Quantisation error curve: A measure of similarity between requantised images

C.-H. Chu and Y.-C. Hsueh

Indexing terms: Image processing, Display instrumentation

A new similarity measure between a digital image and its quantised version is presented. By calculating the quantisation error on each of a sequence of subdomains, we can obtain a quantisation error curve as a similarity measure. This measure outperforms the commonly used measure for estimating the similarity between requantised images.

Introduction: Image requantisation is a necessary step for displaying an image on a display system with different grey-level resolution. For example, when a 256-level image is to be displayed on a monochromatic monitor, it must be requantised to bilevel. If more than two grey levels are available for certain modern graphics devices, then a multilevel quantiser is needed to requantise the given image. Many techniques have been proposed to solve this quantisation problem [1, 2].

However, it is difficult to judge fairly the similarity or dissimilarity between the original image and its quantised image. The sum of the difference of corresponding pixel intensities is commonly used to measure the similarity between two images. However, even when this quantity is small, the difference of two images can be easily distinguished by human eyes in most cases. In other words, the sum of differences is not good enough when used as a similarity measure of two images. Therefore, we propose an improved similarity measure for images. This new measure, referred to as the quantisation error curve (QEC), is based on a similarity criterion proposed by Hsueh *et al.* [3] and is particularly useful for requantised images.

Cardinality distribution and QEC: Let f be an image with N_f grey levels and g be its quantised image with N_g grey levels, both f and g are defined on a finite domain D. Let $\hat{f}$ and $\hat{g}$ be the respective normalised images of f and g. The commonly used algorithms, ordered dither [4] and error diffusion [5], attempt to preserve the following criterion: For any pixel x_0, the average value of $\hat{g}(x)$ when x is near x_0 is approximately equal to $\hat{f}(x_0)$. However, this criterion is not for general graphics display systems as pointed out by Knuth [6]. The cardinality distribution [3], on the other hand, tries to preserve the following property: For any subdomain S of D, $\hat{f}$ and $\hat{g}$ have approximately the same average value in S; that is

$$\sum_{x\in S} \hat{f}(x) \simeq \sum_{x\in S} \hat{g}(x)$$

Based on this more general criterion, we can propose a measure of similarity as follows. First of all, we define the quantisation error of g corresponding to a given subdomain S, written as $QE_S(g)$, by

$$QE_s(g) = \frac{1}{\#(N)} \sum_{x\in N} \left| \sum_{y\in S_x} \hat{f}(y) - \sum_{y\in S_x} \hat{g}(y) \right| \qquad (1)$$

where S_x is the translation of S by x, N is a subset of D such that $\{S_x; x \in N\}$ is a cover of D, and $\#(N)$ denotes the number of pixels in N. When S contains only the origin, QE_S is equal to the mean absolute error (MAE). When $S = D$ and N = {the origin}, QE_S is the difference between the total illuminations of $\hat{f}$ and $\hat{g}$.

Then by choosing a sequence of subdomains $S_0, S_1, ..., S_m$ with $\#(S_i) = 2^i$ for each i = 0, 1, ..., m, we can obtain the quantisation errors $QE_i \equiv QE_{S_i}$ for each i from eqn. 1. The curve linearly approximating the graph $\{(i, QE_i) \mid i = 0, 1, ..., m\}$ will then be called the quantisation error curve (QEC) for g.

Given two quantised images g_1 and g_2 of the original image f. There is no doubt that g_1 is more similar to f than g_2 if $QE_S(g_1) \le QE_S(g_2)$, $\forall S$. Thus, we can use QEC to measure the similarity or dissimilarity between an image and its quantised image. Note that it is time-consuming and impractical to compute QE_S for all possibilities of subdomain S. In our implementation, QE_S is calculated only when S is a square region and N is chosen such that $\{S_x; x \in N\}$ is a partition of D.

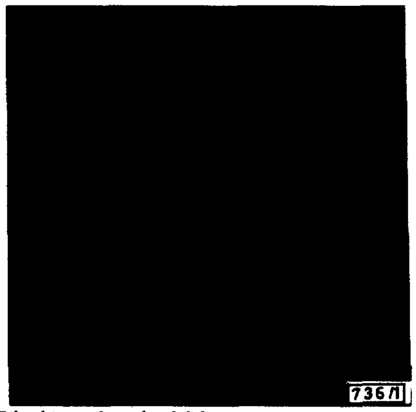

Fig. 1 *Bilevel image by ordered dither*

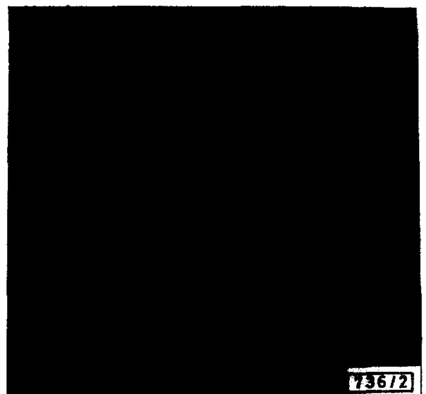

Fig 2 *Bilevel image by dot diffusion*

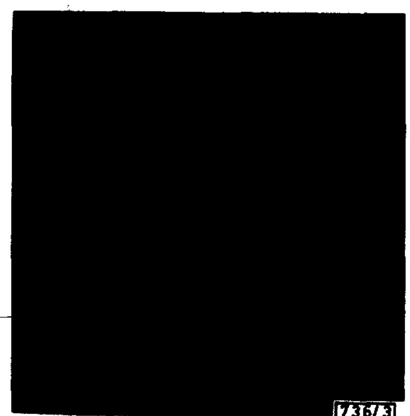

Fig 3 *Bilevel image by cardinality distribution*

Experimental results: Three quantisation algorithms, ordered dither [4], dot diffusion [6] and cardinality distribution [3], are tested for binarisation. The test image has size 256×256 and is digitised into 8 bits per pixel accuracy. This test image contains 12 co-centre circles with different grey levels and the background is fixed at 128. That is, the circle is the only feature in the original image. Figs. 1–3 are the binary images quantised by ordered dither, dot diffusion and cardinality distribution, respectively. Quantisation error curves for these three quantised images are plotted in Fig. 4; here S has size $2^M \times 2^M$. From the quantised images, it can be seen that the image in Fig. 3 captures more clear circles than those in Figs. 1 and 2. This result agrees with the QEC in Fig. 4. It is noted that three quantisation errors are very close when $M = 0$. That is, we cannot quantitatively distinguish those images if only MAE is employed.

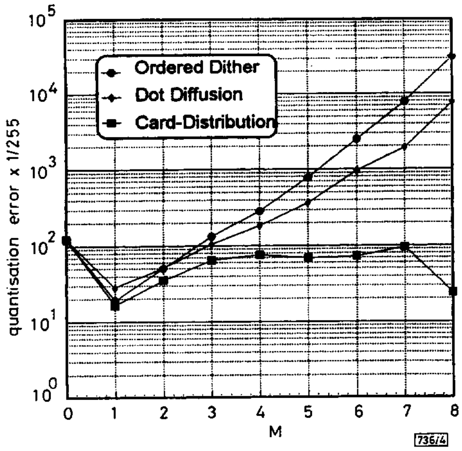

Fig. 4 *QEC for different quantisers*

Conclusion: We have proposed the QEC to measure the similarity or dissimilarity between an image and its quantised counterpart. QEC is based on the following criterion: two similar images must have similar illumination in any subdomains. When the quantisation error with fixed subdomain cannot distinguish the difference of two images, QEC can provide more information.

It should be noted that the QEC can be computed for any two images with the same domain. Thus, the QEC can also be used as a similarity measure for any two images.

 14 January 1994

Electronics Letters Online No: 19940420

C.-H. Chu and Y.-C. Hsueh (*Department of Computer and Information Science, National Chiao Tung University, Hsinchu, Taiwan 30050, Republic of China*)

Y.-C. Hsueh: Corresponding author

References

1 STOFFEL, J.C., and MORELAND, J.F.: 'A survey of electronic techniques for pictorial image reproduction', *IEEE Trans.*, 1981, **COM-29**, pp. 1898–1924

2 ULICHNEY, R.: 'Digital halftoning' MIT Press, Cambridge, Mass., 1987

3 HSUEH, Y.C., CHERN, M.G., and CHU, C.H.: 'Image requantization by cardinality distribution', *Comput. & Graphics*, 1991, **15**, pp. 397–405

4 BAYER, B.E.: 'An optimum method for two-level rendition of continuous-tone pictures', IEEE Int. Conf. on Communications, 1973, Vol. 1, pp. (26-11)–(26-15)

5 FLOYD, R.W., and STEINBERG, L.: 'An adaptive algorithm for spatial grey scale',Society for Information Display Digest, 1975, Vol. SID 75, pp. 36–37

6 KNUTH, D.E.: 'Digital halftones by dot diffusion', *ACM Trans.*, 1987, **6**, (4), pp. 245–273

Capacities of equivalent channels in multilevel coding schemes

J. Huber and U. Wachsmann

Indexing terms: Channel coding, Codes and coding

The channel capacity of an M-ary digital transmission scheme $M > 2$ can in principle be achieved by multilevel codes together with multistage decoding, if the rates of the component codes are equal to the capacities of the equivalent channels for each coding level.

Multilevel coding is a well known approach to designing power- and bandwidth-efficient digital communication systems based on M-ary modulation schemes $M = 2^n$, $n \in \mathbf{N}$, $n > 1$, such as ASK, PSK, QAM etc., see for example [1, 3, 5]. Usually, labels $\vec{c}_m = (c_m^0, c_m^1, \ldots, c_m^q, \ldots, c_m^{n-1})$ with binary components c_m^q are mapped to the elements a_m, $m \in \{1, 2, \ldots, M\}$ of the signal set $\mathbf{A}$ of the modulation scheme via a set partitioning procedure, see for example [6]. Up to n individual binary codes $\mathbf{C}^q$, $q \in \{0, 1, \ldots, n-1\}$ with rates R^q may be applied to the binary sequences $\langle c_{m_\mu}{}^q \rangle = \langle \ldots, c_{m-1}{}^q, c_{m0}{}^q, \ldots \rangle$, $\mu \in \mathbf{Z}$ at each coding level q. We restrict the discussion to binary component codes for simplicity and conciseness. It was often proposed to choose the component codes by maximising the minimum squared Euclidean distance d_{min}^2 between sequences of signal points for given overall rate

$$R = \sum_{q=0}^{n-1} R^q$$

codeword length and decoder complexities, see for example [1].

Multilevel codes can be efficiently decoded by multistage decoding, i.e. individually decoding of each component code $\mathbf{C}^q$ starting from $q = 0$ and using the decisions of prior stages. Its performance loss compared to overall MLSE is surprisingly small. On the other hand, it can be observed that for the Ungerboeck set partitioning strategy [6], errors at the lowest coding level predominate. The reason for this effect is a tremendous increase in the number of nearest neighbour error events due to the multiple representation of the binary symbol c^0 by 2^{n-1} different signal points a_m. Thus, we propose to choose the rates R^q from the capacities of the equivalent channels for the different coding levels which can be calculated very simply.

Let $\mathbf{A}_{c^0c^1\ldots c^{q-1}}$ be the subset of all signal points with labels $(c^0, c^1, \ldots, c^{q-1}, x^q, x^{q+1}, \ldots, x^{n-1})$ with arbitrary $x^i \in \{0, 1\}$ at level q of the set partitioning tree and $C(\mathbf{A}_{c^0c^1\ldots c^{q-1}})$ the channel capacity of this subset of signal points. It is inherent to multilevel coding that all signal points $a_m \in \mathbf{A}$ have equal *a priori* probabilities because the components c^q of the labels are assumed to be chosen independently and equiprobably at each level. We assume that the multilevel code is based on a *regular* binary set partitioning tree, i.e. the set partitioning is carried out in such a way that all 2^q subsets $\mathbf{A}_{c^0c^1\ldots c^{q-1}}$ at level q have the same capacities $C(\mathbf{A}_{c^0c^1\ldots c^{q-1}})$. For the AWGN channel, a partitioning tree is regular if the average Euclidean distance spectrum from any point in $\mathbf{A}_{c^0c^1\ldots c^{q-1}c^q}$ to all points in $\mathbf{A}_{c^0c^1\ldots c^{q-1}\bar{c}^q}$ is equal for all subsets $\mathbf{A}_{c^0c^1\ldots c^{q-1}}$. This property is obviously given for subsets which only differ in translation and rotation. The memoryless channel is characterised by M conditioned probability density functions $f_{y|a_m}(y)$ of the channel output variable y for given input symbol a_m.

(i) *Lemma*: The capacity C^0 of the equivalent binary channel for coding level 0 is given by

$$C^0 = C(\mathbf{A}) - C(\mathbf{A}_0) = C(\mathbf{A}) - C(\mathbf{A}_1) \qquad (1)$$

(ii) *Proof*: C^0 is defined to be (see for example [2]

$$C^0 = \frac{1}{2} \sum_{c^0=0}^{1} \int_Y f_{y|c^0}(y) \log_2 \left(\frac{f_{y|c_0}(y)}{f_y(y)} \right) dy \qquad (2)$$

The integral has to be evaluated over the space Y of the channel output variables y. The density of the channel output variable y for given input c^0 is given by

$$f_{y|c^0}(y) = \frac{2}{M} \sum_{a_m \in \mathbf{A}_{c^0}} F_{y|a_m}(y) \qquad c^0 \in \{0, 1\} \qquad (3)$$

The capacity of any signal (sub-)set $\mathbf{B}$ with $|\mathbf{B}|$ equiprobable elements is