Nonlinear Analysis: Real World Applications 12 (2011) 246–253

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Environmental tax policy, habit formation and nonlinear dynamics

Hung-Ju Chen [a,*], Ming-Chia Li [b,1]

[a] Department of Economics, National Taiwan University, 21 Hsu-Chow Road, Taipei 100, Taiwan

[b] Department of Applied Mathematics, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan

ARTICLE INFO

Article history:
Received 19 November 2008
Accepted 10 June 2010

Keywords:
Chaos
Environmental quality
Habit formation
Bifurcation
Topological entropy
Entropic chaos
Multidimensional perturbation

ABSTRACT

In this paper, we study the impact of environmental tax policy on the dynamic property in an environment-growth model (John and Pecchenino 1994) [3]. We assume that the government levies consumption tax and uses the tax revenue to improve environmental quality. We show that the economic dynamics can be represented by a first-order difference equation in environmental quality when there is no habit formation of environmental quality. If agents have habit formation of environmental quality, the economic dynamics will be represented by a second-order difference equation in environmental quality. In both cases, chaotic and cyclical fluctuations may exist if agents' preference towards environmental quality, the maintenance efficiency relative to degradation and the tax rate are sufficiently low. However, the economy undergoes transformation from complex dynamics to simple dynamics as the tax rate increases. Furthermore, in the presence of habit formation of environmental quality, an increase in the degree of habit formation lowers the possibility of complex dynamics.

1. Introduction

The controversy of the interplay between environmental quality and economic growth causes great debate on the pros and cons of environmental taxation. Supporters of environmental taxation demonstrate that the enforcement of environmental taxation can prevent environmental quality from deterioration and is beneficial to economic growth. On the other hand, opponents of environmental taxation argue that it increases the production cost which will hurt economic growth.

In the literature of environment and natural resources, there have been many studies analyzing the link between environmental quality and economic development.[2] In these studies, the economic transition can be represented by a state variable called environmental quality. Among them, the model developed by John and Pecchenino [3] was the first one where an environment-growth model was constructed based on an overlapping generations setting to provide a theoretical explanation of observed correlations between environmental quality and economic growth. They showed that multiple equilibria may exist and overmaintenance of the environment, analogous to dynamically inefficient overaccumulation of capital, may emerge. Based on the John–Pecchenino model, Gutierrez [11] assumed that households do not care directly about the environment quality, but the deterioration of environmental quality makes them incur health costs when they become old to study the possibility of dynamic inefficiency. The John–Pecchenino model was also used by Wendner [12] and Ono [13] to examine the effects of environmental policy.

It has been well known that complex dynamics can easily emerge in an overlapping generations model (see [14–16]). Hence, with the dissatisfaction that most studies of environment-growth model only concentrated on the local property

* Corresponding author. Tel.: +886 2 23519641x457; fax: +886 2 23582284.
E-mail addresses: hjc@ntu.edu.tw (H.-J. Chen), mcli@math.nctu.edu.tw (M.-C. Li).

1 Tel.: +886 3 5712121x56463; fax: +886 3 5131223.

2 See [1–10].

1468-1218/$ – see front matter
doi:10.1016/j.nonrwa.2010.06.011

around the steady state, Zhang [17] studied the global property of the economic dynamics of the John–Pecchenino model. He argued that although environmental quality is deteriorated by households' consumption activities, it can be improved by households' investments in environmental quality. Thus, cycles and chaotic motion in the sense of Li and Yorke [18] may exist under certain conditions.

In this paper, we introduce the habit formation of environmental quality and consumption tax into the John–Pecchenino model. Unlike traditional assumption about habit formation of consumption in the literature, we assume that households have habit formation of environmental quality. By introducing the habit formation of environmental quality, we show that the John–Pecchenino model can be extended from a one-dimensional dynamical system to a two-dimensional dynamical system (that is, the economic transition is governed by a second-order difference equation in environmental quality). Besides, we also assume that not only households can make private investments to improve environmental quality, but also government can commit itself to the environment preservation. We assume that government levies consumption tax and uses the tax revenue as public investments for environmental improvement. This allows us to study the effects of fiscal policy.

We develop an overlapping generations model where households concern about their consumptions and environmental quality. Furthermore, we assume that households get used to the environment while they grew up and will compare environmental quality in their old age with the one when they were young. Households allocate their income between savings (consumptions) and investments for environment improvement. However, for each unit of consumption, households need to pay consumption tax to the government. The government will use the tax revenue to enhance environment quality.

The focus of this paper is to study the impact of consumption tax and habit formation of environmental quality. We find that when there is no habit formation of environmental quality, the economic dynamics can be represented by a nonlinear first-order difference equation which has been wildly explored in the literature of economic dynamics. However, when there is habit formation of environmental quality, the economic dynamics is represented by a second-order difference equation. Because of the difficulties that economists may encounter when studying the global property of an economic model represented by a high-dimensional dynamical system, there were very few studies constructing economic models which generate difference equations of order greater than one.[3] In the literature of economic dynamics and chaos, economists tended to verify the existence of Li–Yorke chaos in economic models because of its convenience for verification,[4] but recent studies started exploring the presence of other types of chaos.[5] Besides, the Li–Yorke theorem works only for one-dimensional dynamical systems and hence, it is not suitable to study the possibility of Li–Yorke chaos when there is habit formation of environmental quality. In order for our analysis to be consistent in both cases (without and with habit formation of environmental quality), we follow [23] to study the possibility of entropic chaos. In both cases, we find that cyclical fluctuations and entropic chaos may exist if agents' preference towards environmental quality, the maintenance efficiency relative to degradation and the tax rate are sufficiently low. The economy moves from complex to simple dynamics as the tax rate increases. This result implies that government can use fiscal policies to affect the dynamic behavior of the economy. We also show that in the presence of habit formation of environmental quality, an increase in the degree of habit formation lowers the possibility of chaotic dynamics. Furthermore, the required consumption tax rate to avoid complex dynamics in an economy with habit formation of environmental quality is lower than then the one in an economy without habit formation of environmental quality.

In the methodological point of view, this study also contributes to the literature of complex dynamics by providing a technique to study the dynamic behavior in high-dimensional dynamical systems. Our goal is to provide sufficient conditions for the occurrence of entropic chaos in the high-dimensional dynamical system if the dynamics is complex. To achieve this goal, we first examine the possibility of the occurrence of chaotic dynamics for a reduced one-dimensional dynamical system without habit formation. Then we apply the multidimensional perturbation result of Li and Malkin [24] and Juang et al. [25] to verify the occurrence of chaotic dynamics in the high-dimensional system with habit formation; also refer to [26]. Our methodology can be easily applied to other economic models which are high dimensional; moreover, it is free from the constraint of invertibility of the system, which sometimes becomes an obstruction for researchers in proving chaos for high-dimensional dynamical systems.

The remainder of the paper is organized as follows. In the next section, we develop an environment-growth model with habit formation of environmental quality and environmental taxation. The dynamic property of the economy and policy implications are analyzed in Section 3. The final section concludes.

2. The model

We consider an infinite-horizon, discrete-time overlapping generations model where agents live for two periods, corresponding to young and old age. Each old agent (a parent) gives birth to a young agent (a child). Hence, there is no population growth and we normalize the population size to one.

3 A two-dimensional overlapping generations model which generates complex dynamics are studied by Medio and Negroni [19] and Yokoo [16]. In order to produce endogenous fluctuations, Medio and Negroni [19] incorporated Leontief and the CES production function and Yokoo [16] took the accumulation of government debt into consideration.

4 According to Li and Yorke [18], period three implies chaos. For economic studies of Li–Yorke chaos, see [20–22].

5 For example, Boldrin et al. [21] and Mitra [23] studied the presence of ergodic chaos and entropic chaos, respectively.

We use variable E_t to represent the index of environmental quality in period t. This index includes all concerns about environmental problems, such as the inverse of the diffusion of toxic compounds in the soil, atmosphere and water, the inverse of the greenhouse gases (such as carbon dioxide, methane, nitrous oxide and chlorofluorocarbons emissions), the inverse of dispersion of radioactivity and so on. Agents have identical preferences and derive utility from their old age. Old agents care about their consumptions (c_{t+1}) and environmental quality (E_{t+1}). One may think environmental quality as an indicator of the quality of life. Furthermore, environmental quality when agents were young has an externality impact on the preference in their old age because agents have habit formation of the lifestyle characterized by environmental quality. That is, old agents have habits of environmental quality and will compare current environment quality with the one when they were young. The utility function is represented as:

$$u(c_{t+1}, E_{t+1}, E_t) = \log c_{t+1} + \eta \log(E_{t+1} - \phi E_t), \tag{1}$$

where $\eta > 0$ measures agents' preference toward environment quality and $0 \le \phi < 1$ measures the degree of habit formation of environment quality.

Each agent is endowed with one unit of time. Young agents devote all of the time for work to earn a real wage rate (w_t) and old agents use the time for leisure. Young agents decide how to allocate w_t between savings (s_t) for the old-age consumptions and investments (m_t) to promote environmental quality. The budget constraint for young agents is:

$$s_t + m_t = w_t. \tag{2}$$

When old agents consume, they need to pay the consumption tax with the rate of τ for each unit of consumptions. Using $R_{t+1} = 1 + r_{t+1}$ to represent the gross rate of returns for savings, the budget constraint for old agents is:

$$(1 + \tau)c_{t+1} = R_{t+1}s_t. \tag{3}$$

We assume that government runs a balanced budget and uses the tax revenue (T_t) to improve environmental quality (environmental taxation). The budget constraint for the government is:

$$T_t = \tau c_t. \tag{4}$$

The evolution of environmental quality follows:

$$\begin{aligned} E_{t+1} &= (1 - b)E_t - \beta c_t + \gamma(m_t + T_t) \\ &= (1 - b)E_t - (\beta - \gamma\tau)c_t + \gamma m_t, \end{aligned} \tag{5}$$

where $b \in (0, 1)$ is the autonomous evolution of environmental quality. This indicates that when there is no economic activity, environmental quality depreciates by the rate of b in every period. With economic activity, parameter $\beta > 0$ measures the deterioration of environmental quality caused by consumption. However, households and the government can use some resources to enhance (or to maintain) environmental quality and $\gamma > 0$ measures the maintenance efficiency. Eq. (5) implies that there are two sources of investments to improve environmental quality: one comes from private investment (m_t) which is the optimal decision by households and the other one comes from public investment ($T_t = \tau c_t$).

Net output per worker (y_t) is produced by a constant-returns-to-scale production function $y_t = f(k_t) - \delta k_t$, where $f(k_t) = Ak_t^{\alpha}$, $k_t > 0$ is the capital per worker, $\alpha \in (0, 1)$ is the capital share of output, $A > 0$ is the total factor productivity and $\delta \in [0, 1]$ is the depreciation rate of capital. The competitive behavior of firms will equalize the factor prices of labor and capital to their respective marginal product:

$$w_t = f(k_t) - k_t f'(k_t) = (1 - \alpha)f(k_t), \tag{6}$$

$$r_t = f'(k_t) - \delta. \tag{7}$$

Given the initial condition and the tax rate τ, a perfect foresight equilibrium comprises the sequences of individuals' decisions $\{c_t, s_t, m_t\}_{t=1}^{\infty}$, the stock of capital per worker $\{k_t\}_{t=1}^{\infty}$, the factor prices $\{w_t, r_t\}_{t=1}^{\infty}$ and the quality of the environment $\{E_t\}_{t=1}^{\infty}$ such that: (i) the household maximization problem will be solved by maximizing the utility function subject to Eqs. (2), (3) and (5) and the non-negativity constraints of c_t and m_t; (ii) the factor prices satisfy Eqs. (6) and (7); (iii) the goods market clears, $k_{t+1} = s_t$; (iv) the evolution of environmental quality follows Eq. (5); and (v) the government maintains a balanced budget.

Solving for the optimization problem for households, we can obtain:

$$c_{t+1} = \frac{R_{t+1}}{\eta\gamma(1 + \tau)}(E_{t+1} - \phi E_t). \tag{8}$$

Substituting Eq. (8) into Eq. (3) and applying the clearing condition of goods market, the result implies that:

$$k_{t+1} = s_t = \frac{E_{t+1} - \phi E_t}{\eta\gamma}. \tag{9}$$

Combining Eqs. (2), (5) and (8), the evolution of environmental quality becomes:

$$E_{t+1} = (1 - b)E_t - (\beta - \gamma\tau)\frac{R_t}{\eta\gamma(1 + \tau)}(E_t - \phi E_{t-1}) + \gamma\left[w_t - \frac{E_{t+1} - \phi E_t}{\eta\gamma}\right]. \tag{10}$$

In the next section, we study the dynamic behavior of the economy.

3. Chaotic dynamics

Combining Eqs. (6), (7), (9) and (10), the transitional dynamics of environment quality is governed by:

$$\begin{aligned} E_{t+1} &= \frac{\eta}{1+\eta}\left\{\left[1-b-\frac{(\beta-\gamma\tau)(1-\delta)}{\eta\gamma(1+\tau)}+\frac{\phi}{\eta}\right]E_t+\frac{\phi(\beta-\gamma\tau)(1-\delta)}{\eta\gamma(1+\tau)}E_{t-1}\right.\\ &\quad\left.+\frac{\left[A\gamma(1-\alpha)-\frac{A\alpha(\beta-\gamma\tau)}{1+\tau}\right]}{(\eta\gamma)^{\alpha}}(E_t-\phi E_{t-1})^{\alpha}\right\}\\ &= a_0E_t+a_1E_{t-1}+a_2(E_t-\phi E_{t-1})^{\alpha}, \end{aligned} \tag{11}$$

where

$$a_0 = a_0(\phi,\tau) = \frac{\eta}{1+\eta}\left[1-b-\frac{(\beta-\gamma\tau)(1-\delta)}{\eta\gamma(1+\tau)}+\frac{\phi}{\eta}\right],$$

$$a_1 = a_1(\phi,\tau) = \frac{\phi(\beta-\gamma\tau)(1-\delta)}{(1+\eta)\gamma(1+\tau)}, \quad \text{and}$$

$$a_2 = a_2(\phi,\tau) = \frac{\eta^{1-\alpha}}{1+\eta}\frac{A\left[\gamma(1-\alpha)-\frac{\alpha(\beta-\gamma\tau)}{1+\tau}\right]}{\gamma^{\alpha}}.$$

Eq. (11) shows that the transitional dynamics of the economy can be represented by a difference equation in environmental quality.

3.1. No persistent habits of environmental quality

We first study the case where there is no habit formation of environmental quality ($\phi = 0$). If government does not levy any tax on consumption ($\tau = 0$), Eq. (11) will be reduced to a one-dimensional dynamical system $E_{t+1} = f_0(E_t)$ considered by Zhang [17]:

$$E_{t+1} = a_0(0,0)E_t + a_2(0,0)E_t^{\alpha} \equiv f_0(E_t). \tag{12}$$

If government levies consumption tax, the economic dynamics will then be represented by the following one-dimensional dynamical system:

$$E_{t+1} = a_0(0,\tau)E_t + a_2(0,\tau)E_t^{\alpha} \equiv f_\tau(E_t). \tag{13}$$

It is straightforward to show that the map f_τ satisfies the following properties:

- $f_\tau(0) = f_\tau(\hat{E}) = 0$, where $\hat{E} \equiv (-a_2(0,\tau)/a_0(0,\tau))^{1/(1-\alpha)}$ is the upper bound for the quality E;
- f_τ is C^1-*unimodal*. That is, f_τ is continuously differentiable and there exists $\bar{E} = (-\alpha a_2(0,\tau)/a_0(0,\tau))^{1/(1-\alpha)} \in (0,\hat{E})$ such that f_τ is strictly increasing on $[0,\bar{E})$ and strictly decreasing on $(\bar{E},\hat{E}]$;
- $\lim_{E\to 0^+}(f_\tau)'(E) = +\infty$;
- the unique positive steady state is given by $E^* = [a_2(0,\tau)/(1-a_0(0,\tau))]^{1/(1-\alpha)}$;
- $(f_\tau)'(E^*) = \alpha + (1-\alpha)a_0(0,\tau) < 1$ if $a_0(0,\tau) < 0$;
- If $a_0(0,\tau) \in \left[\frac{\alpha^{-\alpha/(1-\alpha)}}{\alpha-1}, 0\right]$, then f_τ maps $[0,\hat{E}]$ into itself.

Following the methodology of de Melo and van Strien [27] and Zhang [17], we have the following theorem:

Theorem 1. *For the dynamics of Eq. (13), the following properties hold:*

1. *If* $a_0(0,\tau) \in \left(\frac{\alpha}{\alpha-1}, 0\right)$ *then for all* $E_0 \in (0,\hat{E})$, $\lim_{t\to\infty} E_t = E^*$ *and* E^* *is a stable node;*
2. *If* $a_0(0,\tau) \in \left(\frac{1+\alpha}{\alpha-1}, \frac{\alpha}{\alpha-1}\right)$ *then for all* $E_0 \in (0,\hat{E})$, $\lim_{t\to\infty} E_t = E^*$ *and* E^* *is a stable spiral;*
3. *If* $a_0(0,\tau) \in \left(\frac{\alpha^{-\alpha/(1-\alpha)}}{\alpha-1}, \frac{1+\alpha}{\alpha-1}\right)$ *then there is a two-period cycle and the set of* $E_0 \in (0,\hat{E})$ *such that* $\lim_{t\to\infty} E_t = E^*$ *is at most countable;*
4. *As* $a_0(0,\tau)$ *decreases, the dynamics undergoes* a sequence of period-doubling bifurcations, *that is, for each* $n \geq 2$, *there exists a value* $a_n^* \in \left(\frac{\alpha^{-\alpha/(1-\alpha)}}{\alpha-1}, \frac{1+\alpha}{\alpha-1}\right)$ *such that an attracting cycle of period* 2^n *emerges if* $a_0(0,\tau) = a_n^*$;
5. *As* $a_0(0,\tau)$ *further decreases, there exists a value* $\bar{a}_\tau \in \left(\frac{\alpha^{-\alpha/(1-\alpha)}}{\alpha-1}, \frac{1+\alpha}{\alpha-1}\right)$ *such that a period-three cycle emerges if* $a_0(0,\tau) < \bar{a}_\tau$.

Note that $a_0(0,\tau) = \frac{\eta}{1+\eta}\left[1-b-\frac{(\beta-\gamma\tau)(1-\delta)}{\eta\gamma(1+\tau)}\right]$ and $\frac{d(a_0(0,\tau))}{d\tau} > 0$. Then Theorem 1 indicates that when τ is large enough, there exists a stable steady state. As τ decreases, cycles start to emerge. Chaotic dynamics may exist if τ is sufficiently low. Theorem 1 also provides the sufficient conditions for the occurrence of different types of dynamic behaviors of Eq. (13). The numerical results are given in Fig. 1.

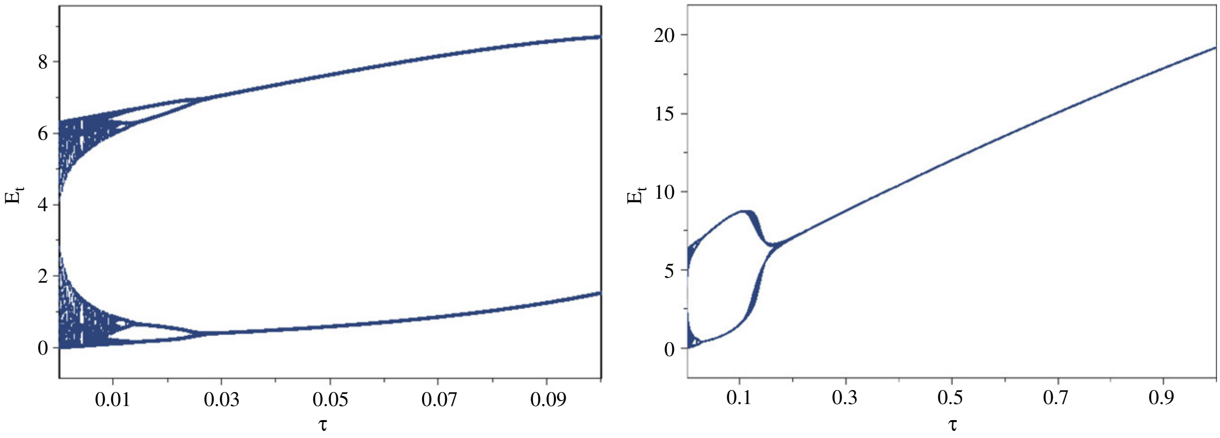

Fig. 1. Bifurcation diagrams at different scales of τ with $b = 0.9$, $\delta = 0.5$, $\beta = 4.285$, $\gamma = 0.7$, $\alpha = 1/9$, $A = 100$, and $\phi = 0$.

Traditionally, researchers concentrated on the study of Li–Yorke chaos because of its easiness to verify (see Item 5 of Theorem 1). However, the Li–Yorke theorem works only for one-dimensional dynamical systems. In order for our analysis to be consistent in both one- and two-dimensional dynamical systems, we follow [23] to examine the possibility of entropic chaos in dynamical systems with different orders. Based on [28], the definition of topological entropy and entropic chaos are given as follows:

Definition 1. Let $g : X \to X$ be a continuous map on the space X with metric d. For $n \in \mathbb{N}$ and $\epsilon > 0$, a set $S \subset X$ is called an (n, ϵ)-*separated set for* g if for every pair of points $x, y \in S$ with $x \neq y$, there exists an integer k with $0 \leq k < n$ such that $d(g^k(x), g^k(y)) > \epsilon$; here we denote the identity function by g^0, and inductively denote $g^k = g \circ g^{k-1}$ for a positive integer k. The *topological entropy of* g is defined to be

$$h_{\text{top}}(g|X) = \lim_{\epsilon\to 0, \epsilon>0} \limsup_{n\to\infty} \frac{\log\left(\max\{\#(S) : S \subset X \text{ is an } (n, \epsilon)\text{-separated set for } g\}\right)}{n},$$

where $\#(S)$ is the cardinality of elements of S.

We say that g exhibits *entropic chaos* on X if $h_{\text{top}}(g|X) > 0$.

Topological entropy describes the total exponential complexity of the orbit structure with a single number in a rough but expressive way. The topological entropy is positive for chaotic systems and is zero for non-chaotic systems.

In the following theorem, we give a sufficient condition for the presence of entropic chaos of Eq. (13) from the perspective of τ.

Theorem 2. *There exists a value* $\bar{a}_0 \in \left(\frac{\alpha^{-\alpha/(1-\alpha)}}{\alpha-1}, \frac{1+\alpha}{\alpha-1}\right)$ *such that if* $\phi = 0$ *and* $a_0(0, 0) < \bar{a}_0$ *then for any* τ *close to zero, Eq.* (11) *exhibits entropic chaos.*

Proof. Let $\phi = 0$ and $\tau = 0$. Then Eq. (11) becomes $E_{t+1} = f_0(E_t)$, where f_0 is given in Eq. (12). By Item 5 of Theorem 1, there exists $\bar{a}_0 \in \left(\frac{\alpha^{-\alpha/(1-\alpha)}}{\alpha-1}, \frac{1+\alpha}{\alpha-1}\right)$ such that if $a_0(0, 0) < \bar{a}_0$ then f_0 has a period-three cycle and hence the topological entropy of f_0 is positive. Allowing $\tau > 0$, we get that Eq. (11) becomes $E_{t+1} = f_\tau(E_t)$, where f_τ is given in Eq. (13). Then the family of functions f_τ is continuous in τ, each f_τ is an unimodal map whose critical point is nondegenerate. By the continuity of topological entropy (e.g., Theorem 9.1 in Chapter II of [27]), we have that the topological entropy of f_τ is positive for all small τ. □

Note that $a_0(0, 0) = \frac{\eta}{1+\eta}\left[1 - b - \frac{\beta(1-\delta)}{\eta\gamma}\right]$. Theorem 2 implies that entropic chaos is likely to emerge if agents' preference towards environmental quality (η), the maintenance efficiency relative to degradation (γ/β) and the consumption tax rate (τ) are sufficiently low. This is because when η, γ/β and τ are low, households will choose to consume more and both households and government will invest less in the improvements of environmental quality. This will hurt the evolution of environmental quality and the sustainable development level will not be able to achieved.

3.2. Persistent habits of environmental quality

We now turn to study an economy where households have habits of environmental quality ($\phi > 0$). We first assume that there is no tax ($\tau = 0$) and study the impact of ϕ on the global property of the economic dynamics. Under this situation, the economic transition of the economy can be represented by the following difference equation of order two:

$$E_{t+1} = a_0(\phi, 0)E_t + a_1(\phi, 0)E_{t-1} + a_2(\phi, 0)(E_t - \phi E_{t-1})^\alpha \equiv g_\phi(E_{t-1}, E_t). \quad (14)$$

Eq. (14) shows that with habit formation of environmental quality, the economic dynamics is represented by a two-dimensional dynamical system. To examine the existence of entropic chaos in a two-dimensional dynamical system, we use the technique recently developed by Juang et al. [25]. In the following theorem, we give a sufficient condition for the presence of entropic chaos from the perspective of the degree of habit formation.

Theorem 3. *Let $\bar{a}_0$ be the same as in Theorem 2. If $\tau = 0$ and $a_0(0,0) < \bar{a}_0$, then for any ϕ close to zero, Eq. (11) exhibits entropic chaos.*

Proof. In the proof of Theorem 2, we have known that if $a_0(0,0) < \bar{a}_0$, then $h_{\text{top}}(f_0) > 0$, where f_0 is given in Eq. (12). Let $\tau = 0$. Then Eq. (11) becomes $E_{t+1} = g_\phi(E_{t-1}, E_t)$, where g_ϕ is given by Eq. (14). Since $a_1(0,0) = 0$, $g_0(E_{t-1}, E_t) = f_0(E_t)$.

Let $a_0(0,0) < \bar{a}_0$. Then there exists a unique $\bar{E} > 0$ such that $(f_0)'(\bar{E}) = 0$. Let $B = f_0(f_0(\bar{E}))$ and $C = f_0(\bar{E})$. Then $0 < B < C$. For $\eta < 1$, define $\Phi_\phi : [B, C]^3 \to \mathbb{R}$ by

$$\Phi_\phi(E_{t-1}, E_t, E_{t+1}) = E_{t+1} - g_\phi(E_{t-1}, E_t).$$

Let Y_ϕ be the set of solutions of the difference equation

$$\Phi_\phi(E_{t-1}, E_t, E_{t+1}) = 0, \tag{15}$$

i.e., the set of bisequences $\underline{E} = (E_t) = (E_1, E_2, E_3, \ldots)$ such that for any integer $t \geq 2$,

1. $E_{t-1} \in [B, C]$; and
2. three consecutive components E_{t-1}, E_t, E_{t+1} of $\underline{E}$ satisfy Eq. (15).

Let σ be the shift map on Y_ϕ, i.e., $(\sigma(\underline{E}))_t = E_{t+1}$ for all $t \geq 1$. The function Φ_ϕ is C^1 on $[B, C]^3$ for each ϕ, and the function $\phi \mapsto \Phi_\phi$ is continuous on [0, 1], and for $i = 1, 2, 3$ the function $\phi \mapsto \partial_i \Phi_\phi$ is continuous on [0, 1], where $\partial_i \Phi_\phi$ is the partial derivative of Φ_ϕ with respect to the ith variable. Now letting $\phi = 0$ we have the limit function $\Phi_\phi(E_{t-1}, E_t, E_{t+1}) = E_{t+1} - f_0(E_t)$. By Theorem 4 in Appendix, for all ϕ near 0, there is a closed σ-invariant subset Γ_ϕ of Y_ϕ in the product topology, such that $h_{\text{top}}(\sigma|\Gamma_\phi) > 0$. Therefore, the dynamics of the economy system exhibits entropic chaos. □

Theorem 3 demonstrates that when the government does not enforce consumption tax, chaotic dynamics may exist if agents' preference towards environmental quality, the maintenance efficiency relative to degradation and the degree of habit formation are sufficiently low.

Finally, we study the most complicated case where both habit formation of environmental quality and environmental taxation exist. In this case, the dynamic behavior of the economy is represented by Eq. (11). The following theorem gives the criterion for the uniqueness and stability of the nontrivial steady state.

Theorem 4. *For the dynamics of Eq. (11), the following properties hold:*

1. *$a_2(\phi,\tau)[1 - a_0(\phi,\tau) - a_1(\phi,\tau)] > 0$ if and only if there is a nontrivial steady state $E^* = \left[\frac{(1-\phi)^\alpha a_2(\phi,\tau)}{1-a_0(\phi,\tau)-a_1(\phi,\tau)}\right]^{1/(1-\alpha)}$; in this case, E^* is the unique nontrivial steady state;*
2. *If $a_2(\phi,\tau)[1 - a_0(\phi,\tau) - a_1(\phi,\tau)] > 0$ then E^* is asymptotically stable if and only if*

$$\min\{1 - B - A, 1 + B - A, A + 1\} > 0,$$

where

$$A = a_1(\phi,\tau) - \frac{\alpha\phi[1 - a_0(\phi,\tau) - a_1(\phi,\tau)]}{1-\phi},$$

$$B = a_0(\phi,\tau) - \frac{\alpha[1 - a_0(\phi,\tau) - a_1(\phi,\tau)]}{1-\phi}.$$

Proof. Letting $E_{t-1} = E_t = E_{t+1} = E$ and solving Eq. (11) for E, we get that $E = E^*$ and hence Item 1 follows. For Item 2, we consider the two-dimensional map $(E_{t-1}, E_t) \mapsto (E_t, E_{t+1})$, where E_{t+1} is given by Eq. (11). Then the derivative of the map at the fixed point (E^*, E^*) is given by the matrix $\begin{bmatrix} 0 & 1 \\ A & B \end{bmatrix}$ with the characteristic polynomial $x^2 - Bx - A$. Since the asymptotic stability of the fixed point (E^*, E^*) is determined by the spectral radius of its derivative, the Schur–Cohn criterion (refer to [29]) gives us the result of Item 2. □

Notice that if $\phi = 0$ and $a_0(0,\tau) < 0$ then $a_1(0,\tau) = 0$ and hence $\min\{1 - B - A, 1 + B - A, A + 1\} = 0$ implies that $a_0(0,\tau) = (1+\alpha)/(\alpha - 1)$. Therefore, Theorem 4 fits well with Theorem 1.

At the end, we give a sufficient condition for the existence of entropic chaos of Eq. (11).

Theorem 5. *Let $\bar{a}_0$ be the same as in Theorem 2. If $a_0(0,0) < \bar{a}_0$, then for either any τ or any ϕ close to zero, Eq. (11) exhibits entropic chaos.*

Proof. Similar to the proof of Theorem 3. □

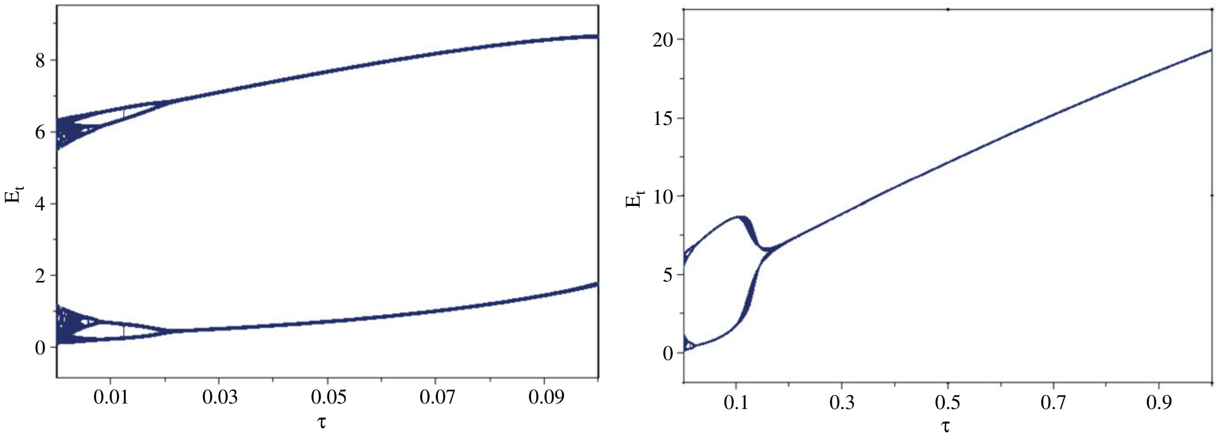

Fig. 2. Bifurcation diagrams at different scales of τ with $b = 0.9$, $\delta = 0.5$, $\beta = 4.285$, $\gamma = 0.7$, $\alpha = 1/9$, $A = 100$, and $\phi = 0.01$.

Theorem 5 indicates that when agents' preference towards environmental quality, the maintenance efficiency relative to degradation, the degree of habit formation of environmental quality and consumption tax rate are sufficiently low, the chaotic dynamics will exist. On the other hand, the dynamic behavior becomes periodic or simple when the degree of habit formation of environmental quality or the tax rate is sufficiently high. This is because there are two forces to affect the dynamic pattern of E_t. Eq. (5) indicates that consumption deteriorates environmental quality while private and public environmental investments improve or maintain environmental quality. The dynamics of E_t is driven by the interaction between these two effects. An increase in the degree of habit formation of environmental quality will raise private environmental investments while an increase in the consumption tax rate will raise public environmental investments. Hence, both will lower the possibility of chaotic dynamics.

Fig. 2 presents bifurcation diagrams with varying τ for $\phi = 0.01$. It shows that the economy undergoes from complex dynamics to simple dynamics as τ increases. If τ is sufficiently large, there will exist a stable steady state. Comparing Fig. 2 with Fig. 1, we find that the presence of habit formation of environmental quality will reduce the possibility of chaotic motion (that is, the range of τ for the emergence of chaos is smaller when there are habits of environmental quality). This implies that a lower consumption tax rate is required to avoid complex dynamics and to achieve the sustainable equilibrium when habit formation of environmental quality presents.

4. Conclusion

In this paper, we develop an environment-growth model with habit formation of environmental quality and consumption tax. Our results show that in the presence of habit formation of environmental quality, the economy undergoes from chaotic to simple dynamics, through periodic dynamics, as the degree of habit formation increases. Furthermore, the presence of habit formation affects the impact of environmental policies on the dynamic behavior of the economy. In an economy with habit formation of environmental quality, the required consumption tax rate enforced by the government to prevent the economy from trapping in complex dynamics is lower than in an economy without habit formation.

Our results illustrate that we cannot only focus our research on the local property for an environment-growth model and a more careful study of the global property is needed. We also show that habit formation of environmental quality plays an important role in determining the dynamic property of the economy. Hence, it is worthy in the future research to empirically measure the degree of habit formation of environmental quality.

Acknowledgements

Both authors are grateful to the referees for their comments and suggestions. The first author (Chen) would like to acknowledge the financial support provided by the National Science Council (grant number: NSC 95-2415-H-002-018) and the Program for Globalization Studies at the Institute for Advanced Studies in Humanities, National Taiwan University (grant number: 95R0064-AH03-03). The second author (Li) was partially supported by NSC grant 96-2115-M-009-004-MY3.

Appendix

The following theorem is a trivial consequence of Theorem 1 of [25] in which difference equation of order N for a general setting was studied; also refer to [24].

Theorem 6. *Consider a difference equation of order two in the form*

$$\Phi_\phi(E_{t-1}, E_t, E_{t+1}) = 0, \quad t \geq 2, \tag{16}$$

where $\phi \in [0, 1]$ is a parameter and the real-valued function Φ_ϕ is defined on a cube $[B, C]^3 \subset \mathbb{R}^3$ with constants $0 < B < C$. Assume that (i) *Φ_ϕ is C^1 on $[B, C]^3$ for each $\phi \in [0, 1]$;* (ii) *the function $\phi \mapsto \Phi_\phi$ is continuous on $[0, 1]$; and* (iii) *for $i = 1, 2, 3$, the function $\phi \mapsto \partial_i \Phi_\phi$ is continuous on $[0, 1]$, where $\partial_i \Phi_\phi$ is the partial derivatives of Φ_ϕ with respect to the ith variable. Suppose that for $\phi = 0$, the difference equation (16) reduces to a difference equation of order one in the form $E_{t+1} - \varphi(E_t) = 0$, $t \geq 1$, where $\varphi : [B, C] \to \mathbb{R}$ is a C^2 function with positive topological entropy. Let Y_ϕ be the set of solutions for (16), i.e. the set of sequences $\underline{E} = (E_1, E_2, E_3, \ldots)$ such that for any $t \geq 2$,*

1. *$E_{t-1} \in [B, C]$; and*
2. *three consecutive components E_{t-1}, E_t, E_{t+1} of $\underline{E}$ satisfy (16).*

Let σ be the shift map on Y_ϕ, i.e., $(\sigma(\underline{E}))_t = E_{t+1}$ for all $t \geq 1$.

Then there exists $\epsilon > 0$ such that for any $0 < \phi < \epsilon$, there is a closed σ-invariant subset Γ_ϕ of Y_ϕ in the product topology such that $h_{\text{top}}(\sigma|\Gamma_\phi) > 0$.

References

[1] A. Beltratti, G. Chichilnisky, G. Heal, Sustainable growth and the green golden rule, NBER Working Paper. No. 4430, 1993.
[2] O. Tahvonen, J. Kuuluvainen, Economic growth, pollution and renewable resources, Journal of Environmental Economics and Management 24 (1993) 101–118.
[3] A. John, R. Pecchenino, An overlapping generations model of growth and the environment, Economic Journal 104 (1994) 1393–1410.
[4] A. John, R. Pecchenino, D. Schimmelpfennig, S. Schreft, Short-lived agents and the long-lived environment, Journal of Public Economics 58 (1995) 127–141.
[5] A.L. Bovenberg, S. Smulders, Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model, Journal of Public Economics 57 (1995) 369–391.
[6] A.L. Bovenberg, S. Smulders, Transitional impacts of environmental policy in an endogenous growth model, International Economic Review 37 (1996) 861–893.
[7] A.L. Bovenberg, R.A. de Mooij, Environmental tax reform and endogenous growth, Journal of Public Economics 63 (1997) 207–237.
[8] E.O'N. Fisher, C. van Marrewijk, Pollution and economic growth, Journal of International Trade and Economic Development 7 (1998) 55–69.
[9] A.L. Bovenberg, B.J. Heijdra, Environmental tax policy and intergenerational distribution, Journal of Public Economics 67 (1998) 1–24.
[10] A. Grimaud, Pollution permits and sustainable growth in a Schumpeterian model, Journal of Environmental Economics and Management 38 (1999) 249–266.
[11] M.-J. Gutierrez, Dynamic inefficiency in an overlapping generation economy with pollution and health costs, DFAEII-University of the Basque Country, Preprint, 2005.
[12] R. Wendner, Frames of reference, the environment, and efficient taxation, Economics of Governance 6 (2005) 13–31.
[13] T. Ono, Environmental tax policy in a model of growth cycles, Economic Theory 22 (2003) 141–168.
[14] J.-M. Grandmont, On endogenous competitive business cycles, Econometrica 53 (1985) 995–1045.
[15] P. Michel, D. de la Croix, Myopic and perfect foresight in the OLG model, Economics Letters 67 (2000) 53–60.
[16] M. Yokoo, Chaotic dynamics in a two-dimensional overlapping generations model, Journal of Economic Dynamics and Controls 24 (2000) 909–934.
[17] J. Zhang, Environmental sustainability, nonlinear dynamics and chaos, Economic Theory 14 (1999) 489–500.
[18] T. Li, J. Yorke, Period three implies chaos, American Mathematical Monthly 82 (1975) 985–992.
[19] A. Medio, G. Negroni, Chaotic dynamics in overlapping generations models with production, in: W.A. Barnett, A.P. Kirman, M. Salmon (Eds.), Non-Linear Dynamics and Economics, Cambridge University Press, 1996, pp. 3–44.
[20] R. Day, Irregular growth cycles, American Economic Review 72 (1982) 406–414.
[21] M. Boldrin, K. Nishimura, T. Shigoka, M. Yano, Chaotic equilibrium dynamics in endogenous growth models, Journal of Economic Theory 96 (2001) 97–132.
[22] S. Auray, F. Collard, P. Feve, Money and external habit persistence: a tale for chaos, Economics Letters 76 (2002) 121–127.
[23] T. Mitra, A sufficient condition for topological chaos with an application to a model of endogenous growth, Journal of Economic Theory 96 (2001) 133–152.
[24] M.-C. Li, M. Malkin, Topological horseshoes for perturbations of singular difference equations, Nonlinearity 19 (2006) 795–811.
[25] J. Juang, M.-C. Li, M. Malkin, Chaotic difference equations in two variables and their multidimensional perturbations, Nonlinearity 21 (2008) 1019–1040.
[26] M.-C. Li, M.-J. Lyu, P. Zgliczynski, Topological entropy for multidimensional perturbations of snap-back repellers and one-dimensional maps, Nonlinearity 21 (2008) 2555–2567.
[27] W. de Melo, S. van Strien, One-Dimensional Dynamics, Springer-Verlag, NY, 1993.
[28] C. Robinson, Dynamical Systems: Stability, Symbolic Dynamics, and Chaos, 2nd ed., CRC Press, Boca Raton, FL, 1999.
[29] B.-S. Du, S.-R. Hsiau, M.-C. Li, M. Malkin, An improved stability criterion with application to the Arneodo-Coullet-Tresser map, Taiwanese Journal of Mathematics 11 (2007) 1369–1382.