	3.193	3.065	0.090	0.078	0.077	0.076
3.130	3.126	3.101	3.142	0.075	0.082	0.078	0.084
3.036	3.148	3.191	3.189	0.085	0.079	0.082	0.079
3.150	3.198	3.173	3.145	0.081	0.076	0.084	0.078
3.088	3.131	3.061	3.116	0.082	0.076	0.082	0.079
3.083	3.131	3.151	3.208	0.076	0.083	0.080	0.082
3.178	3.105	3.055	3.168	0.083	0.075	0.077	0.083
3.227	3.188	3.122	3.241	0.072	0.079	0.077	0.088
3.151	3.181	3.221	3.128	0.081	0.085	0.077	0.081
3.098	3.179	3.235	3.230	0.079	0.075	0.082	0.078
3.214	3.035	3.147	3.259	0.081	0.080	0.078	0.071
3.159	3.159	3.209	3.167	0.085	0.077	0.086	0.079
3.219	3.123	3.186	3.178	0.080	0.076	0.081	0.075
3.268	3.180	3.230	3.181	0.078	0.077	0.082	0.078
3.167	3.172	3.245	3.164	0.080	0.084	0.074	0.089
3.199	3.149	3.152	3.112	0.081	0.083	0.075	0.082
3.142	3.221	3.109	3.165	0.081	0.088	0.078	0.087

Table 8. WDMs data of 100 measurements for PDL and IL

WDMs IL (dB)				WDMs PDL (dB)			
0.692	0.627	0.665	0.594	0.070	0.073	0.070	0.068
0.621	0.615	0.609	0.552	0.078	0.071	0.076	0.070
0.639	0.601	0.643	0.625	0.078	0.078	0.077	0.075
0.591	0.532	0.594	0.603	0.073	0.073	0.071	0.069
0.589	0.574	0.619	0.632	0.072	0.078	0.071	0.077
0.577	0.584	0.544	0.592	0.075	0.072	0.073	0.070
0.589	0.606	0.613	0.596	0.071	0.074	0.074	0.075
0.597	0.653	0.547	0.600	0.075	0.072	0.078	0.074
0.573	0.577	0.644	0.555	0.071	0.079	0.074	0.077
0.603	0.623	0.621	0.615	0.074	0.072	0.072	0.078
0.556	0.584	0.632	0.599	0.075	0.073	0.074	0.072
0.608	0.625	0.555	0.582	0.076	0.074	0.072	0.075
0.532	0.560	0.583	0.629	0.071	0.073	0.075	0.071
0.605	0.578	0.614	0.622	0.072	0.075	0.072	0.066
0.558	0.543	0.604	0.579	0.069	0.072	0.069	0.070
0.596	0.602	0.602	0.656	0.074	0.078	0.074	0.072
0.646	0.614	0.538	0.606	0.070	0.081	0.070	0.069
0.587	0.599	0.616	0.569	0.077	0.074	0.073	0.072
0.576	0.663	0.644	0.604	0.075	0.075	0.074	0.070
0.611	0.575	0.632	0.631	0.073	0.071	0.067	0.079
0.581	0.634	0.591	0.627	0.074	0.074	0.073	0.074
0.583	0.644	0.615	0.626	0.071	0.073	0.078	0.077
0.620	0.638	0.558	0.550	0.067	0.071	0.078	0.072
0.538	0.615	0.579	0.588	0.076	0.070	0.070	0.072
0.590	0.593	0.564	0.628	0.075	0.069	0.076	0.076

Table 9. Calculations for process capability of the couplers and WDMs

Code	Characteristic	USL	$\bar{x}$	$\hat{d}^*$	$\hat{\sigma}^*$	$\hat{C}_{PUj}$	L_C
C1	Couplers IL	3.40	3.160	0.240	0.1561	1.538	1.348
C2	Couplers PDL	0.10	0.080	0.020	0.0117	1.709	1.501
W1	WDM IL	0.70	0.601	0.099	0.0957	1.034	0.901
W2	WDM PDL	0.08	0.073	0.007	0.0090	0.778	0.671

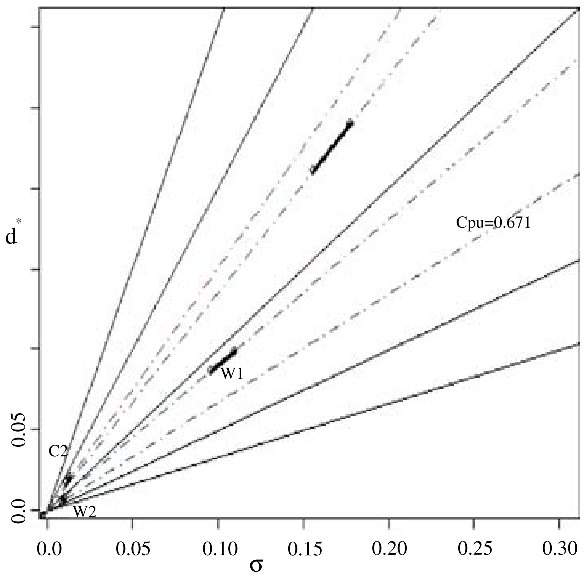

Fig. 7. (d^*, σ^*) plot control chart

We take two samples each of size 100, for the single mode couplers and WDMs from a stable (under statistical control) process in the factory, and measure the two critical product quality characteristics, the insertion loss (IL), and the polarization dependent loss (PDL). These 100 measurements are displayed in Tables 7 and 8, respectively.

For the characteristics IL and PDL of couplers and WDMs, under the Shapiro-Wilk test for normality, the result confirms that all the p-value > 0.1. That is, it is reasonable to assume that the process data collected from the factory are normally distributed. The upper specification limit, the calculated sample mean, location departure, sample standard deviation, the estimated $\hat{C}_{PUj}$ and lower confidence bound L_C for the couplers and WDMs are summarized in Table 9.

Characteristic defect control

Four pairs of the estimated $\hat{d}^*$ and $\hat{\sigma}^*$ values are plotted on the chart. This chart clearly shows the status of each single product characteristic. From the (d^*, σ^*)-plot chart, we can conclude quickly whether those product characteristics are capable or not depending on the plotted points and the yield line. The chart displays the quality status (defect) and directions for improvement for each single characteristic. We can make the following conclusions and recommendations for those four characteristics

Table 10. Calculations for overall yield index

Characteristic	$\hat{C}^T_{PU}$	NCPPM	LCB	NCPPM
Coupler	1.528	2.28	1.385	16.26
WDM	0.732	14046	0.649	25767

The plotted point C1 corresponding to characteristic IL of coupler is between two yield line corresponding to $C_{PU} = 1.25$ and $C_{PU} = 1.45$. For the lower confidence bound (L_C), 1.348, the corresponding NCPPM is 26.27. This shows that the process is "satisfactory" for characteristic IL of coupler.

The plotted point C2 corresponding to characteristics PDL of coupler is above the yield line $C_{PU} = 1.45$, and the lower bound of which show that $\hat{C}_{PU}$ are greater than the upper bound of C_{PU} value is 1.501, which is equivalent to 3.35 NCPPM of the process. Stringent control for characteristic C2 could be reduced since the process is "excellent."

The plotted points W1 and W2 corresponding to characteristics IL and PDL of WDM are below the yield line $C_{PU} = 1.00$, which show both processes, W1 and W2, are "incapable." In fact, the lower confidence bounds of C_{PU} are 0.901 and 0.671, which correspond to 3436 and 22 057 NCPPM, respectively. Thus, both characteristics IL and PDL of WDM are candidates for high-priority quality improvement effort focus. Under the six-sigma program, the quality improvement effort could focus on the reduction of process variability and the increase of the deviation from the upper specification to improve the process quality.

Overall process yield analysis

The sample estimates of C^T_{PU} and the percentile bootstrap method lower confidence bound of C^T_{PU} for the single mode coupler and WDM can be calculated by (9) and (10), respectively. The results are summarized in Table 10.

Table 10 displays the manufacturing capabilities and its corresponding NCPPM for the coupler and WDM processes using the estimated $\hat{C}^T_{PU}$ values (uncorrected) and the lower confidence bounds LCB (corrected). The modified C^T_{PU} obtained using percentile bootstrap method is certainly more reliable than the estimated $\hat{C}^T_{PU}$ index values (an approach widely used in current industrial applications), since the sampling errors are considered in the LCB approach. In fact, as the sample estimate $\hat{C}^T_{PU}$ may overestimate the true capability (overall process yield), it conveys unreliable and misleading information, which should be avoided in factory applications.

7 Conclusion

The couplers and wavelength division multiplexers have been widely used in high-speed, high-volume image data transmission systems to provide sufficient bandwidth and smaller channel spacing for greater throughput. Process capability indices have been widely used in the manufacturing industry providing numerical measures on process precision, process accuracy, and process performance. Capability measurements for processes with single characteristics has been investigated extensively, but is comparatively neglected for processes with multiple characteristics. In this paper, we developed a method for measuring manufacturing capability of processes producing couplers and wavelength division multiplexers, where multiple quality characteristics are involved. The proposed method measures the process capability of reproducing product items meeting the manufacturing specifications, including the polarization dependent loss and the insertion loss characteristics, which are critical in fiber-optic transmission quality. The proposed approach provided reliable information, and makes it possible for the engineers to perform single characteristic defect control and overall process yield analysis for products with multiple characteristics.

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