Nonlinear Transceiver Designs in MIMO Amplify-and-Forward Relay Systems

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***Abstract*—Various linear transceiver design methods have been developed in three-node amplify-and-forward (AF) multiple-input–multiple-output (MIMO) relay systems. Nonlinear designs in such systems, however, have yet to be investigated. In this paper, we propose nonlinear transceiver designs for a linear source and relay precoded system with the QR successive-interference-cancellation (SIC) receiver and another linear source and relay precoded system with the minimum-mean-squared-error (MMSE) SIC receiver. Our designs minimize the criterion of the block error rate, which is a complicated function of the source and relay precoders. Solving the two precoders simultaneously is not feasible. To overcome the difficulties, we first resort to the primal decomposition approach, i.e., transferring the original optimization to a subproblem and a master problem and solving the two precoders individually. However, since two power constraints are mutually coupled, the decomposition cannot actually be conducted. We then propose a unitary structure for the source precoder and show that the power constraints can be decoupled. As a result, the source precoder can be solved as a function of the relay precoder in the subproblem. With a proposed relay precoder structure, the master problem can further be transferred to a scalar-valued concave optimization problem. A closed-form solution can finally be derived by the Karuch–Kuhn–Tucker (KKT) conditions. Simulations show that the proposed transceivers can significantly outperform the existing linear transceivers.**

***Index Terms*—Amplify-and-forward (AF), Karuch–Kuhn–Tucker (KKT) conditions, minimum-mean-squared-error successive interference cancellation (MMSE-SIC), multiple-input–multiple-output (MIMO) relay, precoder design, primal decomposition, QR successive interference cancellation (QR-SIC), transceiver design.**

I. Introduction

COOPERATIVE communication has garnered significant interest since it provides a practical means for coverage extension, capacity boost, and reliability enhancement in wireless networks [1]–[14]. The main idea behind cooperative systems is to deploy relays at some strong shadowing area. In this manner, signals from the source can be transmitted to the destination by the source-to-destination link (direct link) and the source-to-relay and relay-to-destination link (relay link). Two main relaying protocols have been developed: 1) amplify and forward (AF) and 2) decode and forward (DF). In AF, the relays receive the signals from the source and retransmit them to the destination with signal amplification only [9]–[11], [14]. In DF, the relays decode the received signals, reencode the information bits, and then retransmit the resultant signals to the destination. It is well known that the AF protocol has lower complexity and processing delay. In this paper, we consider the transceiver design with AF protocol.

Recently, the multiple-input–multiple-output (MIMO) technique has been introduced to cooperative systems. With multiple antennas deployed at each node, a MIMO relay system is constructed [8]–[14]. Capacity bounds for a single-relay MIMO channel were first addressed in [8]. Similar to a conventional MIMO system, the precoding technique can be applied in a MIMO relay system for further performance improvement. For example, the relay precoder in [9] and [10] was designed to enhance the overall channel capacity. Note that in most of those approaches, only the relay link is considered. It was shown that the capacity can further be increased if the direct link is further taken into account [10]. Apart from the capacity, the minimum-mean-squared-error (MMSE) is an alternative criterion that has been considered [11], [12]. Therein, a relay precoder was designed to minimize the mean square error of the receiver. More recently, the joint source and relay precoder designs have been studied in [13] and [14]. In these approaches, the two precoders are jointly designed for the direct and relay links via the MMSE [13] and the bit-error-rate (BER) criteria, respectively [14].

As far as we know, existing transceiver designs in AF MIMO relay systems all consider linear precoders and linear receivers. As well known in conventional MIMO systems, precoding with nonlinear receivers is superior to that with linear receivers. Since a MIMO relay system can be formulated as a conventional MIMO system, precoding with nonlinear receivers can also provide better performance. In this paper, we consider nonlinear transceiver designs in AF MIMO relay systems. Our transceiver consists of a linear source precoder, a linear relay precoder, and a nonlinear receiver. Two well-known nonlinear receivers, namely, QR successive interference cancellation (SIC) and MMSE-SIC, are considered in our designs. Note that the transceiver design here is different from that in point-to-point MIMO systems. This is because two precoders (the source and relay precoders), two coupled power constraints (the source and relay power constraints), and three channels (the source-to-rely, rely-to-destination, and source-to-destination channels) have to be considered simultaneously.

In the design of communication systems, the BER or the symbol error rate (SER) is the typical criterion we usually minimize. However, for nonlinear MIMO receivers, the SER is a cumbersome function of the source and relay precoders. It is

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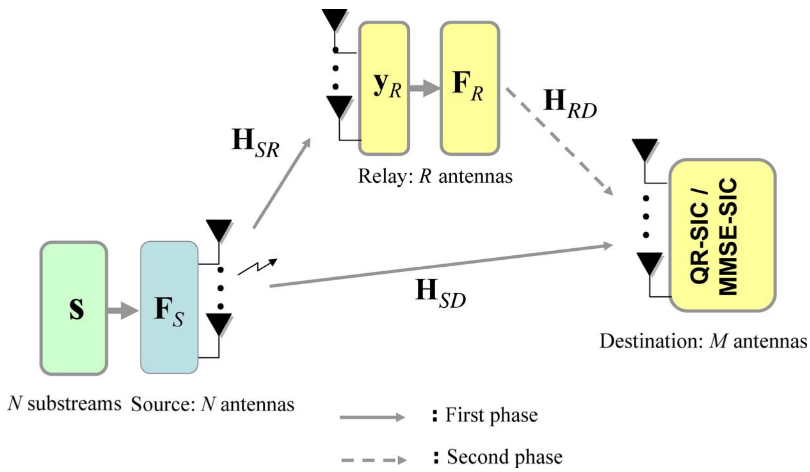

Fig. 1. Linear source and relay precoded AF MIMO relay system with SIC-based receiver.

very difficult to optimize the precoders with the SER. To derive tractable solutions, instead of using SER, we propose using the block error rate (BLER) as our design criterion. As we will see, the BLER criterion is much easier to work with. Our method starts with the primal decomposition method, decomposing the optimization problem into a subproblem and a master problem. The source precoder, which is expressed as a function of the relay precoder, is first solved in the subproblem. Then, the relay precoder is solved in the master problem [19]. However, since the two power constraints are coupled, this decomposition cannot actually be conducted. To overcome the difficulty, we propose using a unitary source precoder and show that the power constraints can be decoupled. With our formulation, the subproblems with the QR-SIC and the MMSE-SIC receivers can easily be solved by the geometric mean decomposition (GMD) and uniform channel decomposition (UCD) methods, respectively. In the master problem, only the relay precoder remains. Unfortunately, this problem is still difficult to solve since the optimization is not a convex problem. We then propose a precoder structure being able to translate the problem to a standard scalar-valued concave optimization problem. Using the Karuch–Kuhn–Tucker (KKT) conditions, we can finally obtain closed-form solutions for the relay and source precoders.

This paper is organized as follows: Section II formulates the precoded AF MIMO relay system with the QR-SIC or MMSE-SIC receiver. Section III describes the proposed design methods and shows how the precoders can be solved. Section IV evaluates the performance of the proposed nonlinear transceivers, whereas Section V draws conclusions.

II. System Models

A. Systems With Linear Source and Linear Relay Precoders

We consider a three-node AF MIMO relay precoding system in which N, R, and M antennas are placed at the source, the relay, and the destination, respectively, as shown in Fig. 1. All the channels are assumed to be flat fading. We consider the general two-phase transmission protocol [9]–[14], avoiding interference between direct and relay links. In the first phase, the source signal vector $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is multiplied by a linear source precoder $\mathbf{F}_S \in \mathbb{C}^{N\times N}$ and then transmitted to the relay and the destination. The received signal vectors at the relay and the destination can be expressed as

$$\mathbf{y}_R = \mathbf{H}_{SR}\mathbf{F}_S\mathbf{s} + \mathbf{n}_R \tag{1}$$
$$\mathbf{y}_{D,1} = \mathbf{H}_{SD}\mathbf{F}_S\mathbf{s} + \mathbf{n}_{D,1} \tag{2}$$

where $\mathbf{H}_{SR} \in \mathbb{C}^{R\times N}$ and $\mathbf{H}_{SD} \in \mathbb{C}^{M\times N}$ are the channel matrices corresponding to the source-to-relay and source-to-destination channels, respectively, $\mathbf{n}_R \in \mathbb{C}^{R\times 1}$ is the received noise vector at the relay, and $\mathbf{n}_{D,1} \in \mathbb{C}^{M\times 1}$ is the first-phase received noise vector at the destination. The (i,j)th element of $\mathbf{H}_{SR}$ represents the channel gain between the jth transmit antenna and the ith receive antenna of the source-to-relay link; a similar definition is applied to the other links. In addition, the ith element of the noise vector denotes the noise received at the ith antenna.

In the second phase, the received signal vector at the relay is multiplied by the relay precoding matrix and then transmitted to the destination. Therefore, the received signal vector at the destination can be expressed as

$$\begin{aligned}\mathbf{y}_{D,2} &= \mathbf{H}_{RD}\mathbf{F}_R\mathbf{y}_R + \mathbf{n}_{D,2}\\ &= \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\mathbf{F}_S\mathbf{s} + (\mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2})\end{aligned} \tag{3}$$

where $\mathbf{F}_R \in \mathbb{C}^{R\times R}$ is the precoding matrix at the relay, $\mathbf{H}_{RD} \in \mathbb{C}^{M\times R}$ is the channel matrix corresponding to the relay-to-destination channel, and $\mathbf{n}_{D,2} \in \mathbb{C}^{M\times 1}$ is the second-phase received noise vector at the destination. Here, we assume that each element in $\mathbf{n}_{D,1}$ has a zero-mean circularly symmetric Gaussian distribution and that all the elements are independent identically distributed (i.i.d.). The same assumption is applied for $\mathbf{n}_{D,2}$ and $\mathbf{n}_R$. As a result, the received signal vectors $\mathbf{y}_{D,1}$ and $\mathbf{y}_{D,2}$ can be combined into a single vector, which is denoted as $\mathbf{y}_D \in \mathbb{C}^{2M\times 1}$. Then, we have

$$\mathbf{y}_D := \begin{bmatrix}\mathbf{y}_{D,1}\\ \mathbf{y}_{D,2}\end{bmatrix} = \mathbf{H}\mathbf{F}_S\mathbf{s} + \mathbf{n} \tag{4}$$

where

$$\mathbf{H} = \begin{bmatrix}\mathbf{H}_{SD}\\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}\end{bmatrix} \tag{5}$$
$$\mathbf{n} = \begin{bmatrix}\mathbf{n}_{D,1}\\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2}\end{bmatrix}. \tag{6}$$

Here, $\mathbf{H}$ is the equivalent channel matrix, and $\mathbf{n}$ is the equivalent noise vector at the destination. To provide sufficient degrees of freedom for signal recovery, we assume that $N \leq \{\min\{M,N\} + \min\{M,N,R\}\}$ and rank $(\mathbf{H}) = N$. It is noteworthy that the noise received at the relay is transformed by the relay precoder and the relay-to-destination channel. In addition, the equivalent channel matrix in (4) is a function of the relay precoder $\mathbf{F}_R$. This scenario is different from that considered in conventional point-to-point MIMO systems.

Denote the noise power at the destination and the relay as $\sigma_{n,d}^2$ and $\sigma_{n,r}^2$, respectively, and assume that the transmitted symbols $\mathbf{s}$ are i.i.d. with zero mean and covariance matrix $\mathbf{R}_S = E[\mathbf{s}\mathbf{s}^H] = \sigma_s^2 \mathbf{I}_N$, where $\mathbf{I}_N$ is an $N \times N$ identity matrix, E is the expectation operator, and σ_s^2 is the transmitted symbol power. Then, we have the covariance matrix of the equivalent noise vector as

$$\mathbf{R}_n = E[\mathbf{n}\mathbf{n}^H] = \begin{bmatrix} \sigma_{n,d}^2 \mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \sigma_{n,r}^2 \mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2 \mathbf{I}_M \end{bmatrix}. \quad (7)$$

From (7), it is clear that the equivalent noise vector is not white. To facilitate later analysis, we first apply a whitening operation to $\mathbf{y}_D$. Letting $\mathbf{W}$ be a whitening matrix and multiplying (4) with $\mathbf{W}$, we can have

$$\tilde{\mathbf{y}}_D := \mathbf{W}\mathbf{y}_D = \tilde{\mathbf{H}}\mathbf{F}_S\mathbf{s} + \tilde{\mathbf{n}} \quad (8)$$

where $\tilde{\mathbf{H}} = \mathbf{W}\mathbf{H}$, and $\tilde{\mathbf{n}} = \mathbf{W}\mathbf{n}$. Due to the whitening operation, $E[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H] = E[\mathbf{W}\mathbf{n}\mathbf{n}^H\mathbf{W}^H] = \mathbf{I}_{2M}$. From (7) and (8), we can then obtain the whitening matrix as

$$\mathbf{W} = \begin{bmatrix} \sigma_{n,d}^{-1}\mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \left(\sigma_{n,r}^2 \mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2 \mathbf{I}_M\right)^{-1/2} \end{bmatrix}. \quad (9)$$

The equivalent channel matrix after the whitening process can then be written as

$$\tilde{\mathbf{H}} = \mathbf{W}\mathbf{H} = \begin{bmatrix} \sigma_{n,d}^{-1}\mathbf{H}_{SD} \\ \left(\sigma_{n,r}^2 \mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2 \mathbf{I}_M\right)^{-1/2} \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}. \quad (10)$$

From (8), we can see that an AF MIMO relay system can be regarded as a MIMO system with the channel matrix shown in (10). The difference lies in that the channel in (10) is a function of the relay precoder. Since $\mathbf{F}_R$ is unknown, existing design methods in MIMO systems cannot directly be applied.

B. QR-SIC Receiver and Its SER

To derive the QR-SIC receiver, we first factorize the equivalent channel of the precoded system by QR decomposition, i.e., $\tilde{\mathbf{H}}\mathbf{F}_S = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is a $2M \times 2M$ unitary matrix, and $\mathbf{R}$ is a $2M \times N$ upper triangular matrix. Equation (8) can then be rewritten as

$$\hat{\mathbf{y}}_D = \mathbf{Q}^H\tilde{\mathbf{y}}_D = \mathbf{Q}^H\mathbf{Q}\mathbf{R}\mathbf{s} + \mathbf{Q}^H\tilde{\mathbf{n}} = \mathbf{R}\mathbf{s} + \hat{\mathbf{n}} = \underbrace{\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N} \\ 0 & r_{2,2} & \cdots & r_{2,N} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & r_{N,N} \\ 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}}_{:=\mathbf{R}} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{bmatrix} + \underbrace{\begin{bmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \vdots \\ \hat{n}_N \\ \hat{n}_{N+1} \\ \vdots \\ \hat{n}_{2M} \end{bmatrix}}_{:=\hat{\mathbf{n}}} \quad (11)$$

where $E[\hat{\mathbf{n}}\hat{\mathbf{n}}^H] = \mathbf{I}_{2M}$. Note that the equivalent channel for QR factorization here includes the source precoder. Signal detection in a QR-SIC receiver can then be conducted as

$$\begin{aligned} &\text{for } i = N : -1 : 1 \\ &\quad \hat{s}_i = \text{Dec}\left[\left(\hat{y}_i - \sum_{j=i+1}^{N} r_{i,j}\hat{s}_j\right) \Big/ r_{i,i}\right] \\ &\text{end} \end{aligned} \quad (12)$$

where Dec$[\cdot]$ denotes the decision operation, $\hat{y}_i$ denotes the ith element of $\hat{\mathbf{y}}_D$, and $\hat{s}_j$ denotes the estimation of s_j. Since noise is whitened and its distribution is Gaussian, the corresponding error rate for $\hat{s}_i$ can be approximated as [20]

$$P_{e,i}^{qr-sic} \simeq 1 - Pr\left(P_{c,i}^{qr-sic} | P_{c,N}^{qr-sic}, \ldots, P_{c,i+1}^{qr-sic}\right) = 1 - \prod_{j=i}^{N}\left(1 - \alpha Q(\sqrt{\beta \text{SNR}_j})\right) \quad (13)$$

where α and β are constants depending on the modulation scheme, $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-y^2/2}dy$, $\text{SNR}_i = \sigma_s^2|r_{i,i}|^2$ denotes the SNR of the ith layer estimated signal, $P_{e,i}^{qr-sic}$ denotes the error rate of $\hat{s}_i$, and $P_{c,i}^{qr-sic}$ denotes the correct rate of $\hat{s}_i$. For the m quadrature-amplitude-modulation (m-QAM) scheme, α and β are 4 and $3/(m-1)$, respectively [15]. Equation (13) indicates that in the backward detection scheme, s_i is correctly detected only if all the previous symbols are correctly detected (i.e., $\hat{s}_j = s_j$, $j > i$). From (11), it is apparent that $r_{i,j}$ is a complicated function of $\mathbf{F}_S$ and $\mathbf{F}_R$.

C. MMSE-SIC Receiver and Its SER

The MMSE-SIC receiver detects the desired symbol by two steps: 1) interference suppression and 2) cancellation. To see the operation, we first rewrite (8) as

$$\tilde{\mathbf{y}}_D := \tilde{\mathbf{H}}\mathbf{F}_S\mathbf{s} + \tilde{\mathbf{n}} = \widehat{\mathbf{H}}\mathbf{s} + \tilde{\mathbf{n}}. \quad (14)$$

We see that $\widehat{\mathbf{H}}$ is a function of $\mathbf{F}_S$ and $\mathbf{F}_R$. With the received model in (14), the ith layer symbol is detected after an inner product operation with a suppression vector $\mathbf{v}_i$ is conducted. Using (14) and the MMSE criterion, we can have the suppressing vector $\mathbf{v}_i$ for the ith layer signal as [17]

$$\mathbf{v}_i = \left(\sum_{j=1}^{i} \widehat{\mathbf{h}}_j \widehat{\mathbf{h}}_j^H + \sigma_s^{-2}\mathbf{I}\right)^{-1} \widehat{\mathbf{h}}_i, \qquad i = 1, \ldots, N \quad (15)$$

where $\widehat{\mathbf{H}} := \widetilde{\mathbf{H}}\mathbf{F}_S = [\widehat{\mathbf{h}}_1, \ldots, \widehat{\mathbf{h}}_N]$. Equation (15) implicitly indicates that the symbols are backward detected from $i = N$ to $i = 1$. The suppression vector can also be found via QR decomposition [17]. Consider the following decomposition:

$$\begin{bmatrix} \widehat{\mathbf{H}} \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \mathbf{Q}_1\mathbf{R}_1 = \begin{bmatrix} \mathbf{Q}_1^u \\ \mathbf{Q}_1^l \end{bmatrix} \mathbf{R}_1 \quad (16)$$

where $\mathbf{R}_1 \in \mathbb{C}^{(2M+N)\times N}$ is an upper triangular matrix with positive diagonal elements, and $\mathbf{Q}_1 \in \mathbb{C}^{(2M+N)\times(2M+N)}$ is a unitary matrix. Note that $\mathbf{Q}_1^u \in \mathbb{C}^{2M\times(2M+N)}$ and $\mathbf{Q}_1^l \in \mathbb{C}^{N\times(2M+N)}$ are not unitary matrices. The suppression vector in (15) can be obtained by

$$\mathbf{v}_i = \mathbf{R}_1^{-1}(i,i)\mathbf{Q}_1^u(:,i), \qquad i = 1, \ldots, N \quad (17)$$

where $\mathbf{R}_1(i,i)$ denotes the ith diagonal element of $\mathbf{R}_1$, and $\mathbf{Q}_1^u(:,i)$ denotes the ith column of $\mathbf{Q}_1^u$. Similar to QR-SIC, once s_i is detected, its interference to other signal components is then subtracted. If the error propagation effect in subtraction is ignored, then the following equivalence holds for each i [17]:

$$\sigma_s^{-2}(1 + \mathrm{SINR}_i) = \mathbf{R}_1^2(i,i), \qquad i = 1, \ldots, N \quad (18)$$

where

$$\mathrm{SINR}_i = \widehat{\mathbf{h}}_i^H \left(\sum_{j=1}^{i-1} \widehat{\mathbf{h}}_j \widehat{\mathbf{h}}_j^H + \sigma_s^{-2}\mathbf{I}_{2M}\right)^{-1} \widehat{\mathbf{h}}_i, \qquad i = 1, \ldots, N \quad (19)$$

denotes the signal-to-interference-plus-noise ratio (SINR) of the ith layer estimated signal in the MMSE-SIC receiver. It is noteworthy that the equivalence in (18) is only valid in a backward detection scheme. Similar to (13), the SER for the ith layer can thus be expressed as

$$P_{e,i}^{\text{mmse-sic}} \simeq 1 - \prod_{j=i}^{N} \left(1 - \alpha Q(\sqrt{\beta \mathrm{SINR}_j})\right). \quad (20)$$

III. Proposed Joint Source/Relay Precoder Design

A. Design Criterion

From (13) and (20), we can find that the SER of each substream is a complicated function of the source and relay precoders. Therefore, it is difficult to design the precoders by minimizing the average SER, even with a numerical method. To overcome the problem, here, we propose using the BLER criterion for our design. A transmit signal vector is referred to a signal block. If at least an element in the signal vector is in error, then the whole signal block is deemed in error. The BLER is then defined as

$$P_{\text{bler}} = 1 - P_c \quad (21)$$

where P_c is the block correct rate. From (13) and (20), we see that a symbol is correctly detected only if all the previous symbols are correctly detected. Since s_1 is the last symbol to be detected, if s_1 is correctly detected, then the whole block is correctly detected. Thus, P_c can be either $(1 - P_{e,1}^{qr-sic})$ or $(1 - P_{e,1}^{\text{mmse-sic}})$.

Theorem 1: The maximum P_c in (21) can be achieved if the source and relay precoders are designed such that the SINR_i's, for all i, are equal and maximized.

Proof: Define

$$P_c(\mathrm{SINR}_1, \ldots, \mathrm{SINR}_N) = \prod_{i=1}^{N} (1 - f(\mathrm{SINR}_i)) \quad (22)$$

where $f(\mathrm{SINR}_i) = \alpha Q(\sqrt{\beta \mathrm{SINR}_i})$. Since the second derivative of $f(\mathrm{SINR}_i)$ is positive [15], $f(\mathrm{SINR}_i)$ is a convex function of SINR_i. In addition, define

$$F = \ln P_c = \sum_{i=1}^{N} \ln\left(1 - f(\mathrm{SINR}_i)\right). \quad (23)$$

Since f is convex, $\ln(1 - f)$ will be concave. The summation of concave functions is also a concave function. Therefore, F is also a concave function [19]. By Jenssen's inequality, we have

$$F = \sum_{i=1}^{N} \ln\left(1 - f(\mathrm{SINR}_i)\right) \leq N \ln\left(1 - f\left(\frac{1}{N}\sum_{i=1}^{N} \mathrm{SINR}_i\right)\right) \quad (24)$$

where the equality holds when $\mathrm{SINR}_i = \mathrm{SINR}_j = (1/N)\sum_{i=1}^{N} \mathrm{SINR}_i$. As a result, we can design the source and relay precoders so that the SINR_j's, for all j, are equal and maximized. In this case, P_{bler} is minimized. The same result is also held when the SINR in (22) is replaced by SNR.

B. Precoder Design for QR-SIC Receiver

Based on Theorem 1, we can formulate the minimum BLER optimization problem as

$$\begin{aligned} &\max_{\mathbf{F}_S, \mathbf{F}_R} \ \mathrm{SNR}_i \\ &s.t. \quad \mathrm{SNR}_i = \sigma_s^2 |r_{i,i}|^2 \text{ are equal} \qquad \forall i,\ \mathrm{C}_1 \text{ and } \mathrm{C}_2, \\ &\mathrm{C}_1 : \sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\} \leq P_{S,T} \\ &\mathrm{C}_2 : tr\left(\mathbf{F}_R\mathbf{y}_R\mathbf{y}_R^H\mathbf{F}_R^H\right) \\ &\qquad = tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \leq P_{R,T} \end{aligned} \quad (25)$$

where C_1 and C_2 indicate the transmitted power constraints at the source and relay (the maximal available power is $P_{S,T}$

and $P_{R,T}$, respectively). From (25), we can easily find that this problem is not a convex optimization problem, and the optimum solution is difficult to derive. To facilitate the optimization, we resort to the primal decomposition method [19], translating (25) into a subproblem and a master problem. The subproblem is first optimized for the source precoder, and subsequently, the master problem is optimized for the relay precoder. To proceed, we reformulate (25) as

$$\begin{aligned} \max_{\mathbf{F}_S,\mathbf{F}_R} \quad & \mathrm{SNR}_i = \max_{\mathbf{F}_R}\max_{\mathbf{F}_S}\mathrm{SNR}_i \\ s.t. \quad & \mathrm{SNR}_i = \sigma_s^2|r_{i,i}|^2 \text{ are equal} \quad \forall i,\ \mathrm{C}_1 \text{ and } \mathrm{C}_2. \end{aligned} \tag{26}$$

In the subproblem, the relay precoder $\mathbf{F}_R$ is assumed to be given. Then, the optimum $\mathbf{F}_S$ can first be derived as a function of $\mathbf{F}_R$. Therefore, only the relay precoder is required to be determined in the master problem.

To proceed, we first consider the subproblem expressed as

$$\begin{aligned} \max_{\mathbf{F}_S(\mathbf{F}_R)} \quad & \mathrm{SNR}_i \\ s.t. \quad & \mathrm{SNR}_i = \sigma_s^2|r_{i,i}|^2 \text{ are equal} \quad \forall i \\ & \mathrm{C}_1 \quad \text{and} \quad \mathrm{C}_2. \end{aligned} \tag{27}$$

If $\mathbf{F}_R$ is given, then the problem can be solved by the GMD method [16]. From [16], we can have the following decomposition:

$$\tilde{\mathbf{H}} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}\tilde{\mathbf{P}}^H \tag{28}$$

where $\tilde{\mathbf{Q}} \in \mathbb{C}^{2M\times 2M}$ and $\tilde{\mathbf{P}} \in \mathbb{C}^{N\times N}$ are unitary matrices, and $\tilde{\mathbf{R}} \in \mathbb{C}^{2M\times N}$ is an upper triangular matrix having identical diagonal elements given by

$$\tilde{r}_{i,i} = \left(\prod_{k=1}^{N} |r_{k,k}|\right)^{1/N} = \left(\prod_{k=1}^{N} \sigma_{\tilde{\mathbf{H}},k}\right)^{1/N} \quad \text{for all } i = 1,\ldots,N. \tag{29}$$

Here, $\tilde{r}_{i,i}$ is the ith diagonal element of $\tilde{\mathbf{R}}$, and $\sigma_{\tilde{\mathbf{H}},k} > 0$ is the kth nonzero singular value of $\tilde{\mathbf{H}}$. Therefore, the equal-SNR constraint can be satisfied by setting the source precoder as

$$\mathbf{F}_S = \alpha\tilde{\mathbf{P}} \tag{30}$$

where α is a scalar chosen to satisfy the power constraint of the source, i.e., $\mathrm{tr}\{\mathbf{F}_S E[\mathbf{s}\mathbf{s}^H]\mathbf{F}_S^H\} = N\sigma_s^2\alpha^2 \le P_{S,T}$ or, equivalently, $\alpha \le \sqrt{P_{S,T}/(N\sigma_s^2)}$. By the GMD method and the following equivalence:

$$\arg\max_{\alpha,\mathbf{F}_R} \alpha\left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^{1/N} = \arg\max_{\alpha,\mathbf{F}_R} \det\left(\alpha^2\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) \tag{31}$$

we can then reformulate the subproblem (26) as

$$\begin{aligned} &\max_{\mathbf{F}_S(\mathbf{F}_R)} \det\left(\alpha^2\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) \\ &s.t. \quad \mathbf{F}_S = \alpha\tilde{\mathbf{P}},\ \mathrm{C}_1',\ \mathrm{C}_2' \\ &\mathrm{C}_1' :\ N\sigma_s^2\alpha^2 \le P_{S,T} \\ &\mathrm{C}_2' :\ tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\alpha^2\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \le P_{R,T} \end{aligned} \tag{32}$$

where $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ is defined in (10). The equality in (31) is due to the following property:

$$\left(\prod_{k=1}^{N}\sigma_{\tilde{\mathbf{H}},k}\right)^2 = \prod_{k=1}^{N}\lambda_{\tilde{\mathbf{H}},k} = \det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right). \tag{33}$$

With the cost function in (31), as we will see, the solution will be much easier to work with. Since $\tilde{\mathbf{P}}$ is a function of $\tilde{\mathbf{H}}$, it is a function of $\mathbf{F}_R$. As a result, $\mathbf{F}_S$ is a function of $\mathbf{F}_R$. Note that C_1' and C_2' are mutually coupled through the factor α. In addition, note that the objective function is an increasing function of α and the power of $\mathbf{F}_R$. However, as α is increased, the power of $\mathbf{F}_R$ may be decreased (see C_2'). As a result, the optimum solution of α cannot be obtained straightforwardly. The following theorem provides the solution for this problem.

Theorem 2: Considering the optimization (32), the optimum α, which is denoted as α_{opt}, can be obtained as

$$\alpha_{\mathrm{opt}} = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}. \tag{34}$$

Proof: For any feasible $\alpha \le \sqrt{(P_{S,T}/N\sigma_s^2)}$, we can always have the optimum relay precoder $\mathbf{F}_R'$ so that

$$tr\left\{\mathbf{F}_R'\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\alpha^2\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R'^H\right\} = P_{R,T}. \tag{35}$$

This is because if the equality does not hold, we can always adjust the scale of $\mathbf{F}_R'$, yielding another $\mathbf{F}_R'' = \beta\mathbf{F}_R'$ $(\beta \ge 1)$ so that the equality can be held and the objective function increased. Now, if we let $\alpha_1 = \gamma_1\alpha$ and $\alpha_2 = \gamma_2\alpha$ with $\gamma_1 \ge \gamma_2 \ge 1$, the optimum relay precoders then become $\beta_1\mathbf{F}_R'$ and $\beta_2\mathbf{F}_R'$, respectively, where

$$\beta_1 = \sqrt{\frac{P_{R,T}}{tr\left\{\mathbf{F}_R'\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\gamma_1^2\alpha^2\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R'^H\right\}}} \tag{36}$$

$$\beta_2 = \sqrt{\frac{P_{R,T}}{tr\left\{\mathbf{F}_R'\left(\sigma_{n,r}^2\mathbf{I}_R + \sigma_s^2\gamma_2^2\alpha^2\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R'^H\right\}}} \tag{37}$$

are scalars designed to satisfy (35), and $\beta_1 \le \beta_2$. Defining $\mathbf{N}(\alpha,\mathbf{F}_R) = \alpha^2\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, we can find

$$\begin{aligned} \mathbf{N}\left(\alpha_1,\beta_1\mathbf{F}_R'\right) &= \gamma_1^2\alpha^2\sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \gamma_1^2\alpha^2\beta_1^2\mathbf{H}_{SR}^H\mathbf{F}_R'^H\mathbf{H}_{RD}^H \\ &\quad\times\left(\sigma_{n,r}^2\beta_1^2\mathbf{H}_{RD}\mathbf{F}_R'\mathbf{F}_R'^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1} \\ &\quad\times\mathbf{H}_{RD}\mathbf{F}_R'\mathbf{H}_{SR} \\ &\succeq \gamma_2^2\alpha^2\sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD} + \gamma_2^2\alpha^2\beta_2^2\mathbf{H}_{SR}^H\mathbf{F}_R'^H\mathbf{H}_{RD}^H \\ &\quad\times\left(\sigma_{n,r}^2\beta_2^2\mathbf{H}_{RD}\mathbf{F}_R'\mathbf{F}_R'^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1} \\ &\quad\times\mathbf{H}_{RD}\mathbf{F}_R'\mathbf{H}_{SR} \\ &= \mathbf{N}\left(\alpha_2,\beta_2\mathbf{F}_R'\right) \end{aligned} \tag{38}$$

where $\mathbf{X} \succeq \mathbf{Y}$ means $\mathbf{X} - \mathbf{Y}$ is a positive semidefinite matrix. By (38), we can have $\det(\mathbf{N}(\alpha_1,\beta_1\mathbf{F}_R')) \ge \det(\mathbf{N}(\alpha_2,\beta_2\mathbf{F}_R'))$, which implies that under the relay power constraint, the optimum relay precoder will increase the objective function along with the increase of α. As a result, we have $\alpha_{\rm opt} = \sqrt{P_{S,T}/N\sigma_s^2}$.

There are two implications in (34). First, it indicates that $\alpha_{\rm opt}$ can maximize the SINR at the relay, reducing the noise enhancement effect at the relay node. Second, the resultant relay power constraint is no longer a function of the source precoder, and it only has to be considered in the master problem. In other words, the two power constraints are decoupled now.

Substituting (34) into (32) and ignoring $P_{S,T}/\sigma_s^2 N$ in the objective function, we then have the master optimization as

$$\begin{aligned} &\max_{\mathbf{F}_R} \det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right)\\ &s.t.\\ &\mathrm{C}_2'': tr\left\{\mathbf{F}_R\left(\frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H + \sigma_{n,r}^2\mathbf{I}_R\right)\mathbf{F}_R^H\right\} \le P_{R,T}. \end{aligned} \tag{39}$$

As seen, the singular values of $\tilde{\mathbf{H}}$ are involved in (39), and a direct maximization in (39) is still difficult. To solve (33), we use the Hardamard inequality, which is described in the following lemma.

Lemma 1 [18]: Letting $\mathbf{M} \in \mathbb{C}^{N\times N}$ be a positive definite matrix, then

$$\det(\mathbf{M}) \le \prod_{i=1}^{N} \mathbf{M}(i,i) \tag{40}$$

where $\mathbf{M}(i,i)$ denotes the ith diagonal element of $\mathbf{M}$. The equality in (40) holds when $\mathbf{M}$ is a diagonal matrix. If we let $\mathbf{M} = \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, then it turns out that when $\mathbf{M}$ is diagonalized, the objective function in (33) is maximized. Unfortunately, from (10), we can see that $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ is a summation of two separated matrices, that one of them does not depend on $\mathbf{F}_R$, and that the diagonalization cannot directly be conducted. The following lemma suggests a feasible way to overcome the problem.

Lemma 2 [18]: Letting $\mathbf{A} \in \mathbb{C}^{N\times N}$ be a positive matrix and $\mathbf{B} \in \mathbb{C}^{N\times N}$, then

$$\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A})\det(\mathbf{I}_N + \mathbf{A}^{-1/2}\mathbf{B}\mathbf{A}^{-1/2}). \tag{41}$$

In using (41), we let $\mathbf{B} = \mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H \times (\sigma_{n,r}^2\mathbf{H}_{RD} \times \mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}$ and $\mathbf{A} = \sigma_{n,d}^{-2} \times \mathbf{H}_{SD}^H\mathbf{H}_{SD}$. We have the following equivalence:

$$\begin{aligned} &\arg\max_{\mathbf{F}_R} \det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right)\\ &= \arg\max_{\mathbf{F}_R} \det\left(\mathbf{A}+\mathbf{B}\right)\\ &= \arg\max_{\mathbf{F}_R} \det\\ &\quad \times \left(\mathbf{I}_N + \sigma_{n,d}^2\mathbf{H}_{SR}'^H\mathbf{F}_R^H\mathbf{H}_{RD}^H\right.\\ &\quad\quad \times \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\\ &\quad\quad \left.\times \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}'\right) \end{aligned} \tag{42}$$

where $\mathbf{H}_{SR}' = \sigma_{n,d}\mathbf{H}_{SR}(\mathbf{H}_{SD}^H\mathbf{H}_{SD})^{-(1/2)}$. Here, $\det(\mathbf{A})$ is ignored since it is not a function of $\mathbf{F}_R$. Equation (42) suggests a feasible way to diagonalize the matrix in the objective function. Consider the following singular value decomposition (SVD):

$$\mathbf{H}_{RD} = \mathbf{U}_{RD}\mathbf{\Sigma}_{RD}\mathbf{V}_{RD}^H \tag{43}$$

$$\mathbf{H}_{SR}' = \mathbf{U}_{SR}'\mathbf{\Sigma}_{SR}'\mathbf{V}_{SR}'^H \tag{44}$$

where $\mathbf{U}_{RD} \in \mathbb{C}^{M\times M}$ and $\mathbf{U}_{SR}' \in \mathbb{C}^{R\times R}$ are left singular matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}'$, respectively; $\mathbf{\Sigma}_{RD} \in \mathbb{R}^{M\times R}$ and $\mathbf{\Sigma}_{SR}' \in \mathbb{R}^{R\times N}$ are the diagonal singular value matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}'$, respectively; and $\mathbf{V}_{RD}^H \in \mathbb{C}^{R\times R}$ and $\mathbf{V}_{SR}'^H \in \mathbb{C}^{N\times N}$ are the right singular matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}_{SR}'$, respectively. Substituting (43) and (44) into (42), we can reformulate the optimization (39) as

$$\begin{aligned} &\max_{\mathbf{F}_R} \det(\mathbf{M}')\\ &s.t. \quad \mathrm{C}_2''\\ &\mathbf{M}' = \left(\mathbf{I}_N + \sigma_{n,d}^2\mathbf{\Sigma}_{SR}'^H\mathbf{U}_{SR}'^H\mathbf{F}_R^H\mathbf{V}_{RD}\mathbf{\Sigma}_{RD}^H\right.\\ &\quad \times \left(\sigma_{n,r}^2\mathbf{\Sigma}_{RD}\mathbf{V}_{RD}^H\mathbf{F}_R\mathbf{F}_R^H\mathbf{V}_{RD}\mathbf{\Sigma}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\\ &\quad \left.\times \mathbf{\Sigma}_{RD}\mathbf{V}_{RD}^H\mathbf{F}_R\mathbf{U}_{SR}'^H\mathbf{\Sigma}_{SR}'\right). \end{aligned} \tag{45}$$

From (45), we can then find a relay precoder structure diagonalizing $\mathbf{M}'$. Let

$$\mathbf{F}_{R,opt}^{qr-sic} = \mathbf{V}_{RD}\mathbf{\Sigma}_R\mathbf{U}_{SR}'^H \tag{46}$$

where $\mathbf{\Sigma}_R$ is a diagonal matrix with the ith diagonal element $\sigma_{r,i}$, $i = 1,\ldots,\kappa$, which has yet to be determined. Here, $\kappa = \min\{N,R\}$. Letting $\sigma_{\mathrm{rd},i}$ and $\sigma_{\mathrm{sr},i}'$ be the ith diagonal element of $\mathbf{\Sigma}_{RD}$ and $\mathbf{\Sigma}_{SR}'$, respectively, substituting (43), (44), and (46) into (45), and taking the ln operation to the cost function, we can then rewrite (45) as

$$\begin{aligned} &\max_{p_{r,i},1\le i\le\kappa} \sum_{i=1}^{\kappa} \ln\left(1 + \frac{p_{r,i}\sigma_{n,d}^2\sigma_{\mathrm{rd},i}^2\sigma_{\mathrm{sr},i}'^2}{p_{r,i}\sigma_{n,r}^2\sigma_{\mathrm{rd},i}^2 + \sigma_{n,d}^2}\right)\\ &s.t.\\ &\sum_{i=1}^{\kappa} p_{r,i}\left(\frac{P_{S,T}}{N}\sigma_{\mathrm{sr},i}'^2\mathbf{D}_{SR}'(i,i) + \sigma_{n,r}^2\right) \le P_{R,T},\\ &p_{r,i} \ge 0 \end{aligned} \tag{47}$$

where $p_{r,i} = \sigma_{r,i}^2$ and $\mathbf{D}_{SR}' = \sigma_{n,d}^{-2}\mathbf{V}_{SR}'^H(\mathbf{H}_{SD}^H\mathbf{H}_{SD})\mathbf{V}_{SR}'$, with $\mathbf{D}_{SR}'(i,i)$ being the ith diagonal element of $\mathbf{D}_{SR}'$. The cost function now is simplified to a function of scalar parameters. Since the cost function and the inequalities are all concave for $p_{r,i} \ge 0$ [19], (47) is a standard concave optimization problem. As a result, the optimum solutions $p_{r,i}$, $i = 1,\ldots,\kappa$ can be solved by means of KKT conditions given as in (48), shown at the bottom of the next page, where μ is chosen to satisfy the power constraint in (47), and $[y]^+ = \max\{0,y\}$. Substituting (48) into (46), we can finally obtain the optimum relay precoder. With the relay precoder, $\tilde{\mathbf{H}}$ in (10) can be obtained, and the optimum source precoder can thus be derived via (30).

C. Precoder Design for MMSE-SIC Receiver

As in the previous section, we also adopt the minimum BLER criterion for our criterion. From Theorem 1, we can have the optimization problem for the MMSE-SIC receiver as

$$\begin{aligned}&\max_{\mathbf{F}_S,\mathbf{F}_R} \text{SINR}_i\\ &s.t.\\ &\text{SINR}_i \text{ are equal} \qquad \forall i,\ \text{C}_1,\ \text{C}_2.\end{aligned} \tag{49}$$

Constraints C_1 and C_2 are the power constraints defined in (25). As we can see from (18), SINR_i is also a complicated function of $\mathbf{F}_S$ and $\mathbf{F}_R$, and the optimum solution for this problem is also difficult to obtain. To facilitate the optimization process, we first seek for an alternative objective function for maximization.

Proposition: The following optimizations are equivalent:

$$\begin{aligned}&\max_{\mathbf{F}_S,\mathbf{F}_R,\text{SINR}_i \text{ are equal} \quad \forall i} \text{SINR}_i\\ &= \max_{\mathbf{F}_S,\mathbf{F}_R,\text{SINR}_i \text{ are equal} \quad \forall i} \ln\det\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{F}_S\right).\end{aligned} \tag{50}$$

Proof: The result can directly be obtained from [17].

As we will see, the cost function with the determinate operation in (50) is easier to work with. Similar to the previous case, we propose using the primal decomposition approach [19] to split our problem into a subproblem and a master problem. The subproblem is first optimized for the source precoder, and subsequently, the master problem is optimized for the relay precoder.

1) Proposed Subproblem Optimization: From (49) and (50), we can formulate the subproblem as

$$\begin{aligned}&\max_{\mathbf{F}_S(\mathbf{F}_R)} \ln\det\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{F}_S\right)\\ &s.t.\\ &\text{SINR}_i \text{ are equal} \qquad \forall i,\ \text{C}_1 \text{ and } \text{C}_2.\end{aligned} \tag{51}$$

Since two power constraints are mutually coupled and SINR_i's $i = 1, \ldots, N$ are nonlinear functions of $\mathbf{F}_S$ and $\mathbf{F}_R$, the subproblem cannot be solved. Similar to the previous design, we propose using a unitary structure for the source precoder, i.e.,

$$\mathbf{F}_S = \alpha\mathbf{U}_S \tag{52}$$

where $\alpha \in \mathbb{R}$, and $\mathbf{U}_S$ is an unitary matrix. The unitary constraint for $\mathbf{F}_S$, as we will see, can greatly simplify the whole problem.[1] From (52), we can then reformulate the subproblem as

$$\begin{aligned}&\max_{\alpha,\mathbf{U}_S(\mathbf{F}_R)} \ln\det\left(\sigma_s^{-2}\mathbf{I}_N + \alpha^2\mathbf{U}_S^H\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}\mathbf{U}_S\right)\\ &s.t.\\ &\text{SINR}_i \text{ are equal} \qquad \forall i,\ \text{C}_1' \text{ and } \text{C}_2'\end{aligned} \tag{53}$$

where the two power constraints become functions of α. Using the result of Theorem 2, i.e., (34), we can obtain the optimal α, which is denoted by α_{opt}, as

$$\alpha_{\text{opt}} = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}. \tag{54}$$

Substituting (54) into (53), we can reformulate (53) as

$$\begin{aligned}&\max_{\mathbf{U}_S(\mathbf{F}_R)} \ln\det\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\mathbf{U}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{U}_S\right)\\ &s.t.\\ &\text{SINR}_i \text{ are equal} \qquad \forall i.\end{aligned} \tag{55}$$

Note here that the relay power constraint is moved to the master problem since it is now only a function of $\mathbf{F}_R$. From (54), we see that the task is to find $\mathbf{U}_S$ in (52) such that all SINR_i's are equal. The problem can be solved by the UCD method proposed in [17], which is subsequently described. From SVD, we can have $\tilde{\mathbf{H}} = \mathbf{U}_{\tilde{\mathbf{H}}}\mathbf{\Sigma}_{\tilde{\mathbf{H}}}\mathbf{V}_{\tilde{\mathbf{H}}}^H$. Let

$$\mathbf{F}_S = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{U}_S = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{V}_{\tilde{\mathbf{H}}}\mathbf{U}_S' \tag{56}$$

where $\mathbf{U}_S'$ is also a unitary matrix to be determined. Substituting the results into (16), we can rewrite it as

$$\begin{aligned}\begin{bmatrix}\tilde{\mathbf{H}}\mathbf{F}_S\\ \sigma_s^{-1}\mathbf{I}_N\end{bmatrix} &= \begin{bmatrix}\mathbf{U}_{\tilde{\mathbf{H}}}\mathbf{\Sigma}_{\tilde{\mathbf{H}}}\sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{U}_S'\\ \sigma_s^{-1}\mathbf{I}_N\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{U}_{\tilde{\mathbf{H}}} & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_S'^H\end{bmatrix}\begin{bmatrix}\sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{\Sigma}_{\tilde{\mathbf{H}}}\\ \sigma_s^{-1}\mathbf{I}_N\end{bmatrix}\mathbf{U}_S'.\end{aligned} \tag{57}$$

Applying the GMD [16] on $\begin{bmatrix}\sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{\Sigma}_{\tilde{\mathbf{H}}}\\ \sigma_s^{-1}\mathbf{I}_N\end{bmatrix}$, we have

$$\begin{bmatrix}\sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{\Sigma}_{\tilde{\mathbf{H}}}\\ \sigma_s^{-1}\mathbf{I}_N\end{bmatrix} = \mathbf{Q}_2\mathbf{R}_2\mathbf{P}_2^H \tag{58}$$

[1]Note that the original UCD precoder in the point-to-point MIMO system is not restricted to having the unitary structure. In addition, the optimization problem in the point-to-point MIMO problem has only one power constraint.

$$p_{r,i} = \left[\sqrt{\frac{\mu}{\sigma_{\text{rd},i}^2\left(\frac{P_{S,T}}{N}\sigma_{\text{sr},i}'^2\mathbf{D}_{\text{sr}}'(i,i) + \sigma_n^2\right)\left(\sigma_{n,r}^2\sigma_{n,d}^{-2}\sigma_{\text{sr},i}'^{-2} + 1\right)} + \frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{\text{rd},i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma_{\text{sr},i}'^2} + 1\right)^2}} - \frac{1 + \frac{\sigma_{n,d}^2\sigma_{\text{sr},i}'^2}{2\sigma_{n,r}^2}}{\sigma_{\text{rd},i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2} + \sigma_{\text{sr},i}'^2\right)}\right]^+ \tag{48}$$

where $\mathbf{Q}_2 \in \mathbb{C}^{(2M+N)\times(2M+N)}$, $\mathbf{P}_2 \in \mathbb{C}^{N\times N}$ are unitary matrices, and $\mathbf{R}_2 \in \mathbb{C}^{(2M+N)\times N}$ is the upper triangular matrix with equal diagonal elements. Substituting (58) into (57) and using (16), we then have

$$\begin{bmatrix} \widehat{\mathbf{H}} \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\boldsymbol{\Sigma}_{\tilde{\mathbf{H}}}\mathbf{U}_{\tilde{\mathbf{H}}}\mathbf{U}'_S \\ \sigma_s^{-1}\mathbf{I}_N \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{U}_{\tilde{\mathbf{H}}} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}'^{H}_S \end{bmatrix}\mathbf{Q}_2}_{:=\mathbf{Q}_1}\underbrace{\mathbf{R}_2\mathbf{P}_2^H\mathbf{U}'_S}_{:=\mathbf{R}_1}. \tag{59}$$

Therefore, if we let $\mathbf{U}'_S = \mathbf{P}_2$, $\mathbf{R}_1$ will be equal to $\mathbf{R}_2$. This way, the diagonal elements in $\mathbf{R}_1$ will become all equal. This makes the SINR of each layer signal the same, which can be checked by (18). The optimum source precoder can then be expressed as

$$\mathbf{F}_S = \sqrt{\frac{P_{S,T}}{\sigma_s^2 N}}\mathbf{V}_{\widetilde{\mathbf{H}}}\mathbf{P}_2. \tag{60}$$

Substituting (60) in (51), we then have the master problem described as follows.

2) Proposed Master Problem Optimization: With (60), the master problem can be reformulated as

$$\max_{\mathbf{F}_R} \ln\det\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) \quad s.t.\ \mathrm{C}''_2. \tag{61}$$

As we can see from (61), $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ is also a complicated function of $\mathbf{F}_R$, and therefore, the relay precoder cannot easily be solved. However, we can use the method proposed in Section III-B to find the optimum relay precoder. To use Lemma 2, we let $\mathbf{A}' = (N/P_{S,T})\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}$ and $\mathbf{B} = \mathbf{H}_{SR}^H\mathbf{F}_R^H\mathbf{H}_{RD}^H \times (\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M)^{-1} \times \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR}$. We then have the following equivalence:

$$\begin{aligned} &\arg\max_{\mathbf{F}_R}\det\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) \\ &= \arg\max_{\mathbf{F}_R}\det\left(\mathbf{A}' + \mathbf{B}\right) \\ &= \arg\max_{\mathbf{F}_R}\det \\ &\quad \times \Big(\mathbf{I}_N + \sigma_{n,d}^2\mathbf{H}''^H_{SR}\mathbf{F}_R^H\mathbf{H}_{RD}^H \\ &\qquad \times \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1} \\ &\qquad \times \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}''_{SR}\Big) \end{aligned} \tag{62}$$

where $\mathbf{H}''_{SR} = \mathbf{H}_{SR}(N/P_{S,T}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD})^{-1/2}$. As a result, we can have the SVD of $\mathbf{H}''_{SR}$ as

$$\mathbf{H}''_{SR} = \mathbf{U}''_{SR}\boldsymbol{\Sigma}''_{SR}\mathbf{V}''^H_{SR} \tag{63}$$

where $\mathbf{U}''_{SR} \in \mathbb{C}^{R\times R}$, $\boldsymbol{\Sigma}''_{SR} \in \mathbb{R}^{R\times N}$, and $\mathbf{V}''^H_{SR} \in \mathbb{C}^{N\times N}$ are, respectively, the left singular matrix, the diagonal singular value matrix, and the right singular matrix of $\mathbf{H}''_{SR}$. Here, $\sigma''_{\mathrm{sr},i}$ is the ith diagonal element of $\boldsymbol{\Sigma}''_{SR}$. Similar to (46), we can have the optimum relay precoding structure, which is denoted as $\mathbf{F}_{R,opt}^{\mathrm{mmse-sic}}$

$$\mathbf{F}_{R,opt}^{\mathrm{mmse-sic}} = \mathbf{V}_{RD}\boldsymbol{\Sigma}_R\mathbf{U}''^H_{SR} \tag{64}$$

where $\boldsymbol{\Sigma}_R$ is a diagonal matrix with the ith diagonal element $\sigma_{r,i}$, $i = 1, \ldots, \kappa$, which has yet to be determined. The optimization in (61) can also be expressed as a scalar-valued concave optimization problem given by

$$\begin{aligned} &\max_{p_{r,i},1\le i\le\kappa} \sum_{i=1}^{\kappa}\ln\left(1 + \frac{p_{r,i}\sigma_{n,d}^2\sigma_{\mathrm{rd},i}^2\sigma''^2_{\mathrm{sr},i}}{p_{r,i}\sigma_{n,r}^2\sigma_{\mathrm{rd},i}^2 + \sigma_{n,d}^2}\right) \\ &s.t.\ \sum_{i=1}^{\kappa} p_{r,i}\left(\frac{P_{S,T}}{N}\sigma''^2_{\mathrm{sr},i}\mathbf{D}''_{\mathrm{sr}}(i,i) + \sigma_{n,r}^2\right) \le P_{R,T}, \\ &\quad p_{r,i} \ge 0 \end{aligned} \tag{65}$$

where $p_{r,i} = \sigma_{r,i}^2$, and $\mathbf{D}''_{SR} = \mathbf{V}''^H_{SR}(N/P_{S,T}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD})\mathbf{V}''_{SR}$. The problem in (65) is exactly the same as that in (47), except for the parameters $\sigma''^2_{\mathrm{sr},i}$ and $\mathbf{D}''_{SR}$. As a result, the optimum solutions $p_{r,i}$, $i = 1, \ldots, \kappa$, can be solved as in (66), shown at the bottom of the page. Substituting (66) into (64), we can finally obtain the optimum relay precoder. With the relay precoder, $\tilde{\mathbf{H}}$ in (10) can be obtained, and the optimum source precoder can thus be derived via (60).

IV. Simulation Results

We report simulation results evaluating the performance of the proposed precoded AF MIMO relay systems. In the simulations, the elements of each channel matrix are assumed to be i.i.d. complex Gaussian random variables with zero-mean and same variance. The channel-state information (CSI) of all links is also assumed to be available at all nodes. For the first set of simulations, we let $N = R = M = 4$, $\mathrm{SNR}_{\mathrm{sr}} = 20$ dB, $\mathrm{SNR}_{\mathrm{sd}} = 5$ dB, and $\mathrm{SNR}_{\mathrm{rd}}$ be varied. The modulation scheme is 4-QAM. Seven systems are compared, namely, 1) unprecoded system with MMSE receiver (U-U-MMSE); 2) linear relay precoded system with MMSE receiver (U-L-MMSE) [11], [12]; 3) unprecoded system with QR-SIC receiver (U-U-QR-SIC); 4) linear source and relay precoded system with MMSE

$$p_{r,i} = \left[\sqrt{\frac{\mu}{\sigma_{\mathrm{rd},i}^2\left(\frac{P_{S,T}}{N}\sigma''^2_{\mathrm{sr},i}\mathbf{D}''_{\mathrm{sr}}(i,i) + \sigma_n^2\right)\left(\sigma_{n,r}^2\sigma_{n,d}^{-2}\sigma''^{-2}_{\mathrm{sr},i} + 1\right)} + \frac{\frac{\sigma_{n,d}^4}{4\sigma_{n,r}^4}}{\sigma_{\mathrm{rd},i}^4\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2\sigma''^2_{\mathrm{sr},i}} + 1\right)^2}} - \frac{1 + \frac{\sigma_{n,d}^2\sigma''^2_{\mathrm{sr},i}}{2\sigma_{n,r}^2}}{\sigma_{\mathrm{rd},i}^2\left(\frac{\sigma_{n,r}^2}{\sigma_{n,d}^2} + \sigma''^2_{\mathrm{sr},i}\right)}\right]^+ \tag{66}$$

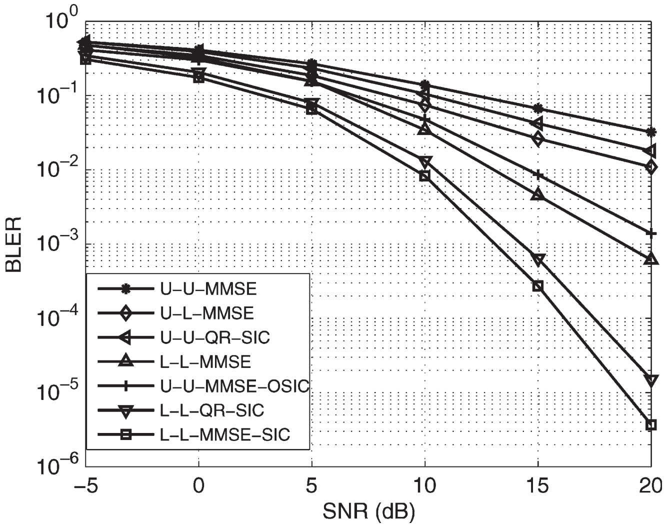

Fig. 2. BLER performance comparison for existing unprecoded/precoded and proposed precoded systems (4-QAM, $N = R = M = 4$, $\mathrm{SNR_{sr}} = 20$, and $\mathrm{SNR_{sd}} = 5$ dB).

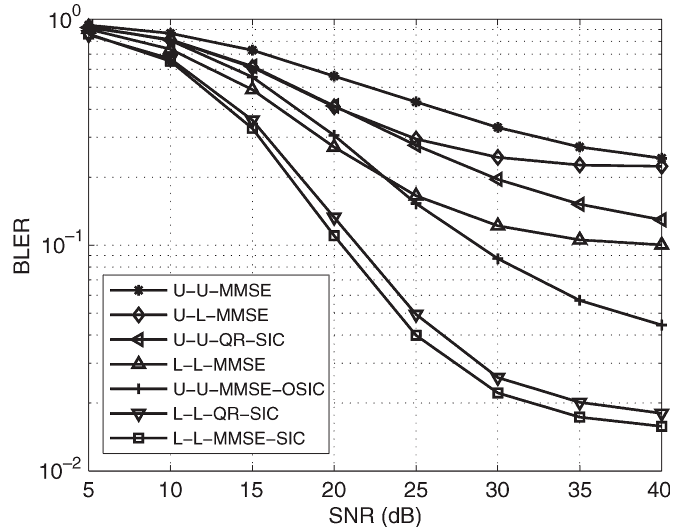

Fig. 3. BLER performance comparison for existing unprecoded/precoded and proposed precoded systems (16-QAM, $N = R = M = 4$, $\mathrm{SNR_{sr}} = 20$, and $\mathrm{SNR_{sd}} = 5$ dB).

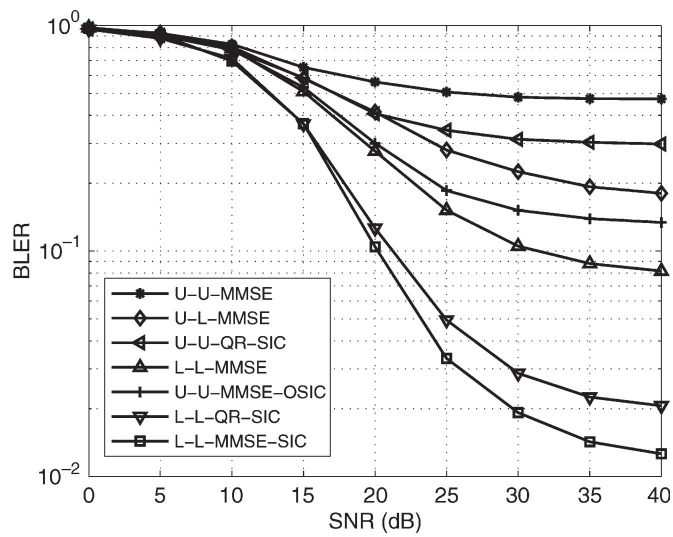

Fig. 4. BLER performance comparison for existing unprecoded/precoded and proposed precoded systems (16-QAM, $N = R = M = 4$, $\mathrm{SNR_{sd}} = 5$, and $\mathrm{SNR_{rd}} = 20$ dB).

receiver [14] (L-L-MMSE); 5) unprecoded system with MMSE ordered SIC receiver (U-U-MMSE-OSIC); 6) proposed linear source and relay precoded system with QR-SIC receiver (L-L-QR-SIC); and 7) proposed linear source and relay precoded system with MMSE-SIC receiver (L-L-MMSE-SIC). Note that MMSE-OSIC will perform better than MMSE-SIC due to the ordering operation. As a result, we will use it in unprecoded systems. In the proposed precoded systems, however, all SINR_j's, for all j, are equal. The performance of the MMSE-SIC and MMSE-OSIC receivers will be the same. Fig. 2 shows the simulated BLER for the systems previously mentioned, respectively. From this figure, we see that the performance of unprecoded systems are limited: either the linear or nonlinear receivers. For the MMSE receiver, the performance can be improved by the relay precoders (U-L-MMSE) and further enhanced by the source and relay precoded precoders (L-L-MMSE). The performance of the proposed precoded systems is much better than that of the other systems. The proposed precoded system with the MMSE-SIC receiver is slightly better than that with the QR-SIC receiver.

In the second set of simulations, the simulation setup remains the same except that the modulation scheme is changed to 16-QAM. Fig. 3 shows the BLER comparison for all the systems we consider. Since the modulation order is higher, the performance of all systems degrades. When the SNR of the relay-to-destination link is high, the significance of the relay precoder is reduced. This explains why the performance of the linear relay precoded system with the MMSE receiver is similar to that of unprecoded systems. From the figure, we also see that the unprecoded system with the MMSE-OSIC receiver is better than the linear source and relay precoded system with the MMSE receiver in high SNR regions. This is due to the fact that nonlinear receivers can provide higher diversity gain [17]. Similar to the previous case, the proposed linear source and relay precoded systems with the nonlinear receivers are better than the other precoded or unprecoded systems. The precoded system with the MMSE-SIC receiver still outperforms the precoded system with the QR-SIC receiver. However, the performance gap is smaller.

For the third set of simulations, we compare the performance of the aforementioned systems under a different SNR environment. We let $\mathrm{SNR_{sd}} = 5$ dB, $\mathrm{SNR_{rd}} = 20$ dB, and $\mathrm{SNR_{sr}}$ be varied. Fig. 4 shows the simulation results for the BLER. As we can see, the relay precoded system with the MMSE receiver outperforms the unprecoded systems with linear and nonlinear QR-SIC receivers at high SNR regions. This is because the performance is dominated by the links of the source to destination and the relay to destination when $\mathrm{SNR_{sr}}$ is high. As a result, the additional relay precoder can improve the overall link quality. Unlike the previous case, the performance of the unprecoded system with the MMSE-OSIC receiver is inferior to the source-and-relay precoded system with the MMSE receiver. This is because when the SNR of the source-to-relay link is sufficiently high, a MIMO relay system is degenerated to a

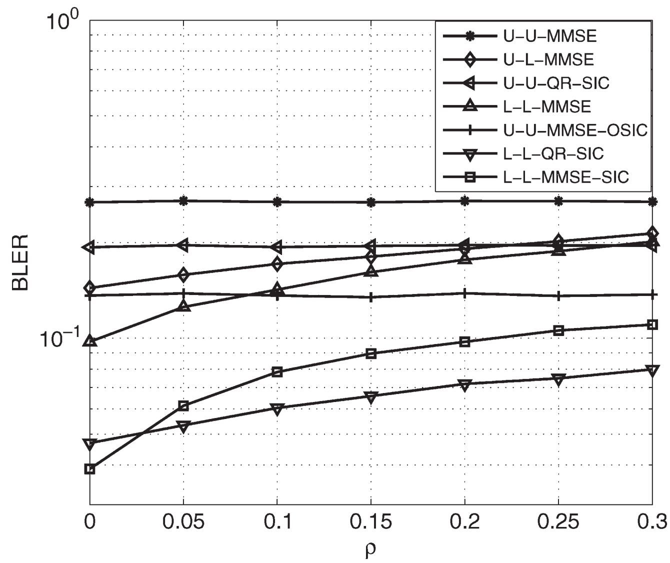

Fig. 5. BER performance comparison for existing unprecoded/precoded and proposed precoded systems under imperfect CSI (16-QAM, $N = R = M = 4$, $\mathrm{SNR_{sd}} = 5$, $\mathrm{SNR_{sr}} = 25$ dB, and $\mathrm{SNR_{rd}} = 20$ dB).

TABLE I
COMPUTATIONAL COMPLEXITY OF EXISTING AND PROPOSED TRANSCEIVER DESIGNS

Transceivers	Precoders
(a) U-U-MMSE	None
(b) U-L-MMSE	$O(N^3 + RN^2 + MR^2 + R^3)$
(c) U-U-QR-SIC	None
(d) L-L-MMSE	$O(N^3 + RN^2 + MR^2 + R^3 + MN^2 + N^3)$
(e) U-U-MMSE-OSIC	None
(f) L-L-QR-SIC	$O(N^3 + RN^2 + MR^2 + MN^2 + R^3)$
(g) L-L-MMSE-SIC	$O(N^3 + RN^2 + MR^2 + MN^2 + R^3)$

MIMO system. As a result, the significance of the precoders is increased. In addition, as expected, the precoded systems with nonlinear receivers outperform the unprecoded systems and the precoded systems with linear precoders and linear receivers.

In the last set of simulations, we evaluate the performance of the proposed transceivers under imperfect CSI. As in the formulation in [21], the channel $\hat{\mathbf{H}}$ used in the design is related to the true channel $\mathbf{H}$ via the equation $\hat{\mathbf{H}} = \sqrt{1-\rho}\mathbf{H} + \sqrt{\rho}\Delta\mathbf{H}$, where $\Delta\mathbf{H}$ models the channel error, and the coefficient ρ characterizes the level of the error. The elements of $\Delta\mathbf{H}$ are modeled as i.i.d. Gaussian variables with zero mean and the same variance. The modulation scheme is 16-QAM, and we let $\mathrm{SNR_{sd}} = 5$ dB, $\mathrm{SNR_{sr}} = 25$ dB, $\mathrm{SNR_{rd}} = 20$ dB, and ρ be varied. Fig. 5 shows the simulation results. As we can see, the BLER of the precoded systems increases as ρ increases, particularly for the precoded system with the MMSE-SIC receiver. The precoded system with the QR-SIC receiver is less affected. Note that the CSI is assumed to be perfectly known at the destination for unprecoded systems, and their BLERs are not affected by the value of ρ.

In Table I, we summarize the computational complexity of the various transceiver design methods we consider, in which the computational complexities are measured in terms of floating-point operations for the precoder design. As we can see, the proposed methods mainly involve SVD, matrix multiplication, matrix inversion, and UCD/GMD decomposition. A relay precoder is considered in (b), whereas both source and relay precoders are considered in (d), (f), and (g). The precoder in (d) is obtained via an iterative approach, and therefore, its computational complexity is higher than that of the proposed methods [see Table I (f) and (g)].

V. CONCLUSION

In this paper, we have studied two nonlinear transceiver designs in AF MIMO relay systems. In our system, a linear precoder is used at the source, a linear precoder is used at the relay, and a QR-SIC or MMSE-SIC receiver is used at the destination. Although the design problems can easily be formulated as optimization problems, the solutions are very difficult to derive. To overcome this, we propose decomposing the original problem into two simpler subproblems via the primal decomposition technique. Using the GMD/UCD method, the proposed unitary source precoder, and the proposed relay precoder structure, we can then transfer the problems into scalar convex optimization problems. The closed-form solutions can finally be derived by the KKT conditions. Simulations show that the proposed precoded systems can significantly outperform the existing precoded and unprecoded AF MIMO relay systems.

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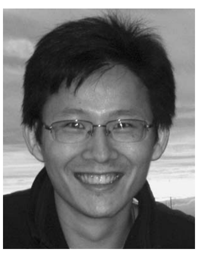

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