A Statistic Approach for Deriving the Short Message Transmission Delay Distributions

Hui-Nien Hung, Yi-Bing Lin, *Fellow, IEEE*, Ming-Kun Lu, and Nan-Fu Peng

***Abstract*—Short Message Service (SMS) is one of the most popular mobile data services. This paper develops a web-based short message system and analyzes the short message transmission delays based on 40 000 measured data collected from commercial operation. We investigate the distributions of SMS round-trip transmission delays. These distributions are roughly approximated by truncated normal, gamma, log-normal, and Weibull distributions with various parameters, which can be utilized in theoretical analysis. Furthermore, with the nonparametric fitting approach, we use the rejection method to develop random number generators for these delays, which can be used in any simulation models that employ SMS transmission delays.**

***Index Terms*—Mobile telecommunications network, random number generator, short message service, transmission delay.**

I. Introduction

SHORT MESSAGE SERVICE (SMS) is considered to be one of the most successful mobile data services today. Since 2002, more than 80 billion short messages have been delivered in China annually. Several SMS-based internet platforms have been developed to support Internet applications [5], [6]. Based on these platforms, services can be flexibly and efficiently developed.

An example of the short message service network architecture is illustrated in Fig. 1. In this architecture, the short message is first delivered from a message sender to a *Short Message Service Center* (SM-SC) [Fig. 1, ①]. The message sender can be a *Mobile Station* (MS) or a wireline short message input device [Fig. 1, ②]. The SM-SC is connected to the mobile network through a specific *Mobile Switching Center* (MSC) called the short message service gateway MSC (SMS GMSC) [Fig. 1, ③]. By querying a mobility database called the *Home Location Register* (HLR) [Fig. 1, ④], the SMS GMSC locates the target MSC of the message receiver and forwards the message to that MSC [Fig. 1, ⑤]. The target MSC broadcasts the message to the base stations, and the base stations page the destination MS [Fig. 1, ⑥]. Note that a short message is delivered by the GSM SDDCH control channel [3]. That is, SMS does not occupy any traffic channel that carries voice. Therefore, a MS engaged in conversation can still receive short messages.

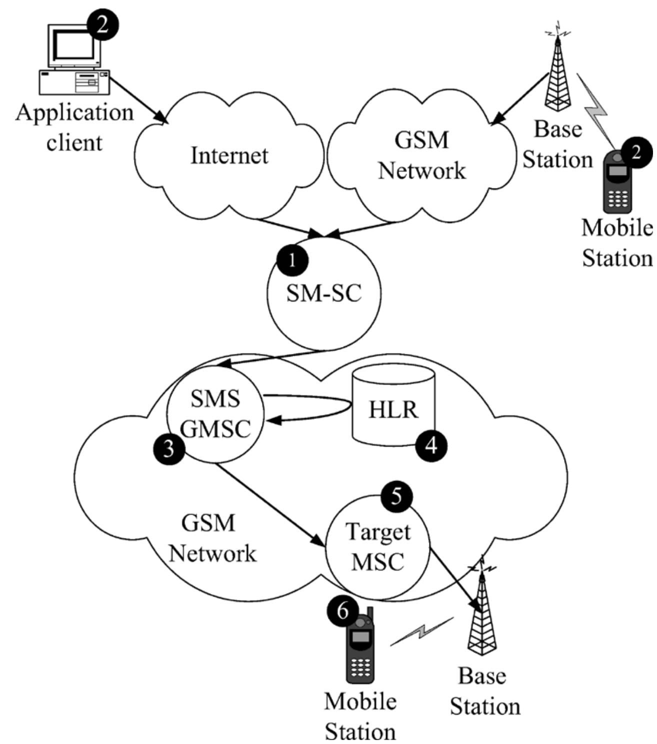

Fig. 1. GSM short message service network architecture.

In a joint project between FarEasTone Telecom. Corporation and National Chiao Tung University (NCTU), we developed a web-based short message system called NCTU-SMS. With a user-friendly *Graphical User Interface*, NCTU-SMS allows users to input short messages through a website. Several commercial web-based SMS services are now available. Among them, a version of NCTU-SMS was the first system utilized by the Taiwan government (specifically, the E-Land County) for civilian services. This system can, for example, notify a citizen that his/her driver's license should be renewed. At NCTU, students who take a personal communication course will receive their test scores and the final grades through short messages generated by NCTU-SMS.

Based on the operation of NCTU-SMS, this paper analyzes some interesting statistics of SMS. Specifically, we investigate the distributions of round-trip transmission delays between the web-based SMS interface and MSs. These distributions are roughly approximated by truncated normal, gamma,

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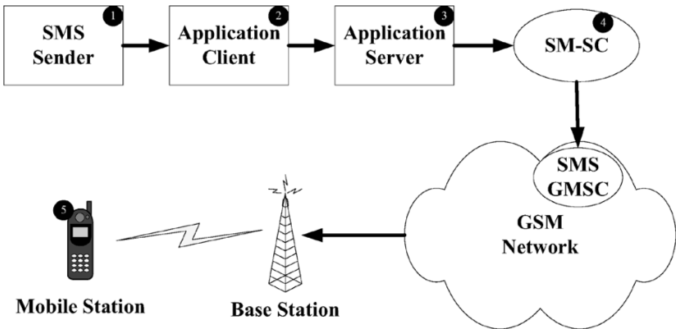

Fig. 2. NCTU-SMS architecture.

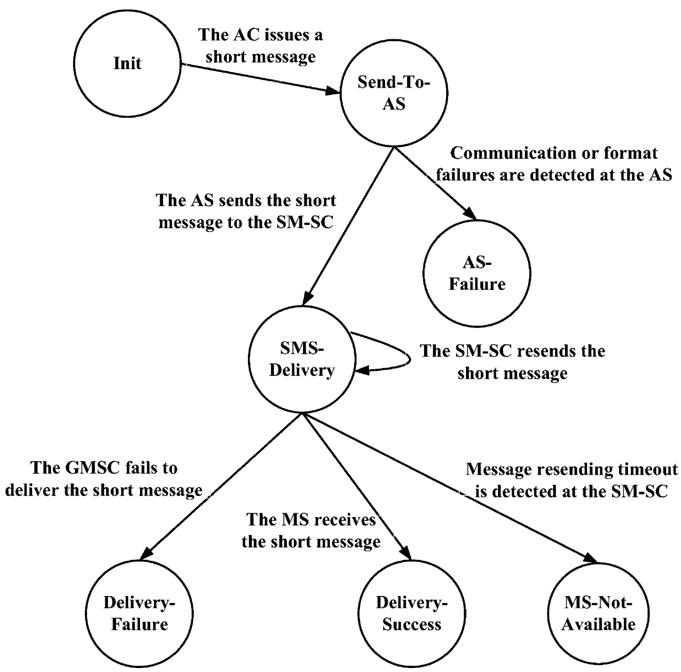

Fig. 3. Delivery state tree (AC: application client; AS: application server).

log-normal, and Weibull distributions with various parameters, which can be utilized in analytic analysis. Furthermore, with the nonparametric fitting approach, we use the rejection method to develop the random number generators of these delays, which can be used in any simulation models that involve SMS delivery.

II. NCTU Short Message System

The NCTU-SMS architecture is illustrated in Fig. 2, which consists of an *application client*, an *application server*, and a SM-SC. A delivery state tree (see Fig. 3) is used to indicate the status of every short message delivery. The delivery state of a short message is recorded in the application client. To send a short message, the following steps are executed.

Step 1) A user [Fig. 2, ①] issues a short message through the web-based GUI. Specifically, the user inputs the destination phone number and types the message text. When the user presses the OK button, the application client [Fig. 2, ②] generates a message delivery record that records the 1) destination MS number (i.e., the mobile phone number); 2) time T_i when the message is issued; 3) current delivery state; 4) time T_s associated with the delivery state; and 5) other parameters.

At this step, the delivery state is Init and $T_s = T_i$.

Step 2) *Steps 2 and 3*) The application client forwards the short message to the application server [Fig. 2, ③], and the delivery state is changed to $\mathsf{Send-To-AS}$ (where AS represents application server). If the communications link between the application client and the application server is disconnected, the application client sets the delivery state to $\mathsf{AS-Failure}$. Otherwise, the application server receives the message delivery request from the application client and performs format checking (e.g., the destination phone number format, the message text format, and so on). If format check fails, an error message is sent back to the application client. In this case, the application client updates T_s to the current time, the delivery state is set to $\mathsf{AS-Failure}$, and the procedure exits. If the format check is a success, then an acknowledgment message is sent back to the application client, and the application server forwards the short message to the SM-SC [Fig. 2, ④]. When the application client receives the acknowledgment, it updates T_s and sets the delivery state to $\mathsf{SMS-Delivery}$.

Step 3) *Step 4*) When the SM-SC receives the short message, it delivers the message following the standard SMS procedure illustrated in Fig. 1. Note that the message may not be actually delivered if errors (e.g., the HLR cannot identify the destination MS) occur. In this case, an error message is sent back to the application client. The application client updates T_s to the current time, the delivery state is set to $\mathsf{Delivery-Failure}$, and the procedure exits. In the normal case, the message will be sent to the destination MS, and the next step is executed.

Step 4) *Step 5*) When the destination MS receives the short message, an acknowledgment is sent back to the application client. The application client updates T_s to the current time, the delivery state is set to $\mathsf{Delivery-Success}$, and the procedure exits. For some reasons, the short message may not be received by the destination MS (e.g., the destination MS is power off). In this case, the short message will be periodically resent until either the destination MS receives the message or the SM-SC gives up. In the latter case, an error message is sent back to the application client. The application client updates T_s, sets the delivery state to $\mathsf{MS-Not-Available}$, and exits the procedure.

From the above description, if the delivery is successful, the final delivery state is $\mathsf{Delivery-Success}$. If the delivery fails, the final delivery state is either $\mathsf{AS-Failure}$, $\mathsf{Delivery-Failure}$, or $\mathsf{MS-Not-Available}$.

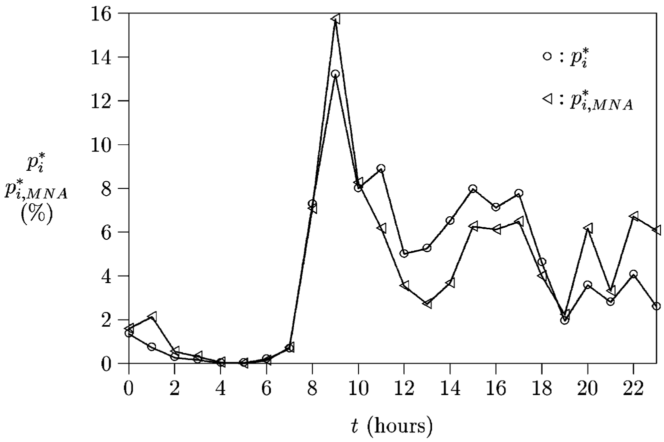

Fig. 4. T_i^* and $T_{i,\mathrm{MNA}}^*$ distributions.

TABLE I
NUMBERS OF DELIVERIES WITH VARIOUS FINAL STATES

State	AS-Failure	Delivery-Failure
Number	423	1649
Percentage	0.92%	3.57%
State	**MS-Not-Available**	**Delivery-Success**
Number	4278	39,780
Percentage	9.27%	86.23%

III. STATISTICS FOR SMS DELIVERY

Several interesting statistics can be derived from the delivery records in the application client; for example, the hourly distribution T_i^* of T_i (the time when a short message is issued), where

$$T_i^* = T_i \bmod 24.$$

We are also interested in the distribution of the issued times $T_{i,\mathrm{MNA}}^*$ for those failed SMS deliveries with the final state MS – Not – Available (MNA). Let $p_i^*(t)$ be the probability that a SMS is issued during the tth hour (where $t = 0, 1, 2, \ldots, 23$). Similarly, let $p_{i,\mathrm{MNA}}^*(t)$ be the probability that a MS – Not – Available SMS is issued during the tth hour. Fig. 4 plots the p_i^* and $p_{i,\mathrm{MNA}}^*$ curves. The p_i^* curve indicates that most short messages are issued during the work hours (8:00 am–5:00 pm), with a major peak at 9:00 am. It implies that many SMS users tend to send short messages immediately after they enter the offices. It is also apparent by observation that the number of issued short messages drops during the lunch break. The $p_{i,\mathrm{MNA}}^*$ curve follows the same trend as the p_i^* curve. It is interesting to observe that $p_{i,\mathrm{MNA}}^* > p_i^*$ during 8:00 pm–1:00 am. This phenomenon can be explained by the following human behavior. We first note that in NCTU-SMS, the short messages are dropped after a resent period of 8 h. Therefore, the MS – Not – Available short messages are dropped around the 8th h after they are issued. Our experience indicates that many people turn off their MSs during 8:00 pm–9:00 am [2]. Therefore, many short messages issued during 8:00 pm–1:00 am will be dropped. During 9:00 am, a large number of short messages are issued in this hour. We observe a nontrivial phenomenon that $p_{i,\mathrm{MNA}}^* > p_i^*$.

From the measured data we collected, Table I shows the numbers of deliveries with various final states. For the deliveries with the AS – Failure state, they failed due to sender mistakes (e.g., inputting the incorrect phone number formats). For the deliveries with the MS – Not – Available state, they failed due to unavailability of the receivers. Therefore, to consider the failures result from the SMS delivery system, we should only consider the Delivery – Failure short messages, and this system failure probability is 3.57%.

The most important statistics are the round-trip transmission delays T_D of short messages, which are defined as

$$T_D = T_s - T_i.$$

Fig. 5 illustrates the T_D probability distribution for successful deliveries, which is normalized in the rage [1,10] (the normalization factor is 1.1). Figs. 6 and 7 illustrate the T_D probability distribution for failed deliveries. Fig. 6 indicates that most Delivery – Failures will be detected within 10 s. Fig. 7 indicates

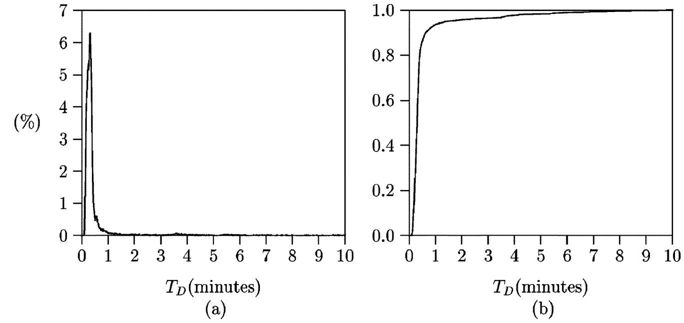

Fig. 5. T_D distribution (Delivery – Success; the normalized factor is 1.1). (a) Probability density function. (b) Cumulative distribution function.

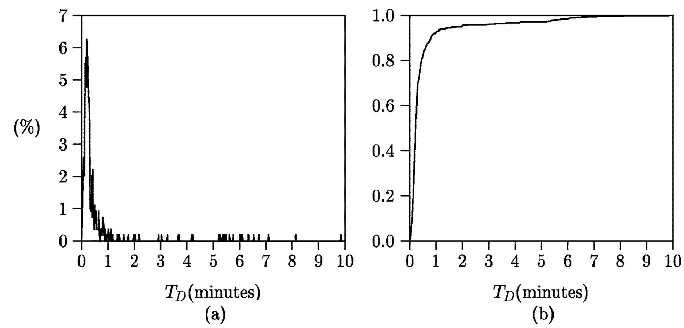

Fig. 6. T_D distribution for Delivery – Failure. (a) Probability density function. (b) Cumulative distribution function.

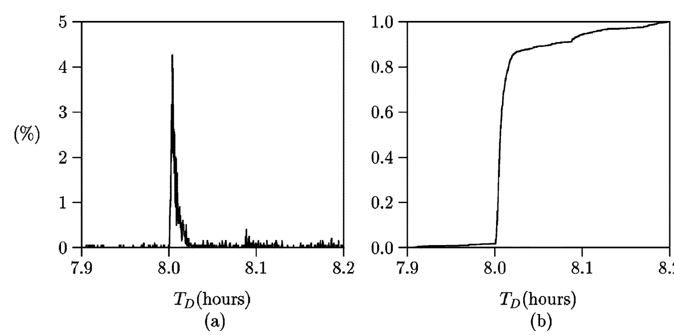

Fig. 7. T_D distribution for MS – Not – Available. (a) Probability density function. (b) Cumulative distribution function.

TABLE II
PARAMETER VALUES FOR THE T_D DISTRIBUTIONS

T_D Distribution	Delivery-Failure	Delivery-Success
Truncated normal	$\mu = 12, \sigma = 6$	$\mu = 19, \sigma = 6$
Truncated gamma	$t = 4, \lambda = 0.28$	$t = 10, \lambda = 0.5$
Truncated log-normal	$\mu = 2.6, \sigma = 0.52$	$\mu = 2.9, \sigma = 0.35$
Truncated Weibull	$\beta = 2.3, \eta = 15$	$\beta = 3.4, \eta = 20$

that most MS − Not − Availables are reported within 15 min after the 8-h timeout.

IV. Round-Trip Transmission Delay Distributions

Based on 40 000 measured data for T_D, this section derives the round-trip transmission delay distributions for both the Delivery − Failure and the Delivery − Success times, respectively. The MS − Not − Available time distribution can be derived using the same approach and the details are omitted. We choose the data within the first 150 s since most of the measured data are distributed within this period. In this section, both parametric fitting and nonparameter fitting approaches are utilized to approximate the round-trip transmission delay distributions.

A. Parametric Fitting Approaches

We consider several truncated distributions to fit the T_D distributions, including the truncated normal, gamma, log-normal, and Weibull distributions. These distributions are widely used in telecommunications studies [4]. The normal distribution has the density function

$$f_N^*(x) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right) e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad \text{for} \quad -\infty < x < \infty.$$

The gamma distribution has the density function

$$f_G^*(x) = \frac{\lambda e^{-\lambda x}(\lambda x)^{t-1}}{\Gamma(t)}, \quad \text{for} \quad 0 \le x < \infty$$
$$\text{where} \quad \Gamma(t) = \int_{y=0}^{\infty} e^{-y} y^{t-1} dy.$$

The log-normal distribution has the density function

$$f_L^*(x) = \left(\frac{1}{\sqrt{2\pi}\sigma x}\right) e^{-\frac{(\log x-\mu)^2}{2\sigma^2}}, \quad \text{for} \quad 0 \le x < \infty.$$

The Weibull distribution has the density function

$$f_W^*(x) = \left(\frac{\beta x^{\beta-1}}{\eta^\beta}\right) e^{-\left(\frac{x}{\eta}\right)^\beta}, \quad \text{for} \quad 0 \le x < \infty.$$

In our study, we consider truncated distributions in the range 1–150 s. For example, the truncated normal density function is

$$f_N(x) = \left(\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\int_{y=1}^{150} f_N^*(y)dy\sqrt{2\pi}\sigma}\right), \quad \text{for} \quad 1 \le x \le 150.$$

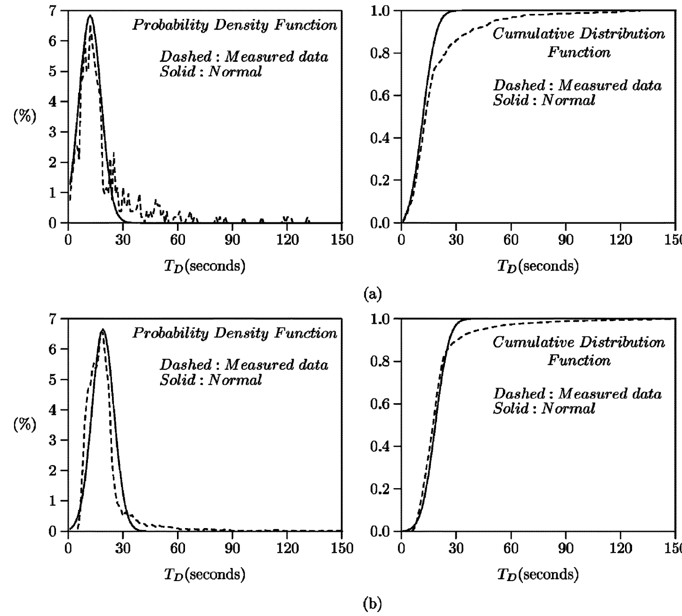

Fig. 8. The normal approximations of the T_D distributions. (a) PDF and CDF of Delivery − Failure. (b) PDF and CDF of Delivery − Success.

TABLE III
SKEWNESS, KURTOSIS, AND KOLMOGOROV–SMIRNOV (K-S) TESTS FOR TRUNCATED NORMAL DISTRIBUTION

Test	Delivery Failure	Delivery Success
skewness value	3.10683	4.30594
p-value	$< 10^{-4}$	$< 10^{-4}$
kurtosis value	12.61896	24.5495
p-value	$< 10^{-4}$	$< 10^{-4}$
K-S statistics	0.2409	0.2537
p-value	0	0

To generate the T_D distributions based on the measured data, the parameter values of the truncated normal, gamma, log-normal, and Weibull distributions are determined according to the *method of moments*, the *maximum likelihood estimates*, and the *trimmed mean and variance method* [1]. By using the statistic tools such as MATLAB and S-PLUS, we obtain the parameter values of the truncated distributions, which are summarized in Table II. Based on the parameter values in Table II, we plot the Delivery − Failure and Delivery − Success time distributions for normal approximations in Fig. 8 (the figures for other distributions provide similar information and are omitted). We apply the *Kolmogorov-Smirnov (K-S) goodness of fit test*, the *skewness test*, and the *kurtosis test* [1], [7] to see if the truncated distributions provides good approximations to the measured data. The skewness test checks the asymmetry of the distribution. For symmetrical distributions such as normal, the skewness value is zero. The kurtosis test checks the "tail" of a distribution. If the distribution has a heavy tail, it will have large kurtosis value. For a normal distribution, the kurtosis value is zero. The K-S goodness of fit test measures the absolute distance between the empirical distribution of the measured data and the test distribution. If the distance is too large, it implies that the empirical distribution cannot be approximated by the test distribution. The skewness, the kurtosis, and the

TABLE IV
KOLMOGOROV–SMIRNOV (K-S) STATISTICS FOR TRUNCATED GAMMA, LOG-NORMAL, AND WEIBULL DISTRIBUTIONS

Gamma	Delivery Failure	Delivery Success
K-S statistics	0.0783	0.1065
p-value	0.0032	0
Log-normal	Delivery Failure	Delivery Success
K-S statistics	0.0918	0.1172
p-value	0.00029	0
Weibull	Delivery Failure	Delivery Success
K-S statistics	0.1501	0.1899
p-value	0	0

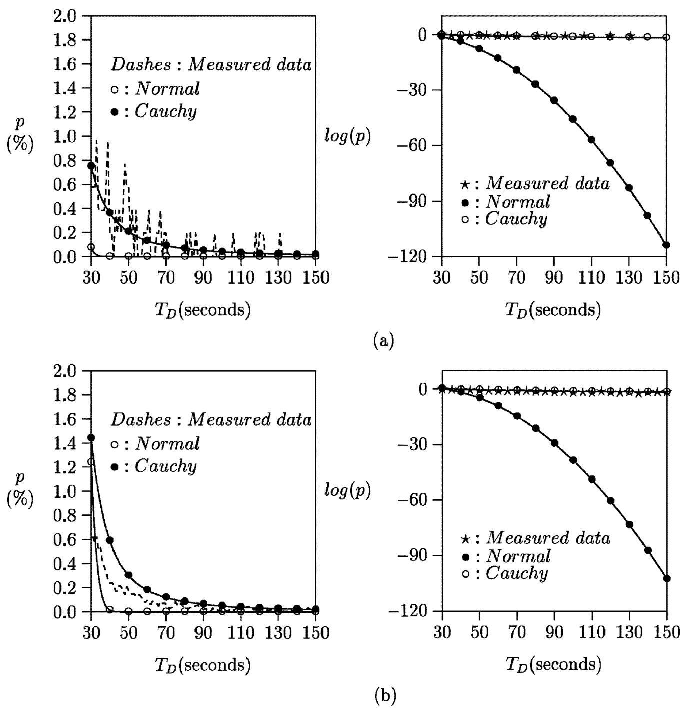

Fig. 9. Normal and cauchy approximations of the T_D tail distributions in the range [30 150]. (a) Delivery − Failure. (b) Delivery − Success.

K-S test results can be converted into the *p-values* [7]. When the p-value is less than 0.05, the test is rejected. A test is typically accepted if the p-value is larger than 0.15. From these tests for truncated normal distributions (see Table III), we find that the transmission delay distributions are not symmetrical and have heavy tails. The p-values are less than 10^{-4}, which indicates that these distributions cannot be approximated by the normal distributions. The K-S statistics and the p-values for truncated gamma, log-normal, and Weibull distributions are listed in Table IV. The table indicates that the p-values of the above tests are less than 0.01 for all truncated distributions under consideration. Fig. 9 indicates that the histograms of the measured data have heavy tails much longer than the truncated normal distribution. Same phenomenon is observed for truncated gamma, log-normal, and Weibull distributions (the corresponding curves are not presented). The Cauchy tail fits the measured data better, which will be elaborated in Section IV-B. Therefore, the truncated distributions can only be used for primary studies, which produce quick but less accurate T_D transmission delays.

B. Nonparametric Fitting Approach

To accurately approximate the T_D distributions, we consider nonparametric fitting [9]. We first use the *kernel density estimator* as first approximation, and then improve the accuracy by the *variable-bandwidth kernel estimator*. The kernel density estimator $f^*(x)$ with *Epanechnikov kernel function* $\theta(y)$ is expressed as

$$f^*(x) = \left(\frac{1}{h}\right) \sum_{i=1}^{150} n_i \theta\left(\frac{x-i}{h}\right). \quad (1)$$

In (1), n_i is the proportion that $T_D = i$ s (which is computed from the measured data), and h is the bandwidth. By using the *cross validation method* (see [9], Sec. 2.1.3), we compute $h = 1.75$. The Epanechnikov kernel function is expressed as

$$\theta(x) = 0.75\left[1 - \max\left(-1, \min(x,1)\right)^2\right].$$

Equation (1) is susceptible to bumpiness in the tails, since it does not adapt to local variations in smoothness. To resolve this issue, we generalize the kernel density estimator by using narrower windows for the contribution of values associated with regions of high density and broader windows for values associated with regions of low density (i.e., the tail part of the distribution). The variable-bandwidth kernel estimator is expressed as

$$f_{\rm npf}(x) = \sum_{i=1}^{150}\left[\frac{n_i}{h(x_i)}\right]\theta\left(\frac{x-i}{h(x_i)}\right), \quad \text{where} \quad h(x_i) = \frac{\sqrt{f(x_i)}}{h^*}. \quad (2)$$

In (2), the function $f(x_i)$ is unknown. Therefore, a "pilot" estimate is usually used to substitute for $f(x_i)$. In our example, (1) is used for the substitution. In (2), the value for h^* can be computed by using the *cross validation method* (see [9], Ch. 2.1.3), or can be observed directly from the measured data. We choose $h^* = 1.5$ for approximating the Delivery − Failure time distribution and $h^* = 2.0$ for the Delivery − Success time distribution. Based on this equation, we use the *rejection method* (see [7], Ch. 5.2) to implement a random number generator (RNG) for approximating the T_D distributions. The procedure for the rejection method is executed through the following steps. Let $F_{\rm npf}$ be the probability distribution with the $f_{\rm npf}$ density. We will prove that this procedure generates random numbers with the $F_{\rm npf}$ distribution in Appendix A.

The Random Number Generation (RNG) Procedure

Step 1) Consider the Cauchy density function [8]

$$g(x) = \frac{1}{s\pi\left[1 + \left(\frac{x-m}{s}\right)^2\right]} \quad (3)$$

where $-\infty < x < \infty$. We choose this function for two reasons. First, it is much easier to generate a random number from a Cauchy distribution than the $F_{\rm npf}$ distribution directly. Second, because of the heavy tail property, the Cauchy distribution can fit the tail part of $f_{\rm npf}(x)$ much better than other distributions such as normal or gamma distributions (see Fig. 9). The parameter values of $g(x)$ [i.e., s and m in (3)] are determined based on the trimmed mean and variance method [1] used in Section IV-A. For the Delivery − Failure time distribution, we obtain $s = 10$ and $m = 12$. For the

Delivery − Success time distribution, $s = 10$ and $m = 19$. In the rejection method, we need to find a constant C such that the following inequality holds:

$$C \cdot g(x) \geq f_{\rm npf}(x), \quad \text{where} \quad 1 \leq x \leq 150. \tag{4}$$

At the end of this section, we will describe how to find an appropriate C value.

Step 2) Based on (3), generate a Cauchy random number Y by the *inverse transform algorithm* (see [7], Ch. 5.1).

Step 3) Generate a random number U from a uniform distribution over the interval (0,1). If

$$U \leq \frac{f_{\rm npf}(Y)}{Cg(Y)} \tag{5}$$

we accept the random number Y. Otherwise, return to Step 2 to regenerate Y.

By executing the above procedure, the final value Y we accept will have the $f_{\rm npf}$ density. Now we show how to choose the C value. Suppose that a random number Y^* is generated at Step 2. Let p^* be the probability that such Y^* is rejected at Step 3; then

$$\begin{aligned} p^* &= \int_{Y^*=-\infty}^{\infty} g(Y^*)\left[1 - \frac{f_{\rm npf}(Y^*)}{Cg(Y^*)}\right] dY^* \\ &= 1 - \frac{1}{C}. \end{aligned} \tag{6}$$

If p^* is small, then it is more likely that a generated Y is accepted at Step 3, and the RNG procedure is more efficient. In (4), we have

$$\int_{x=1}^{150} f_{\rm npf}(x)dx = 1 \quad \text{and} \quad \int_{x=1}^{150} g(x)dx \leq 1.$$

By integrating both sides of (4), it is apparent that $C \geq 1$. From (6), we have $p^* \to 0$ if $C \to 1$. Therefore, when the constant C is closer to 1, the random number generator is more efficient (i.e., the generated Y values are more likely to be accepted at Step 3).

We rewrite (4) as

$$C \geq \max_{1 \leq x \leq 150}\left[\frac{f_{\rm npf}(x)}{g(x)}\right]. \tag{7}$$

Therefore, the minimum value for C is $\max_{1 \leq x \leq 150}[f_{\rm npf}(x)/g(x)]$. From (1), (2), (3), and (7), the constant C can be computed as

$$C = \begin{cases} 1.916, & \text{for the Delivery-Failure distribution} \\ 2.513, & \text{for the Delivery-Success distribution} \end{cases}. \tag{8}$$

Note that if a normal or gamma density is chosen as $g(x)$, the $[f_{\rm npf}(x)/g(x)]$ value will be very large over the interval (50,150), and therefore, the C value is large. For the normal distribution, the constant C will be larger than 1000 (which means that if we use normal $g(x)$, then Step 3 will be executed for many times before a generated Y is accepted). We use the above procedure to generate 100 thousand random numbers for Delivery − Failure times and the Delivery − Success times, respectively. Fig. 10 and Table V show that the generated random numbers fit the T_D distribution very well.

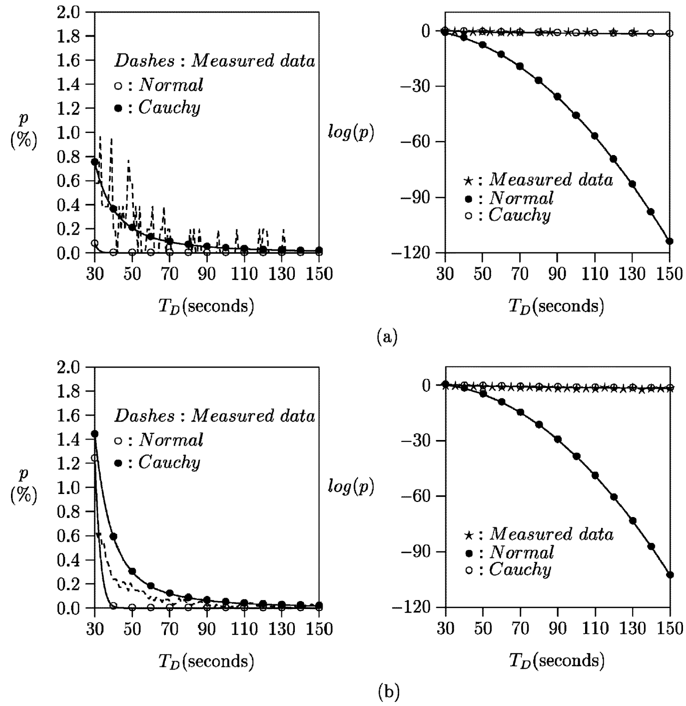

Fig. 9. Normal and cauchy approximations of the T_D tail distributions in the range [30 150]. (a) Delivery − Failure. (b) Delivery − Success.

TABLE V
KOLMOGOROV–SMIRNOV (K-S) STATISTICS FOR THE DATA GENERATED FROM THE RNG

RNG	Delivery Failure	Delivery Success
K-S statistics	0.0408	0.0143
p-value	0.4022	0.3343

V. CONCLUSION

Based on a commercially operated web-based short message service (SMS) system called NCTU-SMS, this paper investigated the statistics of SMS delivery. We collected the round-trip delays for 40 000 successful and failed SMS deliveries. Then we used both parametric and nonparametric fitting methods to derive the SMS round-trip delay (T_D) distributions. With the parametric fitting method, existing probability distributions such as normal, gamma, log-normal, and Weibull are used to approximate the T_D distributions, which can be used in analytic modeling for primary SMS studies. With nonparametric fitting method, we developed a procedure for random number generation, which accurately captures the T_D behavior. The developed random number generators can be used in simulation models for accurate analysis of SMS delivery.

APPENDIX

A. Correctness Proof for the RNG Procedure

This appendix proves that the RNG procedure correctly generates the random numbers with the $f_{\rm npf}$ density.

Theorem: The RNG procedure produces random numbers with the $f_{\rm npf}$ density if and only if inequality (4) holds.

Proof: $\Leftarrow$) We prove that if (4) holds, then the RNG procedure produces random numbers with the $f_{\rm npf}$ density. Consider the RNG procedure in Section IV-B. Let p^* be the probability that a random number generated at Step 2 of the RNG procedure is rejected based on (5). From (6), we have derived $p^* = 1 - (1/C)$. Suppose that a random number Y generated at Step 2 is accepted. For $1 \leq j \leq 150$, let p_j be the probability that $Y \leq j$, which can be expressed as

$$\begin{aligned} p_j &= \sum_{i=1}^{\infty} \Pr\left[\text{On iteration } i, Y \leq j \text{ and } Y \text{ is accepted}\right] \\ &= \sum_{i=1}^{\infty}\left\{\left(1-\frac{1}{C}\right)^{i-1}\left\{\int\limits_{Y=1}^{j}\left\{g(Y)\left[\frac{f_{\rm npf}(Y)}{Cg(Y)}\right]\right\}dY\right\}\right\}. \end{aligned} \quad (9)$$

In (9), $[1-(1/C)]^{i-1}$ is the probability that Step 3 does not accept the generated random numbers in the first $i-1$ iterations, and $\int_{Y=1}^{j}\{g(Y)[f_{\rm npf}(Y)/Cg(Y)]\}dY$ is the probability that, on the ith iteration, the generated random number Y is accepted and its value is less than or equal to j. Then (9) can be expressed as

$$\begin{aligned} p_j &= \sum_{i=1}^{\infty}\left(1-\frac{1}{C}\right)^{i-1}\left(\frac{1}{C}\right)\left[\int\limits_{Y=1}^{j} f_{\rm npf}(Y)dY\right] \\ &= \sum_{i=1}^{\infty}\left(1-\frac{1}{C}\right)^{i-1}\left[\frac{F_{\rm npf}(j)}{C}\right] \\ &= \left(\frac{F_{\rm npf}(j)}{C}\right)\left[1-\left(1-\frac{1}{C}\right)\right]^{-1} \\ &= F_{\rm npf}(j). \end{aligned} \quad (10)$$

Based on the above equation, the RNG procedure produces random numbers with the $f_{\rm npf}$ density if (4) holds.

$\Rightarrow$) We prove by contradiction. Suppose that (4) dose not hold, then there must exist a subset $S \subset [1, 150]$ such that for any $x \in S$,

$$Cg(x) < f_{\rm npf}(x). \quad (11)$$

Since $0 < U < 1$ at Step 3 of the RNG procedure, (11) implies that (5) always holds for any generated random number Y in S. That is, for $Y \in S$, Y is always accepted at Step 3. Therefore, the probability that Step 3 accepts a generated random number Y will be $\min[(f_{\rm npf}(Y)/Cg(Y)), 1]$. In this case, (6) is rewritten as

$$p^* = \int\limits_{Y=-\infty}^{\infty} g(Y)\left[1-\min\left(\frac{f_{\rm npf}(Y)}{Cg(Y)}, 1\right)\right]dY. \quad (12)$$

It is clear that by deriving (12), we will have $p^* \neq 1 - (1/C)$. Therefore, $p_j \neq F_{\rm npf}(j)$ in (10). In other words, the random numbers generated by the RNG procedure will not have the $f_{\rm npf}$ density. Based on the above argument, we have proven that if the RNG procedure generates random numbers with the $f_{\rm npf}$ density, then (4) must hold.

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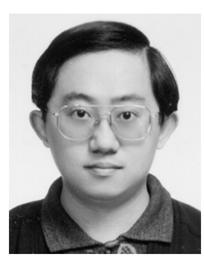

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