Competition, Dedicated Assets, and Technological Obsolescence in Multistage Infrastructure Investments: A Sequential Compound Option Valuation

Yu-Lin Huang and Chia-Chi Pi

***Abstract*—Multistage investments are common in area-wide developments of privatized telecommunication networks and other complex infrastructure systems. They represent an incremental strategy that maintains flexibility in managing market risks, funding needs, and resource constraints in geographical expansions. Though compound options can be used to valuate multistage investments, their valuation is complex when the project in question requires upfront and interim investments in dedicated assets for future expansions. The problem becomes more complex when infrastructure markets are competitive and the investment in question is prone to rapid technological progress, which quickly makes the currently best in-use technology obsolete. This paper develops a European sequential compound call option pricing model to valuate multistage investments and analyze how competition, dedicated assets, and technological obsolescence influence the value of flexibility in this incremental strategy.**

***Index Terms*—Dedicated assets, multistage investment, sequential compound option, technological obsolescence, valuation.**

I. Introduction and Background

DEDICATED assets play an important technological role in long-term contracts. Williamson [1] defined dedicated assets as “discrete additions to generalized capacity that would not be put in place but for the prospect of selling a large amount of product to a particular customer” (p. 532). When deploying special-purpose technologies for future expansions, dedicated assets are prone to expropriation risk. Reciprocal arrangements are necessary to attract dedicated asset investments and ensure the continuity of long-term trading relations.

Dedicated assets are prevailing in multistage infrastructure investments, and reciprocal arrangements are common in build-operate-transfer (BOT) projects. Huang [2, p. 103] found that BOT concession agreements have a system of “concerted reciprocity,” e.g., to protect the concessionaires’ rights to serve, to manage market risk, and to assure full capital recovery. Reciprocity is also important for privatized infrastructure firms. These firms are usually protected by licenses or long-term exclusive concessions. To prevent the abuse of exclusivity rights, the firms are subject to rate-of-return (ROR) or price-cap regulation. According to Rees and Vickers [3], ROR regulation is a form of guarantee forcing regulators to commit to future prices that ensure full remuneration of service costs.

In monopolistic markets with perfect regulation, the presence of dedicated assets is irrelevant in investment valuation. As Schmalensee [4] indicated in an idealized ROR regulation model that assumes no competition and allows full capital recovery, the net present value of all investments is invariant to depreciation schedules. However, promoting competition is a primary policy objective of infrastructure privatization. As Guislain [5, p. 228] observed, many privatized telecommunication firms have long-term concessions with time-bound exclusivity rights. This temporary exclusivity is designed to recover stranded sunk assets and phase out internal cross subsidizations. Once these tasks are complete, existing networks become open to competition. In addition, Huang and Pi [6] found that dedicated asset investments in area-wide infrastructure networks usually occur well before actual expansions. Since future expansions are uncertain, the recovery of dedicated assets comes with the risk of interruption and premature termination. This risk further increases if assets are prone to competition and technological obsolescence. According to Crew and Kleindorfer [7], competition can render full capital recovery impossible if the economic depreciation rate exceeds the accounting depreciation rate in ROR regulation, or if technological progress causes market prices to fall below the price ceiling in price-cap regulation. Newbery [8] suggested that price-cap regulation by total element long-term incremental cost (TELRIC) requires a sensible depreciation rate to compensate for technological obsolescence.

Therefore, competition, dedicated assets, and obsolescence warrant more attention in competitive markets with imperfect regulations. This paper proposes a multifold European sequential compound call option (SCCO) pricing model to tackle these issues. The sequential compound option concept is straightforward for multistage investments with voluntary expansion and abandonment rights: investing in one area gives the concessionaire an option to invest in the next area. The concessionaire will invest in the next area only if that area’s underlying asset value exceeds a critical level. This arrangement is typical of incremental geographical expansions in infrastructure developments. It can be viewed as a one of the strategic options addressed by

Manuscript received April 2, 2009; revised July 17, 2009, and October 5, 2009; accepted February 10, 2010. Date of publication June 1, 2010; date of current version January 19, 2011. Review of this manuscript was arranged by Department Editor J. Sarkis.

The authors are with the Department of Civil Engineering, National Chiao Tung University, Shin Chu 300, Taiwan (e-mail: huangyulin2000@hotmail.com; jenniferbee.cv95g@nctu.edu.tw).

Digital Object Identifier 10.1109/TEM.2010.2048910

Myers [9]. According to Merton [10], a warrant on a portfolio of stocks is less valuable than a portfolio of warrants on the stocks. Hence, multistage investments are potentially more valuable than one-stage investments because they retain a sequence of options in future expansions.

Researchers have recently applied real-option approaches for valuating complex infrastructure investments. For instance, Huang and Chou [11] developed a real-option approach to valuate the minimum revenue guarantees (MRG) of BOT projects. They found intricate interactions between the MRG and the option to abandon. The finding is consistent with Trigeorgis' [12] work on real-option interactions. Wand and Min [13] evaluated the interrelations of power generation projects. Damnjanovic *et al.* [14] evaluated interconnectivity and flexibility in toll road expansions. Huang and Pi [6] evaluated multistage BOT projects requiring upfront and interim investments in dedicated assets.

Unlike the continuous investments assumed in theoretical real-option models (e.g., Majd and Pinkyck [15], and Pindyck [16]), the geographical expansions of infrastructure systems or networks are discrete in time. Dixit and Pindyck [17] presented a solution to a discrete-time two-stage investment problem using dynamic programming and simulation. Huang and Pi [6] extended the two-stage problem to n-stage, and solved the n-stage problem as an n-fold SCCO studied by Thomassen and Van Wouwe [18], Lajeri-Chaherli [19], Agliardi and Agliardi [20], and Lee *et al.* [21].

This paper extends Huang and Pi's [6] work to consider the influences of competition and technological obsolescence on project value. The extension treats competition and obsolescence explicitly as SCCO risk parameters, and derives the sensitivities of the SCCO value regarding these parameters. The extension provides new insights in valuation and concession designs. It applies not only for stand-alone BOT projects, but also for area-wide infrastructure network developments in privatized infrastructure markets.

The remainder of this paper is as follows. Section II discusses the model setup and assumptions. Section III presents a pricing solution to n-fold European SCCOs. Section IV discusses the properties of the solution. Section V presents a numerical analysis using the solution. Section VI discusses implications to infrastructure concession and regulatory designs. Section VII offers a conclusion.

II. Model Setup and Assumptions

This section outlines the setup and assumptions of the proposed SCCO pricing model, including 1) measure of technological obsolescence; 2) measure of competition level; 3) foldwise constant parameters; and 4) European-style SCCOs.

A. Measure of Technological Obsolescence

Economic depreciation is a general measure of technological obsolescence. Hotelling [22] defined economic depreciation as the rate of decrease in asset value. Hulten and Wykoff [23] indicated that a decrease in asset value comprises two effects: vintage and aging. The vintage effect reflects technological obsolescence in the presence of new vintages with superior production efficiency and quality. The aging effect reflects deterioration in asset value due to wear and tear of the in-use asset.

The vintage and aging effects differ substantially in various kinds of assets or cohorts of assets. The aging effect dominates vintage effect in many building and civil engineering structures. For instance, on the basis of depreciation charge and net book value of fixed assets, Cowan [24] calculated that the average asset life in the water industry is over 40 years. More generally, Hulten and Wykoff [23] estimated that the annual depreciation rate of nonresidential structures is 3%. In contrast, the vintage effect dominates in the presence of technological progress. As Newbery [8, p. 338] indicated, technology evolves so rapidly that the cost of the currently least-cost option soon becomes out of date in telecom privatization. In R&D projects, the vintage effect determines the asset value so much that obsolescence risk can be modeled directly as depreciation rate (see, e.g., Berk *et al.* [25]). Finally, like competition, changing environmental, quality, and safety regulations can reinforce the vintage effect if they require early replacement of existing assets.

Researchers have suggested that the dividend payout rate in financial options can be viewed as the depreciation rate in real options (see, e.g., Remer *et al.* [26] and Lee *et al.* [21]). This implies that the depreciation rate can be formally incorporated in the underlying asset-value process of real options. However, according to Hulten and Wykoff [23], there are several plausible economic depreciation patterns. Dixit and Pindyck [17, p. 199] showed that each assumed depreciation pattern has a specific stochastic asset-value process, and Ross [27] developed three option-pricing models based on alternative dividend payout policies.

This paper assumes a *constant percentage rate* in economic depreciation. Denote a constant depreciation rate by q, and the initial value of the underlying asset by V. The instantaneous change in the underlying asset value of an investment is then given by

$$dV = -qV\,dt. \tag{1}$$

The solution to this partial differential equation is as follows:

$$V(t) = V(0)\exp(-qt). \tag{2}$$

In this solution, propositional depreciation produces an exponential decay in asset value. Hulten and Wykoff [23] indicated that a cohort of assets exhibits near-exponential decay even though individual assets can exhibit different depreciation patterns. Domes [28] showed that efficiency decay in capital investment streams also exhibits near-geometric patterns. Therefore, exponential decay is a reasonable approximation for infrastructure networks or systems because their asset bases generally contain many kinds of assets.

B. Measure of Competition Level

To capture the influence of competition, further assume that the underlying asset value of an investment follows the stochastic differential equation in Black and Scholes [29]:

$$dV_t = \mu V_t\,dt + \sigma V_t\,dW_t \tag{3}$$

where V_t denotes the stochastic asset value and W_t denotes the standard Brownian motion. The drift term of this S.D.E. is μV_t, where μ is a constant expected ROR in V_t. The diffusion term is σV_t, where σ is a constant standard deviation of the return.

According to Merton [10], [30], asset-return volatility is a sufficient statistic for comparing the underlying business risks of individual securities that have lognormal return distributions. In the context of the capital asset pricing model (CAPM), Beaver *et al*. [31] indicated that individual securities with a higher than average beta (i.e., the sensitivity of individual security returns to stock market returns) tend to have a higher than average variance of return associated with diversifiable risk factors. For infrastructure firms, Alexander *et al*. [32] suggested that competition, regulatory regime, industry structure, and operation diversity are important risk factors. They found that infrastructure firms subject to price-cap regulation have average betas higher than firms under ROR regulation, and the asset and equity betas of telecom industries are higher than those of electricity, gas, and water industries. Grout [33] suggested that price-cap regulation produces higher betas because it is less adaptive to cost variations. Gaspar and Massa [34] assumed that firms with greater market power and lower demand elasticity have more stable profit margins because they can pass on cost changes to customers. Gaspar and Massa [34] found that competition significantly increased the profit and equity return volatilities of deregulated airlines, electricity, natural gas, and telecom industries. Therefore, asset-return volatility is an effective measure of competition faced by infrastructure projects or firms. Moreover, the positive correlation between asset-return volatility and the CAPM beta implies that competition tends to increase the cost of capital.

Further assume that economic depreciation does not affect the smooth-passing property of the diffusion term in (3). Incorporating propositional depreciation into (3) gives

$$dV_t = (\mu - q)\, V_t dt + \sigma V_t dW_t. \quad (4)$$

The solution to this stochastic differential equation (S.D.E.) is obtained by Ito's Lemma (see, e.g., Shreve [35])

$$V_t = V_0 \exp\left\{\sigma W_t + \left[(\mu - q) - \frac{1}{2}\sigma^2\right] t\right\}. \quad (5)$$

This solution has three important properties. First, economic depreciation reduces the expected ROR by an exponential decay factor e^{-qt}, which agrees with the assumed depreciation pattern. Second, competition can only affect the asset-return volatility because $V_0 \exp\{\sigma W_t - (1/2)\sigma^2 t\}$ is an exponential martingale (i.e., its conditional expectation, given the currently available information, is equal to its current value). Finally, the asset-return volatility is a sufficient measure of competition and other business risk because the return is log-normally distributed with constants μ, q, and σ.

C. Foldwise Constant Parameters

To produce a flexible valuation model, this study assumes that the values of parameters replicating the stochastic asset value are time-dependent and foldwise constant. For instance, it is reasonable to assume that the depreciation rate changes as the asset base changes from one stage to another, and the change in depreciation rate is not stochastic. Agliardi and Agliardi [36] showed that the stochastic interest rate does not guarantee the uniqueness of critical value that triggers the next investment. Rubinstein [37] further noted that if the asset return follows a geometric Brownian motion, it will have a deterministic mean equal to the risk-free return. This is fundamental to the risk-neutral pricing approach in Section III.

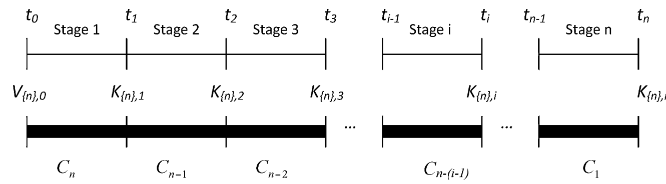

Fig. 1. Modeling multistage investment as an n-fold SCCO.

D. European-Style SCCOs

This paper treats multistage investments in the context of European-style call options. In the European model, the concessionaire can decide to invest only *at* the option maturity dates. This setting reflects the practice of stipulating in concession contracts that, to meet public interests, the construction schedules of privatized works must be fixed in advance according to prudent preconstruction planning and engineering (see, e.g., Huang [2, p. 103]).

The European model is flexible in handling infrastructure valuation problems. First, a simple multistage investment that requires no dedicated assets and faces a low level of market or completion risk can be modeled as *a stand-alone n-fold European SCCO*. Fig. 1 shows this model setup, where V_0 is the present underlying asset value, K_i is the ith SCCO exercise price, and C_i is the ith SCCO value. The present value of the nth fold SCCO gives the present value of multistage investment, denoted by C_n.

Second, when dedicated assets are necessary, Huang and Pi [6] showed that a multistage investment can be modeled as *a vanilla European call option plus a portfolio of European SCCOs*. Fig. 2 shows this model setup, where $K_{\{j\},i}$ denotes the allocation of the ith stage dedicated asset investment to the subsequent stage $j, \forall 1 \leq i < j \leq n$. The total present value of the constituent call options, or $\sum_{i=1}^{n} C_{\{i\},i}$ shows the investment value. Since dedicated asset investments are allocated as exercise prices of downstream SCCOs, their impacts on value can be assessed analytically.

To study the possibility of premature exercise, it is also possible to model multistage investments as American-style SCCOs. According to Merton [10], the propositional dividend payout will always produce a positive probability of optimal premature exercise at a sufficient underlying stock price. Since this study assumes propositional depreciation, it follows that the European model will result in a low bound of multistage investment value if the underlying asset value turns out be larger than expected.

According to Roll [38], Geske and Johnson [39], and Carr [40], American options can be valuated as a portfolio of

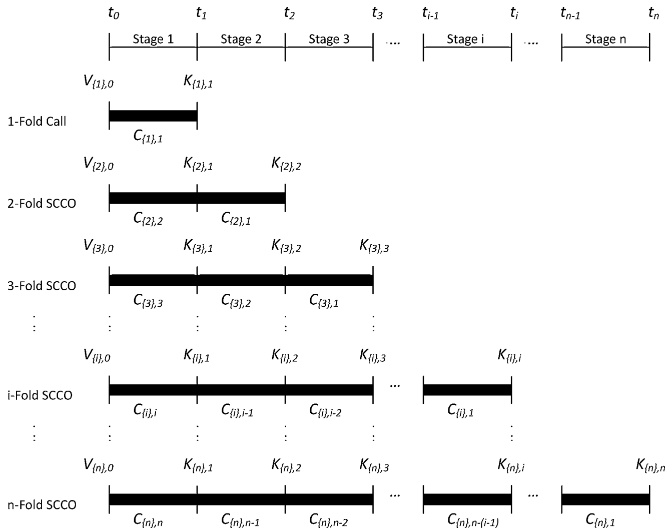

Fig. 2. Modeling n-stage investment requiring dedicated asset investments [6].

European compound options. This valuation is derived from the "pseudo-American option," whose time to maturity is artificially subdivided into a finite number of intervals. It is also possible to first subdivide each stage's construction period into finite intervals (e.g., one year), and simulate staged construction schedules using *a sequence of stand-alone pseudo-American call options*. With these optimal schedules, one can further valuate the entire investment by European SCCOs.

III. Closed-Form Solution

Based on the aforementioned analysis, this section derives a pricing solution to n-fold European SCCOs. The notations for SCCO are the followings:

$C_{\{n\},n}(V,t_0)$	Time t_0 value of an n-fold SCCO.
r_τ	Foldwise constant risk-free interest rate.
σ_τ	Foldwise constant asset-return volatility.
q_τ	Foldwise constant depreciation rate.
$V(t_n,n)$	Time t_n value of the underlying asset of $C_{\{n\},n}(V,t_0)$.
$K_{\{n\},i}$	Time t_i value of the exercise price of $C_{\{n\},n}(V,t_0)$.
$\bar{V}_{i,\{n\}}$	ith equivalent asset value (EAV) of $C_{\{n\},n}(V,t_0)$.
$g_{\{n\},i}$	ith value of factor g of $C_{\{n\},n}(V,t_0)$.
$h_{\{n\},i}$	ith value of factor h of $C_{\{n\},n}(V,t_0)$.
$\rho_{i,j}$	Correlation coefficient between variates i and j.
$N_m(\cdot)$	m-variate normal cumulative distribution function.

Note that the EAV is the critical asset value that triggers the next investment. The EAV is found at $C_{\{n\},n-i}(V,t_i)=K_{\{n\},i}$ using the boundary condition

$$C_{\{n\},n-(i-1)}\left(V,t_i\right)=\max\left[0,C_{\{n\},n-i}\left(V,t_i\right)-K_{\{n\},i}\right],\quad \text{for } 1\le i<n. \tag{6}$$

That is, for an n-stage investment, the concessionaire will decide to invest in the ith stage at time t_i only if $C_{\{n\},n-i}(V,t_i)\ge K_{\{n\},i}$. Although the stochastic interest rate does not guarantee the uniqueness of $\bar{V}_{i,\{n\}}$, Agliardi and Agliardi [36] and Lee *et al*. [21] showed that it is guaranteed when the interest rate and other parameters have deterministic values.

To obtain a solution, assume that there is no arbitrage in frictionless markets (e.g., no transaction cost; see Ross ([41]). Risk-neutral pricing then gives the present value ($t=0$) of an n-fold SCCO as follows:

$$C_n\left(V,t_0\right)=e^{-r_1\tau 1}E^*\left\{\frac{\max\left[0,C_{n-1}\left(V,t_1\right)-K_1\right]}{F_0}\right\} \tag{7}$$

where E^* denotes a conditional expectation operator. Equation (7) states that given information available at time t_0, denoted by F_0, there is a risk-neutral probability such that the discounted underlying asset value of the SCCO is a martingale. Applying the idiosyncratic asset value process in (4) gives the following theorem.

Theorem (The n-fold European SCCO pricing formula):

$$\begin{aligned} &C_{\{n\},n}(V,t_0)\\ &=V_{\{n\},0}e^{-\sum_{u=1}^{n}q_u\tau_u}N_n\{[g_{\{n\},i}]_{n\times 1};[\rho_{\{n\},i,j}]_{n\times n}\}\\ &\quad-\sum_{m=1}^{n}K_{\{n\},m}e^{-\sum_{u=1}^{m}r_u\tau_u}N_m\{[h_{\{n\},i}]_{m\times 1};[\rho_{\{n\},i,j}]_{m\times m}\}\end{aligned} \tag{8}$$

where $\forall\ 1\le i\le n$

$$g_{\{n\},i}=\frac{\ln\left(V_{\{n\},0}/\overline{V}_{i,\{n\}}\right)+\sum_{u=1}^{i}\left(r_u-q_u+(1/2)\sigma_u^2\right)\tau_u}{\sqrt{\sum_{u=1}^{i}\sigma_u^2\tau_u}},$$

$$h_{\{n\},i}=\frac{\ln\left(V_{\{n\},0}/\overline{V}_{i,\{n\}}\right)+\sum_{u=1}^{i}\left(r_u-q_u-(1/2)\sigma_u^2\right)\tau_u}{\sqrt{\sum_{u=1}^{i}\sigma_u^2\tau_u}}$$

and the correlation matrix is symmetrical, or $\rho_{i,j}=\rho_{j,i}$. $\rho_{i,j}=1, \forall i=j$, and $\rho_{i,j}=\sqrt{(\sum_{u=1}^{i}\sigma_u^2\tau_u)/(\sum_{u=1}^{j}\sigma_u^2\tau_u)}\ \forall 1\le i<j\le n$.

Proof: The proof of this formula is by induction: since a SCCO is a call option on a call option, it follows that if (8) is true for an n-fold SCCO, then it must be true for an $(n+1)$-fold SCCO. Huang and Pi [6] provided a detailed outline of the proof of a similar solution, which assumes a propositional dividend payout.

IV. Properties of the Multifold European SCCO Value

Geske [42] showed that the value of the twofold compound call option decreases with exercise price, but increases with the underlying asset value, variance of asset return, risk-free interest rate, and time to expiration. Geske verified that under certain technical conditions, most results agree with Merton's [10] rational option pricing theory. Agliardi and Agliardi [20] extended Geske's analysis to incorporate time-dependent parameters. Thomassen and Van Wouwe [18] extended Geske's analysis to n-fold SCCOs. Lee *et al*. [21] further extended this analysis to generalized n-fold sequential compound options.

This study focuses on the sensitivity of $C_{\{n\},n}\,(V, t_0)$ to three parameters: asset-return volatility, exercise price, and depreciation rate. According to Thomassen and Van Wouwe [18] and Lee *et al*. [21], SCCO value increases monotonically with asset-return volatility. This implies that the value of flexibility created by multistage investment is higher in more competitive business environments. The following derives the partial derivatives of $C_{\{n\},n}\,(V, t_0)$ with respect to exercise price and depreciation rate.

Proposition 1: From the pricing formula (8)

$$\sum_{l=1}^{n} \frac{\partial C_{\{n\},n}(V,t_0)}{\partial K_{\{n\},l}} = -\sum_{l=1}^{n} e^{-\sum_{u=1}^{l} r_u \tau_u} N_l\{[h_{\{n\},i}]_{l\times 1}; [\rho_{\{n\},i,j}]_{l\times l}\}. \quad (9)$$

Proof: See Appendix A.

Proposition 2: From (8)

$$\sum_{l=1}^{n} \frac{\partial C_{\{n\},n}(V,t_0)}{\partial q_l} = -\sum_{l=1}^{n} \tau_l V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u \tau_u} N_n\{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}. \quad (10)$$

Proof: See Appendix B.

Given these partial derivatives, the following corollary provides two new results.

Corollary: $C_{\{n\},n}\,(V, t_0)$ is as follows:

a) strictly decreasing and convex in K_l, for $1 \le l \le n$;

b) strictly decreasing and convex in q_l, for $1 \le l \le n$.

Proof: See Appendix C.

Corollary a) indicates that, other things being equal, the n-fold European SCCO value is a strictly decreasing function, as well as a convex function, of its exercise price. This implies that increasing dedicated asset investment will reduce project value. Likewise, Corollary b) implies that obsolescence will also reduce project value. However, the relationships between σ, q, and K are not straightforward. The signs of related partials, e.g., $\partial^2 C_{\{n\},n}\,(V, t_0)\,/\partial q \partial \sigma$ and $\partial^2 C_{\{n\},n}\,(V, t_0)\,/\partial q \partial K$, are ambiguous. Scenario analysis is recommended to analyze the interplays of these parameters on project value case by case.

V. Numerical Implementation

This section presents an algorithm structure and a numerical example to illustrate how to implement the SCCO pricing model for multistage projects involving dedicated asset investments. The structure of the numerical example follows closely the valuation framework in Fig. 2. Huang and Pi [6] provide a real-world application of the framework for multistage BOT projects.

TABLE I
Base-Case Valuation Model

Option	$V_{\{j\},i}$	$K_{\{j\},i}$				
		$t_1 = 5$	$t_2 = 10$	$t_3 = 15$	$t_4 = 20$	$t_4 = 25$
1-fold Call Option	24	17				
2-fold SCCO	24	0.75	17			
3-fold SCCO	24	0.75	1	17		
4-fold SCCO	24	0.75	1	1.5	17	
5-fold SCCO	24	0.75	1	1.5	3	20
Total	120	20	20	20	20	20

A. Algorithm Structure

A MATLAB program is written to support the implementation on the basis of three major steps.

1) Specify risk parameters that influence the SCCO values of the n-stage project, including S, K, T, n, r_τ, q_τ, and σ_τ.

2) For $j = 2, 3, \ldots, n$, find the EAVs of the $\{j\}$-fold SCCO of the project. From (6), the EAVs are given by

$$\overline{V}_{i,\{j\}} = \begin{cases} K_{\{j\},i} & \forall i = j \\ \text{The asset value } V \text{ given} & \forall 1 \le i < j. \\ \quad C_{\{j\},j-i}(V,t_i) - K_{\{j\},i} = 0 & \end{cases}$$

First, find $\overline{V}_{j,\{j\}} = K_{\{j\},j}$, and calculate $g_{\{j\},j}$, $h_{\{j\},j}$, and $\rho_{i,j}$. Then, work backward to find $\overline{V}_{j-1,\{j\}}$, $\overline{V}_{j-2,\{j\}}, \ldots, \overline{V}_{1\{j\}}$ by (8). MATLAB's *while loop* is used to find the asset values by a tolerance of 10^{-6}. The *mvncdf(X)* function is used to find the cumulative probabilities of the multivariate normal cumulative distributions.

3) Calculate $C_{\{j\},j}$, the value of the $\{j\}$-fold SCCO, and repeat Step 2 to find the total value of the n-stage project, denoted as χ_n, given by

$$\chi_n = C_{\{1\},1}\,(V, t_0) + \sum_{j=2}^{n} C_{\{j\},j}\,(V, t_0)$$

where $C_{\{1\},1}$ is the value of a plain vanilla European call option of the first-stage investment. The vanilla option value is given by an extension to Black and Scholes' [29] classical pricing model using the dynamic asset process of (4).

B. Base-Case Valuation Problem

Table I summarizes the structure of the base-case numerical example. The hypothetical five-stage project is assumed to have dedicated asset investments, and thus, treated as a onefold plain vanilla European call option plus four European SCCOs, as explained in Section II and Fig. 2. The project's total construction period is 25 years, and each stage requires 5 years to complete. Each stage has a nominal investment cost of 20, and 15% of each investment is allocated to dedicated asset investment ("*K ratio*") for later stage(s). Once complete, each stage generates a present value of 24.

Table II summarizes other base-case parameter values: risk-free rate 4%, inflation rate 3%, depreciation rate 5%, and

TABLE II
BASE-CASE PARAMETER VALUES

Parameter	Value
Number of stage (n)	5
Underlying asset value, each stage (V)	24
Nominal construction cost, each stage (K)	20
Dedicated asset investment ratio, each upstream stage	15%
Inflation rate (f)	3%
Risk-free interest rate (r)	4%
Asset return volatility (σ)	0.5
Depreciation rate (q)	5%

TABLE III
BASE-CASE INVESTMENT VALUES

Option	$V_{\{j\},i}$	$\bar{V}_{i,\{j\}}$					$C_{\{j\},i}$	
K=15%								
1-fold	24	--					9.51	27.64%
2-fold	24	6.14	--				8.55	24.86%
3-fold	24	5.52	7.01	--			7.01	20.38%
4-fold	24	6.05	6.75	8.52	--		5.43	15.79%
5-fold	24	7.56	8.24	9.77	13.14	--	3.90	11.33%
					Total		34.41	100.00%
K=20%								
1-fold	24	--					9.83	28.89%
2-fold	24	6.79	--				8.58	25.21%
3-fold	24	6.51	7.80	--			6.86	20.17%
4-fold	24	7.44	8.03	9.56	--		5.18	15.22%
5-fold	24	9.65	10.34	11.95	15.34	--	3.58	10.51%
					Total		34.04	100.00%
K=25%								
1-fold	24	--					10.17	30.11%
2-fold	24	7.31	--				8.62	25.51%
3-fold	24	7.40	8.44	--			6.73	19.93%
4-fold	24	8.76	9.20	10.42	--		4.96	14.67%
5-fold	24	11.68	12.38	14.04	17.38	--	3.30	9.78%
						Total	33.79	100.00%
K=30%								
1-fold	24	--					10.53	31.31%
2-fold	24	7.73	--				8.67	25.78%
3-fold	24	8.22	8.96	--			6.62	19.66%
4-fold	24	10.02	10.28	11.16	--		4.76	14.14%
5-fold	24	13.70	14.38	16.06	19.30	--	3.07	9.11%
					Total		33.64	100.00%

asset-return volatility 0.5. The depreciation rate mimics a higher level of obsolescence than in nonresidential structures (i.e., 3%). The asset-return volatility reflects a competitive business environment. By comparison, Gaspar and Massa [34] estimated that the average idiosyncratic volatility of the U.S. telecom industries increased from 0.087 to 0.223 immediately after deregulation.

C. Investment Values

The first panel of Table III summarizes the base-case investment values. The fourth column of the panel shows that the vanilla call option value is 9.51, the SCCO portfolio value is 24.9, and the total project value is 34.41. The higher order SCCOs are less valuable because their EAVs are higher, as shown in the third column. EAVs are sensitive to changes in the *K ratio*. When the *K ratio* moves from 15% to 30%, the EAVs increase substantially, and the SCCO values decrease accordingly. The increase in *K ratio* reduces the total value from 34.41 to 33.64, even though the vanilla option value increases due to a reduction in its exercise price.

D. Value of Flexibility

Based on Merton [10, Theorem 7], this study defines the value of flexibility of a multistage investment as the value of a portfolio of options on the underlying assets *minus* the value of an option on the assets. Hence, the value is given by the total value of an n-stage project, or χ_n minus the value of the project implemented by a single stage. The single-stage option can be valuated as a plain vanilla call option explained in Step 3 of the algorithm structure using the n-stage's corresponding underlying asset value and exercise price.

For the numerical case, first calculate the value of the single-stage option, which is assumed to complete at year five. Since the base-case construction costs of the five-stage project include a 3% inflation rate, to ensure economic equivalence, the exercise price of the vanilla option is given by the present value formula using the base-case construction costs as cash flows and the inflation rate as discount rate. The adjusted exercise price is 76.05 at maturity (i.e., year five). With the present underlying asset value of 120, the plain vanilla option value is calculated as 50.50.

Second, since the vanilla option does not consider economic depreciation, the base-case value needs to be recalculated by a zero depreciation rate. That is, specify $q_\tau = 0$ in Step 1 of the algorithm structure. The adjustment reduces the EAVs substantially, and increases the total investment value from 34.41 to 87.01. As a result, the value of flexibility of the adjusted five-stage option is 36.51 (=87.01 − 50.50), which is positive and consistent with Merton's theory. Note that the value of flexibility will be destroyed if depreciation is considered. The following sensitivity and scenario analyses will examine this issue more closely.

By comparison, one can treat the project as a simple SCCO by ignoring the presence of dedicated assets, and recalculate the value of flexibility. As explained in Section II, the simple SCCO treats dedicated assets as pure sunk costs, meaning that each fold has an exercise price of 20. By the same algorithm, the value of the simple SCCO is calculated as 82.01 under zero depreciation rate. The corresponding value of flexibility is 31.51 (=82.01 − 50.5), which is lower than that of the adjusted base case. This comparison shows that ignoring the existence of dedicated assets can produce substantially different valuation outcomes.

E. Sensitivity Analysis

1) *Increasing q:* Applying Corollary b) directly, Fig. 3 further plots the base-case value against the plain vanilla option

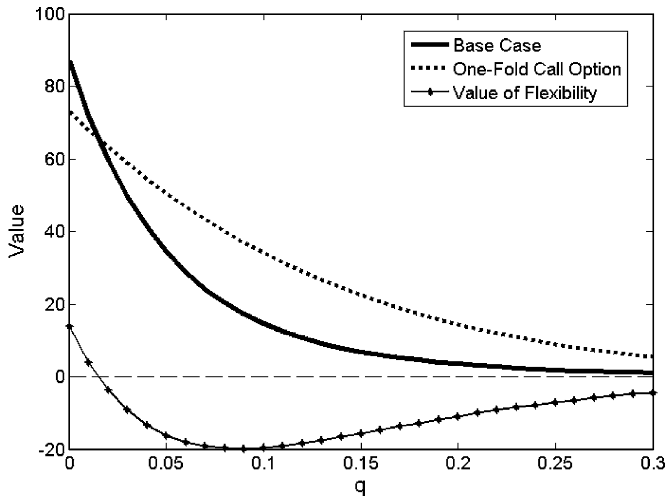

Fig. 3. Sensitivity with respect to q.

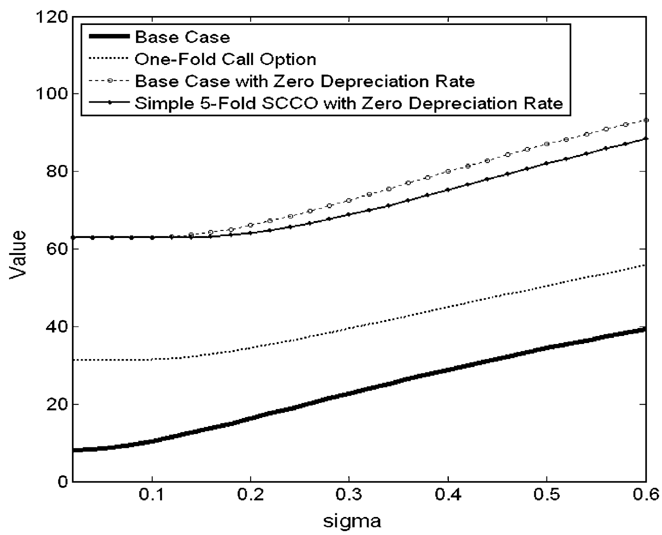

Fig. 4. Sensitivity with respect to σ.

value in terms of increasing q. Increasing q reduces value, and the value of flexibility turns negative when q is over 2%. Other things being equal, rapid technological progress can render the flexibility of multistage investments worthless.

2) *Increasing σ:* Fig. 4 plots the vanilla option value against all other values in terms of increasing σ. It shows that multistage investment is more valuable when the asset return is more volatile, meaning that flexibility is more valuable in more competitive environment. The value of flexibility remains positive and stable in the adjusted base case and the simple SCCO, but the original base-case value is below the vanilla option value due to depreciation. This confirms that obsolescence can destroy value. The adjusted base case produces a higher value of flexibility than the simple SCCO, since it links dedicated assets to future expansions. Future expansions are only valuable if their underlying asset values can justify their exercise prices, including allocated dedicated asset costs.

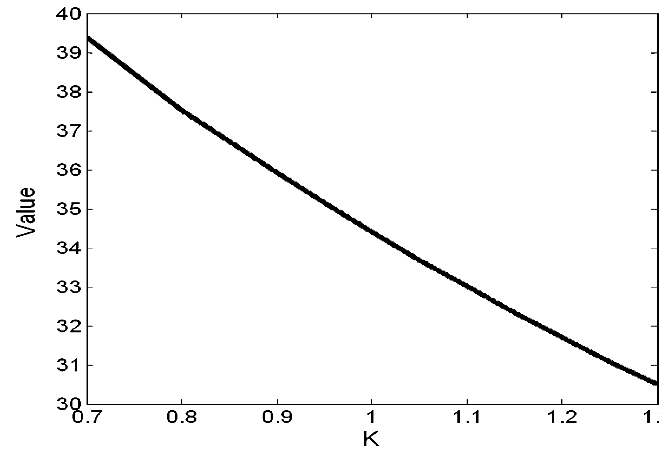

Fig. 5. Sensitivity with respect to K.

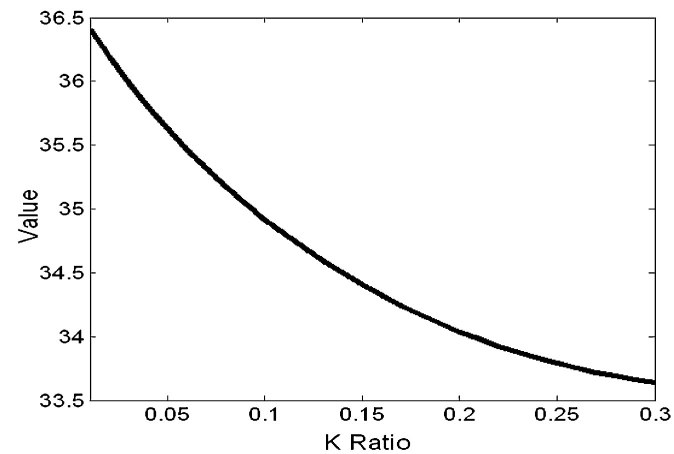

Fig. 6. Sensitivity with respect to the *K ratio*.

3) *Increasing K or the K ratio:* Applying Corollary a), Fig. 5 shows that increasing K in the base case reduces value. In general, increasing (decreasing) embedded sunk costs reduces (increases) values. For instance, the eighth column of Table III shows that as the *K ratio* increases from 15% to 30%, the vanilla option value *increases* and the SCCO portfolio value *decreases*. Fig. 6 further shows that the total value decreases monotonically when the *K ratio* increases in the base case. This means that the decrease in the SCCO portfolio value surpasses the increase in the vanilla option value.

F. Scenario Analysis

1) *Changing σ and the K ratio:* In Fig. 6, the sensitivity of the total project value with respect to the *K ratio* is relatively small. Fig. 7 further shows that as the σ approaches 0.3, the investment value becomes little variant to the *K ratio*. Other things being equal, the influence of σ dominates the influence of the *K ratio* in competitive or risky business markets. Note that the total value increases monotonically in the *K ratio* when the σ value is less than 0.3. This means

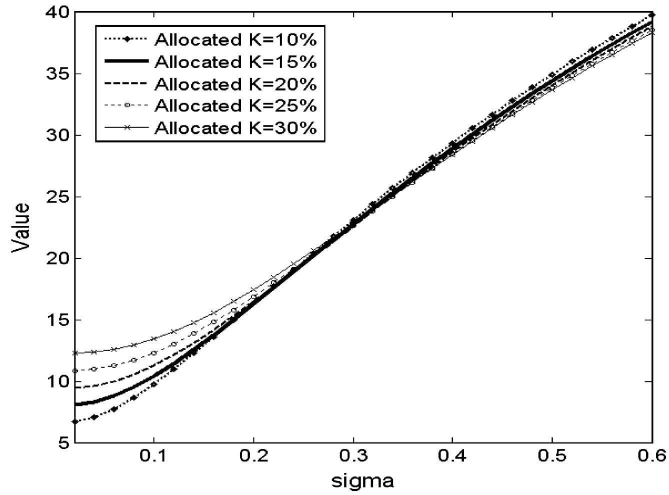

Fig. 7. Scenario analysis with respect to the *K ratio* and σ.

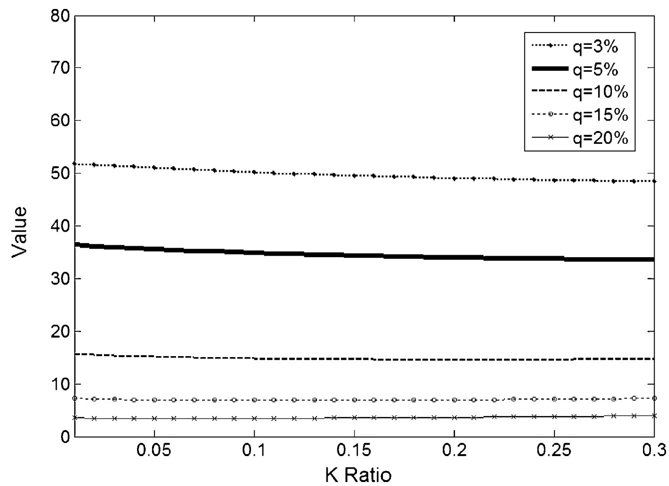

Fig. 8. Scenario analysis with respect to the *K ratio* and q.

that the increase in the plain vanilla option value surpasses the decrease in the SCCO portfolio value.

2) *Changing q and the K ratio:* In Fig. 8, increasing q gradually destroys the investment value, and when the q exceeds 10%, the value becomes almost invariant to the *K ratio*. Thus, the influence of q dominates the influence of the *K ratio* even when the obsolescence rate is modest. For comparison, Hulten and Wykoff [23] estimated that the average annual depreciation rate of computing equipment is about 30%.

3) *Changing σ and q:* In Fig. 9, increasing q destroys the investment value even when the level of σ is high, e.g., 0.6. Hence, the influence of q dominates the influence of σ in the presence of both competition and rapid obsolescence, e.g., in telecom markets.

4) Changing *K, the K ratio, σ, and q:* As in Siegel *et al.* [43], Table IV summarizes the option values per $1 of investment cost under various scenarios. These values increase substantially when the V/K ratio increases from 0.8 to 1.2, implying that the SCCO is more valuable "in the money"

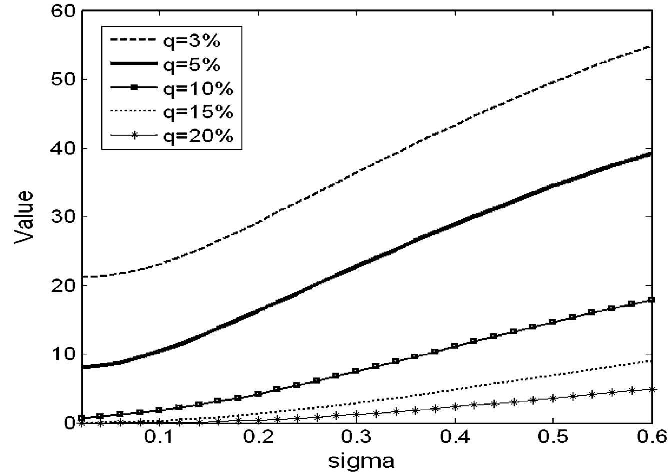

Fig. 9. Scenario analysis with respect to σ and q.

TABLE IV
OPTION VALUES PER $1 OF INVESTMENT COST

K Ratio	*σ=0.4*			*σ=0.5*			*σ=0.6*		
	q=5%	*q=10%*	*q=15%*	*q=5%*	*q=10%*	*q=15%*	*q=5%*	*q=10%*	*q=15%*
V/K=1.2									
10%	0.2935	0.1113	0.0470	0.3492	0.1480	0.0687	0.3972	0.1813	0.0902
15%	0.2888	0.1106	0.0479	0.3441	0.1462	0.0689	0.3920	0.1789	0.0898
20%	0.2859	0.1111	0.0495	0.3404	0.1456	0.0698	0.3881	0.1777	0.0900
25%	0.2844	0.1126	0.0514	0.3379	0.1459	0.0711	0.3852	0.1772	0.0907
30%	0.2840	0.1148	0.0537	0.3364	0.1468	0.0727	0.3832	0.1773	0.0918
V/K=1.0									
10%	0.2181	0.0782	0.0315	0.2692	0.1100	0.0495	0.3132	0.1395	0.0678
15%	0.2135	0.0775	0.0322	0.2642	0.1084	0.0496	0.3082	0.1373	0.0674
20%	0.2106	0.0779	0.0334	0.2606	0.1078	0.0503	0.3044	0.1361	0.0676
25%	0.2090	0.0791	0.0349	0.2582	0.1080	0.0513	0.3016	0.1356	0.0682
30%	0.2087	0.0809	0.0367	0.2567	0.1088	0.0527	0.2997	0.1357	0.0691
V/K=0.8									
10%	0.1486	0.0494	0.0186	0.1935	0.0753	0.0324	0.2324	0.1001	0.0472
15%	0.1443	0.0487	0.0191	0.1887	0.0738	0.0325	0.2277	0.0982	0.0469
20%	0.1415	0.0490	0.0199	0.1853	0.0733	0.0330	0.2241	0.0971	0.0470
25%	0.1400	0.0499	0.0210	0.1830	0.0734	0.0338	0.2215	0.0967	0.0474
30%	0.1396	0.0512	0.0223	0.1817	0.0740	0.0348	0.2197	0.0967	0.0482

(i.e., the underlying asset value is higher than the nominal construction cost). In addition, the option values are more sensitive to the change of the parameter values when the SCCO is "out of the money" (i.e., the underlying asset value is lower than the nominal construction cost). This agrees with Siegel *et al.*'s [43] finding in the valuation of offshore oil properties. However, the option values are destroyed when the q increases from 5% to 15%. Finally, the option values are not monotonic in the *K ratio* when the q is at 10% or 15%. When the q is at 15%, these values tend to increase with the *K ratio*. Therefore, postponing dedicated asset investments tend to increase project values in an environment of rapid technological obsolescence.

VI. Implications to Concession and Regulatory Designs

The foregoing numerical implementation shows that the SCCO model is viable for the valuation of complex multistage infrastructure investments. The results provide some insights in concession and regulatory designs under two common competition policies in infrastructure privatization.

1) *Competition in the field:* When policy makers promote head-to-head competition in the market, e.g., in the telecom sector, investors should have full discretion to develop their investment programs in order to maximize the value of flexibility. In an environment of rapid technological changes, policy markers can focus on reciprocal arrangements to manage dedicated assets and obsolescence risk by, e.g., accelerated depreciation schedule. If the TELRIC is used, then a higher cost of capital can be allowed to compensate for obsolescence risk and maintain project value.
2) *Competition for the field:* When policy makers promote competition for the market by long-term exclusivity rights, and the market is truly monopolistic with trivial demand-side risk, the focus is also on reciprocal arrangements to manage dedicated assets and obsolescence risk. If the market is not monopolistic (e.g., facing intermodal competition in the transportation sector) and/or the demand-side risk is substantial, then policy makers can grant the investor voluntary expansion and abandonment rights to facilitate multistage investments and improve project value.

One crucial question remains: how to justify these reciprocal arrangements if they involve anticompetitive practices? The answer centers on if the arrangements can serve public interests. The following observations derive, largely, from the SCCO analysis.

1) *Long-term exclusivity rights:* Long-term exclusivity rights can limit competition and reduce the value of flexibility. However, according to Gaspar and Massa [34], competition can increase asset-return volatility. Other things being equal, the extent to which exclusivity rights protect an infrastructure firm from competition is the extent to which they will reduce the firm's return volatility. This, in turn, reduces the firm's asset beta, and thus, cost of capital, which eventually reduces service price and serve public interests.
2) *Voluntary expansion and abandonment rights:* These rights are basic tools to facilitate multistage investment in BOT projects. Based on the principle of reciprocity, the grant of these rights should stipulate on the concessionaire's acceptance of a lower required ROR. That is, other things being equal, to the extent that expansion and abandonment rights increase the value of flexibility, the cost of capital should be reduced accordingly.
3) *Commitment for future prices and capital recovery:* According to Alexander *et al*. [32], ROR regulation produces lower asset betas than price-cap regulation because the former is more adaptive to cost changes, and thus, can reduce revenue volatility. If revenue volatility can be reduced, then according to Brealey and Myers [44], the impact of operating leverage (i.e., the commitment to fixed costs) on asset beta is reduced. If the project in question involves dedicated asset and technological obsolescence, then the reduction in asset beta is not trivial because both of them increase operating leverage. Thus, other things being equal, to the extent that government commitment for future prices reduces revenue volatility is the extent it will reduce asset beta. Moreover, if full capital recovery is assured by embedding in the rate base all dedicated assets and other fixed costs, as in Schmalensee [4], then the firm's cost of capital is minimal.

VII. Conclusion

Multistage investment is an incremental strategy in geographical expansions of infrastructure networks or systems. This strategy creates a level of flexibility that is valuable in managing demand-side risks, but faces great challenges when future expansions require upstream investments in dedicated assets prone to obsolescence risk. This paper develops a multifold European SCCO valuation approach to highlight these issues. This approach provides a viable analytical framework for modeling and valuating multistage investments, and offers new insights into the effects of competition, dedicated assets, and obsolescence on value. This study shows that the value of flexibility in multistage investment is substantial in competitive markets, but the value can be totally destroyed by rapid obsolescence. Moreover, dedicated asset investments must be handled carefully, since they influence investment value profoundly.

These results have important implications on infrastructure concession and regulatory designs. The importance of competition and technology in infrastructure expansions cannot be overemphasized. This study supports the practice of facilitating multistage investment by granting exclusivity, expansion, and abandonment rights. The practice can reduce the cost of capital and serve public interests. However, to obtain better outcomes, other reciprocal arrangements are necessary to further mitigate the risks of dedicated assets and obsolescence, e.g., the use of accelerated depreciation schedule in setting infrastructure service prices.

Appendix A

Proof of Proposition 1: For $n = 1$, (8) reduces to a vanilla European call option, and therefore,

$$\begin{aligned}
&\frac{\partial C_{\{1\},1}\left(V,t_0\right)}{\partial K_{\{1\},1}} \\
&= V_{\{1\},0}e^{-q_1\tau_1}\frac{\partial N_1\left\{g_{\{1\},1}\right\}}{\partial K_{\{1\},1}} - K_{\{1\},1}e^{-r_1\tau_1}\frac{\partial N_1\left\{h_{\{1\},1}\right\}}{\partial K_{\{1\},1}} \\
&\quad - e^{-r_1\tau_1}N_1\left\{h_{\{1\},1}\right\} \\
&= V_{\{1\},0}e^{-q_1\tau_1}\frac{1}{\sqrt{2\pi}}e^{-(1/2)g^2_{\{1\},1}}\left(\frac{\partial g_{\{1\},1}}{\partial K_{\{1\},1}}\right) \\
&\quad - K_{\{1\},1}e^{-r_1\tau_1}\frac{1}{\sqrt{2\pi}}e^{-(1/2)h^2_{\{1\},1}}\left(\frac{\partial h_{\{1\},1}}{\partial K_{\{1\},1}}\right) \\
&\quad - e^{-r_1\tau_1}N_1\left\{h_{\{1\},1}\right\}.
\end{aligned}$$

Since $\partial g_{\{1\},1}/\partial K_{\{1\},1} = \partial h_{\{1\},1}/\partial K_{\{1\},1}$ and $g_{\{1\},1} = h_{\{1\},1} + \sqrt{\sigma^2 \tau}$, it follows that

$$\begin{aligned}\frac{\partial C_{\{1\},1}(V,t_0)}{\partial K_{\{1\},1}} &= V_{\{1\},0} e^{-q_1\tau_1} \frac{1}{\sqrt{2\pi}} e^{-(1/2)\left(h_{\{1\},1}+\sqrt{\sigma^2\tau}\right)^2} \left(\frac{\partial h_{\{1\},1}}{\partial K_{\{1\},1}}\right) \\ &\quad - K_{\{1\},1} e^{-r_1\tau_1} \frac{1}{\sqrt{2\pi}} e^{-(1/2)h^2_{\{1\},1}} \left(\frac{\partial h_{\{1\},1}}{\partial K_{\{1\},1}}\right) \\ &\quad - e^{-r_1\tau_1} N_1\left\{h_{\{1\},1}\right\} \\ &= \frac{1}{\sqrt{2\pi}} e^{-(1/2)h^2_{\{1\},1}} e^{-r_1\tau_1} \left(\frac{\partial h_{\{1\},1}}{\partial K_{\{1\},1}}\right) \left[\overline{V}_{1,\{1\}} - K_{\{1\},1}\right] \\ &\quad - e^{-r_1\tau_1} N_1\left\{h_{\{1\},1}\right\}.\end{aligned}$$

By definition, $\overline{V}_{1,\{1\}} = K_{\{1\},1}$, and therefore,

$$\frac{\partial C_{\{1\},1}(V,t_0)}{\partial K_{\{1\},1}} = -e^{-r_1\tau_1} N_1\left\{h_{\{1\},1}\right\}. \tag{A1}$$

Since the $K_{\{n\},i}$, $i = 1, 2, \ldots, n-1$ does not exist in the multivariate normal functions, it follows that

$$\frac{\partial C_{\{n\},n}(V,t_0)}{\partial K_{\{n\},i}} = -e^{-\sum_{u=1}^{i} r_u\tau_u} N_i\{[h_{\{n\},i}]_{i\times 1}; [\rho_{\{n\},i,j}]_{i\times i}\}. \tag{A2}$$

For $i = n$

$$\begin{aligned}\frac{\partial C_{\{n\},n}(V,t_0)}{\partial K_{\{n\},n}} &= V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u\tau_u} \frac{\partial N_n\{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}}{\partial K_{\{n\},n}} \\ &\quad - K_{\{n\},n} e^{-\sum_{u=1}^{n} r_u\tau_u} \frac{\partial N_n\{[h_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}}{\partial K_{\{n\},n}} \\ &\quad - e^{-\sum_{u=1}^{n} r_u\tau_u} N_n\{[h_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}.\end{aligned} \tag{A3}$$

By Lee *et al.*'s [23] Lemma 2

$$\begin{aligned}&\frac{\partial N_n\{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}}{\partial K_{\{n\},n}} \\ &= \sum_{s=1}^{n} f(g_{\{n\},s}) \left(\frac{\partial g_{\{n\},s}}{\partial K_{\{n\},n}}\right) \\ &\quad \times N_{n-1}\left\{\left(\left[\frac{g_{\{n\},i} - g_{\{n\},s}\rho_{\{n\},i,s}}{\sqrt{1-(\rho_{\{n\},i,s})^2}}\right]_{n\times 1}\right)^{(-s,)};\right. \\ &\qquad \left.\left(\left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},i,s}\rho_{\{n\},j,s}}{\sqrt{(1-(\rho_{\{n\},i,s})^2)(1-(\rho_{\{n\},j,s})^2)}}\right]_{n\times n}\right)^{(-s,-s)}\right\} \\ &= \sum_{s=1}^{n} f(g_{\{n\},s}) \left(\frac{\partial g_{\{n\},s}}{\partial K_{\{n\},n}}\right) \\ &\quad \times N_{s-1}\left\{\left[\frac{h_{\{n\},i} - h_{\{n\},s}\rho_{\{n\},i,s}}{\sqrt{1-(\rho_{\{n\},i,s})^2}}\right]_{(s-1)\times 1};\right. \\ &\qquad \left.\left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},i,s}\rho_{\{n\},j,s}}{\sqrt{(1-(\rho_{\{n\},i,s})^2)(1-(\rho_{\{n\},j,s})^2)}}\right]_{(s-1)\times(s-1)}\right\} \\ &\quad \times N_{n-s}\{[g_{\{n\},i,\#s}]_{(n-s)\times 1}; [\rho_{\{n\},i,j,*s}]_{(n-s)\times(n-s)}\}\end{aligned} \tag{A4a}$$

and

$$\begin{aligned}&\frac{\partial N_n\{[h_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}}{\partial K_{\{n\},n}} \\ &= \sum_{s=1}^{n} f(h_{\{n\},s}) \left(\frac{\partial h_{\{n\},s}}{\partial K_{\{n\},n}}\right) \\ &\quad \times N_{s-1}\left\{\left[\frac{h_{\{n\},i} - h_{\{n\},s}\rho_{\{n\},i,s}}{\sqrt{1-(\rho_{\{n\},i,s})^2}}\right]_{(s-1)\times 1};\right. \\ &\qquad \left.\left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},i,s}\rho_{\{n\},j,s}}{\sqrt{(1-(\rho_{\{n\},i,s})^2)(1-(\rho_{\{n\},j,s})^2)}}\right]_{(s-1)\times(s-1)}\right\} \\ &\quad \times N_{n-s}\{[h_{\{n\},i,\#s}]_{(n-s)\times 1}; [\rho_{\{n\},i,j,*s}]_{(n-s)\times(n-s)}\}\end{aligned} \tag{A4b}$$

where the symbol $^*s$ indicates a time shift, such that the correlation matrix starts from time s, or

$$\rho_{\{n\},i,j,*s} = \sqrt{\sum_{u=1+s}^{i} \sigma_u^2\tau_u \Big/ \sum_{u=1+s}^{j} \sigma_u^2\tau_u};$$

$$g_{\{n\},i,\#s} = \left[\ln\left(\frac{\bar{V}_{s,\{n\}}}{\bar{V}_{i,\{n\}}}\right) + \sum_{u=1+s}^{i}\left(r_u - q_u + \frac{1}{2}\sigma_u^2\right)\tau_u\right] \Big/ \sqrt{\sum_{u=1+s}^{i} \sigma_u^2\tau_u},$$

$$h_{\{n\},i,\#s} = \left[\ln\left(\frac{\bar{V}_{s,\{n\}}}{\bar{V}_{i,\{n\}}}\right) + \sum_{u=1+s}^{i}\left(r_u - q_u - \frac{1}{2}\sigma_u^2\right)\tau_u\right] \Big/ \sqrt{\sum_{u=1+s}^{i} \sigma_u^2\tau_u} \ \forall 1 \le s \le i \le n, \quad \text{and}$$

$$\partial g_{\{n\},i}/\partial K_{\{n\},n} = 0, \quad \forall 1 \le i < n.$$

Substituting (A4a) and (A4b) into (A3) gives

$$\begin{aligned}\frac{\partial C_{\{n\},n}(V,t_0)}{\partial K_{\{n\},n}} &= -e^{-\sum_{u=1}^{n} r_u\tau_u} N_n\{[h_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\} \\ &\quad + \hat{C}_{\partial K_{\{n\},n},1} - \hat{C}_{\partial K_{\{n\},n},2}\end{aligned} \tag{A5}$$

where

$$\hat{C}_{\partial K_{\{n\},n,1}} = V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u \tau_u} \frac{1}{\sqrt{2\pi}} e^{-(1/2)(g_{\{n\},n})^2} \left(\frac{\partial g_{\{n\},n}}{\partial K_{\{n\},n}}\right) \times N_{n-1}\left\{\left[\frac{h_{\{n\},i} - h_{\{n\},n}\rho_{\{n\},n,i}}{\sqrt{1-(\rho_{\{n\},n,i})^2}}\right]_{(n-1)\times 1}; \left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},n,i}\rho_{\{n\},n,j}}{\sqrt{(1-(\rho_{\{n\},n,i})^2)(1-(\rho_{\{n\},n,j})^2)}}\right]_{(n-1)\times(n-1)}\right\} \times N_0$$

and

$$\hat{C}_{\partial K_{\{n\},n,2}} = K_{\{n\},n} e^{-\sum_{u=1}^{n} r_u \tau_u} \frac{1}{\sqrt{2\pi}} e^{-(1/2)(h_{\{n\},n})^2} \left(\frac{\partial h_{\{n\},n}}{\partial K_{\{n\},n}}\right) \times N_{n-1}\left\{\left[\frac{h_{\{n\},i} - h_{\{n\},n}\rho_{\{n\},n,i}}{\sqrt{1-(\rho_{\{n\},n,i})^2}}\right]_{(n-1)\times 1}; \left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},n,i}\rho_{\{n\},n,j}}{\sqrt{(1-(\rho_{\{n\},n,i})^2)(1-(\rho_{\{n\},n,j})^2)}}\right]_{(n-1)\times(n-1)}\right\} \times N_0.$$

Since

$$\begin{aligned} f(g_{\{n\},n}) &= 1/\sqrt{2\pi} e^{-(1/2)(g_{\{n\},n})^2} \\ &= 1/\sqrt{2\pi} e^{-(1/2)\left[(h_{\{n\},n}) + \sqrt{\sum_{u=1}^{n} \sigma_u^2 \tau_u}\right]^2} \\ &= 1/\sqrt{2\pi} e^{-(1/2)h_{\{n\},n}^2} \left(\frac{\bar{V}_{n,\{n\}}}{V_{\{n\},0}}\right) e^{-\sum_{u=1}^{n}(r_u - q_u)\tau_u}, \end{aligned}$$

and $\partial g_{\{n\},n}/\partial K_{\{n\},n} = \partial h_{\{n\},n}/\partial K_{\{n\},n}$, $\hat{C}_{\partial K_{\{n\},n,1}} = \hat{C}_{\partial K_{\{n\},n,2}}$, it follows that $\partial C_{\{n\},n}(V,t_0)/\partial K_{\{n\},n} = -e^{-\sum_{u=1}^{n} r_u \tau_u} N_n\{[h_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}$. Denote by K_l a subsequent exercise price. It can be verified by a similar method that for $\forall 1 < l \le n$

$$\frac{\partial C_{\{n\},n}(V,t_0)}{\partial K_{\{n\},l}} = -e^{-\sum_{u=1}^{l} r_u \tau_u} N_l \left\{ \left[h_{\{n\},i}\right]_{l\times 1}; \left[\rho_{\{n\},i,j}\right]_{l\times l} \right\}. \quad \text{(A6)}$$

Summing up (A1) and (A6) gives (9). □

APPENDIX B

Proof of Proposition 2: By (8)

$$\begin{aligned} \frac{\partial C_{\{n\},n}(V,t_0)}{\partial q_1} &= -\tau_1 V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u \tau_u} N_n\{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\} \\ &+ V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u \tau_u} \frac{\partial N_n\{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}}{\partial q_1} \\ &- \sum_{m=1}^{n} K_{\{n\},m} e^{-\sum_{u=1}^{m} r_u \tau_u} \times \frac{\partial N_m\{[h_{\{n\},i}]_{m\times 1}; [\rho_{\{n\},i,j}]_{m\times m}\}}{\partial q_1}. \end{aligned} \quad \text{(B1)}$$

By Lee *et al.*'s [23] Lemma 2

$$\frac{\partial C_{\{n\},n}(V,t_0)}{\partial q_1} = -\tau_1 V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u \tau_u} \times N_n\{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\} + \hat{C}_{\partial q_1,1} - \hat{C}_{\partial q_1,2} \quad \text{(B2)}$$

where

$$\hat{C}_{\partial q_1,1} = V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u \tau_u} \sum_{s=1}^{n} f(g_{\{n\},s}) \left(\frac{\partial g_{\{n\},s}}{\partial q_1}\right) \times N_{s-1}\left\{\left[\frac{h_{\{n\},i} - h_{\{n\},s}\rho_{\{n\},s,i}}{\sqrt{1-\rho_{\{n\},s,i}^2}}\right]_{(s-1)\times 1}; \left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},s,i}\rho_{\{n\},s,j}}{\sqrt{(1-\rho_{\{n\},s,i}^2)(1-\rho_{\{n\},s,j}^2)}}\right]_{(s-1)\times(s-1)}\right\} \times N_{n-s}\{[g_{\{n\},i,\#s}]_{(n-s)\times 1}; [\rho_{\{n\},i,j,*s}]_{(n-s)\times(n-s)}\}$$

and

$$\hat{C}_{\partial q_1,2} = \sum_{m=1}^{n} K_{\{n\},m} e^{-\sum_{u=1}^{m} r_u \tau_u} \sum_{s=1}^{m} f(h_{\{n\},s}) \left(\frac{\partial h_{\{n\},s}}{\partial q_1}\right) \times N_{s-1}\left\{\left[\frac{h_{\{n\},i} - h_{\{n\},s}\rho_{\{n\},s,i}}{\sqrt{1-\rho_{\{n\},s,i}^2}}\right]_{(s-1)\times 1}; \left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},s,i}\rho_{\{n\},s,j}}{\sqrt{(1-\rho_{\{n\},s,i}^2)(1-\rho_{\{n\},s,j}^2)}}\right]_{(s-1)\times(s-1)}\right\} \times N_{n-s}\{[h_{\{n\},i,\#s}]_{(n-s)\times 1}; [\rho_{\{n\},i,j,*s}]_{(n-s)\times(n-s)}\}.$$

The h and the g factors are as defined in Appendix A. The following derivations show that the last two terms in (B2) are equivalent, or $\hat{C}_{\partial q_1,1} - \hat{C}_{\partial q_1,2} = 0 \quad \forall 1 \le s \le n$. First let $\hat{C}_{\partial q_1,3} = 1/\sqrt{2\pi} e^{-(1/2)h_{\{n\},1}^2} e^{-\sum_{u=1}^{1} r_u \tau_u} (\partial h_{\{n\},1}/\partial q_1)$. Since $f(g_{\{n\},s}) = 1/\sqrt{2\pi} e^{-(1/2)h_{\{n\},s}^2} (\overline{V}_{s,\{n\}}/V_{\{n\},0}) e^{-\sum_{u=1}^{s}(r_u - q_u)\tau_u}$ and $\partial g_{\{n\},s}/\partial q_1 = \partial h_{\{n\},s}/\partial q_1$, it follows that for $s = 1$

$$\begin{aligned} \hat{C}_{\partial q_1,1} - \hat{C}_{\partial q_1,2} &= \overline{V}_{1,\{n\}} e^{-\sum_{u=2}^{n} q_u \tau_u} \hat{C}_{\partial q_1,3} N_0 N_{n-1} \\ &\times \{[g_{\{n\},i,\#1}]_{(n-1)\times 1}; [\rho_{\{n\},i,j,*1}]_{(n-1)\times(n-1)}\} \\ &- K_{\{n\},n} e^{-\sum_{u=2}^{n} r_u \tau_u} \hat{C}_{\partial q_1,3} N_0 N_{n-1} \\ &\times \{[h_{\{n\},i,\#1}]_{(n-1)\times 1}; [\rho_{\{n\},i,j,*1}]_{(n-1)\times(n-1)}\} \end{aligned}$$

$$
\begin{aligned}
&- K_{\{n\},n-1} e^{-\sum_{u=2}^{n-1} r_u \tau_u} \hat{C}_{\partial q_1,3} N_0 N_{(n-1)-1} \\
&\times \{[h_{\{n\},i,\#1}]_{((n-1)-1)\times 1}; [\rho_{\{n\},i,j,*1}]_{((n-1)-1)\times((n-1)-1)}\} \\
&- \cdots \\
&- K_{\{n\},2} e^{-\sum_{u=2}^{2} r_u \tau_u} \hat{C}_{\partial q_1,3} N_0 N_{2-1} \\
&\times \{[h_{\{n\},i,\#1}]_{(2-1)\times 1}; [\rho_{\{n\},i,j,*1}]_{(2-1)\times(2-1)}\} \\
&- K_{\{n\},1} \hat{C}_{\partial q_1,3} N_0.
\end{aligned}
$$

Rearranging gets $\hat{C}_{\partial q_1,1} - \hat{C}_{\partial q_1,2} = \hat{C}_{\partial q_1,3} N_0 [\hat{C}_{\partial q_1,4} - K_{\{n\},1}]$, where

$$
\begin{aligned}
\hat{C}_{\partial q_1,4} = &\overline{V}_{1,\{n\}} e^{-\sum_{u=2}^{n} q_u \tau_u} N_{n-1} \\
&\times \{[g_{\{n\},i,\#1}]_{(n-1)\times 1}; [\rho_{\{n\},i,j,*1}]_{(n-1)\times(n-1)}\} \\
&- \sum_{m=2}^{n} K_{\{n\},m} e^{-\sum_{u=2}^{m} r_u \tau_u} N_{n-1} \\
&\times \{[h_{\{n\},i,\#1}]_{(m-1)\times 1}; [\rho_{\{n\},i,j,*1}]_{(m-1)\times(m-1)}\}.
\end{aligned}
$$

The $\hat{C}_{\partial q_1,4}$ is an (n – 1)-fold SCCO starting at time t_1. By definition $\hat{C}_{\partial q_1,4} = K_{\{n\},1}$, and therefore, $\hat{C}_{\partial q_1,1} - \hat{C}_{\partial q_1,2} = 0$. Applying the same method gets $\hat{C}_{\partial q_1,1} - \hat{C}_{\partial q_1,2} = 0$ for $s = 2, 3, \ldots, n$. Accordingly,

$$
\begin{aligned}
\frac{\partial C_{\{n\},n}(V,t_0)}{\partial q_1} = &-\tau_1 V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u \tau_u} \\
&\times N_n \{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}. \quad \text{(B3)}
\end{aligned}
$$

Further denote by q_l a subsequent depreciation rate. One can verify by a similar process

$$
\begin{aligned}
\frac{\partial C_{\{n\},n}(V,t_0)}{\partial q_l} = &-\tau_l V_{\{n\},0} e^{-\sum_{u=1}^{n} q_u \tau_u} \\
&\times N_n \{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\}, \text{ for } 1 \le l \le n. \quad \text{(B4)}
\end{aligned}
$$

Summing up gives (10). □

Appendix C

Proof of Corollary:

1) Since $\sum_{l=1}^{n} \partial C_{\{n\},n}(V,t_0)/\partial K_{\{n\},l} < 0$ in Proposition 1, it follows that $C_{\{n\},n}(V,t_0)$ is a monotonic function of K_l. By a similar method in Appendix A, the following proves that $C_{\{n\},n}(V,t_0)$ is a convex function of K_l:

$$
\begin{aligned}
\sum_{l=1}^{n} \frac{\partial^2 C_{\{n\},n}(V,t_0)}{\partial K_{\{n\},l}^2} = &\frac{1}{K_{\{n\},n}\sqrt{2\pi \sum_{u=1}^{n} \sigma_u^2 \tau_u}} \\
&\times \exp\left(-\frac{1}{2}(h_{\{n\},n})^2 - \sum_{u=1}^{n} r_u \tau_u\right) \\
&\times N_{n-1} \left\{ \left[\frac{h_{\{n\},i} - h_{\{n\},n}\rho_{\{n\},n,i}}{\sqrt{1-(\rho_{\{n\},n,i})^2}}\right]_{(n-1)\times 1}; \right. \\
&\left. \left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},n,i}\rho_{\{n\},n,j}}{\sqrt{(1-(\rho_{\{n\},n,i})^2)(1-(\rho_{\{n\},n,j})^2)}}\right]_{(n-1)\times(n-1)} \right\} \\
&> 0. \quad \text{(C1)}
\end{aligned}
$$

2) Since $\sum_{l=1}^{n} \partial C\,(V,t_0)/\partial q_l < 0$ in Proposition 2, it follows that $C_{\{n\},n}\,(V,t_0)$ is decreasing monotonically in q_l. By a similar method in Appendix B

$$
\begin{aligned}
\sum_{l=1}^{n} \frac{\partial^2 C\,(V,t_0)}{\partial q_l^2} = &\sum_{l=1}^{n} \tau_l^2 V_{\{n\},0} \exp\left(-\sum_{u=1}^{n} q_u \tau_u\right) \\
&\times [N_n \{[g_{\{n\},i}]_{n\times 1}; [\rho_{\{n\},i,j}]_{n\times n}\} + \hat{C}_{\partial q_l^2}] \quad \text{(C2)}
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{C}_{\partial q_l^2} = &\sum_{s=l}^{n} \frac{1}{\sqrt{2\pi \sum_{u-1}^{s} \sigma_u^2 \tau_u}} \exp\left(-\frac{1}{2}(g_{\{n\},s})^2\right) \\
&\times N_{n-s}\{[g_{\{n\},i,\#s}]_{(n-s)\times 1}; [\rho_{\{n\},i,j,*s}]_{(n-s)\times(n-s)}\} \\
&\times N_{s-1} \left\{ \left[\frac{h_{\{n\},i} - h_{\{n\},s}\rho_{\{n\},s,i}}{\sqrt{1-(\rho_{\{n\},s,i})^2}}\right]_{(s-1)\times 1}; \right. \\
&\left. \left[\frac{\rho_{\{n\},i,j} - \rho_{\{n\},s,i}\rho_{\{n\},s,j}}{\sqrt{(1-(\rho_{\{n\},s,i})^2)(1-(\rho_{\{n\},s,j})^2)}}\right]_{(s-1)\times(s-1)} \right\}.
\end{aligned}
$$

Since $\sum_{l=1}^{n} \partial^2 C\,(V,t_0)/\partial q_l^2 > 0$, if follows that $C_{\{n\},n}$ (V,t_0) is convex in q_l. □

References

[1] O. E. Williamson, “Creditable commitments: Using hostages to support exchange,” *Amer. Eco. Rev.*, vol. 73, pp. 519–540, Sep. 1983.

[2] Y. L. Huang, “Project and policy analysis of build-operate-transfer infrastructure developments,” Ph.D. dissertation, Dept. Civil and Environmental Eng., Univ. California at Berkeley, Berkeley, CA, 1995.

[3] R. Rees and J. Vickers, “RPI-X price-cap regulation,” in *The Regulatory Challenge*, M. Bishop, J. Kay, and C. Mayer, Eds. New York: Oxford Univ, Press, 1995, pp. 358–385.

[4] R. Schmalensee, “An expository note on depreciation and profitability under rate-of-return regulation,” *J. Regul. Eco.*, vol. 1, pp. 293–298, 1989.

[5] P. Guislain, *The Privatization Challenge: A Strategic, Legal, and Institutional Analysis of International Experience*. Washington, DC: The World Bank, 1997, pp. 228–242.

[6] Y. L. Huang and C. C. Pi, “Valuation of multistage BOT projects involving dedicated assets: A sequential compound option approach,” *Constr. Manage. Eco.*, vol. 27, no. 7, pp. 653–666, 2009.

[7] M. Crew and P. Kleindorfer, “Economic depreciation and the regulated firm under competition and technological change,” *J. Regul. Eco.*, vol. 4, pp. 51–61, 1992.

[8] D. W. Newbery, *Privatization, Restructuring, and Regulation of Network Utilities*. Cambridge, MA: MIT Press, 2001, pp. 338–341.

[9] S. C. Myers, “Finance theory and financial strategy,” *Midland Corp. Finance J.*, vol. 5, no. 1, pp. 6–13, 1987.

[10] R. C. Merton, “Theory of rational option pricing,” *Bell J. Eco. Manage. Sci.*, vol. 4, pp. 141–183, 1973.

[11] Y. L. Huang and S. P. Chou, “Valuation of minimum revenue guarantee and the option to abandon in BOT infrastructure projects,” *Constr. Manage. Eco.*, vol. 24, no. 4, pp. 379–389, 2006.

[12] L. Trigeorgis, “The nature of option interactions and the valuation of investment with multiple real options,” *J. Financ. Quant. Anal.*, vol. 28, pp. 1–20, 1993.

[13] C. H. Wand and K. J. Min, "Electric power generation planning for interrelated projects: A real option approach," *IEEE Trans. Eng. Manage.*, vol. 53, no. 2, pp. 312–322, May 2006.
[14] I. Damnjanovic, J. Duthie, and S. T. Waller, "Valuation of strategic network flexibility in development of toll road projects," *Constr. Manage. Eco.*, vol. 26, no. 9, pp. 979–990, 2008.
[15] S. Majd and R. S. Pindyck, "Time to build, option value, and investment decisions," *J. Financ. Eco.*, vol. 18, pp. 7–27, 1987.
[16] R. S. Pindyck, "Irreversible investment, capacity choice, and the value of the firm," *Amer. Eco. Rev.*, vol. 79, pp. 969–985, Dec. 1988.
[17] A. K. Dixit and R. S. Pindyck, *Investment under Uncertainty*. Princeton, NJ: Princeton Univ. Press, 1994, ch. 10.
[18] L. Thomassen and M. Van Wouwe, "The *n*-fold compound option," Unpublished research paper 2001–041, Dept. Math. Statist., Univ. Antwerp, Antwerp, Belgium, 2001.
[19] F. Lajeri-Chaherli, "A note on the valuation of compound options," *J. Future Markets*, vol. 22, pp. 1103–1115, 2002.
[20] E. Agliardi and R. Agliardi, "A closed-form solution for multicompound options," *Risk Lett.*, vol. 1, pp. 1–2, 2005.
[21] M. Y. Lee, F. B. Yeh, and A. P. Chen, "The generalized sequential compound options pricing and sensitivity analysis," *Math. Soc. Sci.*, vol. 55, pp. 38–54, 2008.
[22] H. Hotelling, "A general mathematic theory of depreciation," *J. Amer. Statist. Assoc.*, vol. 20, pp. 340–353, 1925.
[23] C. Hulten and F. Wykoff, "Issues on the measurement of economic depreciation: Introductory remarks," *Eco. Inq.*, vol. 34, pp. 10–23, 1996.
[24] S. Cowan, "Privatization and regulation of the water industry in England and Wales," in *Privatization and Economic Performance*, M. Bishop, J. Kay, and C. Mayer, Eds. New York, NY: Oxford Univ. Press, 1994, pp. 112–136.
[25] J. B. Berk, R. C. Green, and V. Naik, "Valuation and return dynamics of new ventures," *Rev. Financ. Stud.*, vol. 17, no. 1, pp. 1–35, 2004.
[26] S. Remer, S. H. Ang, and C. Baden-Fuller, "Dealing with uncertainties in the biotechnology industry: The use of real options reasoning," *J. Comm. Biotech.*, vol. 8, no. 2, pp. 95–105, 2001.
[27] S. M. Ross, *An Elementary Introduction to Mathematical Finance: Options and Other Topics*. Cambridge, MA: Cambridge Univ. Press, 2003, ch. 8.
[28] M. Domes, "Estimating capital efficiency schedules within production functions," *Eco. Inq.*, vol. 34, pp. 78–92, 1996.
[29] F. Black and M. Scholes, "The pricing of options and corporate liabilities," *J. Pol. Eco.*, vol. 81, pp. 637–657, 1973.
[30] R. C. Merton, "On the pricing of corporate debt: The risk structure of interest rates," *J. Finance*, vol. 29, pp. 449–470, May 1974.
[31] W. Beaver, P. Kettler, and M. Scholes, "The association between market determined and accounting determined risk measures," *Account. Rev.*, vol. 45, pp. 654–682, Oct. 1970.
[32] I. Alexander, C. Mayer, and H. Weeds, "Regulatory structure and infrastructure firms: An international comparison," World Bank Policy Research, Working Paper No. 1698, Washington, DC: World Bank, Nov. 30, 1999.
[33] P. Grout, "The cost of capital of regulated industries," in *The Regulatory Challenge*, M. Bishop, J. Kay, and C. Mayer, Eds. New York: Oxford Univ. Press, 1995, pp. 386–407.
[34] J. M. Gaspar and M. Massa, "Idiosyncratic volatility and product market competition," *J. Bus.*, vol. 79, pp. 3125–3152, 2006.
[35] S. E. Shreve, *Stochastic Calculus for Finance II: Continuous-Time Models*. New York: Springer-Verlag, 2003, sec. 4.4.
[36] E. Agliardi and R. Agliardi, "A generalization of Geske formula for compound options," *Math. Social Sci.*, vol. 45, pp. 75–82, 2003.
[37] M. Rubinstein, *A History of the Theory of Investments: My Annotated Bibliography*. Hoboken, NJ: Wiley, 2006, p. 355, note 34.
[38] R. Roll, "An analytic valuation formula for unprotected American call options on stocks with known dividends," *J. Financ. Eco.*, vol. 5, pp. 251–258, 1977.
[39] R. L. Geske and H. E. Johnson, "The American put option valued analytically," *J. Finance*, vol. 39, no. 5, pp. 1511–1524, 1984.
[40] P. Carr, "The valuation of sequential exchange opportunities," *J. Finance*, vol. 43, no. 5, pp. 1235–1256, 1988.
[41] S. A. Ross, "A simple approach to the valuation of risky streams," *J. Bus.*, vol. 51, no. 3, pp. 453–475, 1978.
[42] R. L. Geske, "The valuation of compound options," *J. Financ. Eco.*, vol. 7, pp. 63–81, 1979.
[43] D. R. Siegel, J. L. Smith, and J. L. Paddock, "Valuing offshore oil properties with option pricing models," *Midland Corporate Finance J*, vol. 5, no. 1, pp. 22–30, 1987.
[44] R. A. Brealey and S. C. Myers, *Principles of Corporate Finance*. New York: McGraw-Hill/Irwin, 2003, pp. 237–238.

Yu-Lin Huang received the M.S. and Ph.D. degrees from the University of California, Berkeley, CA.

He is currently an Associate Professor in the Department of Civil Engineering, National Chiao Tung University, Shin Chu, Taiwan. He was an Advisor in the Central Economic Planning Department, Taiwan and the Executive Yuan on Asia Pacific Economic Cooperation and infrastructure privatization issues. He was involved with the Government to draft and pass the current infrastructure privatization law. He has authored or coauthored in the *International Journal of Project Management* and the *Construction Management and Economics*. His research interests include infrastructure investment, privatization, project finance, and project management.

Chia-Chi Pi received the B.S. and M.S. degrees in civil engineering from the National Chiao Tung University, Shin Chu, Taiwan, where she is currently working toward the Ph.D. degree in the Department of Civil Engineering.

She has authored or coauthored in the *Construction Management and Economics*. Her current research interests include project management, financial engineering, and financial economics in the construction field.