A Lagrangean Relaxation-Based Approach for Routing and Wavelength Assignment in Multigranularity Optical WDM Networks

Steven S. W. Lee, Maria C. Yuang, Po-Lung Tien, and Shih-Hsun Lin

***Abstract*—Optical wavelength-division multiplexed (WDM) networks often include optical cross-connects with multigranularity switching capability, such as switching on a single lambda, a waveband, or an entire fiber basis. In addition, it has been shown that routing and wavelength assignment (RWA) in an arbitrary mesh WDM network is an NP-complete problem. In this paper, we propose an efficient approximation approach, called Lagrangean relaxation with heuristics (LRH), aimed to resolve RWA in multigranularity WDM networks particularly with lambda and fiber switches. The task is first formulated as a combinatorial optimization problem in which the bottleneck link utilization is to be minimized. The LRH approach performs constraint relaxation and derives a lower-bound solution index according to a set of Lagrangean multipliers generated through subgradient-based iterations. In parallel, using the generated Lagrangean multipliers, the LRH approach employs a new heuristic algorithm to arrive at a near-optimal upper-bound solution. With lower and upper bounds, we conduct a performance study on LRH with respect to accuracy and convergence speed under different parameter settings. We further draw comparisons between LRH and an existing practical approach via experiments over randomly generated and several well-known large sized networks. Numerical results demonstrate that LRH outperforms the existing approach in both accuracy and computational time complexity, particularly for larger sized networks.**

***Index Terms*—Combinatorial optimization problem, Lagrangean relaxation, multigranularity switching capabilities, routing and wavelength assignment (RWA), wavelength-division multiplexing (WDM).**

I. Introduction

With advances in optical wavelength-division-multiplexing (WDM) technologies [1] and its potential of providing virtually unlimited bandwidth, optical WDM networks have been widely recognized as the dominant transport infrastructure for future Internet backbone networks. To maintain high scalability and flexibility at low cost, WDM networks often include switching devices with different wavelength conversion powers [2], [3] (e.g., no, limited- or full-range), and multigranularity switching capability [4], [5]. In particular, examples of multigranularity optical crossconnects (MG-OXCs) include switching on a single lambda, a waveband (i.e., multiple lambdas), an entire fiber, or a combination of the above.

One major traffic engineering challenge in such WDM networks has been the routing and wavelength assignment (RWA) problem [3], [6]. The problem deals with routing and wavelength assignment between source and destination nodes subject to the wavelength-continuity constraint [7] in the absence of wavelength converters. It has been shown that RWA is an NP-complete problem [7]. Numerous approximation algorithms [3], [6] have been proposed with the aim of balancing the tradeoff between accuracy and computational time complexity. In general, some algorithms [8], [9] focused on the problem in the presence of sparse, limited, or full-range wavelength converters. Some others made an effort to either reduce computational complexity by solving the routing and wavelength assignment subproblems separately [7], or increase accuracy by considering the two subproblems [10] jointly. However, with the multigranularity switching capability taken into consideration, most existing algorithms become functionally or economically unviable.

In this paper, our aim is to resolve the RWA problem in multigranularity WDM networks particularly with fiber switch capable (FSC-OXC) and lambda switch capable (LSC-OXC) devices. It is worth mentioning that, as shown in Fig. 1, an MG-OXC node is logically identical to an individual FSC-OXC node in conjunction with an external separated LSC-OXC node. For ease of illustration, we adopt the separated node form throughout the rest of the paper.

The problem is in short referred to as RWA$^+$. To tackle the problem, we propose an efficient approximation approach, called Lagrangean relaxation with heuristics (LRH). RWA$^+$ is first formulated as a combinatorial optimization problem in which the bottleneck link utilization is to be minimized. The LRH approach performs constraint relaxation and derives a lower-bound solution index according to a set of Lagrangean multipliers generated through subgradient-based iterations. In parallel, using the generated Lagrangean multipliers, the LRH approach employs a new primal heuristic algorithm to arrive at a near-optimal upper-bound solution. With lower and upper bounds, we conduct a performance study on LRH with respect to accuracy and convergence speed under different parameter settings and termination criteria. We further draw comparisons between LRH and an existing practical approach [7] via experiments over randomly generated and several well-known large sized networks. Numerical results demonstrate that LRH

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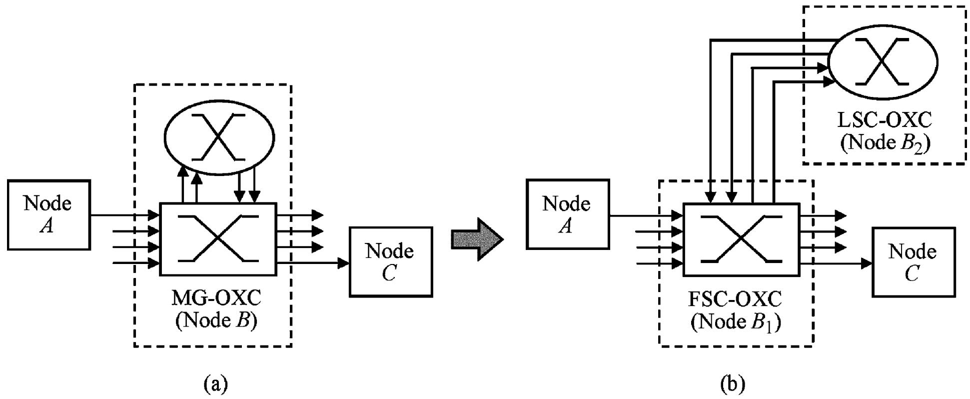

Fig. 1. A combined MG-OXC node and its logically identical separated node form. (a) An MG-OXC node. (b) FSC-OXC and LSC-OXC nodes.

outperforms the existing approach in both accuracy and computational time complexity, particularly for larger sized networks.

The remainder of this paper is organized as follows. In Section II, we first give the RWA$^+$ problem formulation. In Section III, we present the LRH approach and its primal heuristic algorithm. In Section IV, we demonstrate numerical results of the performance study and comparisons under randomly generated and large sized networks. Finally, concluding remarks are made in Section V.

II. RWA$^+$: Problem Formulation

The RWA$^+$ problem is formulated as a linear integer problem stated as follows. Given a physical topology (with FSC-OXCs and LSC-OXCs) and available wavelengths on each link, and requested lightpath demands between all source-destination pairs, determine the routes and wavelengths of lightpaths, such that the maximum number of lightpaths on the most congested link is minimized, subject to the wavelength continuity constraint. For ease of illustration, we assume in the sequel that the number of available wavelengths on each link is the same.

Due to the existence of FSC nodes, a graph transformation is first required. For each FSC node with K input (and output) fibers, it is replaced by a bipartite subgraph with K phantom nodes connecting to input fibers, and another K phantom nodes connecting to output fibers. Besides, there are additional $K \times K$ phantom links connecting the $2K$ phantom nodes. These phantom links describe possible configuration combinations inside an FSC node. For ease of description, we summarize the notation used in the formulation as follows.

Input values:

- N^F set of FSC nodes in the network;
- N^L set of LSC nodes in the network;
- L set of physical optical links;
- L^F set of phantom links within FSC nodes;
- V_n^{in} set of phantom input nodes for node n;
- V_n^{out} set of phantom output nodes for node n;
- W set of available wavelengths on each link; (assumed to be the same for simplicity);
- S set of source-destination (SD) pairs requesting lightpath setup;
- S_n the set of SD pairs where node n is the source node;
- P_{sd} candidate path set for SD pair sd;
- y_{sd} lightpath demand for SD pair sd;
- δ_{pl} $= 1$, if path p includes link l; $= 0$, otherwise;
- σ_{lv} $= 1$, if link l is incident to node v; $= 0$, otherwise;

Decision variables:

- α most congested link utilization (lightpath no./$|W|$);
- x_{pw} $= 1$, if lightpath p uses wavelength w; $= 0$, otherwise;
- z_l $= 1$, if phantom link l is selected; $= 0$, otherwise;

Problem (P):

$$\min \ \alpha$$

subject to

$$\sum_{sd\in S}\sum_{p\in P_{sd}}\sum_{w\in W} x_{pw}\delta_{pl} \le \alpha|W| \quad \forall l \in L \tag{1}$$

$$\sum_{p\in P_{sd}}\sum_{w\in W} x_{pw} = y_{sd} \quad \forall sd \in S \tag{2}$$

$$\sum_{sd\in S}\sum_{p\in P_{sd}} x_{pw}\delta_{pl} \le 1 \quad \forall l \in L, w \in W \tag{3}$$

$$\sum_{sd\in S}\sum_{p\in P_{sd}} x_{pw}\delta_{pl} \le z_l \quad \forall l \in L^F, w \in W \tag{4}$$

$$\sum_{l\in L^F} z_l\sigma_{lv} = 1 \quad \forall v \in V_n^{in}, n \in N^F \tag{5}$$

$$\sum_{l\in L^F} z_l\sigma_{lv} = 1 \quad \forall v \in V_n^{\text{out}}, n \in N^F \tag{6}$$

$$x_{pw} = 0 \text{ or } 1 \quad \forall p \in P_{sd}, sd \in S, w \in W \tag{7}$$

$$0 \le \alpha \le 1 \tag{8}$$

$$z_l = 0 \text{ or } 1 \quad \forall l \in L^F \tag{9}$$

$$\sum_{sd\in S_n}\sum_{p\in P_{sd}} x_{pw}\delta_{pl} \le 1 \quad \forall n \in N^L, \; l \in L \cup L^F, \; w \in W. \tag{10}$$

The objective function is to minimize the highest utilization (α), namely, the utilization on the most congested fiber link with the maximum number of lightpaths passing through. Constraint (1) requires that the number of wavelengths used on every link be less than that of the most congested link. Constraint (2) is the lightpath routing constraint, and restricts the lightpaths demands of all SD pairs to be satisfied. Constraint (3) indicates that for each link, there can be at most one lightpath using each wavelength. Constraints (3) and (7) jointly correspond to the wavelength continuity constraint. In particular, due to FSC nodes, Constraints (5), (6), and (9) delineate the possible configuration of FSC nodes. Constraint (4) states that paths can only

```
Algorithm LRH;
begin
    initialize the Lagrangean multiplier vector s:= 0, q:=0 and r:=0;
    UB:=1 and LB:=0; /*upper and lower bounds on α */
    quiescence_age:=0;
    step size coefficient λ:=2;
    for each k:=1 to Iteration_Number do
    begin
        Solve sub-problem S1;
        Solve sub-problem S2; /* by MSSP Algorithm */
        Solve sub-problem S3;
        Z_dual=Z_s1+Z_s2+Z_s3−∑_{l∈L}∑_{w∈W} q_lw ;  /*Equation (12)*/
        if Z_dual>LB
          then LB:=Z_dual and quiescence_age:=0;
          else quiescence_age:=quiescence_age+1;
        if quiescence_age ≥ Quiescence_Threshold then  λ:=λ/2 and quiescence_age:=0;
        run Primal Heuristic Algorithm;
        if ub<UB then UB:=ub;  /* ub is the newly computed upper bound */
        update the step size and multiplier vector; /* by subgradient method */
    end;
end.
```

Fig. 2. Lagrangean relaxation with heuristics (LRH).

pass through the phantom links determined by (5), (6), and (9). Finally, Constraint (10) is a redundant constraint [11] to Constraints (3) and (4), which is added for optimization purpose.

The problem is NP-complete [7], and is unlikely to obtain an exact solution for realistic networks in real-time. The problem is approximated using the LRH approach presented in the next section.

III. The LRH Approach

The Lagrangean relaxation (LR) method [12]–[16] has been successfully employed to solve complex mathematical problems by means of constraint relaxation and problem decomposition. Particularly for solving linear integer problems, unlike the traditional linear programming approach that relaxes integer into noninteger constraints, the LR method generally leaves integer constraints in the constraint sets while relaxing complex constraints such that the relaxed problem can be decomposed into independent manageable subproblems. Through such relaxation and decomposition, the LR method as will be shown provides tighter bounds and shorter computation time on the optimal values of objective functions than those provided by the linear programming relaxation approach in many instances [14].

Essentially, the original primal problem is first simplified and transformed into a *dual* problem after some constraints are relaxed. If the objective of the primal problem is a minimization (maximization) function, the solution to the dual problem is a lower (upper) bound to the original problem. Such Lagrangean lower bound is a useful by-product in resolving the Lagrangean relaxation problem. Next, due to constraint relaxation, the lower bound solutions generated during the computation might be infeasible for the original primal problem. However, these solutions and the generated Lagrangean multipliers can serve as a base to develop efficient primal heuristic algorithms for achieving a near-optimal upper-bound solution to the original problem. Based on LR, the work reported in [15] and [16] resolved the RWA problems for multifiber WDM networks and WDM networks with limited-range wavelength converters, respectively. To the best of our knowledge, the LR approach is first time used in this paper to resolve an RWA problem for multigranularity WDM networks.

In the sequel, we first give the transformed dual problem and the derivation of the lower bound. We then present the primal heuristic algorithm for obtaining the upper-bound solution.

A. The Dual Problem and Lower Bound

In the relaxation process, Constraints (1), (3), and (4) are first relaxed from the constraint set. As shown in the first line of (11), the three expressions corresponding to the three constraints, are respectively multiplied by Lagrangean multipliers s, q, and r, and then summed with the original objective function. Problem (P) is thus transformed into a dual problem, called Dual_P, given as follows:

Problem (Dual_P):

$$\begin{aligned} Z_{\text{dual}}(\rho) =& \min\Bigg[\alpha + \sum_{l\in L} s_l \left(\sum_{sd\in S}\sum_{p\in P_{sd}}\sum_{w\in W} x_{pw}\delta_{pl} - \alpha|W|\right) \\ &+ \sum_{l\in L}\sum_{w\in W} q_{lw}\left(\sum_{sd\in S}\sum_{p\in P_{sd}} x_{pw}\delta_{pl} - 1\right) \\ &+ \sum_{l\in L^F}\sum_{w\in W} r_{lw}\left(\sum_{sd\in S}\sum_{p\in P_{sd}} x_{pw}\delta_{pl} - z_l\right)\Bigg] \\ =& \min\Bigg[\left(1-\sum_{l\in L} s_l|W|\right)\alpha + \sum_{sd\in S}\sum_{p\in P_{sd}}\sum_{w\in W} \\ &\times\left(\sum_{l\in L}(s_l+q_{lw})\delta_{pl} + \sum_{l\in L^F} r_{lw}\delta_{pl}\right)x_{pw} \\ &- \sum_{l\in L^F}\sum_{w\in W} r_{lw} z_l - \sum_{l\in L}\sum_{w\in W} q_{lw}\Bigg] \qquad (11) \end{aligned}$$

```
Algorithm MSSP;
begin
  for each LSC node src∈N^L do
  begin
    for each wavelength w∈W do   /* initialization */
    begin
      x:=0;  /*decision variable vector*/   π_w:=0;  /*node potential vector*/
      for each link l∈L^F do  cost_lw:=r_lw; /* link cost */   for each link l∈L do  cost_lw:=s_l+q_lw;
    end;
    for each SD pair sd∈|S_src| do
    begin
      dest=destination(sd);
      for each w∈W do begin ready_layer_w:="Unknown"; num-path-setup_sd:=0; end;
      repeat
        for each w∈W do
        begin
          if  ready_layer_w="Unknown" then
          begin
            run Dijkstra's-shortest-path(cost, src, dest) on layer w;
            if the shortest path exists
              then begin  denote the cost increment of taking the path as k_w;
                          ready_layer_w:="Yes";
                   end;
              else ready_layer_w:="No";   /* no more path on the layer for sd */
          end;
        end;
        if there exists a layer (w*) with smallest cost k_w* and ready_layer_w*="Yes"
          then   begin update x_pw*, π_w*, cost_lw* ; /* by SSP algorithm */
                       num-path-setup_sd:= num-path-setup_sd+1;
                       ready_layer_w*:="Unknown";
                 end;
          else  return "infeasible";   /* all ready_layer's are "No" */
      until num-path-setup_sd=y_sd ;
    end;
  end;
end.
```

Fig. 3. MSSP algorithm.

subject to constraints (2), (5)–(10), where $\boldsymbol{\rho} = (\boldsymbol{q}, \boldsymbol{r}, \boldsymbol{s})$ is the nonnegative Lagrangean multiplier vector. To compute the Lagrangean multipliers, we adopt the subgradient method as delineated in the LRH algorithm outlined in Fig. 2. By separating decision variable α, and decision variable vectors $\boldsymbol{x}$ and $\boldsymbol{z}$, Problem (Dual_P) in (11) can be decomposed into three independent subproblems—S1, S2, and S3. Specifically, we have

$$Z_{\text{dual}} = Z_{S1} + Z_{S2} + Z_{S3} - \sum_{l\in L}\sum_{w\in W} q_{lw} \tag{12}$$

where subproblem S1 is given by $Z_{S1}(s) = \min(1 - \sum_{l\in L} s_l|W|)\alpha$, subject to constraint (8); subproblem S2 is given by $Z_{S2}(q,r) = \min(\sum_{sd\in S}\sum_{p\in P_{sd}}\sum_{w\in W}(\sum_{l\in L}(s_l + q_{lw})\delta_{pl} + \sum_{l\in L^F} r_{lw}\delta_{pl})x_{pw})$, subject to constraints (2), (7), and (10); and subproblem S3 is given by $Z_{S3}(r) = \min(-\sum_{l\in L^F}\sum_{w\in W} r_{lw}z_l)$, subject to constraints (5), (6), and (9).

First, subproblem S1 is to determine the decision variable α. Clearly, α is set to 1 if the corresponding cost $1 - \sum_{l\in L} s_l|W|$ is negative; otherwise, α is set to 0. Thus, S1 requires $O(L)$ computation time. Second, subproblem S2 is to compute the decision variable vector $\boldsymbol{x}$. There exist $|S_n|$ (one for each source node) independent problems, each of which is an edge-disjoint-path problem, starting from the given source node and destined to all destination nodes of the SD pairs with nonzero lightpath demands. Due to multiple wavelengths on each link, the network can be viewed as a layered graph with a total of $|W|$ layers, where each layer corresponds to each wavelength. Each layer then contains $(L + L^F)$ links and $(N^L + N^F)$ nodes. Notice that each link can be designated with unit flow capacity and a nonnegative cost, for example, $s_l + q_{lw}$, for each nonphantom link.

Accordingly, the edge-disjoint-path problem for each source corresponds to a minimum-cost flow problem. Ultimately, with $|W|$ layers considered as a whole, the minimum-cost flow problem can be solved by the successive shortest path (SSP) algorithm [14]. However, the integrated problem requires high computational time complexity provided with large values of $|W|$. To reduce the complexity, we employ a modified successive shortest path (MSSP) algorithm as shown in Fig. 3. In the algorithm, we treat each layer graph individually and perform incremental selection of minimum-cost edge-disjoint path (from one layer). The computational complexity of MSSP for each SD pair is $O(k(m + n\log\, n))$, where $m = L + L^F$, $n = N^L + N^F$, and $k = \max\{y_{sd}, |W|\}$. All decision variables x's for S2 can be obtained by repeatedly applying the MSSP algorithm for all sources. Finally, subproblem S3 is to resolve decision variable vector $\boldsymbol{z}$. The problem can be further decomposed into $|N^F|$ (one for each FSC node) independent

```
Algorithm Primal Heuristic;
begin
    for each wavelength w∈W do
    begin
        for each link l∈L^F do if z_l:=1 then cost_lw:=r_lw else cost_lw:=∞;
        for each link l∈L do cost_lw:=s_l+q_lw;
    end;
    for each SD pair sd:=1 to |S| do num-path-setup_sd:=0;
    repeat
        for each SD pair sd:=1 to |S| do
        begin
            if num-path-setup_sd<y_sd then
            begin
                src=source(sd);
                dest=destination(sd);
                run Dijkstra's-shortest-path(cost, src, dest) on each wavelength layer;
                    /* cost is vector of costs of all wavelengths and links*/
                if the shortest path exists then
                begin
                    designate the wavelength associated with the shortest path as w* ;
                    for all links l on the shortest path do
                    begin cost_lw*:=∞;
                        if the number of allocated paths on link l > LB×|W| then
                        for each wavelength w∈W do cost_lw:= cost_lw×Penalty;
                    end;
                end;
                else return "infeasible";
            end;
        end;
    until all SD demand satisfied;
    update upper bound ub;
end.
```

Fig. 4. Primal heuristic algorithm.

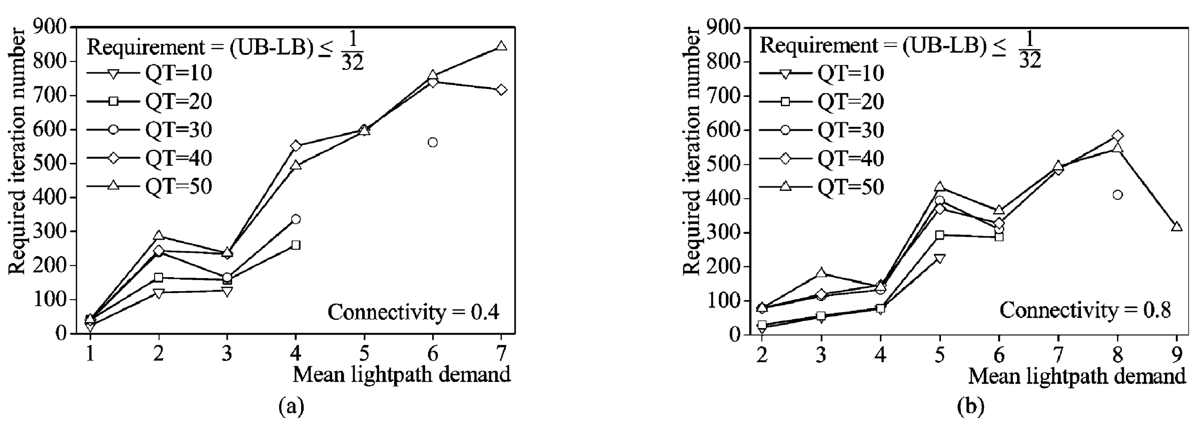

Fig. 5. Convergence speed versus accuracy on the basis of using termination requirement. (a) Sparse network. (b) Dense network.

problems, each of which can be optimally solved by a bipartite weighted matching algorithm. Thus, for an $n \times n$ bipartite graph, the problem requires $O(n^3)$ computation time.

According to the weak Lagrangean duality theorem [14], Z_{dual} in (12) is a lower bound of the original Problem (P) for any nonnegative Lagrangean multiplier vector $\boldsymbol{\rho} = (\boldsymbol{q}, \boldsymbol{r}, \boldsymbol{s}) \geq 0$. Clearly, we are to determine the greatest lower bound. Equation (12) can be solved by the subgradient method, as shown as a part of the LRH approach in Fig. 2. As shown in Fig. 2, the algorithm is run for a fixed number of iterations (i.e., *Iteration_Number*). (Notice that the algorithm can also be driven by given a termination requirement, as will be shown in the next section). In every iteration, the three subproblems (S1–S3) are solved (described above), resulting in the generation of a new Lagrangean multiplier vector value. Then, according to (12), a new lower bound is generated. If the new lower bound is tighter (greater) than the current best achievable lower bound (LB), the new lower bound is designated as the LB. Otherwise, the LB value remains unchanged.

Significantly, if the LB value remains unimproved for a number of iterations that exceeds a threshold, called *Quiescence_Threshold* (QT), the step size coefficient (λ) of the subgradient method is halved, in an attempt to reduce oscillation possibility. Specifically, to update the step size and multiplier

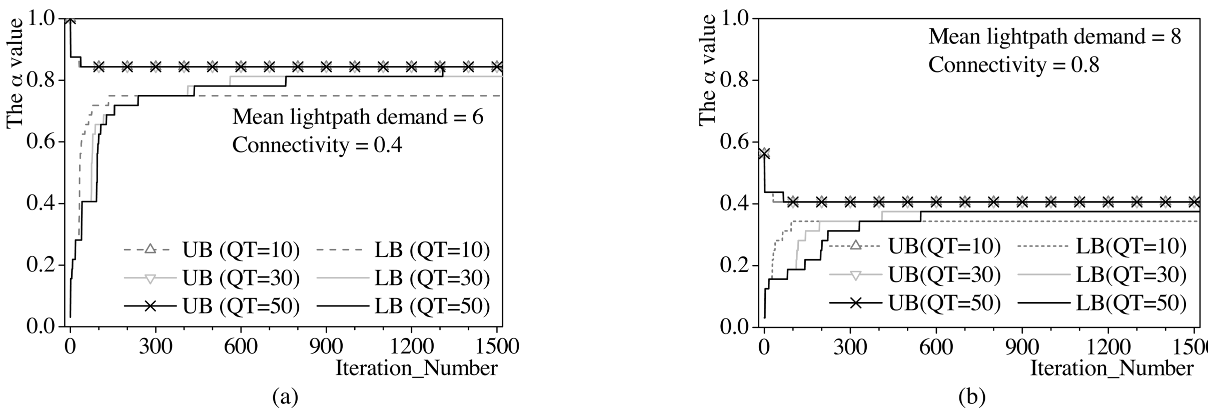

Fig. 6. Convergence speed versus accuracy on the basis of using fixed iteration number. (a) Sparse network. (b) Dense network.

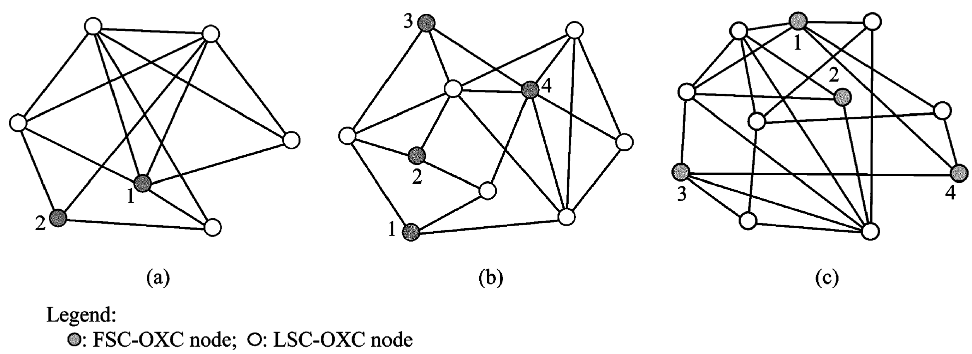

Fig. 7. Network topology. (a) NET1 (seven nodes). (b) NET2 (ten nodes). (c) NET3 (11 nodes).

vector as specified in Fig. 2, the Lagrangean multiplier vector $\boldsymbol{\rho}$ is updated as $\rho_{k+1} = \rho_k + \theta_k b_k$, where θ_k is the step size, determined by $\theta_k = \lambda_k(UB - Z_{\text{dual}}(\rho_k))/\|b_k\|^2$, in which λ_k is the step size coefficient, UB is the current achievable least upper bound obtained from the primal heuristic algorithm described next, and b_k is a subgradient of $Z_{\text{dual}}(\rho)$ with vector size $|L + LW + L^F W|$.

B. The Primal Heuristic Algorithm and Upper Bound

The primal heuristic algorithm in the LRH approach is used to find an updated upper bound ub. Similar to the lower bound case, as given in Fig. 2, if the new upper bound (ub) is tighter (smaller) than the current best achievable upper bound (UB), the new upper bound is designated as the UB.

As shown in Fig. 4, the algorithm first settles the phantom links suggested by the solution to subproblem S3 for all FSC nodes, reducing the problem complexity. The cost of each link is designated as the Lagrangean multipliers previously obtained. Clearly, the cost of unaccepted phantom links are set to ∞, excluding them from subsequent path consideration. The algorithm then repeatedly applies the Dijkstra's shortest path algorithm in an effort to satisfy the lightpath demands of all SD pairs.

At the end of the computation, the costs of those links associated with the selected wavelengths/paths are set to ∞ to prevent the links from being considered by other upcoming iterations. If the number of wavelengths (lightpaths) used on a link is greater than the current tightest lower bound multiplied by $|W|$, indicating potential congestion, the cost of the link is then scaled by multiplying by a constant, referred to as the penalty term. This is to avoid further lightpath setup through this link. The process repeats until either the lightpath demands of all SD pairs are satisfied (i.e., feasible), or there is no remaining resource (i.e., infeasible) in the network.

IV. Experimental Results

We have carried out a performance study on the LRH approach, and drawn comparisons between LRH and the Banerjee–Mukherjee approach [7] via experiments over randomly generated networks. Given the total number of nodes, say n, the greatest possible number of bidirectional links is $C(n, 2)$, where C is the combination operation. Then, for a network with n nodes and connectivity v, it is generated by randomly selecting $C(n, 2) \times v$ out of the $C(n, 2)$ bidirectional links of the network. In the experiments, we used 32 wavelengths on each fiber link (i.e., $|W| = 32$) for all networks.

A. Performance Study

We carried out two sets of experiments over 15-node random networks with two connectivities $v = 0.4$ and 0.8, which correspond to sparse and dense networks, respectively. In the first set of experiments, the LRH algorithm was terminated when the gap between the UB and the LB on α was less than or equal to one out of the maximum number of wavelengths, or the number of iterations exceeds 2000. While the former condition corresponds to reaching a near-optimal upper bound solution, the latter condition represents abnormal termination due to the failure of achieving such accuracy or solution infeasibility. We

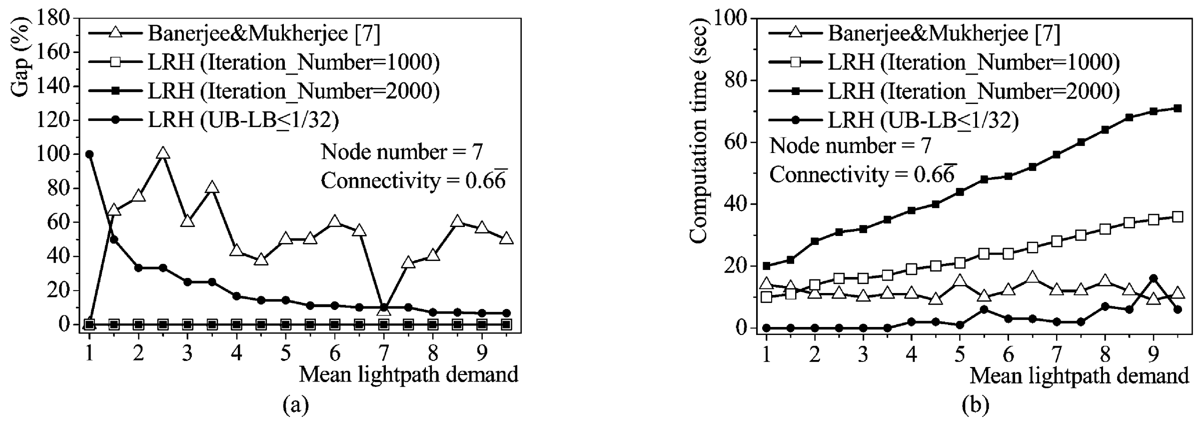

Fig. 8. Comparisons of accuracy and computation time for random network NET1. (a) Accuracy for NET1. (b) Computation time for NET1.

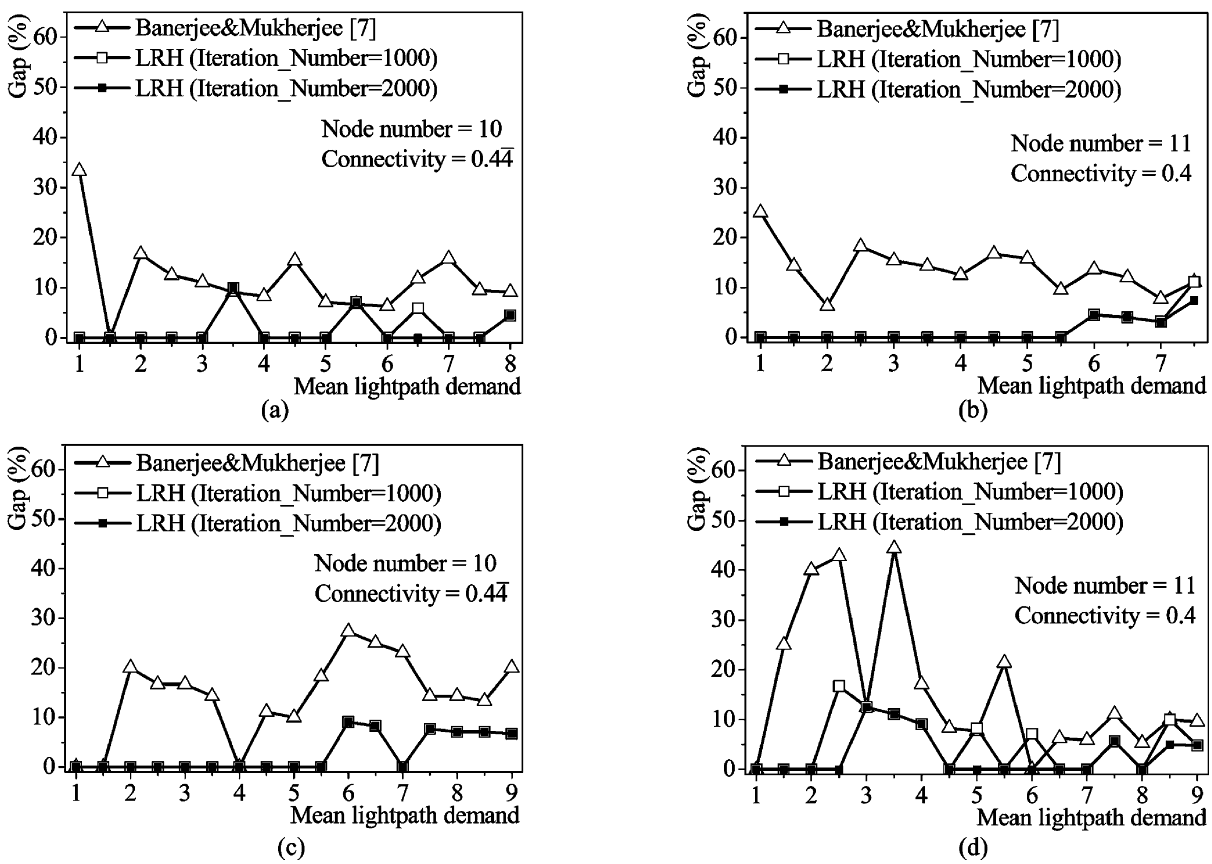

Fig. 9. Comparison of accuracy for random networks NET2 and NET3. (a) NET2 with two FSC nodes. (b) NET3 with two FSC nodes. (c) NET2 with four FSC nodes. (d) NET3 with four FSC nodes.

examine the total number of iterations required as a function of the mean lightpath demand under different QT values. Numerical results are plotted in Fig. 5. Notice that the absence of data under certain demands corresponds to abnormal termination.

First, we observe that the dense network in general requires less number of iterations before reaching a near-optimal solution. Significantly, we discover from the figure that parameter QT plays a key role in the performance tradeoff between convergence speed and accuracy. Smaller values of QT, which imply frequent updates of the subgradient step-size coefficient, yield faster convergence to near-optimal solutions but at the cost of failing to reach accurate solutions under heavier lightpath demands. Greater QT values on the other hand result in completely opposite performance.

In the second set of experiments, the LRH algorithm was terminated when the number of iteration exceeded a predetermined Iteration_Number, ranging from 0 to 1500. Numerical results are displayed in Fig. 6. We study both the lower and upper bounds on α under different QT values. We observe that while the upper bound performance is irrelevant to QT, the lower bound performance is highly dependent on the QT setting in the same manner as above. Specifically, smaller QT values yield faster convergence but only to looser lower bounds, while larger QT values result in tighter lower bounds through gradual con-

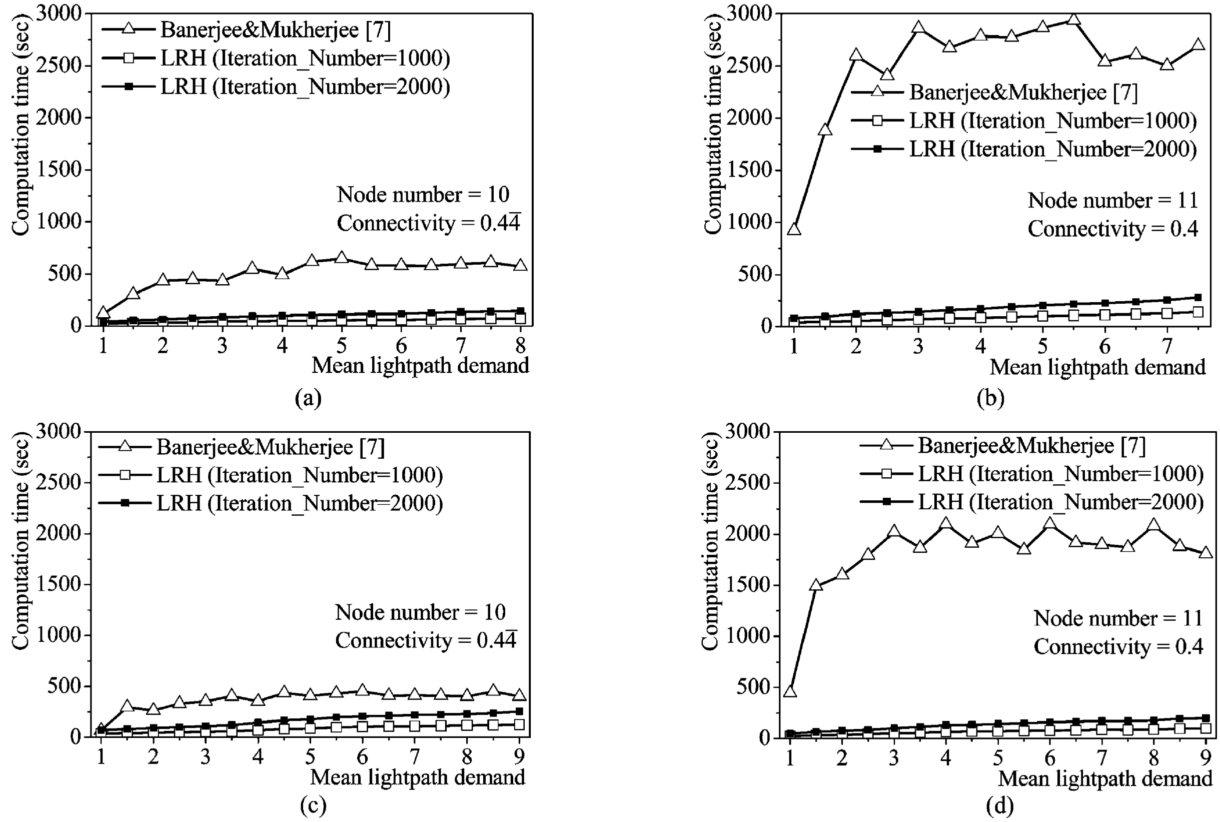

Fig. 10. Comparison of computation time for random networks NET2 and NET3. (a) NET2 with two FSC nodes. (b) NET3 with two FSC nodes. (c) NET2 with four FSC nodes. (d) NET3 with four FSC nodes.

vergence over a larger number of iterations. This fact reveals that, by adjusting the QT value, the LRH approach is capable of balancing the tradeoff between accuracy and efficiency for resolving various types of RWA problems.

B. Performance Comparisons

We further draw comparisons of accuracy and computation time between our LRH approach and a linear programming relaxation (LPR)-based method, i.e., Banerjee–Mukherjee [7]. For generating networks, it is impractical to experiment on networks with smaller numbers of nodes and links. However, for networks with greater than 11 nodes, we experienced that the computation time using the LPR method became unmanageable. Accordingly in the experiment, we considered three random networks, NET1, NET2, and NET3, as shown in Fig. 7. NET1 consists of seven nodes including two FSC nodes, and 14 bidirectional links, corresponding to a connectivity (v) of $0.6\bar{6}$. NET2 consists of ten nodes including two FSC (nodes 1–2) or 4 FSC (nodes 1–4) nodes, and 20 bidirectional links, corresponding to a connectivity (v) of $0.4\bar{4}$. Finally, NET3 consists of 11 nodes including two FSC (nodes 1–2) or four FSC (nodes 1–4) nodes, and 22 bidirectional links, corresponding to a connectivity (v) of 0.4. Results are plotted in Figs. 8–10.

In the computation using our LRH approach, we adopted $\mathrm{QT} = 50$ and three different termination criteria. The three criteria are: Iteration_Number $= 1000, 2000$, and requirement $(UB - LB) \leq 1/32$. The algorithm was written in the C language and operated on a PC running Windows XP with a 2.53 GHz CPU power. In the LPR-based method, by removing constraints (7) and (9), the original integer linear programming (ILP) problem is relaxed to a linear programming (LP) problem. Thus, the solution to the relaxed problem is a legitimate lower bound of the original ILP problem. The upper bound is then obtained according to the randomization procedure proposed in [7]. In the experiment, the LP problem was solved using the *CPLEX* software, operating in the same PC environment. For both approaches, the accuracy is measured in terms of the Gap(%), which is defined as the ratio of the difference of the UB and LB values to the LB value in percentage.

First, we draw comparisons of accuracy and computation time between the LRH approach and the LPR method for random network NET1, as plotted in Fig. 8. Notice that the LRH approach using fixed iteration numbers outperforms the LPR method in accuracy under all lightpath demands. However, it appears that the LRH method using the termination requirement yields a high gap under low demands. This is only due to the magnification of the gap resulting from being divided by a small LB value under low demands. In pafrticular, under demand $= 1$, the algorithm was terminated with $\mathrm{UB} = 2/32$ and $\mathrm{LB} = 1/32$, resulting a 100% gap. Surprisingly, we discover from part of Fig. 8(b) that the LPR method requires less computation time than that of the LRH approach using fixed iterations. This indicates that LPR is an efficient approach particularly for smaller size networks.

For random networks with size over ten nodes (NET2 and NET3) as shown in Figs. 9 and 10, the LPR method yields larger gaps, namely, poorer accuracy, and demands exponentially increasing computation time. In contrast, the LRH ap-

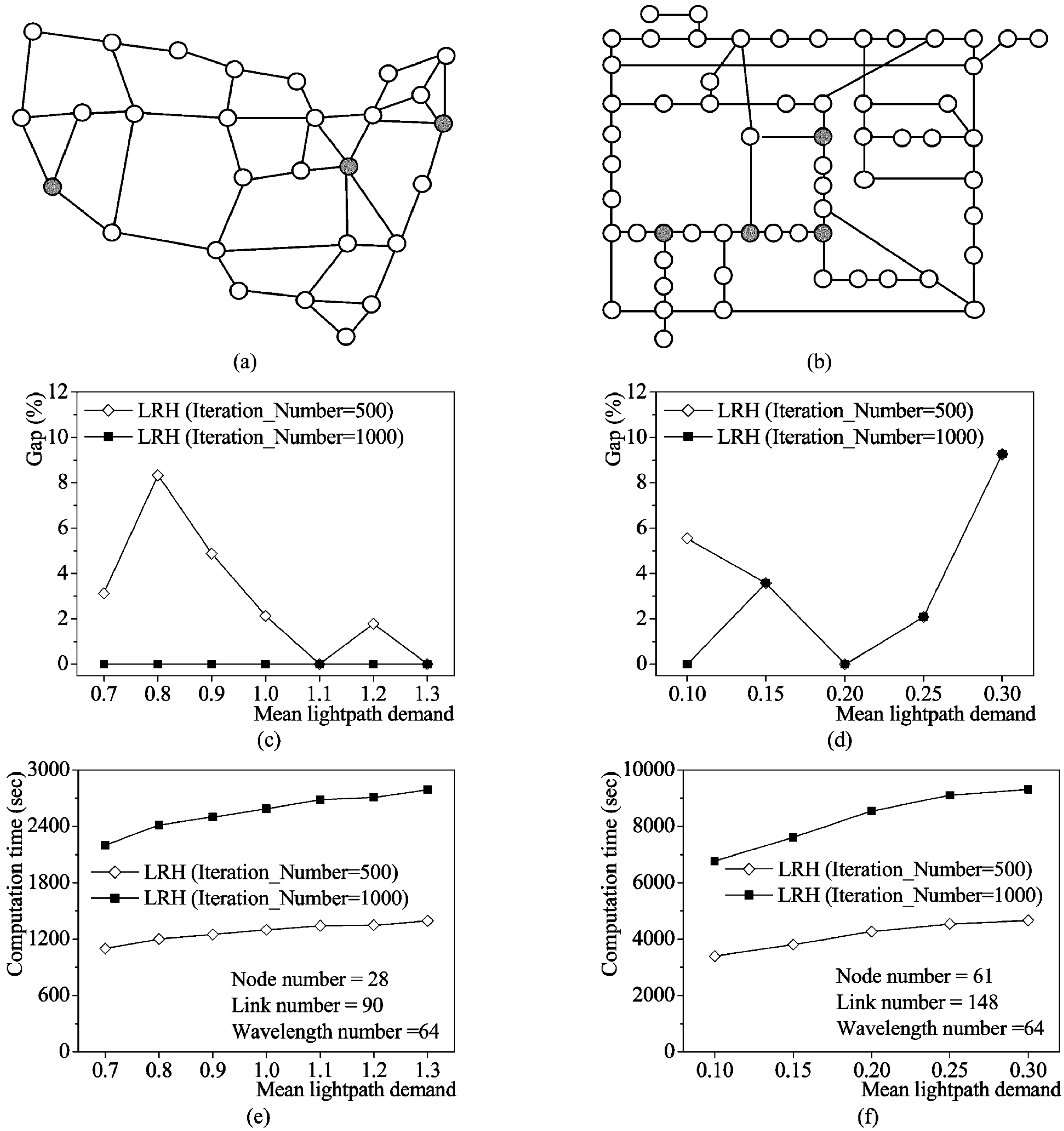

Fig. 11. LRH results for large sized networks. (a) USA network: topology. (b) ARPA network: topology. (c) USA network: computation accuracy. (d) ARPA network: computation accuracy. (e) USA network: computation time. (f) ARPA network: computation time.

proach achieves identical lower and upper bounds, namely, the optimal solutions under several lightpath demand cases. In fact, we discover that both LRH and LPR approaches achieve tight lower bounds. Significantly, the LRH heuristic algorithm arrives at much improved upper bounds due to the use of the Lagrangean multipliers derived upon seeking the Lagrangean relaxation solution. It is worth noticing that the results of the LRH approach using the termination requirement are not shown in Figs. 9 and 10. This is due to its high accuracy and low computation time, yielding impossible plotting within the figures. Specifically, we discover from Fig. 9 that the LRH approach using the 1000 iterations achieves as high accuracy as that using the 2000 iterations under most demand cases. Significantly, the approach using the $(UB - LB) \leq 1/32$ requirement for NET2 reaches the small gap within only a total of (8, 40, 164, 480, 339, 287, 137, 424) iterations for lightpath demands ranging from 1 to 8, respectively.

Furthermore, as shown in Fig. 10, the LRH approach outperforms the LPR method in computation time by at least one order of magnitude under all cases. Notice that the LRH approach using the termination requirement incurs exceptionally low computation times that are equal to (0, 1, 7, 24, 18, 17, 9, 31) for eight lightpath demands, respectively. In this case, compared to the LPR method, the LRH approach offers an improvement of computation time by more than two orders of magnitude.

To observe the performance of our LRH approach for large sized networks, we carried out experiments on two well-known networks, i.e., USA and ARPA, as shown in Fig. 11(a) and (b), respectively. The USA network consists of 28 nodes including 3 FSC nodes and 90 bi-directional links, corresponding to a connectivity (v) of 0.12. The ARPA network has 61 nodes including 4 FSC nodes and 148 bi-directional links, which corresponds to a connectivity (v) of 0.04. There are 64 wavelengths on each fiber for both networks. Numerical results are displayed in Fig. 11(c)–(f).

In the experiment, we adopted QT $= 50$ and two different termination criteria, namely Iteration_Number$=$ 500 and 1000. For the USA network, LRH achieves a guarantee of no more than 8% gap between the upper and lower bounds under both termination criteria. For the ARPA network, the LRH achieves a guarantee of no more than 9.3% gap in less than 9400 s computation time. We particularly observe from Fig. 11(d) that the accuracy of the LRH approach based on the 500-iteration termination criterion is as high as that based on the 1000-iteration termination criterion under most lightpath demand cases. This again demonstrates the superiority of the LRH approach to the RWA$^+$ problem with respect to both computation accuracy and time complexity for large sized networks.

V. Conclusion

In this paper, we have resolved a RWA$^+$ problem using the LRH method, which is a Lagrangean relaxation based approach augmented with an efficient primal heuristic algorithm. With the aid of generated Lagrangean multipliers and lower bound indexes, the primal heuristic algorithm of LRH achieves a near-optimal upper-bound solution. A performance study delineated that the performance tradeoff between accuracy and convergence speed can be manipulated via adjusting the quiescence threshold parameter in the algorithm. We have drawn comparisons of accuracy and computation time between LRH and the LPR-based method, under three random networks. Experimental results demonstrated that, particularly for small to medium sized networks, the LRH approach using a termination requirement profoundly outperforms the LPR method and fixed-iteration-based LRH, in both accuracy and computational time complexity. Furthermore, for large sized networks, i.e., the USA and ARPA networks, numerical results showed that LRH achieves a near optimal solution within acceptable computation time. The above numerical results justify that the LRH approach can be used as a dynamic RWA$^+$ algorithm for small to medium sized networks, and as a static RWA$^+$ algorithm for large sized networks.

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